Deadtime Compensation for Nonlinear Processes

Many industrially important processes feature both nonlinear system dynamics and a process deadtime. Powerful deadtime compensation methods, such as the Smith predictor, are available for linear systems represented by transfer functions. A Smith predictor structure in state space for linear systems is presented first and then directly extended to nonlinear systems. When combined with input/output linearizing state feedback, this Smith-like predictor makes a nonlinear system with deadtime behave like a linear system with deadtime. The control structure is completed by adding an external linear controller, which provides integral action and compensates for the deadtime in the input/output linear system, and an open-loop state observer. Conditions for robust stability with respect to errors in the deadtime and more general linear unstructured multiplicative uncertainties are given. Computer simulations for an example system demonstrate the high controller performance that can be obtained using the proposed method.

Costas Kravaris
Raymond A. Wright
Department of Chemical Engineering
University of Michigan
Ann Arbor, MI 48109

Introduction

The problem of constructing control algorithms that are capable of handling deadtime is a key issue in process control, due to the large number of processes which possess deadtime. Powerful deadtime compensation methods are available in the literature for linear processes which are modeled with a transfer function of the form $G_o(s)e^{-t_d s}$, where $G_o(s)$ is rational. These methods have been motivated by the pioneer work of O. J. M. Smith (1957), who developed the well-known Smith predictor. Since Smith (1957), there have been many modifications and extensions of the original form of the Smith predictor. Reviews of these are in Jerome and Ray (1986) and Wong and Seborg (1986). Deadtime compensation methods with closely related structures include the analytical predictor of Moore et al. (1970), the inferential controller of Brosilow (1979), the Internal Model Control of Morari and coworkers (Garcia and Morari, 1982, Holt and Morari, 1985), the Generalized Analytical Predictor of Wong and Seborg (1986), and Wellons and Edgar (1987). All these are mathematically equivalent to the classical Smith predictor structure; however, they give different interpretations to deadtime compensation and therefore provide more or less clear insights to the design problem.

In this work, linear SISO systems of the form

$$\dot{x} = Ax + bu(t - t_d)$$
$$y = cx \tag{1}$$

will be considered initially and the Smith predictor will be reexamined in state space. It will then be shown that this state-space version of the Smith predictor carries over to SISO nonlinear systems of the form

$$\dot{x} = f(x) + g(x)u(t - t_d)$$
$$y = h(x) \tag{2}$$

For both cases, two assumptions will be necessary:

Assumption A. The process is open-loop stable

Assumption B. The "deadtime-free" part of the process, i.e.

$$\dot{\xi} = A\xi + bu(t)$$
$$y = c\xi \tag{3}$$

in the linear case, or

$$\dot{\xi} = f(\xi) + g(\xi)u(t)$$
$$y = h(\xi) \tag{4}$$

in the nonlinear case, has stable zero dynamics. Alternatively stated, the system of Eq. 3 or 4 has a stable inverse.

Assumption A is central in all Smith predictor approaches and cannot be avoided in nonlinear systems either. Assumption B is central in all available general approaches for nonlinear controller synthesis including input/output linearization techniques (Kravaris and Chung 1987; Kravaris, 1988) and nonlinear Internal Model Control techniques (Economou et al., 1986; Parrish and Brosilow, 1988).

The present paper first briefly reviews the classical Smith predictor for linear output feedback control of linear processes with deadtime. The Smith predictor idea will be subsequently extended to state space for a linear system. A Smith-like predictor for nonlinear systems with deadtime will then be developed along the same lines. This will lead to an extension of the Globally Linearizing Control (GLC) structure to nonlinear processes with deadtime. Following that, robust stability conditions will be given with respect to errors in the deadtime and unstructured linear multiplicative uncertainties. Finally, a simulation example to test the performance of the proposed control methodology will be presented.

The Classical Smith Predictor

Consider a linear process with transfer function $G_o(s)e^{-t_d s}$, where all zeros and poles of $G_o(s)$ are in the left half plane. The classical Smith predictor structure for this system is shown in Figure 1. The Smith predictor simulates the difference between the deadtime-free part of the process model and the (delayed) process model. This corrective signal is added to the measured output signal to predict what the output would have been if there were no deadtime. The prediction y^* is fed to the controller $G_c(s)$. A straightforward calculation gives the closed-loop transfer function

$$\frac{y(s)}{y_{sp}(s)} = \frac{G_c(s)G_o(s)}{1 + G_c(s)G_o(s)}e^{-t_d s} \tag{5}$$

The form of the closed-loop transfer function as well as the interpretation of the feedback signal y^* indicate that *the parameterization of the controller $G_c(s)$ should be chosen in accordance with the deadtime-free part of the model $G_o(s)$. For example,* one can use the synthesis formula (Smith and Corripio, 1985)

$$G_c(s) = \frac{1}{W(s) - 1} \cdot \frac{1}{G_o(s)} \tag{6}$$

where $W(s)$ is the polynomial of desirable closed-loop poles, of degree equal to the relative order, r, of $G_o(s)$. Then, Eq. 5 becomes

$$\frac{y(s)}{y_{sp}(s)} = \frac{1}{W(s)} e^{-t_d s} \tag{7}$$

The particular choice $W(s) = (\epsilon s + 1)^r$ provides critically damped closed-loop response.

