DESY 08-038
CERN-PH-TH/2008-066
ZMP-HH/08-6

# de Sitter vacua in no-scale supergravities and Calabi-Yau string models

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We perform a general analysis on the possibility of obtaining metastable vacua with spontaneously broken $\mathcal{N} = 1$ supersymmetry and non-negative cosmological constant in the moduli sector of string models. More specifically, we study the condition under which the scalar partners of the Goldstino are non-tachyonic, which depends only on the Kähler potential. This condition is not only necessary but also sufficient, in the sense that all of the other scalar fields can be given arbitrarily large positive square masses if the superpotential is suitably tuned. We consider both heterotic and orientifold string compactifications in the large-volume limit and show that the no-scale property shared by these models severely restricts the allowed values for the 'sGoldstino' masses in the superpotential parameter space. We find that a positive mass term may be achieved only for certain types of compactifications and specific Goldstino directions. Additionally, we show how subleading corrections to the Kähler potential which break the no-scale property may allow to lift these masses.

# 1 Introduction

It is widely believed that the existence of four-dimensional de Sitter (dS) vacua in low energy compactifications of string theory entails the presence of extended energy sources, such as D-branes, contributing to the vacuum energy density. This is motivated in part by the observation that smooth compactifications of 10-D and 11-D supergravities do not admit solutions to Einstein's equations characterised by both a positive cosmological constant and a stable ground state [1–3]. It has become clear, however, that this class of no-go theorems can be circumvented by including localised sources and/or taking into account higher order corrections in $\alpha'$ or the string coupling $g_s$ in the low energy analysis. In ref. [4] it was indeed shown that in type-IIB string theory compactified on Calabi-Yau orientifolds with D-branes wrapping around cycles and nontrivial background fluxes a potential is generated for many of the scalar fields (moduli) present in the four-dimensional $\mathcal{N} = 1$ supergravity. Including non-perturbative contributions all moduli can be stabilised but, generically, in a supersymmetric ground state which is either anti-de Sitter or Minkowski [5–9] whereas a positive cosmological constant necessarily requires the breaking of supersymmetry. For the 'uplifting' from a supersymmetric vacuum to a dS vacuum a variety of mechanisms has been proposed and studied. For example, in ref. [5] it was shown that the joint contribution of non-perturbative effects and an explicit supersymmetry-breaking term induced by anti-D3 branes can lead to a dS vacuum with fine-tuned cosmological constant and stable volume modulus. Alternatively, there have been attempts to construct metastable vacua where supersymmetry is broken spontaneously either by $D$- or $F$-terms [10–27].

Interestingly, there are no known examples of metastable vacua with spontaneously broken supersymmetry produced only by the volume moduli –or Kähler moduli– in the absence of $\alpha'$ and worldsheet instanton corrections to the Kähler potential. At first sight this fact is a bit counter-intuitive. The superpotentials available in flux compactifications and/or compactifications on generalised geometries are sufficiently generic [28] that one could expect no serious obstacle towards this end. Nevertheless, it was shown in ref. [29] that for $\mathcal{N} = 1$ supergravities describing string compactifications with a single volume modulus $T$ and a no-scale Kähler potential

$$K = -3\log(T + \bar{T}), \tag{1.1}$$

stationary points of a positive scalar potential $V$ generated only by $F$-terms are always characterised by the existence of at least one tachyonic direction, independently of the superpotential $W = W(T)$. This result was made more precise in ref. [30] and extended to more general situations, and in particular to the class of compactifications in which the

Kähler geometry spanned by the moduli is factorised into one or several sub-manifolds of constant curvature. More precisely, it was shown that also for the no-scale Kähler potential

$$K = -\sum_i n_i \log(T^i + \bar{T}^i), \qquad \text{with} \qquad \sum_i n_i = 3, \tag{1.2}$$

stationary points of a positive scalar potential $V$ have at least one tachyonic direction independently of $W$. Moreover, this tachyonic direction was shown to become marginally flat only when the superpotential $W$ is chosen in such a way that $V = 0$. Similar results were derived in ref. [31] for coset manifolds arising in orbifold compactifications. This new class of no-go theorems –which relies only on the properties of the Kähler potential– raises the natural question about the role of the volume moduli in the construction of metastable vacua in more generic string compactifications where the Kähler geometry spanned by the moduli becomes nontrivial.

The purpose of this paper is twofold. First we refine the previous analysis of four-dimensional $\mathcal{N} = 1$ supergravities given in refs. [30–32] by emphasising that the crucial quantity to study in order to achieve vacuum metastability is the mass of the scalar superpartners of the Goldstino. We show that all of the other scalar fields can be made arbitrarily massive by appropriately choosing the superpotential. However, this is not the case for the two sGoldstinos since in the limit of global supersymmetry the Goldstino is exactly massless and therefore the sGoldstinos can never get a mass from the superpotential. Instead their masses are generated by the supersymmetry breaking mechanism with their mass-difference being of the order of the supersymmetry breaking scale. As a consequence their masses are not necessarily positive. It is precisely this fact which is at the heart of the problem of identifying locally stable dS vacua. From this discussion it is also immediately clear that a positive sGoldstino mass is a necessary condition for the metastability of any dS vacua and, furthermore, this condition does not depend on the superpotential but only on the form of the Kähler potential. This observation considerably simplifies the search for a viable dS ground state.

The second aspect of this paper concerns an analytical study of specific classes of $\mathcal{N} = 1$ supergravities which appear as the low energy limit of string compactifications. We show that there exist entire classes of compactifications which do not admit any metastable dS vacua, irrespectively of the superpotential or the vacuum expectation values that the moduli may acquire. For instance, we show that de Sitter vacua are excluded in the case of $K3$ fibrations regardless of the number of moduli or their vacuum expectation values. On the other hand, we also identify particular classes of compactifications in which the necessary conditions are indeed fulfilled and thus viable dS vacua should exist. Let us stress here that we do not minimise any explicit potential. Rather we study the condition for the existence of dS vacua and show that irrespectively of the superpotential this condition is not easily satisfied. We think that this is the reason for the difficulties encountered in constructing explicit metastable de Sitter vacua in low energy compactifications of string theory.

The organisation of this paper is as follows. In Section 2, we start by reviewing the conditions under which a generic supergravity model with chiral multiplets admits viable vacua with spontaneously broken supersymmetry and non-negative cosmological constant. Then in Section 3 we apply the resulting condition to the class of models where the Kähler potential satisfies either the no-scale property or a more restrictive homogeneity property respected by large-volume scenarios of string theory. In Sections 4 and 5 we study the large-volume limit of heterotic and orientifold models respectively and derive in each case the form of the metastability condition. There we also apply our general results to classes of models where the metastability condition can be studied analytically and show explicitly that a positive square mass may be achieved only for certain types of compactifications and particular Goldstino directions. We also study the effect of (subleading) $\alpha'$ corrections to the Kähler potential and show that they contribute to the sGoldstino masses and can render them positive even for those models where it is not possible at leading order. Finally, in Section 6 we present our conclusions.

# 2 Metastable vacua in supergravity

In this section, we briefly review and extend the strategy that was presented in refs. [30–32] to study the stability of non-supersymmetric vacua in general supergravity models with $\mathcal{N} = 1$ supersymmetry in four dimensions.[1] We assume that vector multiplets play a negligible role in the dynamics of supersymmetry breaking, and focus thus on theories with only chiral multiplets.

Recall first that the most general two-derivative Lagrangian for a supergravity theory with $n$ chiral superfields is entirely defined by a single arbitrary real function $G$ depending on the corresponding chiral superfields $\Phi^i$ and their conjugates $\bar{\Phi}^{\bar{\imath}}$. Derivatives with respect to $\Phi^i$ and $\bar{\Phi}^{\bar{\jmath}}$ are denoted by lower indices $i$ and $\bar{\jmath}$. Using Planck units where $M_P = 1$, the function $G$ can be decomposed in terms of a real Kähler potential $K$ and a holomorphic superpotential $W$ in the following way:

$$G(\Phi, \bar{\Phi}) = K(\Phi, \bar{\Phi}) + \log W(\Phi) + \log \bar{W}(\bar{\Phi}) \,. \tag{2.1}$$

[1] A similar strategy has also been used in ref. [33] to explore the statistics of supersymmetry breaking vacua in certain classes of string models.

The quantities $K$ and $W$ are however defined only up to Kähler transformations acting as $K \to K + f + \bar{f}$ and $W \to W e^{-f}$, where $f$ is an arbitrary holomorphic function of the superfields. The bosonic part of the action takes the form:

$$S = \int \sqrt{-g} \left[ \frac{1}{2} R - g_{i\bar{\jmath}} \partial \phi^i \partial \bar{\phi}^{\bar{\jmath}} - V(\phi, \bar{\phi}) \right]. \tag{2.2}$$

The Kähler metric $g_{i\bar{\jmath}} = K_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} K$ is used to raise and lower indices, and defines a Kähler geometry for the manifold spanned by the scalar fields. It is assumed to be positive definite, such that the scalar's kinetic energy is positive. The potential takes the following simple form:

$$V = e^G (G^i G_i - 3). \tag{2.3}$$

The auxiliary fields of the chiral multiplets are fixed by their equations of motion to be $F^i = m_{3/2} G^i$ with a scale set by the gravitino mass $m_{3/2} = e^{G/2}$. Whenever $F^i \neq 0$ on the vacuum, supersymmetry is spontaneously broken, and the direction $G^i$ in the space of chiral fermions defines the Goldstino which is absorbed by the gravitino in the process of supersymmetry breaking.

## 2.1 Condition for metastability

Supersymmetry-breaking metastable vacua with non-negative cosmological constant are associated to local minima of the potential at which $F^i \neq 0$ and $V \geq 0$. These vacua can be classified by looking at stationary points with $V' = 0$, imposing that the value of the potential should not be negative, $V \geq 0$, and finally requiring that the Hessian matrix should be positive definite: $V'' > 0$.

The derivatives of the potential (2.3) are most conveniently computed by using the covariant derivative $\nabla_i$ defined by the Kähler metric $g_{i\bar{\jmath}}$, and the associated Riemann curvature tensor $R_{i\bar{\jmath}m\bar{n}}$. The first derivative is just $V_i = \nabla_i V$, and the stationarity conditions $V_i = 0$ read

$$e^G \left( G_i + G^k \nabla_i G_k \right) + G_i V = 0. \tag{2.4}$$

The second derivatives of the potential can also be computed by using covariant derivatives, since the extra connection terms vanish by the stationarity conditions. There are two different $n$-dimensional blocks, $V_{i\bar{\jmath}} = \nabla_i \nabla_{\bar{\jmath}} V$ and $V_{ij} = \nabla_i \nabla_j V$, and these are found to be given by the following expressions:[2]

$$V_{i\bar{\jmath}} = e^G \left( G_{i\bar{\jmath}} + \nabla_i G_k \nabla_{\bar{\jmath}} G^k - R_{i\bar{\jmath}m\bar{n}} \, G^m G^{\bar{n}} \right) + \left( G_{i\bar{\jmath}} - G_i G_{\bar{\jmath}} \right) V, \tag{2.5}$$

$$V_{ij} = e^G \left( 2 \nabla_i G_j + G^k \nabla_i \nabla_j G_k \right) + \left( \nabla_i G_j - G_i G_j \right) V. \tag{2.6}$$

[2]Our conventions for the Riemann tensor are given by eq. (A.5) in the Appendix.

