NBER WORKING PAPER SERIES

# DEBT AND SENIORITY: AN ANALYSIS OF THE ROLE OF HARD CLAIMS IN CONSTRAINING MANAGEMENT

Oliver Hart
John Moore

Working Paper No. 4886

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 1994

This paper is part of NBER's research program in Corporate Finance. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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## ABSTRACT

We argue that long-term debt has a role in controlling management's ability to finance future investments. A company with high (widely-held) debt will find it hard to raise capital, since new security holders will have low priority relative to existing creditors. Conversely for a company with low debt. We show there is an optimal debt-equity ratio and mix of senior and junior debt if management undertakes unprofitable as well as profitable investments. We derive conditions under which equity and a single class of senior long-term debt work as well as more complex contracts for controlling investment behavior.

Oliver Hart
Department of Economics
Littauer Center 220
Harvard University
Cambridge, MA 02138
and NBER

John Moore
London School of Economics
Houghton Street
London WC2A 2AE
UNITED KINGDOM

Since the time of Modigliani and Miller's famous irrelevance theorem, economists have devoted much effort to relaxing the theorem's assumptions in order to understand the real-world trade-offs between debt, equity and other corporate financial instruments. In particular, literatures have developed that explain the issuance of debt by public companies as an attempt to reduce taxes (see, e.g., Franco Modigliani and Merton Miller (1963) and Miller (1977)); as a signalling device (see, e.g., Hayne Leland and David Pyle (1977) and Stephen Ross (1977)); as a way of completing markets (see, e.g., Joseph Stiglitz (1974) and Franklin Allen and Douglas Gale (1988)); and as an attempt to raise funds without diluting the value of equity (see Stewart Myers and Nicholas Majluf (1984)).[1]

While each of these approaches has provided important insights, none has been entirely successful in explaining the choice of financial structure. In particular, these approaches cannot explain the types of debt claims observed in practice. As one of us has argued elsewhere (see Oliver Hart (1993)), under the maintained hypothesis of most of the literature that management is not self-interested, the first-best can be achieved by making all of a firm's debt soft; that is, all debt should be junior (management should be given the right to issue unlimited amounts of additional debt senior to existing debt) and postponable (all debt should be in the form of payment-in-kind (PIK) bonds, which give management the right to postpone debt payments at management's direction). The reason is that, by issuing such soft claims, the firm can take advantage of all the tax and market completion benefits of debt without incurring any bankruptcy or financial distress costs. Efficient investment choices can be ensured by putting management on an incentive scheme that rewards it according to the firm's total (net) market value, rather than just the value of equity. Such an incentive scheme also avoids conflicts of interest between shareholders and creditors (of the type stresed by Michael Jensen and William Meckling (1976)).[2]

In reality, firms issue considerable amounts of "hard" (senior, nonpostponable) debt.[3] That is, we do not appear to live in the idealized world described above. Presumably, the reason is that managers are self-interested in practice (in this paper, we do not distinguish between management and the board of directors). Among other things, managers have goals, such as the pursuit of power and perquisites, that are not shared by investors. "Hard" debt then has an important role to play in curbing managerial excess.[4] First, nonpostponable, short-term debt forces managers to disgorge funds that they might otherwise use to make unprofitable but empire-building investments, and to trigger liquidation in states of the world where the firm's assets are more valuable elsewhere. Second, senior long-term debt prevents managers from financing unprofitable investments by borrowing against future earnings. Hard debt may be put in place either by the company's founders before the company goes public, or by management itself in response to a hostile takeover bid.

The role of short-term debt in forcing management to disgorge free cash flow has been stressed by Jensen (1986), although he does not analyze it formally.[5] In addition, Jensen emphasizes the benefits of debt, but has little to say about the costs. The role of long-term debt in constraining self-interested management from raising new capital has not been analyzed at all, as far as we know. The purpose of this paper is to provide an analysis of the costs and benefits of debt, with a particular emphasis on long-term debt.

We consider a public company, consisting of assets in place and new investment opportunities (along the lines of Myers (1977); in Myers' work, however, management is not self-interested). The company's security structure is chosen at date 0, an investment decision is made by management at date 1, and funds are paid out to investors at date 2 (there is symmetric information throughout). We assume that management's empire-building tendencies are sufficiently strong that it will always undertake the new investment if it can, even if the investment has negative net present value. In order to focus on the role of long-term debt, we assume that the firm's going concern value exceeds its liquidation value, and that the company's date 1 earnings are insufficient to finance the investment internally. Under these assumptions, we show that the optimal level of short-term debt is zero. However, (senior) long-term debt is important in constraining management's ability to raise new funds. The trade-off for investors is the following. If the company has little or no long-term debt, management will find it easy to finance some negative net present value projects by borrowing against (that is, diluting) future earnings from assets in place. That is, there will be <u>overinvestment</u>. On the other hand, if the company has a large amount of senior long-term debt, management will be unable to finance some positive net present value projects because earnings from assets in place are over-mortgaged (there is debt-overhang in the sense of Myers (1977)). That is, there will be <u>underinvestment</u>.

We use this trade-off to determine the company's optimal debt-equity ratio and to derive a number of comparative statics results concerning the relationship between the debt-equity ratio and the mean and ex ante variance of the return on assets in place and on new investments.[6] In fact, it turns out that it is sometimes optimal for the company to issue a more sophisticated set of claims than just senior debt and equity; in particular, to issue classes of debt of different seniorities, with covenants allowing (limited) dilution of each class. This is observed in practice and is analyzed in the paper. Finally, we show that our theory is consistent with the "two most striking facts about corporate finance" (see Myers (1990)): profitability and financial leverage are negatively correlated, and increases in leverage raise market value.

It is useful to note some aspects of our theory of debt that distinguish it from other agency theories in the literature. First, in most of the literature, debt is equivalent to an incentive contract with management of the form: "If you (the manager) do not pay $D_t$ dollars to security-holders at date t, then the company goes bankrupt, i.e. you lose your job." Given this, it is unclear why an incentive contract of this form would not be employed directly. In contrast, in our model, debt is not equivalent to a contingent firing. Rather, debt regulates the manager's ability to raise capital, making this sensitive to new information available to the market at date 1 when investment decisions are taken. Since we suppose this information to be observable but not verifiable, there is no standard incentive scheme that duplicates the optimal debt contract.

Second, much of the literature derives the optimal debt-equity ratio under the assumption that the company can issue only standard debt and equity claims. In contrast, Section III considers the case where the company can issue arbitrary claims (in some cases we show that the extra degree of freedom will not be used). Thus the paper can be seen as a contribution to the emerging literature on optimal security design (see the recent survey by Harris and Raviv (1992)).

The paper is organized as follows. In Section I, we lay out the basic model, and obtain a sufficient condition for the optimal level of short-term debt to be zero. Section II contains a number of results about the optimal level of long-term debt. In Section III, we consider more general security structures. Finally, Section IV concludes.

## I. The Model

We use the following model, first laid out by Myers (1977).

Consider a firm consisting of assets in place and new investment opportunities, which exists at three given dates (see Figure 1).

FIGURE 1 NEAR HERE

At date 0 the firm's financial structure is chosen. At date 1 the assets in place yield a return of $y_1$ and a new investment opportunity costing i appears. At this time, the firm can be liquidated, yielding L (in addition to the $y_1$ already realized). We take both investment and liquidation to be zero-one decisions. If the firm is not liquidated, at date 2 the assets in place yield a further return $y_2$ and the new investment opportunity -- if it was taken at date 1 -- yields r. At this date the firm is wound up, and receipts are allocated to security holders.

The firm is run by a single manager. This manager decides whether to take the new investment opportunity. The variables $y_1$, $y_2$, i, r, L are typically uncertain as of date 0; however, their probability distribution is common knowledge. All uncertainty about $y_1$, $y_2$, i, r and L is resolved at date 1, and there is symmetric information throughout. However, $y_1$, $y_2$, i, r and L, although observable, are not verifiable. In other words, these variables cannot be the basis of an enforceable, contingent contract.[7] Assume also, for simplicity, a zero interest rate, and that investors are risk neutral.