It is important to mention a common misconception in many of the past applications of the Smith predictor idea, i.e., trying to design $G_c(s)$ on the basis of the deadtime-free part of the model $G_o(s)$ *regardless of the level of error in deadtime.* This has, of course, led to poor designs. For a discussion on this subject, see Laughlin and Morari (1987) and Morari and Doyle (1986), who advocate the use of the Internal Model Control configuration as providing a more transparent controller design framework.

A formal application of the Doyle-Stein robust stability criterion (Doyle and Stein, 1981) to the *overall system* of Figure 1 gives the following condition for robust stability

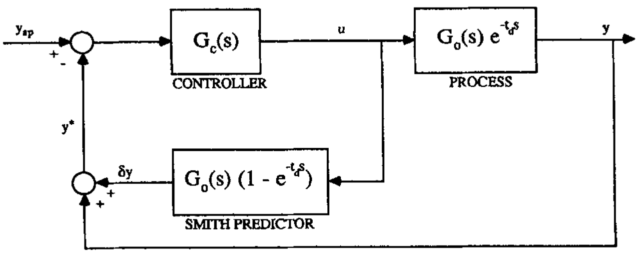

Figure 1. Classical Smith Predictor structure.

$$\left|\frac{G_c(i\omega)G_o(i\omega)}{1 + G_c(i\omega)G_o(i\omega)}\right| < \frac{1}{\lambda(\omega)} \tag{8}$$

where $\lambda(\omega)$ is an upper bound of the multiplicative uncertainty of the *overall* process, including errors in the deadtime. For the particular controller parameterization of Eq. 6, condition 8 becomes

$$|W(i\omega)| > \lambda(\omega) \tag{9}$$

Extension of the Smith Predictor Idea to State Space

In this section, the deadtime compensation problem for linear systems will be reexamined in state space. The aim of this section is *not* to develop another version of the Smith predictor for linear processes with deadtime, but rather to understand deadtime compensation in state space in a way which carries over to nonlinear systems.

Consider a linear process with deadtime of the form

$$\dot{x} = Ax + bu(t - t_d)$$
$$y = cx \tag{1}$$

where det $(sI - A)$ and c Adj $(sI - A)b$ have all roots in the open left-half plane. If the process is deadtime-free ($t_d = 0$) and is subject to the static state feedback $u = v - Kx$, the closed-loop transfer function is given by

$$\frac{y(s)}{v(s)} = \frac{c\,\mathrm{Adj}\,(sI - A)b}{\det\,(sI - A) + K\,\mathrm{Adj}\,(sI - A)b} \tag{10}$$

and a block diagram is shown in Figure 2. If the process has deadtime ($t_d \neq 0$), something similar to the classical Smith pre-

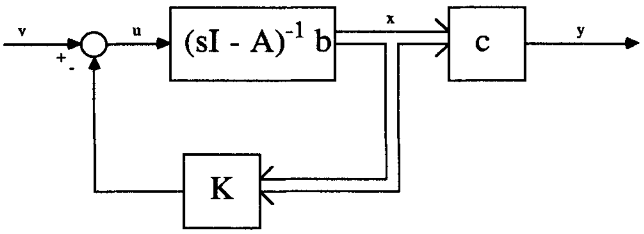

Figure 2. State feedback for linear systems without deadtime.

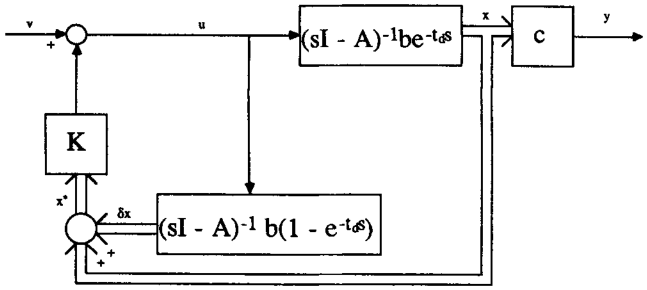

Figure 3. Smith Predictor structure in state space for linear systems.

dictor can be done in state space, i.e., predict what the *states* would have been if there were no deadtime. This can be achieved by adding a corrective signal to the state measurements obtained by simulating the difference between the delayed and nondelayed states (See Figure 3). The closed-loop transfer function becomes

$$\frac{y(s)}{v(s)} = \frac{c\,\mathrm{Adj}\,(sI - A)b}{\det (sI - A) + K\,\mathrm{Adj}\,(sI - A)b} e^{-t_d s} \tag{11}$$

which is the same as in the deadtime-free process except for the factor $e^{-t_d s}$. Nothing can be done about this factor, since it would require a noncausal state feedback for the delayed process to produce a nondelayed response.

From the closed-loop transfer function (Eq. 11) it is clear that with this structure it is possible to select the closed-loop poles for the delayed process using some type of *pole placement formula for the deadtime-free part of the process. For example,* if it were desired to place the closed-loop poles at the system zeros and at the roots of the polynomial

$$\sum_{k=0}^{r} \beta_k s^k = 0 \tag{12}$$

where r is the relative order of the process, then

$$K = \frac{1}{\beta_r c A^{r-1} b} (\beta_r c A^r + \cdots + \beta_1 c A + \beta_0 c) \tag{13}$$

would be chosen. This would make the closed-loop transfer function

$$\frac{cA^{r-1}b}{\beta_r s^r + \cdots + \beta_1 s + \beta_0} \tag{14}$$

for a deadtime-free process, or

$$\frac{cA^{r-1}b}{\beta_r s^r + \cdots + \beta_1 s + \beta_0} e^{-t_d s} \tag{15}$$

for a process with deadtime.