The metastability condition is then the requirement that the whole $2n$-dimensional Hessian mass matrix $M^2$ should be positive definite, where

$$M^2 = \begin{pmatrix} V_{i\bar{\jmath}} & V_{ij} \\ V_{\bar{\imath}\bar{\jmath}} & V_{\bar{\imath}j} \end{pmatrix} . \tag{2.7}$$

It is clear that for a fixed Kähler potential $K$, most of the eigenvalues of this mass matrix can be made positive and arbitrarily large by suitably tuning the superpotential $W$. More precisely, the $n-1$ chiral multiplets that are orthogonal to the Goldstino multiplet can acquire a large overall supersymmetric mass contribution from $W$, which can overcome the mass splitting of order $m_{3/2}$ induced by supersymmetry breaking, and lead to positive square masses for the scalar field components. The Goldstino multiplet, on the other hand, cannot receive any supersymmetric mass contribution from $W$, since in the limit of rigid supersymmetry its fermionic component must be massless. The mass splitting of order $m_{3/2}$ induced by supersymmetry breaking can then potentially make the square mass of the scalar field component negative.

From a more technical point of view, this conclusion can be obtained by recalling that derivatives of $G$ with mixed holomorphic and antiholomorphic indices depend only on $K$, while quantities like $G_i$, $\nabla_i G_j$ and $\nabla_i \nabla_j G_k$ depend also on $W$, and more precisely on $(\log W)_i$, $(\log W)_{ij}$ and $(\log W)_{ijk}$. Keeping $K$ fixed and tuning $W$, one can then vary in an arbitrary way these quantities. This allows to adjust first the quantities $\nabla_i \nabla_j G_k$ to set the block $V_{ij}$ to zero, and next the quantities $\nabla_i G_j$ to make most of the eigenvalues of $V_{i\bar{\jmath}}$ positive. On top of that, one still has the freedom of arbitrarily choosing $G_i$. The only restriction in the second step comes from the fact that the projection of $V_{i\bar{\jmath}}$ along the Goldstino direction $G^i$ is actually fixed by the stationarity condition (2.4), and can therefore not be adjusted. This means that the square masses of the two sGoldstinos cannot be arbitrarily shifted by adjusting $W$, and that their value crucially depends on $K$.

In order to study metastability, it is thus sufficient to study the projection of the diagonal block $V_{i\bar{\jmath}}$ of the mass matrix along the Goldstino direction $G^i$. More precisely, we find it convenient to rescale this quantity by the overall mass scale $m_{3/2}^2$ and consider the following parameter:

$$\lambda = e^{-G} \, V_{i\bar{\jmath}} \, G^i G^{\bar{\jmath}} \, . \tag{2.8}$$

Strictly speaking $\lambda$ is a linear combination of eigenvalues of $V_{i\bar{\jmath}}$ with non-negative coefficients in front of them. It therefore defines a natural mass scale $\tilde{m}^2 \equiv e^G \lambda / G^i G_i$ which can be thought of as the mass obtained by projecting $V_{i\bar{\jmath}}$ along the Goldstino direction $G^i$. Accordingly, we identify here $\tilde{m}$ with the mass of the sGoldstinos.

By using eqs. (2.4) and (2.5), one can compute $\lambda$ more explicitly. The result is found to depend only on the parameters $G^i = e^{-G/2} F^i$ defining the direction of supersymmetry breaking, contracted with the metric and the Riemann tensor of the scalar geometry:

$$\lambda = 2\, g_{i\bar{\jmath}} G^i G^{\bar{\jmath}} - R_{i\bar{\jmath}m\bar{n}}\, G^i G^{\bar{\jmath}} G^m G^{\bar{n}} \,. \tag{2.9}$$

For given $K$ and arbitrary $W$, the quantities $G^i$ can be varied but the metric and the Riemann tensor are fixed. One can then look for the preferred direction that maximises $\lambda$.[3] If $\lambda_{\max} < 0$, then one of the sGoldstinos is unavoidably tachyonic, and the vacuum is unstable. If instead $\lambda_{\max} > 0$, then the sGoldstinos can be kept non-tachyonic by choosing $W$ such that the Goldstino direction is close enough to the preferred direction, and more precisely inside a cone for which $\lambda \in [0, \lambda_{\max}]$. As already mentioned, the rest of the scalars can always be given a positive square mass by further tuning $W$. The crucial condition for metastability, which constrains both the Kähler geometry and the supersymmetry breaking direction, is then [30]

$$\lambda > 0 \,. \tag{2.10}$$

## 2.2 Analysis of the metastability condition

The implications of the metastability condition $\lambda > 0$ have been studied in refs. [30, 31] for models with a fixed cosmological constant. But one can actually perform a similar study without specifying the value of the cosmological constant and only requiring that it is non-negative. It is clear from the form of eq. (2.9) that for sufficiently small values of the $G^i$, it would always be possible to find configurations such that $\lambda > 0$, since the quartic term becomes subdominant and the quadratic term is positive. However, in this regime the cosmological constant would necessarily be negative. Whenever some of the $G^i$ are instead of order 1, as required to achieve a non-negative cosmological constant, the quadratic and quartic terms compete, and the existence of configurations with $\lambda > 0$ strongly depends on the form of the curvature tensor. To analyse the rather constrained problem of finding whether there exist vacua with $V \geq 0$ and $\lambda > 0$ it is convenient to rewrite $\lambda$ as the sum of two pieces,

$$\lambda = -\frac{2}{3} e^{-G} V (e^{-G} V + 3) + \sigma, \tag{2.11}$$

where $\sigma$ is defined to be

$$\sigma = \Big[\frac{1}{3}\,(g_{i\bar{\jmath}}\, g_{m\bar{n}} + g_{i\bar{n}}\, g_{m\bar{\jmath}}) - R_{i\bar{\jmath}m\bar{n}}\Big] G^i G^{\bar{\jmath}} G^m G^{\bar{n}} \,. \tag{2.12}$$

[3] See ref. [34] for an algebraic method for finding the minima for a wide class of superpotentials.

As long as $V > 0$ the first term in eq. (2.11) is always negative and its precise value depends only on the length of the vector $G^i$ which determines the cosmological constant. The second term in eq. (2.11) has instead a sign that depends only on the orientation of the vector $G^i$, and not on its length. Therefore, the possibility of finding solutions to the metastability condition $\lambda > 0$ depends exclusively on the sign of $\sigma$. Indeed, starting from any $G^i$ such that $\sigma(G^i) > 0$, one can always tune the superpotential $W$ to rescale $G^i$ by some real factor $r$ to achieve $V(rG^i) = 0$ and thus $\lambda(rG^i) > 0$, proving the existence of Minkowski vacua. Moreover, by slightly increasing $r$ one can make $V(rG^i) > 0$ and still keep $\lambda(rG^i) > 0$, achieving thereby de Sitter vacua. For a fixed value of the gravitino mass scale $m_{3/2} = e^{G/2}$ it is however clear that how big a cosmological constant $V$ can be achieved while keeping $\lambda > 0$ depends on the size of $\sigma$ for the reference situation where $V(G^i) = 0$. The same kind of reasoning tells us that if $\sigma < 0$ for all the possible orientations of $G^i$, then one can never achieve $V \geq 0$ and $\lambda > 0$ simultaneously. We can therefore conclude that the analysis of the sign of the function $\lambda$ for non-supersymmetric vacua with $V \geq 0$ is equivalent to the analysis of the sign of the function $\sigma$ without specifying the value of the cosmological constant. More precisely, the condition for the existence of viable vacua is that

$$\sigma > 0\,. \tag{2.13}$$

It is easy now to check a few well known results concerning the existence of metastable vacua. Consider for instance those models where the Kähler potential is of the canonical form $K = \sum_i |\Phi^i|^2$ for which the Kähler manifold has a vanishing Riemann tensor. In this case one has

$$\sigma = \frac{2}{3}(G^i \bar{G}_i)^2 > 0\,, \tag{2.14}$$

and no obstruction is met towards the construction of metastable vacua. Another simple example is provided by string compactifications described by a single volume modulus $T$ and a no-scale Kähler potential of the form $K = -3\log(T + \bar{T})$. In this case, one finds that

$$\sigma = 0\,, \tag{2.15}$$

independently of the value $G^T$, and thus dS vacua are excluded [29] (see also [35]). Finally, models with separable $K = -3\log(T + \bar{T}) + \sum_i |\Phi^i|^2$ also grant the existence of de Sitter vacua as long as $G^i \neq 0$. If $W$ is separable as well, so that the 2 sectors interact only gravitationally, it is actually possible to uplift any would-be supersymmetric minimum in the $T$ sector with a $\Phi^i$ sector breaking spontaneously supersymmetry well below the Planck scale [30]. See [36] for a generalization to a certain class of non-separable $W$, and [37, 38] for specific examples. On the other hand, for similar models with non-separable

$K = -3\log(T + \bar{T} - 1/3\sum_i |\Phi^i|^2)$, as those considered in ref. [39], the scalar manifold is maximally symmetric and one finds again $\sigma = 0$ [31]. See ref. [27] for a recent general study of this type of uplifting.

Notice that $\sigma$ has the very useful property of being a homogeneous function of degree $(2,2)$ in the variables $(G_i, G_{\bar{j}})$, meaning that

$$G_i \frac{\partial \sigma}{\partial G_i} = G_{\bar{j}} \frac{\partial \sigma}{\partial G_{\bar{j}}} = 2\sigma\,. \tag{2.16}$$

As a consequence of this property, any stationary point of $\sigma$ as a function of $G_i$ leads to $\sigma = 0$. This implies in turn that, at any given point in the Kähler manifold spanned by the chiral fields, the function $\sigma$ can have only one such stationary point, or a degenerate family of them, with $\sigma = 0$. This is due to the fact that if the value of the function becomes non-zero when moving away from such a stationary point, then its first derivative is no longer allowed to vanish again.

