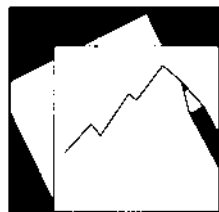


# Working Paper

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## Debt Dilution and Sovereign Default Risk

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and César Sosa Padilla*

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**Debt Dilution and Sovereign Default Risk****Prepared by Juan Carlos Hatchondo, Leonardo Martinez, and César Sosa Padilla<sup>1</sup>**

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**Abstract****This Working Paper should not be reported as representing the views of the IMF.**

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We propose a modification to a baseline sovereign default framework that allows us to quantify the importance of debt dilution in accounting for the level and volatility of the interest rate spread paid by sovereigns. We measure the effects of debt dilution by comparing the simulations of the baseline model (with debt dilution) with the ones of the modified model without dilution. We calibrate the baseline model to mimic the mean and standard deviation of the spread, as well as the external debt level, the mean debt duration and a measure of default frequency in the data. We find that, even without commitment to future repayment policies and without contingency of sovereign debt, if the sovereign could eliminate debt dilution, the number of default per 100 years decreases from 3.10 to 0.42. The mean spread decreases from 7.38% to 0.57%. The standard deviation of the spread decreases from 2.45 to 0.72. Default risk falls in part because of a reduction of the level of sovereign debt (36% of the face value and of 11% of the market value). But we show that the most important effect of dilution on default risk results from a shift in the set of government's borrowing opportunities. Our analysis is also relevant for the study of other credit markets where the debt dilution problem could be present.

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Contents	Page
I. Introduction . . . . .	<a href="#">3</a>
II. The model . . . . .	<a href="#">7</a>
A. The baseline environment. . . . .	<a href="#">7</a>
B. Recursive formulation of the baseline framework. . . . .	<a href="#">9</a>
C. A model without debt dilution. . . . .	<a href="#">10</a>
D. Recursive formulation of the framework without debt dilution . . . . .	<a href="#">11</a>
E. Discussion of the framework without debt dilution. . . . .	<a href="#">12</a>
III. Results . . . . .	<a href="#">12</a>
A. Calibration. . . . .	<a href="#">13</a>
B. Simulation results . . . . .	<a href="#">14</a>
C. Welfare costs of debt-dilution. . . . .	<a href="#">17</a>
D. One-period bonds vs. long-duration bonds without debt dilution. . . . .	<a href="#">18</a>
VII. Conclusions. . . . .	<a href="#">20</a>
References . . . . .	<a href="#">22</a>
Tables	
1. Parameter values . . . . .	<a href="#">13</a>
2. Business cycle statistics . . . . .	<a href="#">15</a>
3. Business cycle statistics without debt dilution . . . . .	<a href="#">19</a>
Figures	
1. Menu of combinations of spreads and next-period debt levels. . . . .	<a href="#">16</a>
2. Consumption compensation. . . . .	<a href="#">18</a>
3. Menu of combinations of spreads and next-period debt levels without dilution. . . . .	<a href="#">20</a>

## I. Introduction

We study the effects of the sovereign debt dilution problem caused by the government's lack of commitment to avoid decreasing the value of debt issued in the past by issuing new debt. The possibility of dilution is a central issue in debt markets and have received considerable attention in both academic and policy discussions (see, for example, Bizer and DeMarzo (1992), Bolton and Jeanne (2009), Bolton and Skeel Jr. (2005), Borensztein et al. (2004), Detragiache (1994), Eaton and Fernandez (1995), Kletzer (1984), Niepelt (2008), Sachs and Cohen (1982), Saravia (forthcoming), Tirole (2002), and UN (2004)). Several theoretical studies describe the benefits from eliminating debt dilution. For instance, Bizer and DeMarzo (1992) show how dilution may lead to equilibria with higher debt levels and higher interest rates implied by higher default probabilities. In the context of sovereign debt markets, it has been argued that the possibility to dilute debt may lead to excessive issuance of short-term debt (Kletzer (1984)), or of debt that is hard to restructure after a default episode (Bolton and Jeanne (2009)), which could increase the likelihood and/or severity of sovereign debt crisis.<sup>1</sup>

Participants in various credit markets have made efforts to mitigate the dilution problem, which is suggestive of the relevance assigned to this issue. First, we observe debt claims with different seniority. This is clear in corporate debt and in collateralized loans to households. In contrast, for sovereign bonds, we are not aware of differences in legal seniority.<sup>2</sup> Second, in some markets debt contracts include covenants intended to limit debt dilution (see Smith and Warner (1979) and Rodgers (1965) for a discussion of debt covenants in corporate debt markets). In sovereign debt contracts it is common to introduce a *pari passu* clause and many contracts also contain negative pledge clauses that prohibit future issuances of collateralized debt. These clauses intend to avoid making new bonds senior to previously issued bonds, but do not protect against dilution caused by future borrowing behavior.<sup>3</sup> The weaker protection against sovereign debt dilution may be due in part to the weak enforcement in sovereign debt markets. This weak enforcement has lead to several proposals to induce more orderly sovereign debt restructurings (see for example Bolton and Skeel Jr. (2005), Borensztein et al. (2004), G-10 (2002), IMF (2003), Krueger and Hagan (2005), and Paulus (2002)). For example, Bolton and

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<sup>1</sup>Detragiache (1994), Sachs and Cohen (1982), and Niepelt (2008) discuss further inefficiencies raised by sovereign debt dilution.