Thus, with the Smith predictor idea, the pole placement problem for a process with deadtime *reduces to* the pole placement problem for the deadtime-free part of the process. Note also that the closed-loop system will be ISE-optimal for step changes in the limit as the roots of the denominator polynomial of Eq. 15 tend to negative infinity.

Remark 1. Any pole placement formula for K will depend on a number of tunable parameters (like $\beta_0, \beta_1, \ldots, \beta_r$ in the previous example) which will have to be tuned taking into account the model uncertainty of the overall system, including errors in the deadtime.

A Smith-Like Predictor for Nonlinear Processes with Deadtime

Consider a nonlinear process without deadtime of the form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u(t) \\ y &= h(x) \end{aligned} \tag{16}$$

that satisfies assumptions A and B stated in the introduction section. In Kravaris and Chung (1987), a static state feedback law is synthesized for input/output linearity with prespecified poles for the input/output system. This result is summarized below:

If r is the relative order of the system of Eq. 16, i.e., the smallest integer for which

$$L_g L_f^{r-1} h(x) \neq 0$$

then the static state feedback

$$u = \Psi(x, v) = \frac{v - \sum_{k=0}^{r} \beta_k L_f^k h(x)}{\beta_r L_g L_f^{r-1} h(x)} \tag{17}$$

makes the input/output behavior of the system follow the equation

$$\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k} = v \tag{18}$$

The parameters β_k must be chosen so that all roots of $\Sigma_{k=0}^{r} \beta_k s^k$ are in the left half-plane; this guarantees input/output stability. Given assumption B, it guarantees internal stability as well (Kravaris, 1988).

Consider now a nonlinear process with deadtime

$$\begin{aligned} \dot{x} &= f(x) + g(x)u(t - t_d) \\ y &= h(x) \end{aligned} \tag{2}$$

that satisfies assumptions A and B of the introduction section. In the present section, input/output linearizing state feedback will be extended to systems with deadtime.

First, it should be observed that, since the system of Eq. 2 has deadtime, it will be impossible to find a causal state feedback that would transform it to a deadtime-free linear system like the one of Eq. 18. The best that can be obtained is a linear $v - y$ system *with deadtime* of the form

$$\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k} = v(t - t_d) \tag{19}$$

In this direction, the restriction that the state feedback be static should be removed and a state feedback *with predictive action* similar to the Smith predictor should be sought.

The state space Smith predictor of the previous section can easily be extended to the nonlinear process. The process model

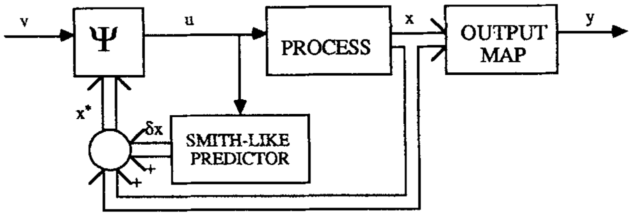

Figure 4. Smith-like Predictor structure in state space for nonlinear systems.

can be used to compute a corrective signal which can then be added to the states measurement signal to predict what the states would have been if there were no deadtime. The predicted states can then be fed to a static feedback controller Ψ. A block diagram of the resulting structure is shown in Figure 4.

Result 1. When the Smith-like predictor simulates

$$\dot{\tilde{x}} = f(\tilde{x}) + g(\tilde{x})u(t)$$
$$\dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u(t - t_d)$$
$$\delta x = \tilde{x}(t) - \hat{x}(t) \tag{20}$$

thus yielding the prediction

$$x^*(t) = x(t) + \delta x(t) \tag{21}$$

and Ψ is given by Eq. 17 with $x = x^*$, then the input/output behavior of the system is governed by

$$\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k} = v(t - t_d) \tag{22}$$

A proof of Result 1 is given in the Appendix.

Remark 2. The closed-loop response (Eq. 22) will be ISE-optimal for a step change in v (i.e., the response will be a delayed step) for $\beta_0 = 1$ and in the limit as the roots of the polynomial $\beta_k s^k + \cdots + \beta_1 s + \beta_0$ tend to negative infinity.

The Globally Linearizing Control (GLC) Structure for Nonlinear Processes with Deadtime

In the case of a deadtime-free process, the Globally Linearizing Control structure (Kravaris and Chung, 1987) consists of applying the input/output linearizing static state feedback

$$u = \Psi(x, v) = \frac{v - \sum_{k=0}^{r} \beta_k L_f^k h(x)}{\beta_r L_g L_f^{r-1} h(x)} \tag{17}$$

in an inner loop and an external linear controller around the linear $v - y$ system (See Figure 5). The external linear controller must possess integral action for "offsetless" control and at the same time must have parameters whose effect on performance and robustness is clearly understood. *For example,* one could use a classical PI

$$K_c\left(1 + \frac{1}{\tau_I s}\right)$$

or, more conveniently,

$$\frac{\sum_{k=0}^{r} \beta_k s^k}{(\epsilon s + 1)^r - 1} \tag{23}$$

The latter arises from the synthesis formula (Smith and Corripio, 1985) with the requirement of critically damped response. The resulting overall closed-loop transfer function is

$$\frac{y(s)}{y_{sp}(s)} = \frac{1}{(\epsilon s + 1)^r} \tag{24}$$

The GLC structure can be extended to nonlinear processes with deadtime in a natural way in view of the nonlinear Smith-like predictor developed in the previous section (See Figure 6). It is important to note that since the linear $v - y$ system has deadtime, the external linear controller must compensate for deadtime as well. However, this is not a problem since the classical Smith predictor can be used to obtain an appropriate parameterization for the external controller:

$$\frac{\sum_{k=0}^{r} \beta_k s^k}{(\epsilon s + 1)^r - e^{-t_d s}} \tag{25}$$

This external controller provides the overall closed-loop transfer function

$$\frac{y(s)}{y_{sp}(s)} = \frac{e^{-t_d s}}{(\epsilon s + 1)^r} \tag{26}$$

The effect of ϵ on performance is clear; in the next section, its effect on robustness will be shown.