Based on this property, it is possible to outline a general and systematic procedure to find out whether $\sigma > 0$ can be achieved in a particular model by only requiring that the set of points $G_i^0$, at which $\sigma$ becomes stationary, is known. Indeed, it is sufficient to study the convexity of the function $\sigma(G^i)$ in the vicinity of $G_i^0$ by scanning all the orientations of $G_i$ away from $G_i^0$ for which $\sigma$ is allowed to grow. If $\sigma(G_i^0) = 0$ is a local minimum then, by the method described before, any direction $G_i \neq G_i^0$ may be rescaled to render a metastable vacuum. If instead $\sigma(G_i^0) = 0$ turns out to be a maximum, then one is forced to exclude the Kähler potential $K$ of the model as a possible candidate to generate metastable vacua. Finally, if $\sigma(G_i^0) = 0$ turns out to be a saddle point, then only a reduced subset of orientations $G_i$ will qualify to render metastable vacua. We should bear in mind, however, that the metric and the Riemann tensor appearing in the definition of $\sigma$ depend on the values of the scalar fields. Therefore, one should also scan over the allowed values of $\phi^i$.

The procedure just described is very useful and in principle simple to implement when the convexity of the function $\sigma$ cannot be determined analytically. This is particularly the case of the class for models appearing in large volume compactifications of string theory. As we show in the next section, the scaling properties respected by the type of Kähler potentials appearing in such scenarios imply two important properties of the function $\sigma$: first, stationary points of $\sigma$ are of the form $G^i \propto K^i$, and second, such points are either of the saddle-point type or maxima. One is then left with the task of determining, by studying the vicinity of $G^i \propto K^i$, which one of these two situations is being dealt with.

# 3 Metastability in large-volume scenarios

We now focus on some generic properties respected by models emerging in large-volume scenarios of string theory. More specifically, we apply the analysis of the previous section to the class of models where the Kähler potential satisfies either the no-scale property or an even more restrictive scaling property.

## 3.1 No-scale models

A common characteristic found in string compactifications is the no-scale property [40]

$$K^i K_i = 3\,, \tag{3.1}$$

which holds for the Kähler moduli parameterising the shape and size of the compactified volume in the large-volume limit. Similarly, it also holds for the complex structure moduli in the large-complex-structure limit. We would then like to study the function $\sigma$ as defined in (2.12) for the particularly relevant class of supergravity models satisfying this no-scale property, in order to understand whether this restriction implies any useful information concerning metastability.

The simplest examples of such no-scale models are certain coset manifolds of the type $SU(p,q)/(U(1)\times SU(p)\times SU(q))$ and $SO(2,2+p)/(SO(2)\times SO(2+p))$, with appropriate constant curvature, arising in orbifold string models. Due to the fact that they are homogeneous and symmetric, these particular spaces lead to a simple form of the Riemann tensor. The implications of the stability condition can then be worked out completely. It was in fact shown in [31] that in these models the maximal value of $\sigma$ is precisely zero, and that this value is obtained for the particular direction $G^i = K^i$, or equivalent directions related to this by the isometries of the space.

In more complicated situations where the curvature is not constant, like in Calabi-Yau models with and without orientifolds, the Riemann tensor takes a more complicated form and the study of the metastability condition becomes substantially more complicated. However, since the property (3.1) is valid at any point of the Kähler manifold, it implies some simple and nontrivial restrictions on the Riemann tensor, and in particular on its contractions with the special vector $K^i$. For instance, taking one derivative of (3.1) one finds

$$K_i + K^k \nabla_i K_k = 0, \tag{3.2}$$

whereas taking two derivatives one deduces the following relations:

$$g_{i\bar{\jmath}} + \nabla_i K_k \nabla_{\bar{\jmath}} K^k - R_{i\bar{\jmath}m\bar{n}} K^m K^{\bar{n}} = 0\,, \tag{3.3}$$

$$2\,\nabla_i K_j + K^k \nabla_i \nabla_j K_k = 0\,. \tag{3.4}$$

Contracting the first of these relations with $K^i K^{\bar{\jmath}}$ and $K^{\bar{\jmath}}$ respectively, one can then derive the relations

$$R_{i\bar{\jmath}m\bar{n}} K^i K^{\bar{\jmath}} K^m K^{\bar{n}} = 6\,, \tag{3.5}$$

$$R_{i\bar{\jmath}m\bar{n}} K^{\bar{\jmath}} K^m K^{\bar{n}} = 2K_i\,. \tag{3.6}$$

These relations are useful to study the function $\sigma$ for this class of models. In order to do so, it is natural to introduce the projector onto the subspace orthogonal to $K^i$, since we know that at least in the particular case of constant curvature manifolds this is the special direction that maximises $\sigma$. Thanks to the no-scale property, this projector is simply

$$P_i^j = \delta_i^j - \frac{1}{3} K_i K^j\,. \tag{3.7}$$

We can then decompose the vector $G_i$ into two independent pieces, one parallel to $K_i$ and parameterised by a numerical coefficient $\alpha$, and one orthogonal to $K_i$ and parameterised by a vector $N_i$ satisfying $N^i K_i = 0$:

$$G_i = \alpha K_i + N_i\,. \tag{3.8}$$

The quantities $\alpha$ and $N^i$ are given by

$$N_i = P_i^j G_j\,, \qquad \alpha = \frac{1}{3} K^i G_i\,. \tag{3.9}$$

The function $\sigma$, as defined in eq. (2.12), may then be expressed in terms of the independent quantities $\alpha$ and $N_i$ in the following way:

$$\begin{aligned}\sigma =\;& 4|\alpha|^2 \left(g_{i\bar{\jmath}} - R_{i\bar{\jmath}m\bar{n}} K^m K^{\bar{n}}\right) N^i N^{\bar{\jmath}} - \left(\bar{\alpha}^2 R_{i\bar{\jmath}m\bar{n}} K^i K^m N^{\bar{\jmath}} N^{\bar{n}} + \text{c.c}\right) \\ & -2\,\left(\bar{\alpha} R_{m\bar{n}i\bar{\jmath}} K^m N^{\bar{n}} N^i N^{\bar{\jmath}} + \text{c.c}\right) \\ & + \Big[\frac{1}{3}\left(g_{i\bar{\jmath}}\, g_{m\bar{n}} + g_{i\bar{n}} g_{m\bar{\jmath}}\right) - R_{m\bar{n}i\bar{\jmath}}\Big] N^i N^{\bar{\jmath}} N^m N^{\bar{n}}\,.\end{aligned} \tag{3.10}$$

Note that this result is at least quadratic in the variables $N^i$. This implies that there is a degenerate family of stationary points for $N^i = 0$ and arbitrary $\alpha$, that is for $G^i \propto K^i$, with value $\sigma = 0$. To say more about the convexity of $\sigma$ at this set of points we still require some more information regarding contractions between $K_i$ and the Riemann tensor. As we will see in the following, this additional information can be obtained by imposing an extra condition generically respected by large-volume string compactifications.

## 3.2 Real homogeneous no-scale models

A more restrictive property characterising large-volume scenarios is that their Kähler potential depends only on the real part of the superfields and exhibits therefore $n$ independent shift symmetries, under which $\delta_i \Phi^j = i\epsilon\delta_i^j$ with constant $\epsilon$. This means in particular that any distinction between holomorphic and antiholomorphic indices can be dropped. Furthermore, it turns out that there exists a coordinate frame where $e^{-K}$ is a homogeneous function of degree 3 in the fields $\Phi^i + \bar{\Phi}^i$. This implies that

$$-(\Phi^i + \bar{\Phi}^i) K_i = 3\,. \tag{3.11}$$

Taking a derivative, it then follows that

$$K^i = -(\Phi^i + \bar{\Phi}^i)\,. \tag{3.12}$$

This equation guarantees, together with the previous one, that the no-scale property $K^i K_i = 3$ is satisfied. But taking a derivative, it also implies that $\partial_i K^j = -\delta_i^j$, which after lowering the indices implies

$$K_{ijm} K^m = 2\, g_{ij}\,. \tag{3.13}$$

Taking another derivative of this, one finds also

$$K_{ijmn} K^m = 3 K_{ijn}\,. \tag{3.14}$$

From these two equations, it follows then that

$$R_{ijmn} K^m = K_{ijn}\,, \tag{3.15}$$

$$R_{ijmn} K^m K^n = R_{imjn} K^m K^n = 2\, g_{ij}\,. \tag{3.16}$$

Finally, contracting these equations with one and two more $K^k$'s and using the no-scale condition, one also recovers the same relations (3.5) and (3.6) holding for general no-scale models.

It is convenient at this point to introduce a new notation to deal with complex quantities such as $G_i$ and $G_{\bar{\imath}}$ in such a way that the bar does not appear on top of the indices. Compared to the usual notation, we introduce the following substitutions: $G_i \to G_i$, $G_{\bar{\imath}} \to \bar{G}_i$, $G^i \to \bar{G}^i$, $G^{\bar{\imath}} \to G^i$. Similarly, for the $N_i$'s we use: $N_i \to N_i$, $N_{\bar{\imath}} \to \bar{N}_i$, $N^i \to \bar{N}^i$, $N^{\bar{\imath}} \to N^i$.

Using eqs. (3.5), (3.6), (3.15) and (3.16), and decomposing as before $G_i = \alpha K_i + N_i$ and $\bar{G}^i = \bar{\alpha} K^i + \bar{N}^i$, one finds that the function $\sigma$ takes in this case the following form:

$$\begin{aligned} \sigma = & -2\left(\alpha \bar{N}^i + \bar{\alpha} N^i\right)\left(\alpha \bar{N}_i + \bar{\alpha} N_i\right) - 2\, K_{imn}\left(\alpha \bar{N}^i + \bar{\alpha} N^i\right) N^m \bar{N}^n \\ & + \Big[\frac{1}{3}\left(g_{ij}\, g_{mn} + g_{in}\, g_{mj}\right) - R_{ijmn}\Big] N^i \bar{N}^j N^m \bar{N}^n\,. \end{aligned} \tag{3.17}$$

This result shows that $\sigma$ has a local maximum with value 0 at $N_i = 0$ at quadratic order in the $N^i$ variables for orientations of $G^i$ characterised by $\alpha\bar{N}^i + \bar{\alpha}N^i \neq 0$. Nevertheless, this does not imply that $\sigma$ is negative definite, because when $\alpha\bar{N}^i + \bar{\alpha}N^i = 0$ the potential is flat at the quadratic and cubic orders and its convexity is determined by the quartic terms in $N_i$. In order to gain further insight it is useful to complete the squares in the variable $\alpha\bar{N}^i + \bar{\alpha}N^i$ and rewrite $\sigma$ in the form

$$\sigma = -2\, s^i s_i + \omega\,, \tag{3.18}$$

where

$$s^i = \alpha\bar{N}^i + \bar{\alpha}N^i + \frac{1}{2}P^{ij}K_{jmn}N^m\bar{N}^n\,, \tag{3.19}$$

$$\omega = \Big[\frac{1}{3}(g_{ij}\, g_{mn} + g_{in}\, g_{mj}) - R_{ijmn} + \frac{1}{2}K_{ijk}P^{kl}K_{lmn}\Big]N^i\bar{N}^j N^m\bar{N}^n\,. \tag{3.20}$$

Observe now that all the dependence on $\alpha$ is contained in the semi-negative definite term $-2s^i s_i$ involving the norm of the vector $s_i$. This fact allows us to eliminate one redundant direction in the superpotential parameter space spanned by the $G^i$'s in the analysis of $\sigma$. Indeed, observe that $\sigma$ can be maximised with respect to $\alpha$ when $\alpha$ is chosen in such a way that $s^i N_i = 0$. Since our interest is to determine whether $\sigma > 0$ can be achieved, this condition fixes $\alpha$ in terms of $N^i$. It also reduces the number of orientations of $G_i$ that need to be analysed in order to deduce the convexity of $\sigma$ about the set of stationary points $G_i \propto K_i$. Notice additionally that in the particular case of two moduli $i = 1, 2$, the condition $s^i N_i = 0$ is equivalent to $s_i = 0$, as there is only one possible direction perpendicular to $K_i$, implying that $s_i$ and $N_i$ are parallel to each other.