<sup>2</sup>It has been argued that loans from institutions such as the International Monetary Fund or the World Bank receive de facto seniority over loans from private agents (see, for example, Saravia (forthcoming)).

<sup>3</sup>Sturzenegger and Zettelmeyer (2006) discuss that in the 2000 sovereign debt restructuring in Ecuador, the government replaced defaulted bonds with new bonds that included a clause specifying that if there was a default within the following 10 years, the government had to give new bonds to the holders of the restructured debt. Sturzenegger and Zettelmeyer (2006) argue that the "effect of this was to offer a (limited) protection of bondholders against the dilution of their claims by new debt holders in the event of default". However, the inclusion of such debt covenants is much more an exception than a rule.

Skeel Jr. (2005) argue for the importance of being able to grant seniority to debt issued while the country is negotiating with holders of debt in default, as observed in corporate bankruptcy procedures. Borensztein et al. (2004) suggest changes in national and international laws that may facilitate the introduction of debt contracts that provide some protection against debt dilution. Overall, it seems clear that existing sovereign debt contracts do not eliminate the risk of debt dilution.

While the studies mentioned in previous paragraphs suggest that debt dilution may be an important source of inefficiencies in debt markets, they do not quantify the effects of dilution.<sup>4</sup> We contribute to the discussion of sovereign debt dilution by providing a measure of its effects on the levels of sovereign debt and default risk.

We measure the effects of dilution through the lens of a baseline sovereign default framework à la Eaton and Gersovitz (1981), similar to the ones used in recent studies.<sup>5</sup> We analyze a small open economy that receives a stochastic endowment stream of a single tradable good. The government's objective is to maximize the expected utility of private agents. Each period, the government makes two decisions. First, it decides whether to default on previously issued debt. Second, it decides how much to borrow or save. The government can borrow (save) by issuing (buying) non-contingent long-duration bonds, as in Hatchondo and Martinez (2009).<sup>6</sup> The cost of defaulting is represented by an endowment loss that is incurred in the default period.

There are three features of this framework that imply inefficiencies that could be important in accounting for the equilibrium levels of debt and sovereign default risk. First, the government cannot commit to its future repayment policy. Second, bond payments are not contingent to income shocks. Third, the government can borrow from multiple lenders and cannot commit to not decrease the value of debt issued in the past with new issuances (debt dilution). These three features represent characteristics of sovereign debt in reality and are standard in sovereign debt models. We study the effects of debt dilution in the presence of the first two features mentioned above (the lack of commitment to future repayment policies and

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<sup>4</sup>Bi (2006) presents a quantitative analysis of a model with one and two-quarter bonds. She studies the effects of making earlier issuances senior to new issuances. She finds that this decreases the default frequency but increases the mean debt level (perhaps because the endogenous borrowing constraint in the model is relaxed by making earlier issuances less risky).

<sup>5</sup>See, for instance, Aguiar and Gopinath (2006), Arellano (2008), Arellano and Ramanarayanan (2010), Bai and Zhang (2006), Benjamin and Wright (2008), Borri and Verdelhan (2009), Boz (2009), Cuadra et al. (forthcoming), Cuadra and Sapriza (2006, 2008), Chatterjee and Eyigungor (2009), D'Erasmus (2008), Hatchondo and Martinez (2009), Hatchondo et al. (2007, 2009, 2010), Lizarazo (2005, 2006), Mendoza and Yue (2008), Sandleris et al. (2009), and Yue (2010). These models share blueprints with the models used in studies of household bankruptcy—see, for example, Athreya (2002), Athreya et al. (2007a,b), Chatterjee et al. (2007a), Chatterjee et al. (2007b), Li and Sarte (2006), Livshits et al. (2008), and Sánchez (2008).