In the case of unavailable state measurements, it will be necessary to use a state observer. The GLC structure is then modified as shown in Figure 7 for a deadtime-free process and as shown in Figure 8 for a process with deadtime. The construction of state observers is briefly reviewed in Kravaris and Chung (1987). It is important to mention a particular type of state observer applicable to open-loop stable processes: the full-order

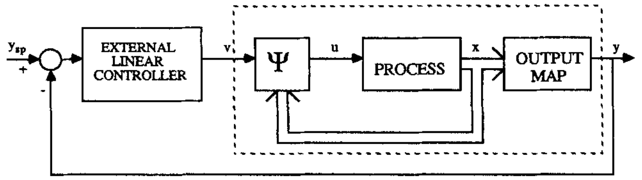

Figure 5. GLC structure.

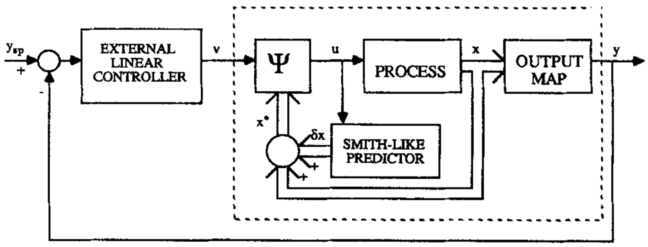

Figure 6. GLC structure for nonlinear processes with deadtime.

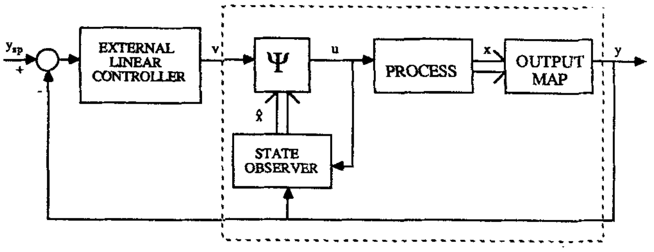

Figure 7. GLC with state observer.

open-loop observer. This involves on-line simulation of the process model in parallel with the process

$$\dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u(t - t_d) \tag{27}$$

i.e., an internal model of the process in the control system. The presence of an open-loop observer does not alter the closed-loop input/output behavior of the system, under the assumption, of course, of a perfect model.

Finally, it should be pointed out that the Smith-like predictor, Eqs. 20 and 21, and the state observer of Eq. 27 can be combined as shown in Figure 9, where the overall state predictor simulates

$$\dot{x}^* = f(x^*) + g(x^*)u(t) \tag{28}$$

i.e., the deadtime-free part of the process.

Robust Stability

In this section, robust stability results for two cases will be presented. First, uncertainty in the value of the deadtime only will be considered, since this has the most critical effect on the closed-loop system. Secondly, the more general case of unstructured linear multiplicative uncertainties will be considered. For both cases, the structure referred to will be that of Figure 9, where Ψ is given by Eq. 17 with $x = x^*$ and the overall state predictor simulates Eq. 28; the transfer function of the external controller will be denoted by $C(s)$.

Errors in the deadtime

Denoting by

- Δt_d = uncertainty in the deadtime with upper bound $(\Delta t_d)_{\max}$

- $$\lambda_d(\omega) = \sup_{0 \le |\Delta t_d| \le (\Delta t_d)_{\max}} |e^{-i\Delta t_d \omega} - 1| = \begin{cases} 2 \sin [\omega(\Delta t_d)_{\max}/2], & \text{for } \omega \le \pi/(\Delta t_d)_{\max} \\ 2, & \text{for } \omega \ge \pi/(\Delta t_d)_{\max} \end{cases} \tag{29}$$

the following result is obtained.

Result 2. The overall closed-loop system of Figure 9 will be stable for all uncertainties

$$|\Delta t_d| \le (\Delta t_d)_{\max} \quad \text{if}$$

$$\left| \frac{C(i\omega)}{\sum_{k=0}^{r} \beta_k (i\omega)^k + C(i\omega)e^{-t_d i\omega}} \right| \le \frac{1}{\lambda_d(\omega)} \tag{30}$$

In particular, for the external controller parameterization of Eq. 25, the condition becomes

$$(\epsilon^2\omega^2 + 1)^{r/2} > \lambda_d(\omega) \tag{31}$$

A proof of Result 2 is given in the Appendix.