In the next two sections we study more concretely the function $\sigma$ for the two relevant cases of heterotic and orientifold compactifications of string theory.

# 4 Heterotic compactifications of string theory

In this section we consider a class of supergravity models which arises in compactifications of the heterotic string on Calabi-Yau threefolds.[4] Let us first discuss some generic features of these compactifications and then continue with specific examples.

## 4.1 General discussion

The moduli of heterotic Calabi-Yau compactifications include the dilaton/axion and the deformations of the Calabi-Yau metric. The latter are divided into deformations of the

[4] Alternatively they can also be viewed as the NS-sector of type II compactifications.

Kähler class and deformations of the complex structure. Locally, the moduli space $\mathcal{M}$ is the product manifold

$$\mathcal{M} = \mathcal{M}^{\text{ks}} \times \mathcal{M}^{\text{cs}} \times \frac{SU(1,1)}{U(1)}\,, \tag{4.1}$$

where $\mathcal{M}^{\text{ks}}$ is the space spanned by the Kähler moduli, $\mathcal{M}^{\text{cs}}$ is spanned by the complex structure moduli while the dilaton/axion are the coordinates of the last factor. $\mathcal{M}^{\text{ks}}$ and $\mathcal{M}^{\text{cs}}$ are special Kähler manifolds in that their Kähler potential can be expressed in terms of a holomorphic prepotential $f = f(\Phi)$. One has [41–43]

$$K = -\log Y, \qquad \text{with} \qquad Y = -2(f + \bar{f}) + (f_k + \bar{f}_{\bar{k}})(\Phi^k + \bar{\Phi}^k)\,, \tag{4.2}$$

where in the large-volume limit $Y^{\text{cs/ks}}$ are given by

$$Y^{\text{cs}} = i\int_X \Omega \wedge \bar{\Omega}\,, \qquad Y^{\text{ks}} = \mathcal{V} \equiv \frac{4}{3}\int_X J \wedge J \wedge J\,. \tag{4.3}$$

Here $\Omega$ and $J$ are, respectively, the holomorphic $(3,0)$-form and the Kähler $(1,1)$-form of the Calabi-Yau threefold. $\mathcal{V}$ is the classical volume in that the equality $Y^{\text{ks}} = \mathcal{V}$ only holds in the large-volume limit, and it is modified by $\alpha'$ and worldsheet-instanton corrections.

There exist various dynamical effects, such as fluxes or gaugino condensates, which can induce a nontrivial superpotential $W$ for the moduli [28]. We do not systematically discuss here all the possible superpotentials but rather assume that most of the moduli are stabilised in a supersymmetric way at high energy scales. In addition we assume that supersymmetry is broken by $F$-terms of the remaining moduli multiplets.[5] This latter sector is the one we want to study in the spirit of Sections 2 and 3. In other words, we want to understand under what conditions the moduli sector can simultaneously break supersymmetry and generate a de Sitter vacuum.

For concreteness, let us focus on the Kähler moduli sector in the large-volume limit and assume that it induces supersymmetry breaking. Of course we could equivalently consider the complex structure moduli in the large-complex-structure limit which –due to mirror symmetry– would lead to an identical analysis.

Since $J$ is harmonic, it can be expanded in a $h^{1,1}$-dimensional basis $w_i, i = 1, \ldots, h^{1,1}$ of the cohomology group $H^{1,1}$ via $J = v^i w_i$. The NS two-form enjoys a similar expansion $B_2 = b^i \omega_i$. The coefficients in these expansions $v^i$ and $b^i$ are scalar fields which combine into the complex coordinates $T^i = v^i + ib^i$. Inserting this into (4.3), one obtains

$$K = -\log \mathcal{V}\,, \qquad \text{with} \qquad \mathcal{V} = \frac{1}{6}\, d_{ijk}\,(T^i + \bar{T}^i)(T^j + \bar{T}^j)(T^k + \bar{T}^k)\,, \tag{4.4}$$

[5]We similarly assume that matter fields are stabilised at supersymmetric points and that their vacuum expectation values remain zero after supersymmetry is broken by the moduli.

where $d_{ijk} = \int_X w_i \wedge w_j \wedge w_k$ are the Calabi-Yau intersection numbers.[6]

Before we continue let us emphasise that such a Kähler potential also appears as a subsector of other string compactifications, for example, in Calabi-Yau compactifications of type IIB with $O5/O9$-orientifold planes [44]. Therefore the following analysis is not only valid for heterotic compactifications but rather for any moduli-sector with a Kähler potential of the form given in eq. (4.4).

In order to compute $\sigma$ let us first recall a few further properties of $K$ (for more details on the following computations we refer the reader to the appendix). Its first derivative reads

$$K_i = -\frac{\mathcal{V}_i}{\mathcal{V}} \ , \qquad \mathcal{V}_i = \frac{1}{2} d_{ijk}(T^j + \bar{T}^j)(T^k + \bar{T}^k) \ . \tag{4.5}$$

The Kähler metric is then given by

$$g_{ij} \ = \ -\frac{\mathcal{V}_{ij}}{\mathcal{V}} + \frac{\mathcal{V}_i \mathcal{V}_j}{\mathcal{V}^2} \ = \ e^K d_{ijk} K^k + K_i K_j \ , \tag{4.6}$$

where the matrix $\mathcal{V}_{ij} = d_{ijk}(T^k + \bar{T}^k)$ has a signature $(1, h^{1,1} - 1)$ for all allowed values of $T^i + \bar{T}^i$, i.e. those values for which $\mathcal{V}$ is positive and the Kähler metric is positive-definite [43]. The inverse metric is conveniently expressed in terms of the matrix $\mathcal{V}^{ij}$ which is defined as the inverse of $\mathcal{V}_{ij}$, i.e. $\mathcal{V}^{ij}\mathcal{V}_{jk} = \delta^i_k$. Using $2\mathcal{V}^{ij}\mathcal{V}_j = T^i + \bar{T}^i = -K^i$ one has

$$g^{ij} = -\mathcal{V}\mathcal{V}^{ij} + \frac{1}{2} K^i K^j. \tag{4.7}$$

From (4.5) and (4.7) it follows that $K$ obeys the the no-scale condition (3.1) and also the homogeneity property (3.11).

Using (4.5)–(4.7) one also easily computes the third derivative of $K$ and its Riemann tensor:

$$K_{ijk} = -e^K d_{ijk} + g_{ij} K_k + g_{ik} K_j + g_{jk} K_i - K_i K_j K_k \ , \tag{4.8}$$
$$R_{ijmn} = g_{ij} g_{mn} + g_{in} g_{mj} - e^{2K} d_{imp} g^{pq} d_{qjn} \ . \tag{4.9}$$

Notice that the specific form of the Riemann tensor holds for any special Kähler manifold with $d_{ijk}$ replaced by the third derivative $f_{ijk}$ of the prepotential [45, 46]. Inserting (4.9) into eq. (2.12) we finally obtain

$$\sigma = -\frac{4}{3}(G^i \bar{G}_i)^2 + e^{2K} G^i G^j d_{ijp} g^{pq} d_{qmn} \bar{G}^m \bar{G}^n. \tag{4.10}$$

[6] This is indeed a special Kähler geometry since $\mathcal{V}$ can be derived from the holomorphic prepotential $f(T) = 1/6\, d_{ijk} T^i T^j T^k$.

As in the last section we can rewrite $\sigma$ in terms of $K_i$ and its orthogonal complement $N_i$ as defined in eqs. (3.8) and (3.9). Inserting (4.8) and (4.9) into (3.19) and (3.20) we arrive at $\sigma = -2s^i s_i + \omega$ with $s_i$ and $\omega$ given by

$$s^i = \alpha \bar{N}^i + \bar{\alpha} N^i - \frac{1}{2} e^K P^{ij} d_{jmn} N^m \bar{N}^n \,, \tag{4.11}$$

$$\omega = \Big(-\frac{4}{3} g_{ij}\, g_{mn} + \frac{1}{3} g_{im}\, g_{jn} + \frac{1}{2} e^{2K} d_{ijp} P^{pq} d_{qmn} + e^{2K} d_{imp} P^{pq} d_{qjn}\Big) N^i \bar{N}^j N^m \bar{N}^n \,. \tag{4.12}$$

Let us recall here that with these expressions it is possible now to study the convexity of $\sigma$ by scanning $N^i$ and keeping $\alpha$ fixed in such a way that $s^i N_i = 0$.

## 4.2 Particular classes of models

We now discuss a few specific classes of Kähler moduli spaces that can be handled analytically. As we shall see, it is possible to obtain examples of models where $\sigma > 0$ for certain directions $G^i$ offering the possibility of generating metastable vacua. Nevertheless, we shall also see that there are entire classes of models for which $\sigma$ is unavoidably negative-definite, implying the existence of at least one tachyonic state in the spectrum which renders the theory unstable independently of the form of the superpotential.

### 4.2.1 Factorisable Kähler manifolds

As our first example we discuss Calabi-Yau threefolds which are $K3$-fibrations over a $\mathbf{P_1}$-base. In the limit of a large $\mathbf{P_1}$ the Kähler potential simplifies and reads [47, 48]

$$K = -\log\Big(\frac{1}{2}\, d_{1ab}(T^1 + \bar{T}^1)(T^a + \bar{T}^a)(T^b + \bar{T}^b) + \dots\Big)\,, \tag{4.13}$$

where $T^1$ parametrises the volume of the $\mathbf{P_1}$-base while the $T^a, a = 2, \dots, h^{1,1}$ are moduli of the $K3$ fibre. The dots indicate further cubic terms which, however, are independent of $T^1$ and therefore subleading in the large $\mathbf{P_1}$-limit. In that limit the Kähler metric is block diagonal ($g_{1a} = 0$) and hence the moduli space factorises into the special Kähler space[7]

$$\mathcal{M}^{\rm ks} \;=\; \frac{SU(1,1)}{U(1)} \times \frac{SO(2, h^{1,1}-1)}{SO(2) \times SO(h^{1,1}-1)}\ . \tag{4.14}$$

The Kähler potential also enjoys the properties

$$K^1 K_1 = 1\ , \qquad K^a K_a = 2\ . \tag{4.15}$$

[7]This also uses the fact that the matrix $d_{1ab}$ has signature $(1, h^{1,1} - 2)$.