<sup>6</sup>With one-period bonds, when the government decides its current borrowing level, the outstanding debt level is zero (either because the government honored its debt obligations at the beginning of the period or because it defaulted on them). Thus, the government does not have the option to dilute the value of debt it issued in previous periods.

the lack of contingency of sovereign debt).<sup>7</sup>

The standard modeling approach for the study of debt dilution is to focus on the effect of seniority clauses in debt contracts. However, it is well known that seniority may not fully eliminate debt dilution and, therefore, comparing equilibria with and without seniority may not be equivalent to comparing equilibria with and without debt dilution (see, for example, Bizer and DeMarzo (1992)). Furthermore, in a model in which the duration of sovereign bonds matches the one observed in the data, imposing a full seniority structure would require a large number of state variables, which would make the model intractable.

We propose a new modeling approach for the study of debt dilution. We present a modification of the baseline default model that eliminates dilution without increasing the dimensionality of the state space: We study a modified model in which the government must pay to existing bondholders a compensation equal to the reduction in the market value of their bonds caused by the government's current debt issuance (we assume the government does not compensate bondholders for bond price declines that are not the result of new debt issuances). Thus, when buying sovereign debt, investors anticipate that the future value of their investment is independent of future issuances, which eliminates the dilution problem.<sup>8</sup>

We measure the effects of debt dilution by comparing the simulations of the baseline model (with debt dilution) with the ones of the modified model without dilution. We impose discipline to our quantitative exercise by calibrating the baseline model targeting the default probability, the level of public external debt, the debt duration, and the mean and standard deviation of the interest rate spread (i.e., the difference between the sovereign bond yield and the risk-free interest rate). We find that, even without commitment to future repayment policies and without contingency of sovereign debt, if the sovereign eliminates debt dilution, the number of default per

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<sup>7</sup>Bolton and Jeanne (2009) argue that it is somewhat of a puzzle that the overwhelming majority of sovereign debts are not GDP indexed. Indexing debt payments to GDP appears to be feasible, desirable, and relatively immune to manipulation (see also Borensztein and Mauro (2004), Durdu (2009), and Sandleris et al. (2009)). Bolton and Jeanne (2009) also argue that sovereigns' willingness to repay has many other determinants besides domestic GDP. Tomz and Wright (2007) show that these other determinants play an important role as predictors of sovereign defaults. For instance, Alfaro and Kanczuk (2005), Cole et al. (1995), Hatchondo et al. (2009), and Guembel and Sussman (2009) discuss how sovereign defaults may be triggered by changes in political circumstances. Richer models that incorporates determinants of sovereign default other than GDP would feature market incompleteness even with GDP-indexed bonds. We follow the most common approach of assuming that GDP shocks are the only shock in the economy and that sovereign debt contracts are not GDP-indexed.

<sup>8</sup>In contrast, with seniority, lenders may need to worry about a decline in the price of their bonds caused by increases in the default probability implied by new issuances. To illustrate this point consider an economy in which there is an explicit seniority structure. Suppose that in case of default, the recovery rate of a senior bond is less than one and the recovery rate is not affected by the issuance of a junior bond (for instance, because the value of the collateral that backs senior debt claims is not affected by the junior debt issuance). The probability of a default on senior bonds may still be affected by the issuance of new—junior—debt (see, for example Bizer and DeMarzo (1992)).

100 years decreases from 3.10 (with debt dilution) to 0.42 (without debt dilution). That is, the dilution problem accounts for 86% of the default risk in the simulations of the baseline model. In the model, default risk is reflected in the interest rate spread. The mean spread decreases from 7.38% to 0.57%. The standard deviation of the spread decreases from 2.45 with debt dilution to 0.72 without debt dilution. Thus, our exercise is indicative of the quantitative importance of debt dilution and supports the view that debt dilution should be a central issue in discussions of sovereign debt management and the international financial architecture.

The reduction in default risk implied by the elimination of dilution occurs in part because of a reduction in debt levels. Without dilution, the mean debt face value is 36% lower and the mean debt market value is 11% lower. However, we find that the most important effect of dilution on default risk results from a shift in the set of government's borrowing opportunities (combinations of borrowing levels and spreads). The equilibrium combinations of debt and spread levels without dilution are not part of the government's choice set with dilution. For the equilibrium debt levels in the economy without dilution, equilibrium spread levels would be about 400 basis points higher in the economy with dilution. This occurs because, with dilution, even if the government would choose in the present period debt levels such that the expected recovery rate—i.e., the fraction of the loan lenders expect to recover—would be close to one *without future issuances*, lenders anticipate that the government will decrease the expected recovery rate with future debt issuances. Thus, lenders would ask for a high spread even for low debt levels (even for debt levels close to zero). In contrast, without dilution, the government could choose low debt levels paying a spread close to zero because lenders anticipate that even if the government were to increase the default probability with future issuances, they would not be affected by these issuances.