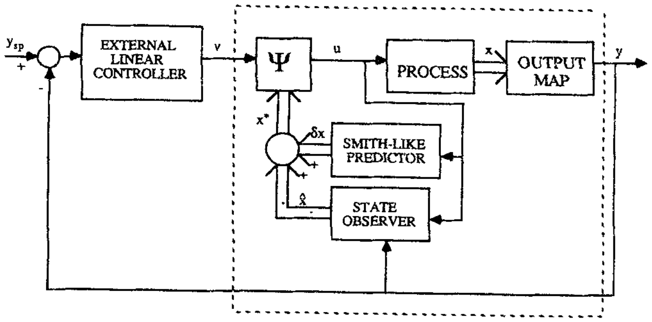

Figure 8. Complete GLC structure for deadtime compensation in nonlinear processes.

Unstructured linear multiplicative uncertainty

Recall that, for linear systems, the multiplicative uncertainty description is of the form:

$$G_p^{\text{true}}(s) - G_p(s) = \bar{l}_m(s)G_p(s) \tag{32}$$

where $G_p^{\text{true}}(s)$ is the true transfer function of the process; $G_p(s)$ is the model transfer function; and $\bar{l}_m(s)$ is the (linear) multiplicative uncertainty. By letting $y = \mu^{\text{true}}(u)$ represent the input/output behavior of the true process and $\mu(u)$ the input/output behavior of the model, it is natural for nonlinear systems to consider uncertainty descriptions of the form

$$\mu^{\text{true}}(u) - \mu(u) = \Lambda_m \mu(u) \tag{33}$$

where Λ_m is a linear time-invariant Volterra operator of the form

$$\Lambda_m(\cdot) = \int_0^t l_m(t - \tau) \cdot (\tau)\, d\tau \tag{34}$$

This form of the above uncertainty description for nonlinear systems is analogous to the multiplicative uncertainty description (Eq. 32) for a linear system, since it is of the general form: (process/model mismatch) = (uncertainty operator)·(model). In nonlinear systems, the description must be in the time domain instead of the Laplace domain; hence, the convolution integral arises. Denoting by $\bar{l}_m(s)$ the Laplace transform of $l_m(t)$, the following result is obtained.

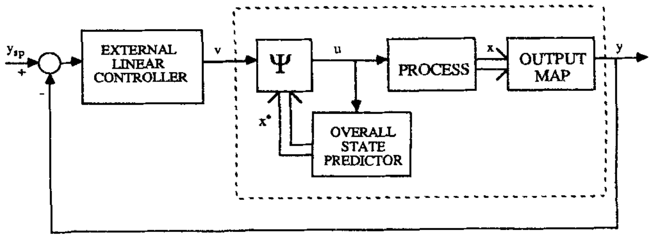

Figure 9. Complete GLC structure with overall predictor for nonlinear processes with deadtime.

Result 3. The overall closed-loop system of Figure 9 will be stable for all uncertainties l_m that satisfy

$$|\bar{l}_m(i\omega)| < \lambda(\omega) \text{ for all } \omega \tag{35}$$

if

$$\left| \frac{C(i\omega)}{\sum_{k=0}^{r} \beta_k(i\omega)^k + C(i\omega)e^{-t_d i\omega}} \right| < \frac{1}{\lambda(\omega)} \tag{36}$$

In particular, for the external controller parameterization of Eq. 25, this condition becomes

$$(\epsilon^2\omega^2 + 1)^{r/2} > \lambda(\omega) \tag{37}$$

A proof of Result 3 is given in the Appendix.

Remark 3. Result 3 actually generalizes Result 2. Indeed, $l_m(t) = \delta(t - \Delta t_d) - \delta(t)$, where $\delta(\cdot)$ is the delta function, corresponds to error in the deadtime.

The above robust stability results show that the method proposed in the previous sections will result in a robust control structure, a necessary feature for practical implementation.

Example

The control method of the previous sections will now be illustrated through a simulation example. The system chosen for study was a nonisothermal CSTR with the consecutive reactions

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

taking place. The state equations for such a system are:

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Ai} - C_A) - k_1 C_A^2 \tag{38}$$

$$\frac{dC_B}{dt} = -\frac{F}{V}C_B + k_1 C_A^2 - k_2 C_B \tag{39}$$

$$\rho C p V\frac{dT}{dt} = \rho C p F(T_i - T) + k_1 C_A^2(-\Delta H_1)V + k_2 C_B(-\Delta H_2)V + Q(t - t_d) \tag{40}$$

with

$$k_1 = A_{10}e^{-E_1/RT} \tag{41}$$

Table 1. Values Used in Simulations

$C_{Ai} = 1\ \text{kmol}\cdot\text{m}^{-3}$	$\Delta H_1 = 418{,}000\ \text{kJ}\cdot\text{kmol}^{-1}\cdot\text{K}^{-1}$
$A_{10} = 11\ \text{m}^3\cdot\text{kmol}^{-1}\cdot\text{s}^{-1}$	$\Delta H_2 = 418{,}000\ \text{kJ}\cdot\text{kmol}^{-1}\cdot\text{K}^{-1}$
$A_{20} = 172.2\ \text{m}^3\cdot\text{kmol}^{-1}\cdot\text{s}^{-1}$	$R = 8.314\ \text{kJ}\cdot\text{kmol}^{-1}\cdot\text{K}^{-1}$
$E_1 = 4{,}180\ \text{kJ}\cdot\text{kmol}^{-1}\cdot\text{K}^{-1}$	$V = 100\ \text{m}^3$
$E_2 = 34{,}833\ \text{kJ}\cdot\text{kmol}^{-1}\cdot\text{K}^{-1}$	$T_i = 25°\text{C}$
$\rho = 1{,}000\ \text{kg}\cdot\text{m}^{-3}$	$T_{sp} = 150°\text{C}$
$C_p = 1\ \text{kJ}\cdot\text{kg}^{-1}\cdot°\text{C}^{-1}$	$(T_{sp})_{final} = 160°\text{C}$
$F = 10\ \text{m}^3\cdot\text{s}^{-1}$	$t_d = 10\ \text{s}$