Although $y_1$, $y_2$, i, r, L are not verifiable, we suppose that the total amount paid out to security-holders is verifiable. Thus securities can be issued at date 0 with claims conditional on the amount that is paid out. However, we do not allow claims to be issued on the return from the investment, r, separately from the return from the assets in place, $y_2$; that is, we rule out project financing.[8] Until Section III,we shall confine attention to the case where the firm issues short-term debt due at date 1, long-term debt due at date 2, and equity; and for the time being we shall suppose that both kinds of debt are senior, in the sense that any new claims issued by the firm at date 1 are entitled to payment only if date 0 debt-holders have been fully paid off. In Section III, we will investigate the role of more sophisticated securities.

We also assume that it is prohibitively costly for the firm to renegotiate with creditors at date 1. Thus if the firm defaults on its short-term debt at date 1, then this triggers bankruptcy, which, in turn, leads to liquidation; i.e., L is realized.[9] (We discuss the no-renegotiation assumption further in Section IV.)

As emphasized earlier, we are interested in a situation where management may carry out some investment projects for power or empire-building reasons even though they are unprofitable.[10] To simplify, we consider the (admittedly) extreme case where the empire-building motive is so strong that no feasible financial incentive payment can persuade the manager not to invest at date 1.[11] At the same time we suppose that the manager's empire-building tendencies are limited to a single, indivisible project. That is, once the project is financed, the manager has no further uses for company funds, i.e. he (or she) cannot or does not wish to employ such funds to make additional investments or to pay for perks or higher salaries (one interpretation is that perks and salaries are adequately controlled by other mechanisms, e.g., incentive schemes).[12]

Given these assumptions about empire-building behavior, the <u>only</u> way to stop the manager from investing in the project is to prevent the necessary funds from being made available at date 1. We also suppose that the manager never liquidates the firm voluntarily at date 1, since this involves a loss of power. In contrast, at date 2 there are no investment opportunities and so the manager is willing to pay out all accumulated funds (one interpretation is that the manager retires and the firm is wound up at date 2).

The financial structure of the firm is chosen at date 0 so as to maximize the aggregate expected return of the security holders; i.e. to maximize the firm's date 0 market value. This may seem an odd assumption given that financial structure decisions are typically made by management (or the board of directors), and we have supposed management to be self-interested. The assumption can be justified in two ways. First, the financial structure choice may be made, prior to a public offering at date 0, by an original owner, who wishes to maximize his total receipts in the subsequent offering (he is about to retire). Second, one can imagine that the firm is initially all equity, and the threat of a hostile takeover at date 0 forces management to choose a new financial structure which maximizes date 0 market value (the hostile bidder is present now, but may not be around at date 1, so management must bond itself now to act well in the future).[13]

Define $d_1$ to be the amount owed at date 1, and $d_2$ to be the amount owed at date 2 -- i.e., $d_1$ and $d_2$ are the face values of short-term and long-term debt respectively. (Of course, at date 0 these debt claims will typically trade for less than their face value because of the risk of default.) In general, the instruments $d_1$ and $d_2$ have distinct roles in curbing the manager's empire building tendencies. Since the role of short-term debt is relatively well understood, in this paper we concentrate on the role of long-term debt. To this end, we now present an assumption which implies that it is optimal to set $d_1 = 0$.

(A1) $y_1 < 1$ and $y_2 \geq L$ with probability 1.

(A1) says that the manager can never finance the investment out of date 1 earnings (there is no free cash flow) and that it is never efficient to liquidate the firm at date 1. (A1) may be plausible for the case of a growth company that, at least initially, requires an injection of new capital to prosper.

To understand why (A1) implies that the optimal $d_1$ is zero, consider the situation facing the manager at date 1. Given the manager's empire-buiding tendencies, his first choice is to invest (if he can raise the funds). His second choice is to maintain the firm as a going concern (if he can't invest, but can pay off his date 1 debts). His third choice is to close the firm down (he does this only if he is forced to default at date 1 and go bankrupt).

Now the firm's total revenues if the manager invests are $y_1 + y_2 + r$, of which $d_1 + d_2$ is mortgaged to the old (senior) creditors. (Recall that all uncertainty is resolved at date 1.) Hence the most the firm can borrow at date 1 is $y_1 + y_2 + r - d_1 - d_2$. It follows that the manager will invest if and only if

(1) $$y_1 + y_2 + r - d_1 - d_2 \geq 1.$$

If (1) is satisfied, the total return to date 0 claim-holders, R, is

$$(2) \qquad R = y_1 + y_2 + r - i,$$

of which date 0 creditors receive $d_1 + d_2$ and shareholders receive the rest.

Notice for future reference the two sources of inefficiency here. Sometimes the manager will invest even though $r < i$, because $y_1 + y_2$ is big relative to $d_1 + d_2$. Othertimes he will be unable to invest even though $r > i$, because $y_1 + y_2$ is small relative to $d_1 + d_2$.

If (1) is not satisfied, the manager will be able to maintain the firm as a going concern as long as

$$(3) \qquad \text{either } y_1 \geq d_1 \text{ or } y_1 + y_2 \geq d_1 + d_2.$$

In the first case, the manager pays the date 1 debt out of current earnings, while, in the second case, he pays it by borrowing out of future earnings. If (3) but not (1) is satisfied, the total return to date 0 claim-holders is

(4) $$R = y_1 + y_2.$$

Finally, if neither (1) nor (3) is satisfied, the firm is liquidated at date 1 and

(5) $$R = y_1 + L.$$

It should now be clear why $d_1 = 0$ is optimal; that is, why $d_1 = 0$ maximizes the expected value of R. Only the sum $d_1 + d_2$ matters in (1) and the second half of (3). However, a low $d_1$ is good in increasing the chance that the first half of (3) is satisfied, i.e. minimizing the likelihood of liquidation (liquidation is undesirable since, given $y_2 \geq L$, (4) and (5) imply that R is higher when the firm survives than when it is liquidated).

We have therefore established

Proposition 1

Assume (A1). Then the date 0 market value of the firm is maximized by setting $d_1 = 0$.[14]

In the next section, we analyse the optimal level of long-term debt.

## II. Long Term Debt

For the remainder of the paper we assume that (A1) holds, and so, by Proposition 1, the optimal $d_1$ is zero. Thus liquidation never occurs at date 1 and L is irrelevant.

Denote by $F(y_1, y_2, i, r)$ the probability distribution of $y_1$, $y_2$, i and r, as of date 0. The return from the existing assets,

$$\int (y_1 + y_2) dF(y_1, y_2, i, r),$$

is fixed, and so in evaluating the effects of different levels of $d_2$ we can focus on the net social return from the new investment, $r - i$. From (1), we know that the investment goes ahead if and only if $y_1 + y_2 + r - i \geq d_2$; and so an optimal $d_2$ solves:

$$\text{(6)} \qquad \text{maximize} \int_{y_1+y_2+r-i \geq d_2} (r - i)\, dF(y_1, y_2, i, r).$$

The basic tradeoff is the following. The benefit of a high $d_2$ is that the returns from the assets in place are mortgaged, which stops management from using them to subsidize bad investment projects. The cost of a high $d_2$ is debt overhang: given that new investors' claims are junior to the debt $d_2$, good projects cannot be undertaken if returns from existing assets are overmortgaged. An optimal $d_2$ strikes the right balance between these two conflicting objectives.

There are four cases in which it is possible to obtain the first-best; these are grouped in Proposition 2.

Proposition 2

(1) If $r$ is greater than $i$ with probability 1, then the first-best can be achieved by setting $d_2$ equal to 0 (all equity).

(2) If $r$ is less than $i$ with probability 1, then the first-best can be achieved by setting $d_2$ large enough.

(3) If $y_1 + y_2$ equals some constant $\bar{y}$ with probability 1, then the first-best can be achieved by setting $d_2$ equal to $\bar{y}$.

(4) If $i$ and $y_1$ are deterministic, and $r \equiv g(y_2)$ where $g(.)$ is a strictly increasing function, then the first-best can be achieved by setting $d_2$ equal to $y_1 + g^{-1}(i)$.

Part (1) of Proposition 2 is immediate. Since there is no danger of the manager making an unprofitable investment, it is best to give him the flexibility to raise as much money as he can at date 1, by having no debt. The investment then always goes ahead, as required in the first-best. Part (2) is the opposite extreme, where no investment is ever profitable. The effect of a large amount of senior (nondilutable) debt $d_2$ is that the manager is never able to raise further funds at t=1, and so the investment never goes ahead.