In order to compute $\sigma$ we observe that (4.6) implies $d_{1ab} = e^{-K} K_1 \, (g_{ab} - K_a K_b)$ which, together with (4.15), leads to

$$e^{2K} \, d_{1ac} \, d^c{}_{1b} = g_{11} \, g_{ab} \ , \qquad e^{2K} d_{ab1} \, d^1{}_{ce} = (g_{ab} - K_a K_b) \, (g_{ce} - K_c K_e) \, . \tag{4.16}$$

Inserting this into (4.10) we obtain

$$\sigma = -\frac{4}{3}(G^1 \bar{G}_1 + G^a \bar{G}_a)^2 + |G_a G^a - (K_a G^a)^2|^2 + 4 \, (G^1 \bar{G}_1)(G^a \bar{G}_a) \, . \tag{4.17}$$

To find an upper bound for this function, we use the inequality $|A \cdot B|^2 \leq |A|^2 |B|^2$ for $A_a = (g_{ab} - K_a K_b) \, G^b$ and $B_a = G_a$. This together with (4.15) yields

$$|G_a G^a - (K_a G^a)^2|^2 \leq (G^a \bar{G}_a)^2 \, . \tag{4.18}$$

As a consequence, the function $\sigma$ given in eq. (4.17) obeys

$$\sigma \leq -\frac{1}{3} \, (2 \, G^1 \bar{G}_1 - G^a \bar{G}_a)^2 \, . \tag{4.19}$$

We see that $\sigma$ is always negative and vanishes along the flat direction where $2G^1 \bar{G}_1 = G^a \bar{G}_a$. This means that the preferred supersymmetry breaking direction is $G^i \propto K^i$ as for models with constant curvature. We conclude that in this class of models one always has a tachyonic sGoldstino, which can at best become massless for Minkowski vacua and for a special Goldstino direction.

Note that the scalar manifold (4.14) associated with these factorisable models is a constant curvature coset manifold. The implications of the metastability condition for this type of models were also studied in ref. [31]. It was in particular shown that the second factor in (4.14) behaves effectively as two copies of the first factor, independently of $h^{1,1}$. This implies that the metastability condition for $K3$ fibrations is analogous to that of models with 3 independent moduli, as in eq. (1.2) with $n_i = 1$, providing an alternative derivation of the fact that $\sigma$ is at best zero in these models.

#### 4.2.2 Two-field models

Another class of models that can be studied analytically are those with only 2 moduli $T^i = v^i + i b^i$, with $i = 1, 2$. To perform this analysis we recall that $\sigma$ may be written as $\sigma = -2 s^i s_i + \omega$ with $s_i$ and $\omega$ given by eqs. (4.11) and (4.12) respectively. In the case of 2 moduli it was shown in Section 3.2 that it is always possible to choose $s_i = 0$, thereby maximising $\sigma$. We are thus left with the task of computing the function $\omega$ and check if $\omega > 0$ is allowed. As can be read from (4.12), the function $\omega$ depends on the variables

$N_i$. Since these are orthogonal to $K^i$, they can be parameterised with a single complex quantity $C$ as

$$(N^1, N^2) = (K_2, -K_1)\, C\,. \tag{4.20}$$

With this definition, one has $N^i N_i = 3 \det g\, |C|^2$.

One first case that we can analyse is the case of models with only diagonal intersection numbers $d_{111}$ and $d_{222}$. In this example the Kähler potential takes the form

$$K = -\log\Big(\frac{1}{6} d_{111}(T^1 + \bar{T}^1)^3 + \frac{1}{6} d_{222}(T^2 + \bar{T}^2)^3\Big)\,. \tag{4.21}$$

Computing the metric and its inverse, and using eqs. (4.20) and (4.21) with (4.12), we find that

$$\omega = \frac{81}{8}\, e^{4K}\, \frac{d_{111}^2 d_{222}^2}{\det g}\, |C|^4\,. \tag{4.22}$$

This result is positive since the metric has to be positive definite. This shows that $\sigma$ can be made positive and that the stability condition can be fulfilled for certain particular directions of $G_i$. As shown for general large-volume scenarios, we find that the point $N^i = 0$, where $G^i \propto K^i$, is indeed a stationary point with $\sigma = 0$. Nevertheless, as can be read off from (4.22), in this case this stationary point is a saddle point, and $\sigma(G^i)$ can actually be made positive along some directions.

By now we have shown that in the case of factorisable Kähler potentials we get $\omega = 0$ and in the case of diagonal intersection numbers we get $\omega > 0$. But one may wonder whether in some cases one can have $\omega < 0$. In order to answer this question, let us consider a model with the following Kähler potential:

$$K = -\log\Big(\frac{1}{2}\, d_{122}\, (T^1 + \bar{T}^1)(T^2 + \bar{T}^2)^2 + \frac{1}{6}\, d_{111}(T^1 + \bar{T}^1)^3\Big)\,. \tag{4.23}$$

Note now that in the limit $d_{111} \to 0$ this Kähler potential becomes of the form (4.13) describing factorisable models, for which the maximal value of $\sigma$ is zero. One can then study how this result is modified in the case where $d_{111} \ll d_{122}$ by performing an expansion in the small parameter

$$\epsilon = \frac{d_{111}}{d_{122}}\,. \tag{4.24}$$

Following now the same strategy as before it is straightforward to find that

$$\omega = \frac{81}{2}\, \epsilon\, e^{4K}\, \frac{d_{122}^4}{\det g}\, |C|^4\,. \tag{4.25}$$

This result can be either positive or negative depending on the sign of $\epsilon$. This implies that $\sigma$ can be positive or must be negative, depending on the sign of $\epsilon$.

Actually, for these two-field models it is possible to compute the function $\omega$ for generic values of all the independent intersection numbers $d_{111}$, $d_{222}$, $d_{122}$ and $d_{112}$. Using the general form for the Kähler potential (4.4) and following the same steps as in the previous examples one finds, after some algebra, that the value of $\omega$ can be cast into the simple form

$$\omega = -\frac{3}{8}\, e^{4K}\, \frac{\Delta}{\det g}\, |C|^4\,, \tag{4.26}$$

where the quantity $\Delta$ is the discriminant of the cubic polynomial defined by $d_{ijk}v^i v^j v^k$ after scaling out one variable, and reads

$$\Delta = -27\Big(d_{111}^2 d_{222}^2 - 3\, d_{112}^2 d_{122}^2 + 4\, d_{111} d_{122}^3 + 4\, d_{112}^3 d_{222} - 6\, d_{111} d_{112} d_{122} d_{222}\Big)\,. \tag{4.27}$$

Since we must require $\det g > 0$, the sign of $\omega$ is fixed by the sign of $\Delta$. Moreover, it becomes now clear that the two categories of models with $\omega > 0$ and $\omega < 0$ are of comparable size and that they merge in the very special class of models with factorisable Kähler geometries, for which $\omega = 0$.

## 4.3 Including $\alpha'$ corrections

So far we have analysed models respecting the no-scale property $K_i K^i = 3$. This property is however violated when $\alpha'$, worldsheet instanton or string loop corrections to the Kähler potential are taken into account, although they are suppressed in the large-volume and weak-coupling limit. It is therefore interesting to study how the bounds on the mass of the sGoldstinos are modified by these effects, particularly for those models in which $\sigma \le 0$ at leading order. For concreteness we here consider only $\alpha'$ corrections, but the effect of other corrections can be studied in a similar way.

When $\alpha'$ corrections are taken into account, the Kähler potential is $K = -\log Y$ where [49]

$$Y = \mathcal{V} + 4\xi\,. \tag{4.28}$$

The quantity $\xi = -\zeta(3)\chi/2$ is a real constant determined by the Euler characteristic of the Calabi-Yau manifold, given by $\chi = 2(h^{1,1} - h^{2,1})$. The geometry is still of the special-Kähler type, with prepotential $f(T) = 1/6\, d_{ijk} T^i T^j T^k - \xi$. However, as mentioned above, $\alpha'$ corrections break the no-scale property (3.1), which is seen from eqs. (A.2) and (A.3) of the appendix with $n = 1$ and $\theta = (3/2)\mathcal{V}/(\mathcal{V} + 4\xi)$.

The natural small dimensionless parameter controlling the effect of $\alpha'$ corrections relative to the leading-order Kähler potential is given by

$$\delta = \frac{4\xi}{\mathcal{V}}\,. \tag{4.29}$$

In the following, we work at leading order in this parameter, which is small when the volume is large. Using eqs. (A.1) and (A.3) with $\theta \simeq 3/2(1-\delta)$, one then finds that

$$K_i K^i \simeq 3 + 6\,\delta\,. \tag{4.30}$$

The Riemann tensor is given by eq. (A.7). The quantities $f_{ijk}$ are as before given by the intersection numbers, whereas the metric $g_{ij}$ and its inverse $g^{ij}$ are affected by the corrections and can be computed from (A.1).

In order to understand how the corrections modify the bounds on the sGoldstino masses, it is useful to compute the function $\sigma(G^i)$ up to second order in the $N^i$'s and at leading order in $\delta$. One finds

$$\begin{aligned}\sigma(G^i) \simeq\ & 120\,\delta\,|\alpha|^4 - 4\,(1-2\,\delta)\,|\alpha|^2 g_{ij} N^i \bar{N}^j \\ & - 2\,(1+9\,\delta)\Big(\alpha^2 g_{ij} \bar{N}^i \bar{N}^j + \text{c.c.}\Big) + \mathcal{O}(N^3)\,.\end{aligned} \tag{4.31}$$

Notice that $\sigma$ continues to be stationary at $N^i = 0$, but its value at that point becomes $\sigma_0 \simeq 120\,\delta\,|\alpha|^4$. If $\chi < 0$ (i.e. $h^{2,1} > h^{1,1}$) then this is positive and the special direction $G^i \propto K^i$ always allows to fulfil the metastability condition.

Up to this point we have left $\alpha$ undetermined. We can however express $|\alpha|^2$ in terms of the vacuum energy density $V = e^G(G^i G_i - 3)$ and gravitino mass scale $m_{3/2} = e^{G/2}$ as $|\alpha|^2 = 1 + V/(3\,m_{3/2}^2) + \mathcal{O}(\delta)$. Inserting this relation back into eq. (4.31) and evaluating at $N^i = 0$ one obtains

$$\sigma_0 \simeq 120\,\delta\left(1 + \frac{V}{3\,m_{3/2}^2}\right)^2. \tag{4.32}$$

This relation can be used to compute the mass scale $\tilde{m}^2 = e^G \lambda / G^i G_i$, as introduced in Section 2.1, at the critical value $G^i \propto K^i$. This is particularly important for models where $\sigma \leq 0$ at leading order, as it then provides a bound on the attainable values of the sGoldstino mass. By inserting eq. (4.32) into eq. (2.11), and specialising to the relevant regime $V/m_{3/2}^2 \ll 1$, one obtains

$$\frac{\tilde{m}^2}{m_{3/2}^2} \simeq 40\,\delta - \frac{2}{3}\frac{V}{m_{3/2}^2}\,. \tag{4.33}$$

It immediately follows that if $\delta \gtrsim V/(60\,m_{3/2}^2)$ then the metastability condition is fulfilled. This gives a criterion on how large $\alpha'$ corrections have to be for given gravitino scale and vacuum energy density in order to admit viable vacua. Notice that under these circumstances the value of the sGoldstino mass is essentially the gravitino mass suppressed by $\alpha'$ corrections. We should bear in mind, however, that other corrections to the Kähler potential could compete against $\alpha'$ corrections and modify this result.