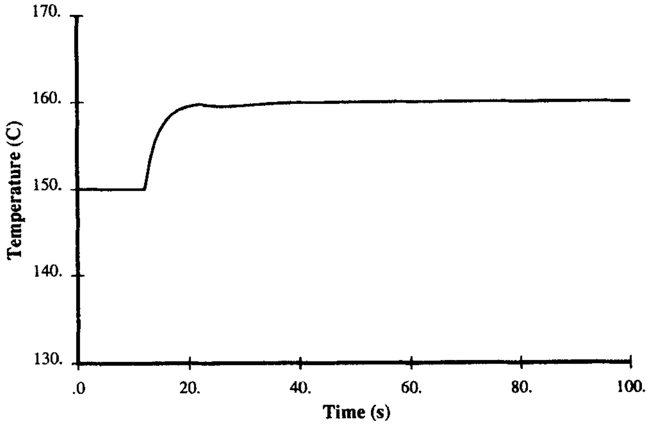

Figure 10. Response with $\beta_0 = 1$, $\beta_1 = 5$, and $\epsilon = 3.5$.

$$k_2 = A_{20}e^{-E_2/RT} \tag{42}$$

The controlled output of the system is the temperature of the reactor, T, and the manipulated input is the heat added to the reactor, Q, which is delayed by t_d. The relative order of the system is clearly 1. The required state feedback for this system is:

$$\begin{aligned} u &= \Psi(C_A, C_B, T, v) \\ &= 1/\beta_1\{\rho C_p V(v - \beta_0 T) - \beta_1[\rho C_p F(T_i - T) \\ &\quad + (-\Delta H_1)Vk_1C_A^2 + (-\Delta H_2)Vk_2C_B]\} \end{aligned} \tag{43}$$

It is assumed that the temperature is measured, but open-loop state observers must be used for the two concentrations. The control structure employed is that of Figure 9. The external controller parameterization of Eq. 25 was used, which for $r = 1$ becomes

$$\frac{\beta_0 + \beta_1 s}{\epsilon s + 1 - e^{-t_d s}}$$

In each simulation, a step change in the temperature setpoint was introduced at time $t = 1$ s. The specific numbers used for each of the parameters are given in Table 1.

Figures 10 and 11 show the response of the system to an increase in the temperature setpoint for a perfect model. In Figure 10, the controller parameters are $\beta_0 = 1$, $\beta_1 = 5$, and $\epsilon = 3.5$.

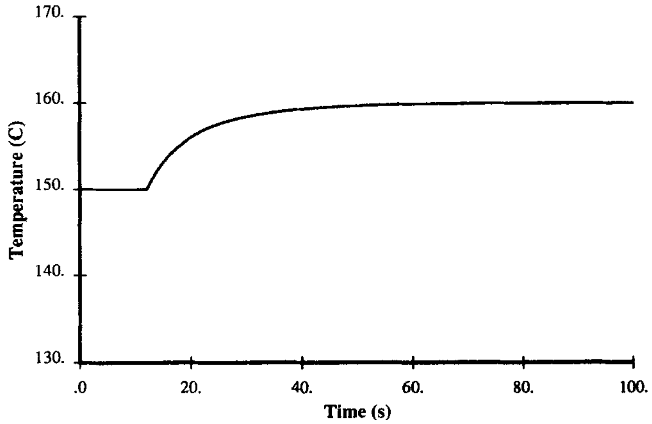

Figure 11. Response with $\beta_0 = 1$, $\beta_1 = 5$, and $\epsilon = 10$.

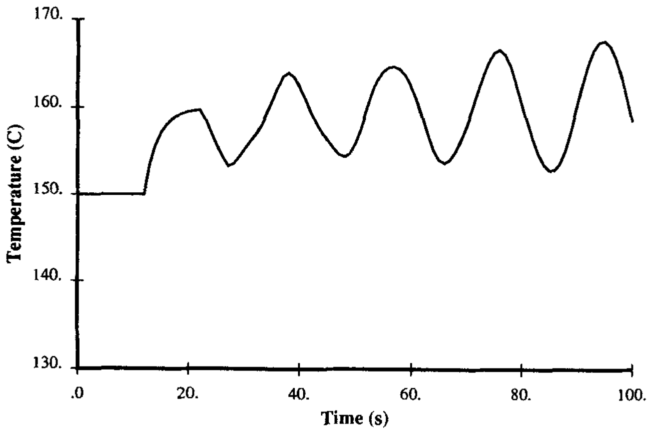

Figure 12. Response with $\beta_0 = 1$, $\beta_1 = 5$, $\epsilon = 3.5$, and $\Delta t_d = 5$.

In Figure 11, the β's have the same values and $\epsilon = 10$. Figures 12 and 13 show the response of the system when a 5-second error in the deadtime is introduced in the controller. The same β's as above were used; in Figure 12, $\epsilon = 3.5$ and in Figure 13, $\epsilon = 10$.