Part (3) reveals quite a lot about the economics of the model. The difficulty faced by those designing the security structure at date 0 is that the uncertain returns $y_1 + y_2$ from existing assets cannot be disentangled from the uncertain returns $r - i$ from new investment. As a result, there is a danger that the manager will be able to finance the new investment at date 1 by borrowing against $y_2$. However, the danger can be avoided if the total return $y_1 + y_2$ from existing assets is fixed, because this return can be mortgaged away at date 0 by issuing senior, nondilutable debt. The manager then invests if and only if r is greater than i. One point to observe here is that the debt $y_1 + y_2$ is riskless; however, this does not mean that it does not play an important role in preventing the manager from making unprofitable investments.

Part (4) is also revealing. If $r \equiv g(y_2)$, and i and $y_1$ are deterministic, then it is again possible in effect to disentangle the returns from the existing assets and returns from the new investment. Given $d_2 = y_1 + g^{-1}(i)$, the manager invests if and only if $y_2 + r$ exceeds $i + g^{-1}(i)$ -- i.e., since $r \equiv g(y_2)$, if and only if r is greater than i.

In general the first-best cannot be achieved. Some insight into the second-best can be obtained by differentiating (6). If F is a continuous distribution, we can express the first-order condition for an interior optimum as

(7) $$E[r-i \mid y_1+y_2+r-i=d_2] = 0.$$

That is, $d_2$ is set at a level such that the marginal investment project just breaks even.

We consider next some comparative static properties. Proposition 3 below shows how $d_2$ changes with the <u>means</u> of $y_1$, $y_2$, i and r. Part (2) of the Proposition uses the following condition.

<u>Condition C</u> The left hand side of ( 7 ), $E[r-i \mid y_1+y_2+r-i=d_2]$, is continuously differentiable on an interval $[\underline{d}_2, \overline{d}_2]$. Moreover, whenever ( 7 ) holds, it is also the case that

(8) $$0 < \frac{\partial}{\partial d_2} E[r-i \mid y_1+y_2+r-i=d_2] < 1.$$

Denote r − i by X and $y_1 + y_2$ by Y. (8) says that $E[X \mid X+Y=d_2]$ increases with $d_2$, but by less than the amount $d_2$ increases. Note that the left hand inequality in Condition C is simply the second order condition for $d_2$ in ( 6 ), which will hold (albeit weakly) if $(r - i)$ and $(y_1 + y_2 + r - i)$ are <u>affiliated</u> (that is, in a loose sense, positively correlated; see Paul Milgrom and Robert Weber (1982), Theorem 5). The right hand inequality will hold if, in addition, $(y_1 + y_2)$ and $(y_1 + y_2 + r - i)$ are affiliated.

<u>Proposition 3</u>

Suppose $d_2$ is an optimum debt level.

(1) If a dollar is added to every realization of $y_1$, or added to every realization of $y_2$, then $d_2 + 1$ is a new optimum debt level.

(2) Assume that Condition C holds, and that $d_2$ is an interior optimum in $[\underline{d}_2, \bar{d}_2]$ (in which case it is unique). If a small amount is added to every realization of i, or subtracted from every realization of r, then the optimum debt level strictly increases.

Proposition 3 is proved in the Appendix. The intuition is straightforward. An increase in $y_1$ gives the manager more cash with which to invest at date 1, and so at the margin his ability to borrow should be constrained: the debt level should rise. Equally, an increase in the mean of $y_2$ implies that the manager can borrow more at date 1 against the return from the existing assets, and so, again, the debt level should rise. An increase in the mean of i, or a decrease in the mean of r, implies that investments are less likely to be profitable, and therefore the manager should be constrained with higher debt.

Notice that part (1) of the Proposition says that $d_2$ in fact moves dollar-for-dollar with increases in $y_1$ or $y_2$. This follows directly from an inspection of program ( 6 ): if a dollar is added to every realization of $y_1$ (or $y_2$) and a dollar is added to $d_2$, then the set of states in which investment goes ahead is unchanged (and the return from investment, $r - i$, is also unchanged).

There are no comparably general results for how the optimal $d_2$ changes with the variances of $y_1$, $y_2$, $i$ and $r$. However, it is easy to compute the optimal $d_2$, and therefore to see the variance effects, in two examples. These examples also illustrate the mean effects from Proposition 3. For simplicity, in both examples we assume that $y_1$ and $i$ are deterministic (with $y_1 < i$).

Example 1

$y_2$ and $r$ are independently and normally distributed with means $\mu_2$, $\mu_r$ and variances $\sigma_2^2$, $\sigma_r^2$.[15] Then by standard distribution theory the LHS of (7) is simply

$$\mu_r - i + \left(\frac{\sigma_r^2}{\sigma_r^2 + \sigma_2^2}\right)[d_2 - y_1 - \mu_2 - \mu_r + i];$$

and hence the optimal debt level $d_2$ is

$$d_2 = y_1 + \mu_2 - \frac{\sigma_2^2}{\sigma_r^2}[\mu_r - i].^{16} \tag{9}$$

Example 2

$y_2$ and r are independently and uniformly distributed on $[\mu_2-s_2, \mu_2+s_2]$, $[\mu_r-s_r, \mu_r+s_r]$ respectively. Assume that $\mu_r - s_r < i < \mu_r + s_r$ (otherwise, by Propositions 2(1) and 2(2), the optimal $d_2$ would be either zero or infinity). Also, to simplify the example, we assume that $s_r < s_2$.

There are two cases to consider: (a) the case where on average new investment is profitable ($\mu_r > i$); and (b) where it is unprofitable ($\mu_r < i$).

In case (a), the optimal debt level $d_2$ is indicated in Figure 2 -- where the support of $y_1 + y_2$ is the horizontal of the rectangle, and the support of $r - i$ is the vertical (note that, since $s_r < s_2$, the rectangle is wider than it is tall). Notice that conditional on $(y_1 + y_2) + (r - i) = d_2$ -- i.e. conditional on lying along the $135^o$ line intersecting the rectangular support -- the expectation of $r - i$ is zero, as required by (7).

FIGURE 2 NEAR HERE

From the figure it is clear that the optimal debt level must satisfy $d_2 - (y_1 + \mu_2 - s_2) = -(\mu_r - s_r - i)$; that is,

$$d_2 = y_1 + \mu_2 - s_2 - \mu_r + s_r + i. \qquad (\mu_r > i) \tag{10}$$

In case (b), a similar line of reasoning leads to

$$d_2 = y_1 + \mu_2 + s_2 - \mu_r - s_r + i. \qquad (\mu_r < i) \tag{11}$$

In both Examples 1 and 2, $d_2$ rises with $y_1$, rises with the mean of $y_2$, falls with i, and rises with the mean of r -- all as per Proposition 3. Also, if new investment is on average profitable (i.e., $\mu_r > i$), then $d_2$ falls with the variance of $y_2$, and rises with the variance of r. However, if new investment is on average unprofitable ($\mu_r < i$), these variance effects are reversed: $d_2$ rises with the variance of $y_2$, and falls with the variance of r.

To understand the variance effects better, note that, in problem (6), $d_2$ is set so that the market's assessment of future total return at date 1 screens the quality of new investment appropriately. If $y_1$ and i are constant (as in Examples 1 and 2), the first-order condition (7) says that the conditional expectation of r, $E[r | y_2 + r = P]$, must equal i when $P = d_2 + i - y_1$. Now as the variance of $y_2$, $\sigma_2^2$, rises relative to the variance of r, $\sigma_r^2$, the fact that $y_2 + r = d_2 + i - y_1$ reveals less information about $r - i$ (we are looking at this from date 0, when r and i are uncertain). In the limit as $\sigma_2^2 \to \infty$ or $\sigma_r^2 \to 0$, the LHS of ( 7 ) becomes simply $\mu_r - i$, and the optimum will not be interior. In the case where new investment is on average profitable ($\mu_r > i$), it is best to give the manager maximum freedom to finance new investment; i.e. to set $d_2 = 0$. In the case where new investment is on average unprofitable ($\mu_r < i$), it is best to give the manager no freedom to finance new investment; i.e. to set $d_2 = \infty$. Put simply, if the manager's ability to raise fresh capital at date 1 is almost entirely determined by the realized returns from existing assets, then there is little point in using a security structure to screen out the bad new investments. One may as well rely on prior (date 0) information -- i.e. whether or not new investments are on average profitable.