# 5 Orientifold compactifications of string theory

## 5.1 General discussion

In contrast to the heterotic string, type IIB Calabi-Yau compactifications give theories with $\mathcal{N}=2$ supersymmetry in 4 dimensions. The RR forms which are present in 10-D type II supergravities lead to additional massless 4-D fields which, together with the geometric moduli, arrange into $\mathcal{N}=2$ supermultiplets. The scalars in the vector multiplets span again a special Kähler manifold $\mathcal{M}^{\mathrm{SK}}$ whereas the scalars in the hypermultiplet span a dual quaternionic manifold $\mathcal{M}^{\mathrm{Q}}$.

One way to obtain a theory with $\mathcal{N}=1$ supersymmetry is to impose an orientifold projection. In type IIA, this involves $O6$-planes while in type IIB one has $O3/O7$ or $O5/O9$-planes. The moduli space in all of these three cases has the form [44, 50, 51]

$$\tilde{\mathcal{M}} = \tilde{\mathcal{M}}^{\mathrm{SK}} \times \tilde{\mathcal{M}}^{\mathrm{Q}} \,, \tag{5.1}$$

where $\tilde{\mathcal{M}}^{\mathrm{SK}}$ is a special Kähler submanifold of the "parent" $\mathcal{N}=2$ moduli space $\mathcal{M}^{\mathrm{SK}}$ while $\tilde{\mathcal{M}}^{\mathrm{Q}}$ is a Kähler submanifold of $\mathcal{M}^{\mathrm{Q}}$. In the large-volume large-complex-structure limit, the $\tilde{\mathcal{M}}^{\mathrm{SK}}$ factor satisfies the no-scale property and the Kähler potential does in fact coincide with the Kähler potential of eq. (4.4). Therefore the analysis of Section 4 holds unmodified for the moduli of $\tilde{\mathcal{M}}^{\mathrm{SK}}$. On the other hand the $\tilde{\mathcal{M}}^{\mathrm{Q}}$ sector, which includes the dilaton, satisfies $K^iK_i = 4$, and if the dilaton is fixed, the latter sector is also no-scale [44]. However, the Kähler potential of $\tilde{\mathcal{M}}^{\mathrm{Q}}$ is different for the three orientifold compactifications.

For concreteness let us focus on type IIB with O3/O7 planes, where the Kähler potential in the large-volume limit reads [44]

$$K_Q = -2\log\mathcal{V} - \log(S+\bar{S})\,, \qquad \text{with} \qquad \mathcal{V} = \frac{1}{48}\, d^{ijk} v_i v_j v_k \ . \tag{5.2}$$

$\mathcal{V}$ is again the classical volume of the Calabi-Yau orientifold, $S$ is the dilaton/axion and the $v_i, i=1,\ldots,h_+^{1,1}$ are the Kähler moduli of the Calabi-Yau orientifold. However the $v_i$ do not appear as components of chiral multiplets in the low energy effective action. Instead, they determine the real part of the Kähler coordinates $T^i = \rho^i + i\zeta^i$ via the quadratic relation[8]

$$\rho^i = \frac{1}{16}\, d^{ijk} v_j v_k \ . \tag{5.3}$$

[8]Strictly speaking there can also be $h_-^{1,1}$ moduli $G$ with couplings specified in [44] which however we neglect during the analysis of this paper.

Due to this relation the Kähler potential of eq. (5.2) cannot explicitly be expressed in terms of the coordinates $T^i$, but is only implicitly defined through eq. (5.3).[9] As in the previous section we assume that the dilaton is fixed to a supersymmetric configuration and focus only on the Kähler moduli.

The metric can be conveniently expressed in terms of

$$d^{ij} \equiv \frac{\partial \rho^i}{\partial v_j} = \frac{1}{8}\, d^{ijk} v_k \ , \qquad d_{ij} \equiv \frac{\partial v_i}{\partial \rho^j} \ . \tag{5.4}$$

Using (5.2) – (5.4), one computes

$$K_i = -\,\frac{1}{2}\, e^{K/2} v_i \ , \qquad d^{ij} = -\,\frac{1}{4}\, e^{-K/2} d^{ijk} K_k \ . \tag{5.5}$$

This in turn determines the Kähler metric and its inverse to be

$$g_{ij} = \frac{1}{2}\, K_i K_j - \frac{1}{4}\, e^{K/2} d_{ij} \, , \qquad g^{ij} = 4\,\rho^i \rho^j - 4\, e^{-K/2} d^{ij} \, . \tag{5.6}$$

One can now check that $K$ satisfies the no-scale property $K^i K_i = 3$ as well as the special identity $K^i = -2\rho^i$, which again results from the fact that $e^{-K}$ is a homogeneous function of degree 3 in $\rho^i$. This can be used to slightly rewrite the inverse metric as

$$g^{ij} = e^{-K} d^{ijk} K_k + K^i K^j \ . \tag{5.7}$$

Notice that this expression for the inverse metric is equal in form to the metric (4.6) of the heterotic case. Similarly, the inverse metric of the heterotic case is equal in form to the metric (5.6) for the orientifold case examined here. As was shown in ref. [52] this property directly follows from the fact that in the orientifold case the Kähler coordinates $T^i$ feature the dual variables $\rho^i$ instead of $v_i$ as the real part.

In order to determine $\sigma$ we need again the third derivatives of the Kähler potential and the Riemann tensor. For this it is convenient to first compute derivatives of $g^{ij}$. Using the above relations we find

$$\begin{aligned} \left[g^{ij}\right]_k &= e^{-K} d^{ijm} g_{mk} - (g^{ij} - K^i K^j) K_k - \delta^i_k K^j - \delta^j_k K^i \, , \\ \left[g^{ij}\right]_{mn} &= -e^{-2K} d^{ijp} g_{pq} d^{qrs} g_{rm} g_{sn} + \delta^i_m \delta^j_n + \delta^i_n \delta^j_m \, . \end{aligned} \tag{5.8}$$

$K_{ijm}$ and the Riemann tensor are expressed in terms of these derivatives as

$$\begin{aligned} K_{ijm} &= -g_{ip} [g^{pq}]_j g_{qm} \, , \\ R_{ijmn} &= -g_{ip} g_{qj} [g^{pq}]_{mn} + g_{ir} [g^{rp}]_m g_{pq} [g^{qs}]_n g_{sj} \, . \end{aligned} \tag{5.9}$$

[9]In order to comply with the standard notation whereby chiral coordinates carry upper indices, we have slightly abused the notation by lowering the indices of $v$ and raising them for the intersection numbers $d$. We have also rescaled the intersection numbers as $d_{ijk} \to d^{ijk}/8$. However we stress that they are exactly the same objects as in the heterotic case.

Inserting (5.8) into (5.9) and using (5.4)–(5.6) we arrive at[10]

$$
\begin{aligned}
K_{ijm} &= e^{-K}\hat{d}_{ijm} - g_{ij}K_m - g_{im}K_j - g_{jm}K_i + K_iK_jK_k \ , \\
R_{ijmn} &= -g_{im}g_{jn} + e^{-2K}(\hat{d}_{ijk}g^{kl}\hat{d}_{lmn} + \hat{d}_{ink}g^{kl}\hat{d}_{ljm}) + g_{in}K_jK_m + g_{jm}K_iK_n \\
&\quad + g_{im}K_jK_n + g_{jn}K_iK_m + g_{ij}K_mK_n + g_{mn}K_iK_j - 3K_iK_jK_mK_n \\
&\quad - e^{-K}(\hat{d}_{imj}K_n + \hat{d}_{imn}K_j + \hat{d}_{inj}K_m + \hat{d}_{nmj}K_i) \ ,
\end{aligned} \tag{5.10}
$$

where we abbreviated

$$
\hat{d}_{ijk} \equiv g_{ip}g_{jq}g_{kl}d^{pql} \ . \tag{5.11}
$$

Inserting (5.10) into (2.12) we finally arrive after some algebra at

$$
\begin{aligned}
\sigma = &\ \frac{2}{3}(G^i\bar{G}_i)^2 + |G^iG_i - (K^iG_i)^2|^2 + 2|K^iG_i|^4 - 4|K^iG_i|^2G^j\bar{G}_j \\
&-2\,e^{-2K}G_i\bar{G}_jd^{ijp}g_{pq}d^{mnq}G_m\bar{G}_n + 2\,e^{-K}d^{ijk}G_i\bar{G}_j(G_kK^n\bar{G}_n + \bar{G}_kK^nG_n)\,.
\end{aligned} \tag{5.12}
$$

It is also possible to write $\sigma$ in terms of the decomposition $G_i = N_i + \alpha K_i$ defined in (3.8). Doing so, one finds the result (3.17) or (3.18), with the quantities $g_{ij}$, $K_{ijk}$ and $R_{ijmn}$ given by eq. (5.10). Again only the few terms transverse to $K^i$ contribute in contractions with $N^i$. The quantities $s_i$ and $\omega$ are obtained by inserting (5.10) into (3.19) and (3.20) and are given by

$$
\omega = \Big(g^{im}g^{jn} - \frac{3}{2}\,e^{-2K}d^{ijp}P_{pq}d^{pmn}\Big)N_i\bar{N}_jN_m\bar{N}_n \,, \tag{5.13}
$$

$$
s_i = \alpha N_{\bar{\imath}} + \bar{\alpha}N_i - \frac{1}{2}e^{-K}P_{ij}d^{jmn}N_mN_{\bar{n}}\,. \tag{5.14}
$$

It is interesting to compare both of these quantities with their heterotic counterparts, given in eqs. (4.11) and (4.12). While $s_i$ of eq. (4.11) is equal in form to the one given here, $\omega$ of eq. (4.12) has essentially the opposite sign to the one shown here, and involves the inverse metric instead of the metric and $e^{-K}$ instead of $e^K$. As we shall see, this result is particularly relevant for models with two moduli, for which $\sigma$ can be maximised by setting $s_i = 0$ and thus the sign of $\sigma$ is determined by $\omega$.

## 5.2 Particular classes of models

As for the heterotic case, we can only make further progress by computing $\sigma$ for specific classes of Calabi-Yau orientifolds. In the following we consider the same examples as in Section 4.2.

[10] This Riemann tensor was also computed in ref. [53].