From the robustness condition (Eq. 31), it follows that robust stability is guaranteed for ϵ bigger than about $0.8\Delta t_d$, where Δt_d is the error in deadtime. This is clearly in agreement with the simulation results. Furthermore, the simulated closed-loop response under perfect model conditions is in agreement with the theoretical results. Finally, the simulations confirm the intuitively understood tradeoff between performance and robustness.

Conclusion

A novel approach for deadtime compensation for nonlinear processes has been developed. The approach structure consists of using a Smith-like predictor and linearizing state feedback which make the nonlinear system with deadtime behave like a linear system with deadtime. The control structure is completed by adding an open-loop state observer and a linear external controller which provides integral action and compensates for the deadtime of the input/output linear system. Conditions for robust stability are given with respect to errors in the deadtime and with respect to general linear unstructured multiplicative uncertainties. Computer simulations confirm theoretical results.

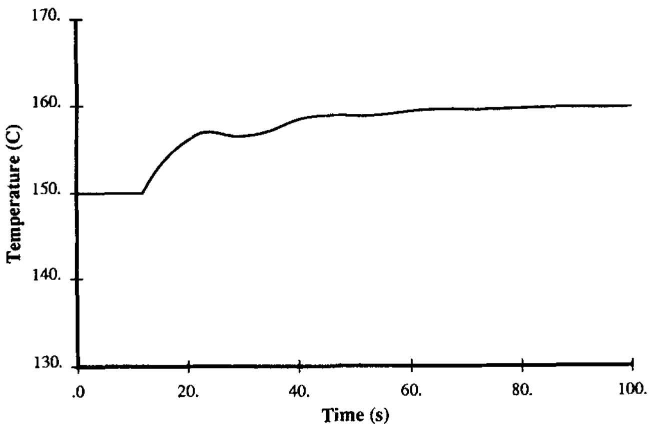

Figure 13. Response with $\beta_0 = 1$, $\beta_1 = 5$, $\epsilon = 10$, and $\Delta t_d = 5$.

Acknowledgment

Financial support from Dow Chemical USA is gratefully acknowledged.

Notation

Adj M = adjugate of matrix M
A, b, c = matrices in the standard state space description of a linear system
C_A = concentration of A in effluent stream
C_{Ai} = inlet concentration of A
C_B = concentration of B in effluent stream
C_p = heat capacity of inlet stream
$C(s)$ = transfer function of external controller in GLC structure
det M = determinant of matrix M
F = flowrate through the CSTR
$f(x), g(x)$ = vector fields characterizing the state model of a nonlinear process
$G_c(s)$ = controller transfer function
$G_o(s)$ = deadtime-free part of process transfer function
$h(x)$ = scalar field determining the state/output map
i = imaginary unit
$L_f^k h(x)$ = kth-order Lie derivative of h with respect to f
l_m = linear multiplicative uncertainty
K = gain vector in linear static state feedback
k_1 = reaction rate constant of first reaction in the example
k_2 = reaction rate constant of second reaction in the example
Q = heat added to the reactor
r = relative order
s = Laplace transform variable
T = temperature of the CSTR
T_i = temperature of inlet stream
t_d = deadtime in the process
t = time
u = manipulated input
V = volume of the CSTR in the example
v = GLC transformed control variable
x = state vector
y = process output
y_{sp} = set point value of process output

Greek letters

β_k = coefficients of characteristic polynomial of the linear $v - y$ system
$-\Delta H_1$ = heat of reaction of first reaction in the example
$-\Delta H_2$ = heat of reaction of second reaction in the example
ϵ = design parameter
$\lambda_d(\omega)$ = upper bound on multiplicative uncertainty corresponding to errors in deadtime only
$\lambda(\omega)$ = upper bound on linear multiplicative uncertainty
Λ_m = linear time-invariant Volterra operator characterizing linear multiplicative uncertainty
μ = input/output behavior of the process
ρ = density of inlet stream

Literature Cited

Brosilow, C. B., "The Structure and Design of Smith Predictor from the Viewpoint of Inferential Control," *Proc. JACC,* Denver (1979).
Doyle, J. C., and G. Stein, "Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis," *IEEE Trans. Auto. Control,* **AC-26,** 44 (1981).
Economou, C., M. Morari, and B. O. Palsson, "Internal Model Control 5. Extension to Nonlinear Systems," *Ind. Eng. Chem. Proc. Des. Dev.,* **25,** 403 (1986).
Garcia, C. E., and M. Morari, "Internal Model Control. 1. A Unifying Review and Some New Results," *Ind. Eng. Chem. Proc. Des. Dev.,* **21,** 308 (1982).