Since the above intuition for the variance effects does not hinge on the particular distributional assumptions of Examples 1 or 2, we suspect that it may be possible to establish a general result about the effects of increasing variance. We do not have such a result to report at this time, however.

## III. General Long-Term Security Structures

In this section, we explore the possibility that more sophisticated long-term security structures than simple debt may be optimal. We start by laying out a quite general class of security structures, and give an example showing how the extra degrees of freedom help. The kind of securities we introduce may at first sight seem rather foreign; but we go on to show that they can be approximated by a conventional class of debt contracts based on seniority. The section ends by considering under what circumstances one would not need the additional flexibility offered by our general security structure, and could instead merely rely on simple debt and equity (as in Section II).

Recall that although $y_1$, $y_2$, i, r are not verifiable, the total amount paid out to security-holders at date 2, denoted by P, is verifiable. Thus securities can be issued at date 0 with claims conditional on P. The most general long-term security structure consists of contingent debt, along the following lines. The firm issues a single class of securities at date 0 with an (enforceable) promise that if P dollars are distributed at date 2, this class will collectively receive O(P) of them, where $0 \le O(P) \le P$. (The "O" in O(P) denotes the old, or original, date 0 security holders.) In addition, management is given permission to issue any new securities it likes at date 1. That is, management can earmark the residual amount $N(P) \equiv P - O(P)$ for new investors at date 1 in the attempt to finance new investment. (The "N" in N(P) denotes the new investors at date 1.) Note that a choice of O(P) close to or far away from P at date 0 constrains the firm more or less in its investment choice at date 1.[17]

General long-term securities like O(P) are not, to our knowledge, observed. However, we show shortly that, under two mild assumptions, any choice of O(P) is equivalent to a package of "standard" securities, consisting of equity and various seniorities of debt. Thus for the moment we stick with the general specification O(P). We continue to assume (A1) throughout this section.[18]

Given N(.) (or equivalently O(.)), consider the position of management at date 1 once $(y_1, y_2, i, r)$ are realized. Since $y_1$ is less than i, the manager can invest only if he can raise $i - y_1$ from the market. If he does invest, $P = r + y_2$, and so the most he can offer the market at date 2 is $P - O(r + y_2) = N(r + y_2)$. It follows that the manager will be able to finance the investment if and only if $N(r + y_2) \geq i - y_1$.

As in ( 6 ) , an optimal security structure at date 0 is represented by a function N(P), which solves:

(12) $$\underset{N(.)}{\text{Maximize}} \int_{N(r+y_2)\geq i-y_1} (r - i)\, dF(y_1, y_2, i, r)$$

subject to

(13) $$0 \leq N(P) \leq P.$$

So far we have allowed the slope of N(P) to be almost arbitrary. With a minor modification in the manager's set of available actions, however, we can restrict N to have a slope between zero and one. Assume that the manager can commit himself at date 1 to lower the return both of the investment project and of the assets in place, e.g. by selling off some fraction of the assets at an artificially low price or by hiring extra workers. Suppose that $N(P^-) > N(P^+)$ for some $P^- < P^+$. Then the firm's date 0 market value can only increase if $N(P^+)$ is raised to equal $N(P^-)$. The reason is that the low value of $N(P^+)$ cannot be effective in deterring management from investing, since if $y_2 + r = P^+$ and $N(P^+) < i - y_1 \le N(P^-)$, the manager will raise the $(i - y_1)$ dollars necessary to invest by committing himself to lower total return from $P^+$ to $P^-$. Thus if $N(P^+)$ is raised to $N(P^-)$, the same investment decisions occur but total return is generally higher since the manager is not encouraged to engage in wastage. An extension of this argument shows that date 0 market value can only increase if $N(P)$ is replaced by $\sup\{N(P^-) | P^- \le P\}$ for each P. This yields a monotonically increasing function N(P).

A similar argument shows that the slope of N can be set less than or equal to one, if the manager can always raise more funds than he needs for the investment project and save the rest at the going rate of interest.[19]

From now on, therefore, when we solve for the optimal security structure we impose the extra constraint that N has slope between 0 and 1:

$$(14) \qquad 0 \le N(P^+) - N(P^-) \le P^+ - P^- \qquad \text{for all } P^- \le P^+.$$

Is all the flexibility afforded by this general security structure useful? Example 3, which generalizes Proposition 2(4) to the case of uncertain i, shows that indeed it is.

Example 3

Suppose $y_1 \equiv 0$, and $r \equiv g(y_2)$, where $g(.)$ is a strictly increasing function. Then one can obtain first-best by putting $N(P) = N$, the (unique) solution to

$$N + g^{-1}(N) = P.$$

(It is straightforward to confirm that (13) and (14) hold.) For a given r (and hence $y_2 = g^{-1}(r)$), the manager can raise up to $N(r + y_2)$ to finance the investment. But by construction, $N(r + y_2) = r$. Moreover, since $y_1 = 0$, the manager needs to raise the full cost i to make the investment -- which means he will be in a position to invest if and only if $r \geq i$ (i.e. the first-best is implemented).

Our next task is to show that our general security structure N(P) can be represented by a "standard package" of securities, comprising equity and noncontingent debt of various seniorities.

A standard package of debt and equity consists of n classes of debt and a single class of equity. The jth class of debt, $j = 1, \ldots, n$, is characterized by an amount $D^j$ collectively owed to class j at date 2 and a maximum additional amount $\Delta D^j$ of indebtedness to class j that the firm can take on at date 1 (i.e. a covenant in the initial debt contract allows the firm to issue new debt at date 1 until the total amount owed class j is $D^j + \Delta D^j$). The classes are ranked by seniority with 1 being the most senior (in the sense that it must be paid off first) and n the most junior. The firm can create an $(n+1)^{th}$ class of debt of any size at date 1, which is junior to all existing debt, but senior to equity; in effect, $\Delta^{n+1} = \infty$.

This description of debt is consistent with what we observe in practice. Firms do issue securities of different seniorities -- the typical order being secured debt, then various priority claims, then unsecured debt, then subordinated debt, and finally equity. Moreover, firms retain the right to issue further securities of comparable or higher seniority, but within prespecified limits. For secured debt, these limits will be determined by the amount of collateral still available; and also possibly by a negative pledge clause, which prohibits the issuance of any new debt with a superior claim to existing unsecured debt or which requires that unsecured creditors be raised to equal status with subsequent claims. For unsecured debt, the freedom to issue further debt is often constrained by covenants specifying upper limits on the ratio of debt to net worth or to tangible assets.[20]

Let us now consider the shape of the function N(P) for the above package of standard debt and equity. Suppose at date 1 the firm issues <u>all</u> the additional debt $\Delta D^1, \ldots, \Delta D^n, \Delta D^{n+1}$ that it is permitted to under the date 0 covenants. How much will these various new issues fetch? Suppose it is known at date 1 that the firm's date 2 pay-out will be P. For $0 \le P \le D^1 + \Delta D^1$, only the most senior class of creditors receive any payment, and the slope of $N(P) = \Delta D^1/(D^1 + \Delta D^1)$; the point is that, in this region, at date 2 every dollar of P is divided in the proportions $\Delta D^1:D^1$ between new and old class 1 creditors. For $D^1 + \Delta D^1 < P \le D^1 + \Delta D^1 + D^2 + \Delta D^2$, the slope of N(P) $= \Delta D^2/(D^2 + \Delta D^2)$. Here, at date 2 class 1 creditors (old and new) are fully paid, and every dollar of the residual, $P - D^1 - \Delta D^1$, is divided in the proportions $\Delta D^2:D^2$ between new and old class 2 creditors; more junior creditors receive nothing. And so on ... See Figure 3.

FIGURE 3 NEAR HERE

It follows from Figure 3 that a standard debt/equity package yields a particular function N(P) that satisfies (13) and (14). It is also clear from Figure 3 that the converse holds, at least approximately: given <u>any</u> function N(P) satisfying (13) and (14), we can find a standard debt/equity package that approximately implements it. Simply approximate the curve N(P) by a piecewise linear graph whose slope always lies between 0 and 1. Such a piecewise linear graph is a representation of some standard debt/equity security package.

The leading example of a standard package is the case of <u>simple debt/equity</u>, which we examined in Section II. Namely, there is a single class of debt that cannot be diluted: $n = 1$, $D^1 > 0$, and $\Delta D^1 = 0$. To marry up with Section II, let $D^1 = d_2$. Then N(P) reduces to Max $(P-d_2, 0)$. That is, for $P \le d_2$, all of P must be given to senior debt-holders and there is none for new investors. On the other hand, for $P > d_2$, the firm can issue junior debt and give $P-d_2$ to new investors.