### 5.2.1 Factorisable Kähler manifolds

We again start with $K3$-fibred Calabi-Yau threefolds where the Kähler potential takes the form

$$K = -2\,\log\Big(\frac{1}{16}\,d^{1ab}v_1 v_a v_b\Big)\,. \tag{5.15}$$

For these intersection numbers $(v_1, v_a)$ can be explicitly determined in terms of the $(\rho^1, \rho^a)$ via (5.3). One finds $v_1 = 2\,(d^{-1}_{1ab}\rho^a\rho^b/\rho^1)^{1/2}$ and $v_a = 4\,d^{-1}_{1ab}\rho^b(\rho^1/d^{-1}_{1cd}\rho^c\rho^d)^{1/2}$. Inserting into (5.15) and using $\rho^i = (T^i + \bar{T}^i)/2$ yields

$$K = -\log\Big[\frac{1}{2}\,d^{-1}_{1ab}(T^1 + \bar{T}^1)(T^a + \bar{T}^a)(T^b + \bar{T}^b)\Big]\,. \tag{5.16}$$

This is exactly the same $K$ as in the heterotic case but with an inverse intersection matrix. In particular, $K$ obeys again $K^1K_1 = 1$ and $K^aK_a = 2$. It is nevertheless instructive to recompute the function $\sigma$ by using the formulae obtained for orientifold models. From eq. (5.7) we first infer $d^{1ab} = e^K K^1(g^{ab} - K^aK^b)$. This allows us to compute

$$\begin{aligned} e^{-2K}G_i\bar{G}_j d^{ijp}d_p^{mn}G_m\bar{G}_n &= (G^a\bar{G}_a - |K^aG_a|^2)^2 + G^aG_a(K^1\bar{G}_1)^2 \\ &\quad + \bar{G}^a\bar{G}_a(K^1G_1)^2 + 2\,G^aG_a|K^1G_1|^2\,, \end{aligned} \tag{5.17}$$

$$\begin{aligned} e^{-K}d^{ijk}G_i\bar{G}_jG_kK^n\bar{G}_n &= 2\,(K^1\bar{G}_1 + K^aG_{\bar{a}})K^1G_1(G^bG_b - |K^bG_b|^2) \\ &\quad + (K^1\bar{G}_1 + K^a\bar{G}_a)K^1\bar{G}_1(G^bG_b - (K^bG_b)^2)\,. \end{aligned} \tag{5.18}$$

Inserting into (5.12) we arrive at

$$\sigma = -\frac{1}{3}(G^a\bar{G}_a + |K^1G_1|^2)^2 + |G^aG_a - (K^aG_a)^2|^2 - (G^a\bar{G}_a - |K^1G_1|^2)^2\,. \tag{5.19}$$

Using the same inequality (4.18) as for heterotic models, and noticing also the simplification $|K^1G_1|^2 = G^1\bar{G}_1$, one finally deduces the same upper bound as before:

$$\sigma \le -\frac{1}{3}(2\,G^1\bar{G}_1 - G^a\bar{G}_a)^2\,. \tag{5.20}$$

Therefore, we arrive at the same conclusion as for heterotic models: in this class of factorisable models the stability bound is always at least marginally violated.

### 5.2.2 Two-field models

As for heterotic models, another class of models where the analysis simplifies are those involving two fields. In such a situation, there is again a single direction $N^i$ orthogonal to $K_i$, which can be parametrised as

$$(N_1, N_2) = (K^2, -K^1)C\,. \tag{5.21}$$

With this definition, one has $N^i N_i = 3/\det g\,|C|^2$. As before using this parametrisation we can compute the value of the quantity $\omega$ defined by (5.13), which provides an upper bound to $\sigma$.

As for the heterotic models we consider first the simplest case of models with only diagonal intersection numbers $d^{111}$ and $d^{222}$. The corresponding Kähler potential is of the form[11]

$$K = -2\,\log\Big(\frac{1}{48}d^{111}v_1^3 + \frac{1}{48}d^{222}v_2^3\Big)\,. \tag{5.22}$$

Using (5.3) one determines $v_1 = 4\,(\rho^1/d^{111})^{1/2}$ and $v_1 = -4\,(\rho^2/d^{222})^{1/2}$ which, when inserted back into (5.22), yields

$$K = -2\log\Big(\frac{\sqrt{2}}{3}(d^{111})^{-1/2}(T^1+\bar{T}^1)^{3/2} - \frac{\sqrt{2}}{3}(d^{222})^{-1/2}(T^2+\bar{T}^2)^{3/2}\Big)\,. \tag{5.23}$$

The function $\omega$ is now easily computed and is found to be

$$\omega = -\frac{81}{8}\,e^{-4K}\,(d^{111})^2(d^{222})^2\,\det g\,|C|^4\,. \tag{5.24}$$

This result is negative and shows that in this case $\sigma \le 0$ for any choice of $G_i$. It is therefore impossible to obtain stable de Sitter vacua in this case. Furthermore, this inequality is saturated only for $N_i = 0$, which corresponds to the configuration $G_i \propto K_i$. The result presented here should be contrasted to the one presented in eq. (4.22).

To understand whether this negative sign for $\omega$ persists or not in more general 2-field models, let us as before consider a small deformation of a factorisable model. The simplest example has non-zero $d^{122}$ and $d^{111}$, and a Kähler potential given by

$$K = -2\log\Big(\frac{1}{16}\,d^{122}v_1v_2^2 + \frac{1}{48}\,d^{111}v_1^3\Big)\,. \tag{5.25}$$

In the limit $d^{111} \ll d^{122}$, in which the model is nearly factorisable, one can expand at leading order in the small parameter

$$\epsilon = \frac{d^{111}}{d^{122}}\,. \tag{5.26}$$

One finds $v_1 = 2\,(d^{122}\rho^1)^{-1/2}\rho^2[1+\epsilon/8(\rho^2/\rho^1)^2]$ and $v_2 = 4\,(d^{122}/\rho^1)^{-1/2}[1-\epsilon/8(\rho^2/\rho^1)^2]$. The Kähler potential can then be rewritten as

$$K = -\log\Big(\frac{1}{2}\frac{1}{d^{122}}(T^1+\bar{T}^1)(T^2+\bar{T}^2)^2 - \frac{1}{24}\frac{d^{111}}{(d^{122})^2}\frac{(T^2+\bar{T}^2)^4}{T^1+\bar{T}^1}\Big)\,. \tag{5.27}$$

[11] In ref. [53] the same manifold was studied as an example where the Riemann tensor of the manifold and its dual manifold do not coincide.

After a straightforward computation, the function $\omega$ is found to be

$$\omega = -\frac{81}{2}\,\epsilon\, e^{-4K}\,(d^{122})^4\,\det g\,|C|^4\,. \tag{5.28}$$

As in the heterotic case we have again that this result can be either positive or negative, depending on the sign of $\epsilon$. This means that also in orientifold compactifications one can have models with $\sigma > 0$ and models with $\sigma < 0$.

Note that the results (5.24) and (5.28) take the same form as (4.22) and (4.25) for heterotic models but with the substitutions $e^K \to e^{-K}$, $\det g \to (\det g)^{-1}$ and a flip in the overall sign. This is due to the fact that in the case of two-field models, where the parametrisations (4.20) and (5.21) can be used, the functions (4.12) and (5.13) get indeed precisely mapped into each other by these substitutions. This map can then be used to infer that also for orientifold models the result for generic intersection numbers $d^{111}$, $d^{222}$, $d^{122}$ and $d^{112}$ should take a simple form, obtained by applying it to the heterotic result (4.26). This leads to the result

$$\omega = \frac{3}{8}\,e^{-4K}\,\Delta\,\det g\,|C|^4\,, \tag{5.29}$$

in terms of the discriminant

$$\begin{aligned}\Delta = -27\Big(&(d^{111})^2(d^{222})^2 - 3\,(d^{112})^2(d^{122})^2 + 4\,d^{111}(d^{122})^3 \\ &+ 4\,d^{222}(d^{112})^3 - 6\,d^{111}d^{112}d^{122}d^{222}\Big)\,.\end{aligned} \tag{5.30}$$

It is not straightforward to verify this result explicitly, because performing the change of variables (5.3) involves in this general case finding the roots of a quartic polynomial. But we were nevertheless able to verify it by brute force with computer assistance. Since we must require $\det g > 0$, the sign of $\omega$ is again determined by the sign of the quantity $\Delta$, which has exactly the same structure as for heterotic models.

It is important to note that the results found for heterotic and orientifold models imply that for any given string compactification with non-zero $\Delta$, one can have either viable heterotic models but no viable orientifold models (if $\Delta < 0$), or vice-versa (if $\Delta > 0$).

## 5.3 Including $\alpha'$ corrections

We now include $\alpha'$ corrections in orientifold compactifications. When these corrections are taken into account, the Kähler potential of eq. (5.2) is modified to $K = -2\log Y - \log(S + \bar{S})$, where [54]

$$Y = \mathcal{V} + \frac{\xi}{2}\left(\frac{S+\bar{S}}{2}\right)^{3/2}. \tag{5.31}$$

One difficulty arises from the fact that these corrections depend on the dilaton which, strictly speaking, now should be considered as a dynamical quantity (this is due to the fact that in the presence of $\alpha'$ corrections the Kähler potential is not factorisable). To simplify the presentation of this section we nevertheless assume that the dilaton can be fixed to a constant value in eq. (5.31), and define the new constant $\tilde{\xi} = (\xi/2)[(S+\bar{S})/2]^{3/2}$.[12] As before, $\alpha'$ corrections break the no-scale property (3.1), which can be seen from eqs. (A.2) and (A.3) of the appendix with $n = 2$ and $\theta = 3\mathcal{V}/(\mathcal{V}+\tilde{\xi})$.

The small dimensionless parameter controlling the relative effect of the $\alpha'$ corrections is in this case given by

$$\tilde{\delta} = \frac{\tilde{\xi}}{8\mathcal{V}}\,. \tag{5.32}$$

We will work at leading order in this parameter. Using the results of the appendix with $\theta \simeq 3(1-8\tilde{\delta})$, one finds then that

$$K_i K^i \simeq 3 + 12\,\tilde{\delta}\,. \tag{5.33}$$

The Riemann tensor, given by eq. (A.5), can be evaluated by using $Y_{ij} = 1/8\, d_{ij}$, $Y_{ijm} = -1/128\, d_{ir} d_{js} d_{mt} d^{rst}$ and $Y_{ijmn} = 24\, Y_{ijs} d^{sr} Y_{rmn}$.