Holt, B. R., and M. Morari, "Design of Resilient Processing Plants V. The Effect of Deadtime on Dynamic Resilience," *Chem. Eng. Sci.*, **40**(7), 1229 (1985).
Jerome, N. F., and W. H. Ray, "High Performance Multivariable Control Strategies for Systems having Time Delays," *AIChE J.*, **32**, 914 (Apr., 1986).
Kravaris, C., "Input/Output Linearization: A Nonlinear Analog of Placing Poles at the Process Zeros," *AIChE J.*, **34**, 1803 (Nov., 1988).
Kravaris, C., and C. B. Chung, "Nonlinear Feedback Synthesis by Global Input/Output Linearization," *AIChE J.*, **33**, 592 (Apr., 1987).
Laughlin, D. L., and M. Morari, "Smith Predictor Design for Robust Performance," *Proc. ACC*, Minneapolis (1987).
Morari, M., and J. C. Doyle, "A Unifying Framework for Control System Design under Uncertainty and its Application for Chemical Process Control," *Proc. 3rd Int. Conf. Chem. Proc. Control*, M. Morari and T. J. McAvoy, eds., Elsevier (1986).
Moore, C. F., C. L. Smith, and P. W. Murrill, "Improved Algorithm for Direct Digital Control," *Instr. and Contr. Sys.*, **43**, 70 (1970).
Parrish, J. R., and C. B. Brosilow, "Nonlinear Inferential Control," *AIChE J.*, **34**, 633 (Apr., 1988).
Smith, O. J. M., "Closer Control of Loops with Dead Time," *Chem. Eng. Prog.*, **53**, 217 (1957).
Smith, C. A., and A. B. Corripio, *Principles and Practice of Automatic Process Control*, Wiley, New York (1985).
Wellons, M. C., and T. F. Edgar, "The Generalized Analytical Predictor," *Ind. Eng. Chem. Res.*, **26**(8), 1523 (1987).
Wong, S. K. P., and D. E. Seborg, "A Theoretical Analysis of Smith and Analytical Predictors," *AIChE J.*, **32**, 914 (June, 1986).

Appendix: Proofs of Results 1, 2 and 3

Proof of Result 1. Under proper initialization of the predictor,

$$x(t) = \hat{x}(t) \text{ and thus from Eq. 21 } x^*(t) = \tilde{x}(t).$$

Hence, $x^*(t) = x(t + t_d)$. Thus, the state feedback law

$$u = \Psi(x^*, v) = \Psi[x(t + t_d), v(t)]$$
$$= \frac{v(t) - \sum_{k=0}^{r} \beta_k L_f^k h[x(t + t_d)]}{\beta_r L_g L_f^{r-1} h[x(t + t_d)]}$$

when substituted into Eq. 2, yields

$$\dot{x} = f(x) + g(x) \frac{v(t - t_d) - \sum_{k=0}^{r} \beta_k L_f^k h[x(t)]}{\beta_r L_g L_f^{r-1} h[x(t)]}$$

whose input/output behavior is governed by Eq. 22. QED

Proof of Result 2. Let t_d^{true} be the true deadtime, t_d the model deadtime, and $\Delta t_d = t_d^{true} - t_d$. Due to the time invariance of the system, it is clear that

$$y(t) = \mu^{\text{true}}[u(t)] = \mu[u(t - \Delta t_d)]$$

Hence

$$\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k} = \sum_{k=0}^{r} \beta_k \frac{d^k}{dt^k} \mu[u(t - \Delta t_d)] \tag{44}$$

But by construction of the compensator in the inner loop of Figure 9 (Eqs. 17 and 28),

$$\sum_{k=0}^{r} \beta_k \frac{d^k}{dt^k} \mu[u(t)] = v(t - t_d)$$

and therefore

$$\sum_{k=0}^{r} \beta_k \frac{d^k}{dt^k} \mu[u(t - \Delta t_d)] = v(t - t_d - \Delta t_d) \tag{45}$$

Combining Eqs. 44 and 45 yields

$$\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k} = v(t - t_d - \Delta t_d)$$

or, in the Laplace domain,

$$\left(\sum_{k=0}^{r} \beta_k s^k\right) y(s) = e^{-t_d s} e^{-(\Delta t_d)s} v(s)$$

or

$$y(s) - \frac{e^{-t_d s}}{\sum_{k=0}^{r} \beta_k s^k} v(s) = (e^{-(\Delta t_d)s} - 1) \frac{e^{-t_d s}}{\sum_{k=0}^{r} \beta_k s^k} v(s)$$

This means that the multiplicative uncertainty of the $v - y$ system is $e^{-\Delta t_d s} - 1$. A formal application of the Doyle-Stein theorem to the closed-loop system leads immediately to the result.

Proof of Result 3. From the uncertainty description (Eq. 33), i.e., $y - \mu(u) = \Lambda_m \mu(u)$, it follows that

$$\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k} - \sum_{k=0}^{r} \beta_k \frac{d^k}{dt^k} \mu(u) = \left(\sum_{k=0}^{r} \beta_k \frac{d^k}{dt^k}\right) \Lambda_m \mu(u)$$

and, since Λ_m is of the form of Eq. 34,

$$\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k} - \sum_{k=0}^{r} \beta_k \frac{d^k}{dt^k} \mu(u) = \Lambda_m \left(\sum_{k=0}^{r} \beta_k \frac{d^k}{dt^k}\right) \mu(u)$$

But by construction of the compensator of the inner loop of Figure 9 (Eqs. 17 and 28),

$$\sum_{k=0}^{r} \beta_k \frac{d^k}{dt^k} \mu[u(t)] = v(t - t_d)$$

and therefore

$$\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k} - v(t - t_d) = \Lambda_m v(t - t_d)$$

or, in the Laplace domain,

$$y(s) - \frac{e^{-t_d s}}{\sum_{k=0}^{r} \beta_k s^k} v(s) = \bar{l}_m(s) \frac{e^{-t_d s}}{\sum_{k=0}^{r} \beta_k s^k} v(s)$$

This means that the multiplicative uncertainty of the $v - y$ system is $\bar{l}_m(s)$. A formal application of the Doyle-Stein theorem to the closed-loop system leads immediately to the result.

Manuscript received Dec. 29, 1988, and revision received June 14, 1989.