In Section II, we proceeded on the assumption that nothing more sophisticated than simple debt/equity need be considered in many instances. It is time to justify that assumption. Obviously, the four special cases given in Proposition 2 are examples where simple debt/equity is optimal, since in each case we are able to obtain the first best. Another case in which we can be sure that simple debt/equity is optimal is where $i$ and $y_1$ are deterministic:

Proposition 4

If $i$ and $y_1$ are deterministic, then simple debt/equity is optimal.

Proof New investment occurs iff $N(P) \geq i - y_1$; i.e., in view of (14), iff $P \geq$ some critical value $P^c$, say. It follows that the optimum can be sustained by a simple debt/equity structure with $\omega_2 = P^c - (i - y_1)$.

Q.E.D.

Note that Proposition 4 justifies our restriction to simple debt/equity in Examples 1 and 2 in Section II.

The following lemma presents a general sufficient condition under which simple debt/equity is optimal when $i$ and/or $y_1$ are stochastic. We assume that the distribution function $F(y_1, y_2, i, r)$ satisfies:

Condition F $F(y_1, y_2, i, r)$ is continuously distributed, and the joint density function of $P = (r + y_2)$ and $N = (i - y_1)$ -- $f(P,N)$ say -- is strictly positive everywhere in the set $T \equiv \{(P,N) | \underline{P} \leq P \leq \bar{P};\ 0 \leq N \leq P\}$, where $\underline{P}$ and $\bar{P}$ are respectively the minimum and maximum possible value of $r + y_2$.

Note that in choosing an optimal security structure $N(P)$, given the constraint (13), set $T$ is the only relevant part of the support of $F(y_1, y_2, i, r)$. For $(P,N) \in T$, we define the following conditional expectation:

$$K(P,N) \equiv E[r-i | r+y_2=P \ \& \ i-y_1=N].$$

Key Lemma Assume Condition F holds. A long-term security structure comprising simple debt $d_2$ and equity is optimal if the following condition holds:

Condition K For any pair $(P^*,N^*) \in T$,

$$K(P^*,N^*) \geq 0 \quad \text{implies} \quad K(P,N) \begin{pmatrix} > \\ \geq \end{pmatrix} 0 \qquad \text{if} \quad 0 \leq N-N^* \begin{pmatrix} < \\ \leq \end{pmatrix} P-P^*;$$

$$K(P^*,N^*) \leq 0 \quad \text{implies} \quad K(P,N) \begin{pmatrix} < \\ \leq \end{pmatrix} 0 \qquad \text{if} \quad 0 \leq N^*-N \begin{pmatrix} < \\ \leq \end{pmatrix} P^*-P.$$

To help shed light on condition K, it is useful to rewrite K(P,N) as $E[r-i \mid y_1+y_2+r-i=P-N \ \& \ i-y_1=N]$; viz., the expected return from the new investment project, conditional on the total net profit equalling P - N and the amount of new financing equalling N. If this conditional expectation is (strictly) increasing in (P-N) and (weakly) increasing in N -- i.e., if the project's profit rises with both the total net profit and the external financing requirement -- then condition K will hold.[21]

The lemma is proved in the Appendix. A rough intuition is as follows. K(P,N(P)) is the expected value of a marginal date 1 investment given a total date 2 payout of P. Condition K implies that there is a cutoff value of P, say $P^*$, such that the expected value of a marginal date 1 investment is negative [resp. positive] if $P < P^*$ [resp. $P > P^*$]. Other things equal, then, one would like to lower N(P) for $P < P^*$, and raise N(P) for $P > P^*$. But we have to contend with the constraints (13 ) and (14 ). It should be clear that a simple security structure comprising debt $d_2 = P^* - N(P^*)$ and equity does a good job of balancing these goals (since $N'(P) = 0$ for $P < P^*$, and $N'(P) = 1$ for $P > P^*$).

The result follows from the lemma.

Proposition 5

Assume Condition F holds. A long-term security structure comprising simple debt $d_2$ and equity is optimal if

(1) i is deterministic;

and (2) $y_1$ is distributed independently of $y_2$ and r;

and (3) $E[r|r+y_2 = P]$ is strictly increasing in P, for $\underline{P} \le P \le \overline{P}$.[22]

Proof Conditions (1) and (2) of the Proposition jointly imply that K(P,N) is independent of N. And together with Condition (3), they imply that K(P,N) is strictly increasing in P. Hence Condition K is satisfied. Now apply the lemma.

Q.E.D.

Note that Condition (3) of Proposition 5 is very natural, and will hold (at least weakly) if r and $r+y_2$ are affiliated. The intuition behind the result is that, when i is fixed, new investment should not occur for low values of $r+y_2$ -- since this signifies low r, on average; whereas the investment should go ahead for high values of $r+y_2$. A simple debt/equity security structure implements this quite well.

Our final result concerns the opposite case to Proposition 5.

Proposition 6

Assume Condition F holds. If

(1) $r$ is deterministic

and (2) $y_2$ is independent of $y_1$ and $i$

and (3) $E(i|i-y_1=N)$ is strictly increasing in $N$, for $\underline{P} \le N \le \bar{P}$,

then it is optimal at date 0 to issue two classes of debt: a negligible amount of senior debt, with an option to borrow a finite amount of additional debt of the same seniority at date 1 ($d_1 \cong 0$, $\Delta d_1 > 0$); and a large amount of a second class of debt with no option to borrow any more ($d_2 = \infty$, $\Delta d_2 = 0$).

In a sense the optimal security structure in Proposition 6 is the obverse of simple debt/equity: the manager can raise the first $\Delta d_1$ of any $P$, but no more ($N(P) = \min \{P, \Delta d_1\}$). The intuition is that, given a fixed $r$, low/high values of $i$ represent good/bad investment opportunities and should be encouraged/discouraged. To this end, the manager is given an "overdraft facility" of $\Delta d_1$.

We do not give a formal proof of Proposition 6, since it is similar to the proofs of the lemma and Proposition 5 -- with Condition K replaced by: $K(P,N)$ independent of $P$ and strictly decreasing in $N$, for $(P,N) \in T$.

## IV. Conclusions and Extensions

In this paper, we have explored the role of long-term debt in preventing self-interested management from financing unprofitable investments. We have shown that in those cases where simple debt and equity are optimal: the higher is the average profitability of a firm's new investment project, the lower will be the level of long-term debt; and the higher is the average profitability of a firm's existing assets (assets in place), the higher will be the level of long-term debt. We have also shown that, in general, it is optimal for a firm to issue classes of debt of different seniorities, with covenants allowing (limited) dilution of each class. Finally, we have derived sufficient conditions for the additional flexibility afforded by different classes not to be useful; that is, for simple debt and equity to be optimal.

It should be noted that some of our predictions are novel. For example, a theory which trades off the tax benefits of debt against the bankruptcy costs of debt would not distinguish between assets in place and new investments, and would predict a positive correlation between profitability and the debt level. In contrast, our theory explains the observed strong negative correlation between profitability and leverage (see Carl Kester (1986) and Myers (1990)), as long as high profitability is associated with new projects; this is Myers' (1990) first "striking fact."[23] Note that we can also explain Myers' second striking fact. Consider a company that for some reason — perhaps historical — has (relatively) little debt, and suppose the company faces the threat of a hostile takeover. Then, according to our theory, the managers may engage in a debt-equity swap — that is, borrow and use the proceeds to pay a dividend or buy back equity — in order to commit themselves not to undertake future (bad) investments (thereby persuading shareholders not to tender to the raider). Under these conditions, increases in leverage and increases in market value will move together.[24]

As noted in the Introduction, however, perhaps a more important difference between our theory and most others in the literature is that other theories cannot explain the fact that firms issue hard (senior, nonpostponable) debt claims, whereas our theory can explain this.

An important assumption that we have made is that a firm cannot renegotiate with its claim-holders at date 1 when a new investment project becomes available. Note that, if renegotiation were costless, there would be no disadvantage in having high debt since if the new project had positive net present value the creditors would always be prepared to renegotiate their claims so as to allow the project to go ahead. Thus in a world of costless renegotiation, it would be optimal to have infinite (or very high) debt, in effect forcing the firm to return to the capital market -- or, to put it another way, to seek permission from its creditors -- for every new investment.