As worked out in Section 4.3 for the case of heterotic compactifications, one may compute $\sigma$ up to second order in $N^i$ and at first order in $\tilde{\delta}$:[13]

$$\begin{aligned}\sigma(G^i) \simeq\; & 105\,\tilde{\delta}\,|\alpha|^4 - 4\,(1+14\,\tilde{\delta})\,|\alpha|^2 g_{ij} N^i \bar{N}^j \\ & - 2\,(1+27\,\tilde{\delta})\Big(\alpha^2 g_{ij} \bar{N}^i \bar{N}^j + \text{c.c.}\Big) + \mathcal{O}(N^3)\,.\end{aligned} \tag{5.34}$$

Again, $\sigma$ is stationary at $N^i = 0$ with a value $\sigma \simeq 105\,\tilde{\delta}\,|\alpha|^4$. Observe that the only difference with respect to the result found for heterotic models, shown in eq. (4.31), is the numerical factor in front of $\tilde{\delta}$. We can now calculate the mass scale $\tilde{m}^2 = e^G \lambda / G^i G_i$ associated to the sGoldstino. By repeating the steps of Section 4.3 and assuming that $V/m_{3/2}^2 \ll 1$ one arrives at

$$\frac{\tilde{m}^2}{m_{3/2}^2} \simeq 35\,\tilde{\delta} - \frac{2}{3}\frac{V}{m_{3/2}^2}\,. \tag{5.35}$$

Similarly to the case of heterotic compactifications, if $\tilde{\delta} \gtrsim 2V/(105 m_{3/2}^2)$ then the metastability condition is fulfilled and the sGoldstino mass becomes of the order of the gravitino mass suppressed by $\alpha'$ corrections. This is for instance the case in the models of ref. [7, 17].

[12] Similar conclusions are obtained in the full computation with a dynamical dilaton by assuming that $S$ is fixed to a supersymmetric configuration $G_S = 0$.

[13] For this computation, the following contractions are needed: $Y_{ij} K^i K^j = 3\,(\theta-1)^{-2}\mathcal{V}$, $Y_{ijm} K^i K^j = Y/2\,(\theta-1)^{-2} K_m$, $Y_{ijmn} K^i K^j K^m K^n = 9\,(\theta-1)^{-4}\mathcal{V}$.

# 6 Conclusions

In this paper we have analysed the role that neutral chiral multiplets have in the construction of 4-D metastable vacua, paying special attention to the generic class of models obtained in large-volume compactifications of string theory. In general, metastable vacua with spontaneously broken supersymmetry are only granted in models where a non-vanishing $F$-term $F^i = m_{3/2}G^i$ exists such that $\sigma(G^i) > 0$, as defined in eq. (2.12). This necessary condition was shown to be equivalent to the requirement of having a positive square mass for the sGoldstinos when the vacuum energy density $V$ is non-negative. Interestingly, this condition was also shown to be sufficient, with the understanding that all of the other scalar fields can be given arbitrarily large positive square masses if the superpotential of the theory is suitably tuned.

In the particular case of large-volume string compactifications the function $\sigma$ respects some severe restrictions. For instance, from the general analysis made in Section 3, we have learned that the set of values $G_i \propto K_i$ corresponds to a family of stationary points of $\sigma$ with $\sigma = 0$. Moreover, they are either saddle points or maxima, depending on the intersection numbers of the particular model. Despite of the difficulties posed by a complete analytical study of the function $\sigma$ we were still able to outline a general procedure to determine whether a particular compactification admits dS vacua. This procedure was introduced first for generic supergravity models in Section 2.2 and then refined in Section 3.2 for the particular case of string compactifications. We believe that such a procedure can be implemented numerically and should be of considerable help in any computer scan of string ground states. We also saw, however, that there are interesting and nontrivial examples of compactifications which can be handled analytically. For $K3$ fibrations, for instance, we showed that $\sigma$ can be at best zero. For 2-field models, on the other hand, the maximal value of $\sigma$ can be non-vanishing, and its sign is controlled by the discriminant $\Delta$ of the cubic polynomial defined by the intersection numbers. Moreover, for $\Delta < 0$ one can find viable heterotic models but no viable orientifold model, and vice-versa for $\Delta > 0$.

The results of this paper are useful for determining which type of configurations within a given model should help in the construction of vacua. We have seen for example that exploring configurations in the superpotential parameter space close to the critical point $G^i \propto K^i$ give a vanishing value for $\sigma$ and that $\alpha'$ corrections can help in obtaining a positive –although suppressed– square mass for the sGoldstinos, independently of whether $\sigma > 0$ is admitted or not at leading order. In fact, one could expect this to be a generic feature of any additional sector which breaks the no-scale property $K^iK_i = 3$ respected by the Kähler moduli sector.

Finally, let us mention here that a strategy similar to the one used in this paper could be used also to study the possibility of constructing successful models of slow-roll inflation within a string-theoretical scenario. This requires finding some direction in field space with small first and second derivatives of the potential. The first condition corresponds approximately to stationarity, whereas the second one requires a small negative mass. The algebraic problem defined by these two conditions is then very similar to the one faced in this paper [55].

# Acknowledgements

This work was partly supported by the German Science Foundation (DFG) under the Collaborative Research Center (SFB) 676, by the European Union 6th Framework Program MRTN-CT-503369 "Quest for unification" and by the Swiss National Science Foundation. LC would like to thank the Institute of Theoretical Physics of the Warsaw University for their hospitality during the final stages of this work. LC is supported by a Maria Curie Transfer of Knowledge Fellowship of the European Community's Sixth Framework Programme under contract number MTKD-CT-2005-029466 (2006-2010). CS thanks the University of Hamburg and DESY for their hospitality during the beginning of this work.

# A Details of Kähler geometries

In this appendix, we collect some useful formulae concerning the geometry of Kähler and special-Kähler manifolds, which are needed in some derivations in the main text.

## A.1 Logarithmic Kähler potentials

Let us consider a Kähler potential of the form $K = -n \log Y$, where $Y$ is some real function of the scalar fields $\phi^i$ and $n$ is a real number. Denoting by $Y^{i\bar{\jmath}}$ the inverse of $Y_{i\bar{\jmath}}$, one easily finds

$$\begin{aligned}
K_i &= -n \frac{Y_i}{Y}\,, \\
g_{i\bar{\jmath}} &= -n \frac{Y_{i\bar{\jmath}}}{Y} + n \frac{Y_i Y_{\bar{\jmath}}}{Y^2} = -n \frac{Y_{i\bar{\jmath}}}{Y} + \frac{1}{n} K_i K_{\bar{\jmath}}\,, \\
g^{i\bar{\jmath}} &= -\frac{Y Y^{i\bar{\jmath}}}{n} + \frac{1}{n} \frac{1}{\theta - 1} Y^{i\bar{r}} Y_{\bar{r}} Y^{\bar{\jmath}s} Y_s = -\frac{Y Y^{i\bar{\jmath}}}{n} + \frac{\theta - 1}{n} K^i K^{\bar{\jmath}}\,, \\
K^i &= -\frac{1}{\theta - 1} Y^{i\bar{r}} Y_{\bar{r}}\,.
\end{aligned} \tag{A.1}$$

The quantity $\theta$ is defined as

$$\theta \equiv \frac{Y_i Y^{i\bar{\jmath}} Y_{\bar{\jmath}}}{Y}\,, \tag{A.2}$$

and controls the value of the contraction defining the no-scale property:

$$K^i K_i = n \frac{\theta}{\theta - 1}\,. \tag{A.3}$$

The third derivatives of $K$ are

$$\begin{aligned} K_{i\bar{\jmath}m} &= -\frac{n}{Y} Y_{i\bar{\jmath}m} + \frac{n}{Y^2}(Y_i Y_{\bar{\jmath}m} + Y_m Y_{\bar{\jmath}i} + Y_{\bar{\jmath}} Y_{im}) - \frac{2n}{Y^3} Y_i Y_{\bar{\jmath}} Y_m\,, \\ K_{i\bar{\jmath}\bar{n}} &= -\frac{n}{Y} Y_{i\bar{\jmath}\bar{n}} + \frac{n}{Y^2}(Y_{\bar{\jmath}} Y_{i\bar{n}} + Y_{\bar{n}} Y_{i\bar{\jmath}} + Y_i Y_{\bar{\jmath}\bar{n}}) - \frac{2n}{Y^3} Y_i Y_{\bar{\jmath}} Y_{\bar{n}}\,. \end{aligned} \tag{A.4}$$

Finally, the Riemann tensor for the Kähler manifold is

$$\begin{aligned} R_{i\bar{\jmath}m\bar{n}} &= K_{i\bar{\jmath}m\bar{n}} - K_{im\bar{r}} g^{\bar{r}s} K_{s\bar{\jmath}\bar{n}} \\ &= \frac{1}{n}(g_{i\bar{\jmath}}\, g_{m\bar{n}} + g_{i\bar{n}}\, g_{m\bar{\jmath}}) - \frac{n}{Y} Y_{i\bar{\jmath}m\bar{n}} - \frac{n}{Y^2}\Big(n Y_{im\bar{s}} g^{\bar{s}r} Y_{r\bar{\jmath}\bar{n}} + \frac{1}{\theta - 1} Y_{im} Y_{\bar{\jmath}\bar{n}}\Big) \\ &\quad + \frac{n^2}{Y^3}(Y_{im} Y_{\bar{\jmath}\bar{n}r} g^{r\bar{s}} Y_{\bar{s}} + Y_{\bar{\jmath}\bar{n}} Y_{im\bar{s}} g^{\bar{s}r} Y_r)\,. \end{aligned} \tag{A.5}$$

## A.2 Special Kähler manifolds

We now consider the case of special Kähler geometries, for which the Kähler potential $K = -\log Y$ itself admits a holomorphic prepotential $f$, in terms of which

$$Y = -2(f + \bar{f}) + (f_k + \bar{f}_{\bar{k}})(\phi^k + \bar{\phi}^k)\,. \tag{A.6}$$

The Riemann tensor simplifies substantially in this case. Indeed, one easily computes $Y_i + Y_{\bar{\imath}} = N_{ij}(\phi^j + \bar{\phi}^{\bar{\jmath}})$ and $Y_{i\bar{\jmath}} = N_{ij}$, where $N_{ij} = f_{ij} + \bar{f}_{\bar{\imath}\bar{\jmath}}$. Combining these two expressions, one gets then $Y^{i\bar{\jmath}}(Y_j + Y_{\bar{\jmath}}) = (\phi^i + \bar{\phi}^{\bar{\imath}})$. Finally, combining this result with $Y_{ij} = f_{ijk}(\phi^k + \bar{\phi}^{\bar{k}})$ and $Y_{ij\bar{k}} = f_{ijk}$, one obtains the relation $Y_{ij\bar{s}} Y^{\bar{s}r}(Y_r + Y_{\bar{r}}) = Y_{ip}$. Using these relations, one finally finds [46]

$$R_{i\bar{\jmath}m\bar{n}} = g_{i\bar{\jmath}}\, g_{m\bar{n}} + g_{i\bar{n}}\, g_{m\bar{\jmath}} - \frac{1}{Y^2} f_{imr} g^{r\bar{s}} \bar{f}_{\bar{s}\bar{\jmath}\bar{n}}\,. \tag{A.7}$$

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