Such an extreme outcome is unrealistic, and there are strong theoretical reasons why. Because investors are wealth-constrained and risk averse, a major corporation will typically be financed by a sizable number of small investors, rather than just a small number of very large ones. But this means that free-rider and hold-out problems are likely to make renegotiation difficult. In particular, if the debt level is too high to allow a positive NPV project to take place, then while it is in the collective interest of creditors to forgive a portion of the debt, it is in any small creditor's interest to refuse to forgive his share since the chance that his decision will affect the outcome is very small.[25] Thus in many cases one would expect the renegotiation process to break down and investment not to occur; moreover the evidence of Gilson et.al. (1990) suggests that renegotiation frequently does fail in practice.[26]

One possible way round the free-rider problem is to include a provision in the initial debt contract that the aggregate debt level can be reduced as long as a majority of creditors approve (i.e. the majority's wishes are binding on the minority). It turns out that the Trust Indenture Act of 1939 makes such a provision illegal in the U.S.for public debt. However, even if it were legal, there are strong theoretical reasons for thinking that it would not solve the problem. For majority rule to work well, individual investors must keep abreast of the firm's progress and have very good information about a firm's investment prospects. This is a very demanding requirement in a complex world where most of investors' time is quite properly allocated to other activities. In other words, our assumption that the profitability of new investment is public information should not be taken literally -- it is meant to apply to the most sophisticated arbitrageur, rather than to the average investor. Thus to make the firm's investment decision depend on a majority vote of average investors would be rather like running the firm by a not very well informed committee -- a procedure whose record of success historically has been less than outstanding.[27]

For these reasons, our assumption that renegotiation is impossible does not seem an unreasonable theoretical simplification for companies with widely-held debt.

There are a number of possible extensions of the analysis. An obvious one is to increase the number of periods. This raises at least two new -- and far from straightforward -- issues. First, in a multiperiod model, management faces the choice of raising capital for investment today or waiting to invest until tomorrow. In order to decide which choice is preferred, one needs to know how management trades off different sizes of empires at different moments in time. In other words, the multiperiod extension requires the specification of an intertemporal managerial utility function, whereas the two period model required only the assumption that management prefers more investment to less.

A second complication is that the interpretation of seniority becomes less clear-cut. To give an example: in what sense does a senior debt claim issued at date 1 with a promise to pay one dollar at date 4 have priority over a junior debt claim issued at date 2 which promises to pay one dollar at date 3? The answer is that it depends on whether the firm goes bankrupt. If it does, the first claim is senior, but if it does not the second claim may be senior because it is paid off first. In other words, the notion of seniority that we have analysed must be enlarged to encompass seniority in an intertemporal sense.

Finally, our analysis has completely ignored the role of shareholder voting and takeovers in a firm's choice of financial structure. Yet voting and takeovers are important restraining forces on management. In future work it is desirable to develop a framework which permits a study of the interplay between debt and the market for corporate control as constraints on managerial behaviour.

FOOTNOTES

*Department of Economics, Harvard University, Cambridge, MA 02138, and Department of Economics, London School of Economics, Houghton Street, London, WC2 2AE, England. This is a major revision of a previous paper, "A Theory of Corporate Financial Structure Based on the Seniority of Claims." We are particularly grateful to Ian Jewitt for simplifying the proof of the Key Lemma in Section III. We would also like to thank Rabindran Abraham, Abhijit Banerjee, Dick Brealey, Lucien Bebchuk, Stu Myers, Fausto Panunzi, David Webb, two anonymous referees and an Associate Editor, Paul Milgrom, for helpful comments. They are, of course, not responsible for the views expressed here. Finally, we acknowledge financial support from the National Science Foundation, the LSE Suntory-Toyota International Centre for Economics and Related Disciplines, the Olin Foundation, and the Center for Energy Policy Research at MIT.

[1]Milton Harris and Artur Raviv (1991) survey these theories. A more recent literature has viewed debt as a way of shifting control rights from corporate insiders to security-holders in certain (bankruptcy) states of the world (see Philippe Aghion and Patrick Bolton (1992) and Oliver Hart and John Moore (1989, 1994)). Control-based theories seem more applicable to smallish, entrepreneurial firms than to public companies (the focus of this paper); in the latter, managers or directors rarely have voting control even when the company is solvent.

[2]For details, see Hart (1993). A related point has been made by Philip Dybvig and Jaime Zender (1991).

[3]Clifford Smith and Jerold Warner (1979) found that in a random sample of eighty-seven public issues of debt registered with the Securities and Exchange Commission between January 1974 and December 1975, more than 90% of the bonds contained restrictions on the issuing of additional debt. Although the strength of such debt covenants declined during the 1980s, it is still very common for new public debt issues to contain some restrictions on new debt. See Kenneth Lehn and Annette Poulsen (1991).

[4]See Sanford Grossman and Oliver Hart (1982).

[5]Some formal analysis is provided by Rene Stulz (1990) and Guozhong Xie (1990).

[6]The trade-off between overinvestment and underinvestment has also been analyzed recently by Elazar Berkovitch and E. Han Kim (1990), Stulz (1990), Xie (1990), and Robert Gertner and David Scharfstein (1991). Stulz and Xie consider models in which high short-term debt is good in that it forces management to pay out funds, but bad because it leads to inefficient piecemeal liquidation in the event of default (with, in Stulz's case, a loss of investment opportunities); Stulz and Xie do not consider the role of long-term debt in preventing the firm from raising new capital. Berkovitch and Kim and Gertner and Scharfstein do consider the role of long-term debt, but assume that managers act on behalf of shareholders; that is, management is (implicitly) assumed not to be self-interested. As we have noted, if management is not self-interested, the first-best can be achieved by putting management on an appropriate incentive scheme and making all of the firm's debt junior and postponable.

[7]For example, a statement in the corporate charter stipulating that management should invest if and only if $r \geq 1$ is unenforceable since a disinterested judge or jury would not know whether $r \geq 1$. A statement that management must pay out all earnings at date 1 (that is, all of $y_1$, whatever $y_1$ may be) is unenforceable for similar reasons.

[8] If project financing were possible, the new investment could be financed as a stand-alone entity, whose merit could be assessed by the market at date 1; and debt levels could be set very high to prevent the manager using funds from the existing assets to subsidize investment. There are several reasons for ruling out project financing. First, it may be that i represents an incremental investment -- e.g., maintaining or improving the existing assets -- and the final return $y_2 + r$ is simply the overall return from the (single) project. Second, it may be that the same management team looks after both the old assets and the new project, and can use transfer pricing to reallocate profits between them; hence the market can keep track only of total profits. Finally, even if project-specific financing is feasible, it is not at all clear that managers will want to finance a project that is not part of their empire since they will not enjoy the private benefits of control (on this, see Shan Li (1993)).

[9] We ignore more sophisticated bankruptcy systems that try to preserve the firm's going-concern value; examples are US Chapter 11 or the procedure discussed in Philippe Aghion et.al. (1992).

[10] This is in the spirit of the early managerial literature of William Baumol (1959), Robin Marris (1964) and Oliver Williamson (1964), as well as of the later work of Jensen (1986). For empirical support, see Gordon Donaldson (1984).

[11]We suppose that the manager has no (or little) initial wealth and so cannot be charged up front for empire-building benefits.

[12]This distinguishes our model from a "pure free cash flow model" of the Jensen (1986) variety. In a pure free cash flow model, the manager always has further uses of company funds and so will squander each dollar of investor returns that is not mortgaged to creditors. Thus in a free cash flow model the value of equity is zero. In contrast, in our model, as the reader will shortly see, the value of equity can be positive.

Note that this is not a critical difference between the two analyses since our main results would still hold under the more extreme Jensen assumptions. A more important difference between the models is that ours explicitly considers the costs as well as the benefits of short and long-term debt.

[13]Both of these scenarios are of course special. We believe that the thrust of our analysis applies also to the case where management chooses financial structure to maximize its <u>own</u> welfare. In the present three date model, this leads to the trivial outcome of no debt (management clearly prefers not to be under pressure from creditors). However, in a model with more periods management may issue (senior) debt voluntarily, since this may be the only way to raise funds from investors concerned that their claims may be diluted if management undertakes bad investments in the future. On this, see Jeffrey Zwelbel (1993).

[14] Notice that we are ruling out the possibility of <u>negative</u> debt. For example, a negative $d_1$ -- in effect a pre-arranged boost to $y_1$ -- allows the manager to make an investment without the need to go to the market at date 1, even when $y_1 < 1$. This may be helpful if the profitable investments are small ones. A negative $d_1$ may be hard to implement, however. It may be impossible to arrange for dispersed creditors to pay in money at date 1; and if the manager is given the money at date 0 he may use it to make an unprofitable date 0 investment.

[15] Strictly speaking, we ought to truncate the distributions of $y_2$ and r so that they are nonnegative. See our 1990 Working Paper for further details.

[16] Example 1 is easily generalised to allow for correlation between $y_2$ and r. If their correlation coefficient is $\rho$, then the optimal $d_2$ is given by

$$d_2 = y_1 + \mu_2 - \left(\frac{\sigma_2^2 + \rho\sigma_2\sigma_r}{\sigma_r^2 + \rho\sigma_2\sigma_r}\right)[\mu_r - 1].$$

Note that the denominator $\sigma_r^2 + \rho\sigma_2\sigma_r$ is positive from the second order condition. Thus, provided the numerator $\sigma_2^2 + \rho\sigma_2\sigma_r$ is positive -- which it will be unless $(-\sigma_r/\sigma_2) < \rho \leq (-\sigma_2/\sigma_r)$ -- the comparative statics results reported in the text continue to hold.

[17] One can think of even more general securities. One possibility is that O(P) could be conditioned on the amount of money raised at date 1. However, our preliminary investigations suggest that the extra degree of freedom would not help.

Another possibility is that O(P) could be sensitive to the <u>market</u> <u>value</u> of securities. The difficulty with this is that there is a tricky bootstraps effect: market values are affected by the manager's actions, which are in turn constrained by the form of O(P).

[18] It is straightforward to confirm that Proposition 1 continues to apply, even when more sophisticated long-term securities are admitted. That is, there is no role for short-term securities -- like short-term debt $d_1$ -- which promise to pay out at date 1.

[19] The argument is as follows. Suppose $P^- < P^+$ and $N(P^+) - N(P^-) > P^+ - P^-$. Then the firm's date 0 market value will not change if $N(P^-)$ is raised to $N(P^+) - P^+ + P^-$. The reason is that if $y_2 + r = P^-$ and $N(P^-) < i - y_1 \leq N(P^+) - P^+ + P^-$, the manager can raise $(i - y_1) + (P^+ - P^-)$ dollars from the market, invest i in the project and save the remaining $(P^+ - P^-)$. This yields a total date 2 return of $P^+$, out of which the manager can repay new security-holders up to $N(P^+) \geq i - y_1 + (P^+ - P^-)$. Again this argument can be extended to show that date 0 market value will be unchanged if N(P) is replaced by $\sup((N(P^+) - P^+ + P) | P^+ \geq P)$. This yields a function N(P) whose slope is less than or equal to one.

[20] For a discussion of covenants used in practice, see Smith and Warner (1979) and Lehn and Poulsen (1991). For an example of a bond prospectus (Potomac Electric Power Co.) with essentially the form of our standard debt/equity package, see Brealey and Myers (1988), pp 591-599.

[21] We are grateful to a referee for pointing this out.

[22] Note that there is no inconsistency between Proposition 5 and Example 3. If $i$ is deterministic <u>and</u> $y_1 = 0$, $r \equiv g(y_2)$, then there is indeterminacy in the optimal security structure.

[23] Measured profitability reflects the profitability of assets in place. However, if the profitabilities of assets in place and new investments are positively correlated, then measured profitability may serve as a proxy for the profitability of new investments.

[24] For details, see Hart (1993). Other bonding theories, such as those in Grossman and Hart (1982) and Jensen (1986), can also explain this observation.

[25] See, for example, Gertner and Scharfstein (1991).

[26] In a study of the companies listed on the New York and American Stock Exchanges that were in severe financial distress during 1978-1987, Gilson et.al. (1990) found that workouts failed more often than 50 percent of the time, and were more likely to fail the larger the number of creditors. See also Gilson (1991).

[27] To put it another way, to the extent that (dispersed) creditors are poorly informed, any debt forgiveness is likely to be insensitive to the ex post realizations of $y_1$, $y_2$, r and i; that is, debt forgiveness will be approximately a fixed amount d'. But then the same outcome could be achieved by setting the original debt level equal to $d_2 - d'$; i.e., debt forgiveness serves no useful role.

# Appendix

In this Appendix, we prove Proposition 3 and the Key Lemma.

## Proof of Proposition 3

Part (1) follows immediately from an inspection of program ( 6 ).

If a small $\varepsilon > 0$ is added to every realization of $i$, the left hand side of ( 7 ) becomes

$$E[r-i-\varepsilon|y_1+y_2+r-i-\varepsilon=d_2] \quad = \quad -\varepsilon + E[r-i|y_1+y_2+r-i=d_2+\varepsilon]$$

$$< \quad E[r-i|y_1+y_2+r-i=d_2] \quad = \quad 0$$

by the right hand inequality in Condition C and by ( 7 ).

Hence, from the left hand inequality in Condition C, it follows that the new debt level strictly exceeds $d_2$. An identical argument can be used for the case where $\varepsilon > 0$ is subtracted from every realization of r.

Q.E.D.

Proof of Key Lemma

Let C be the class of admissible security structures N satisfying ( 13 ) and ( 14 ), and let V(N) denote the integral in ( 12 ). Note that C is convex: $\lambda N + (1-\lambda)M \in C$ for any $N, M \in C$ and any $0 \le \lambda \le 1$. If N is an optimal security structure, then $\partial V(\lambda N+(1-\lambda)M)/\partial\lambda \ge 0$ at $\lambda = 1$. Carrying out the differentiation,

$$(A.1) \qquad \int_{P=\underline{P}}^{\overline{P}} [N(P) - M(P)]\, K(P,N(P))\, f(P,N(P))\, dP \ \ge \ 0.$$

where, by Condition F, the joint density $f(P,N(P)) > 0$ for $\underline{P} \le P \le \overline{P}$.

Let $P^* = \inf \{P | \underline{P} \le P \le \overline{P};\ K(P,N(P)) > 0\}$. Condition K implies that $K(P,N(P)) \le 0$ for all $\underline{P} \le P < P^*$, and $K(P,N(P)) \ge 0$ for all $P^* < P \le \overline{P}$.

Suppose N(P) is not a simple debt/equity security structure in the relevant domain (viz., $\underline{P} \le P \le \overline{P}$). Then construct a security structure M comprising simple debt $d_2$ and equity, where

$$d_2 = P^* - N(P^*).$$

(We know that this $d_2 \ge 0$, since N satisfies ( 13 ).) That is, $M(P^*) = N(P^*)$. In the light of the fact that N satisfies ( 13 ) and ( 14 ), $M(P) \le N(P)$ for all $\underline{P} \le P < P^*$, and $M(P) \ge N(P)$ for all $P^* < P \le \bar{P}$. Thus the left hand side of (A.1) is at most zero.

Moreover, since $N \ne M$, there must be some open interval $S \subset [\underline{P},\bar{P}]$, not containing $P^*$, such that, for all $P \in S$, both (i) $N(P) \ne M(P)$, and (ii) $N(P^*) - N(P) \ne P^* - P$. From Condition K, (ii) implies $K(P,N(P)) \ne 0$ for all $P \in S$. But this means that the left hand side of (A.1) is in fact strictly negative; a contradiction. Hence a simple debt/equity security structure is optimal.

Q.E.D.

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# FIGURES

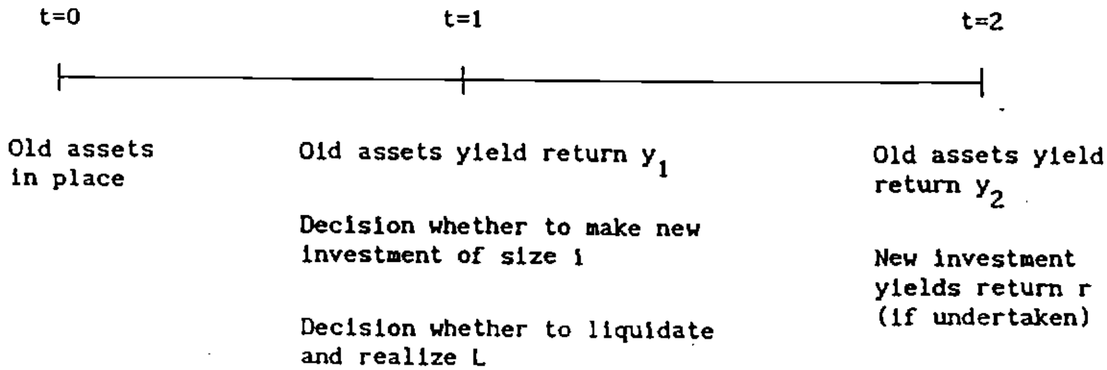
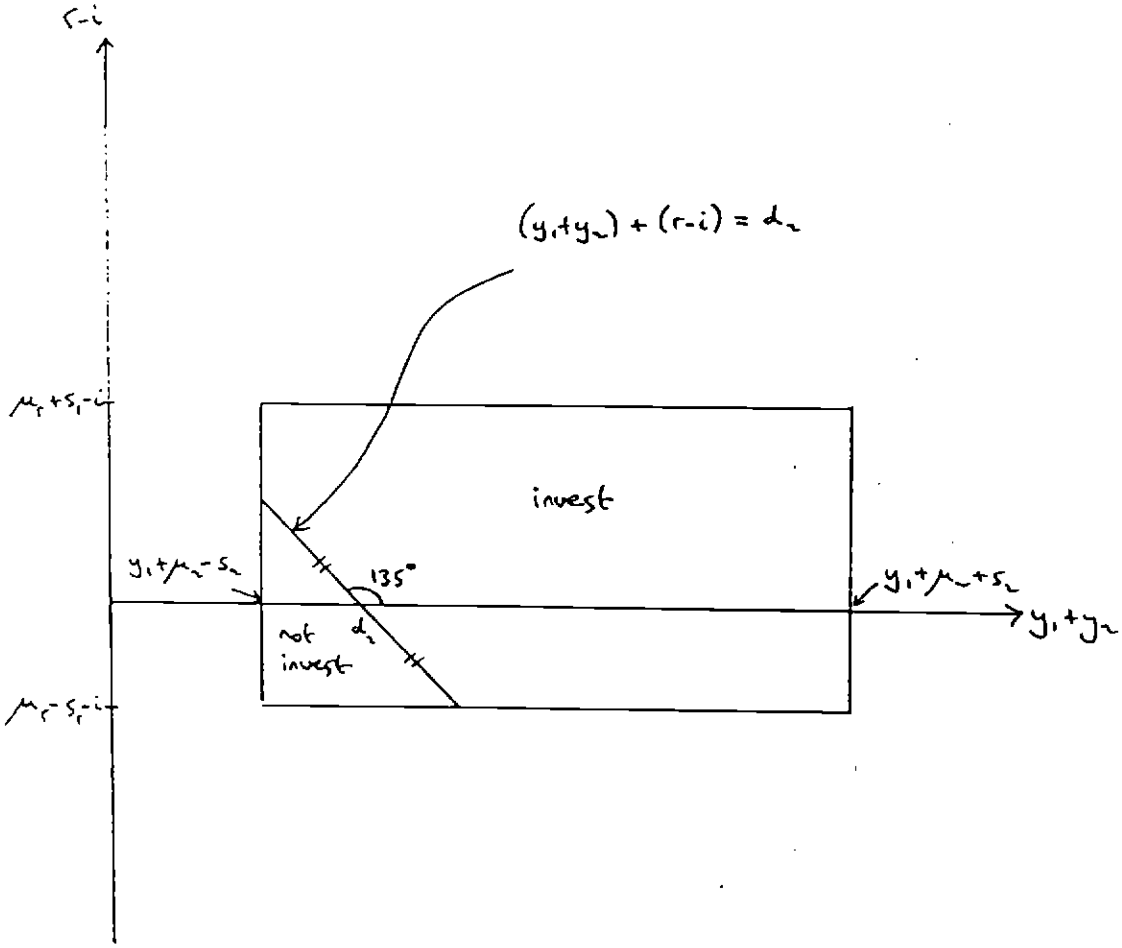
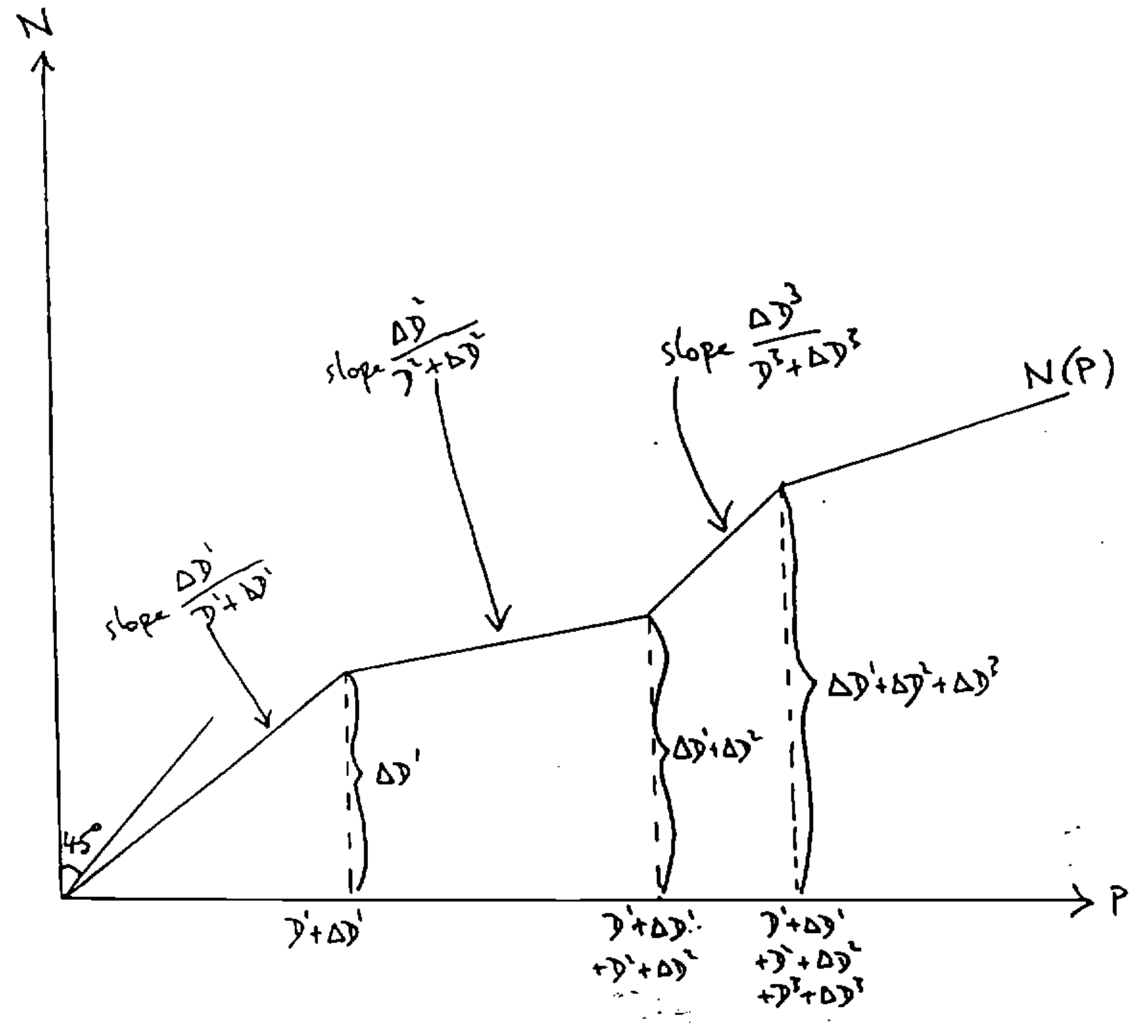



Figure 1



r-i
$(y_1+y_2)+(r-i)=d_2$
$\mu_r+s_1-i$
invest
$y_1+\mu_2-s_2$
135°
$y_1+\mu_2+s_2$
$y_1+y_2$
not invest
$d_2$
$\mu_r-s_r-i$


Figure 2


N
P
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45°
slope $\frac{\Delta D^1}{D^1+\Delta D^1}$
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Figure 3