Understanding the behavior of interest rates in emerging economies is a central issue in academic and policy discussions. Neumeyer and Perri (2005) and Uribe and Yue (2006) argue that the level and volatility of interest rates in emerging economies may play a significant role in accounting for the distinctive features of business cycle dynamics in these economies.<sup>9</sup> Mendoza and Yue (2008) show how, shocks may be amplified through changes in interest rates. In addition, it has also been argued that high debt levels are of particular relevance in emerging economies because the high volatility of their borrowing cost makes them vulnerable to crises that are characterized by sharp contractions in aggregate activity (see, for example, the discussions in Uribe (2006a,b)). Our results indicate that debt dilution plays an important role in accounting for high and volatile interest rates and high debt levels in emerging economies.

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<sup>9</sup>Interest rates in emerging economies are higher and more volatile than in developed economies, interest rates are countercyclical in emerging economies and procyclical or acyclical in developed economies, and emerging economies feature higher output volatility, more countercyclical net exports, and higher consumption volatility than income volatility (see, for example, Aguiar and Gopinath (2007), Alvarez et al. (2009), Boz et al. (2008), Neumeyer and Perri (2005), and Uribe and Yue (2006)).



The rest of the article proceeds as follows. Section II. introduces the model. Section III. presents the results. Section IV. concludes and discusses possible extensions.

## II. The model

We first discuss the baseline model with debt dilution and later introduce a modification to this model that allows us to quantify the role of debt dilution.

### A. The baseline environment

We follow Aguiar and Gopinath (2006) and Arellano (2008), who extended the sovereign default model presented by Eaton and Gersovitz (1981) in order to study its quantitative performance. There is a single tradable good. The economy receives a stochastic endowment stream of this good  $y_t$ , where  $\log(y_t) = (1 - \rho)\mu + \rho \log(y_{t-1}) + \varepsilon_t$ , with  $|\rho| < 1$ , and  $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ .

The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$E \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right],$$

where  $\beta$  denotes the subjective discount factor and the utility function is assumed to display a constant coefficient of relative risk aversion, denoted by  $\gamma$ . That is,  $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ .

Each period, the government makes two decisions. First, it decides whether to default, which implies repudiating all current and future debt obligations contracted in the past. We follow most recent studies of sovereign default by assuming that the recovery rate of debt in default is zero. The default cost is represented by an endowment loss in the default period that, as in Chatterjee and Eyigungor (2009), takes the form of a quadratic loss function  $\phi(y) = d_0 y + d_1 y^2$ .<sup>10</sup> Second, the government decides the number of bonds that it purchases or issues in the current period. We allow for long-duration bonds. As Hatchondo and Martinez (2009), we assume that a bond issued in period  $t$  promises an infinite stream of coupons, which decreases at a constant rate  $\delta$ . In particular, a bond issued in period  $t$  promises to pay one unit of the good in period  $t + 1$  and  $(1 - \delta)^{s-1}$  units in period  $t + s$ , with  $s \geq 2$ .<sup>11</sup>

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<sup>10</sup>As in Hatchondo and Martinez (2009) and Hatchondo et al. (2009), we do not assume that the government is excluded from capital markets after a default episode. Hatchondo et al. (2007) solve a baseline model of sovereign default with and without the exclusion punishment and show that eliminating this punishment only affects significantly the debt level generated by the model.

<sup>11</sup>The Macaulay duration of a bond with this coupon structure is given by  $D = 1 + r^* \frac{1}{\delta + r^*}$ , where  $r^*$  denotes the constant per-period yield delivered by the bond.

It should be emphasized that  $\delta$  is a fixed parameter of the model, it is not allowed to change over time, and it is not chosen by the government. This allows us to study long-duration bonds without increasing the dimensionality of the state space. If one allows the government to choose a different value of  $\delta$  each period, one would have to keep track of how many bonds the government has issued for each possible value of  $\delta$ . For instance, Arellano and Ramanarayanan (2010) study a model in which the government can choose to issue bonds with two possible values of  $\delta$ , which requires to keep track of two state variables to determine the government's liabilities. The computation cost of including additional state variables may be significant (Hatchondo et al. (2010) show that the computation cost of obtaining accurate solutions in default models may be significant and Chatterjee and Eyigungor (2009) explain how the cost increases when long-duration bonds are assumed).

Following Arellano and Ramanarayanan (2010) and motivated by several studies that document that the risk premium is an important component of sovereign spreads and that a significant fraction of the spread volatility in the data is accounted for by volatility in the risk premium (see, for example, Borri and Verdelhan (2009), Broner et al. (2007), Longstaff et al. (2007), and González-Rozada and Levy Yeyati (2008)), we assume that sovereign bond prices satisfy a no arbitrage condition with stochastic discount factor  $M(y', y) = \exp(-r - \alpha\varepsilon' - 0.5\alpha^2\sigma_\varepsilon^2)$ , where  $r$  denotes the risk-free rate at which lenders can borrow or lend. Thus, the risk premium depends on the income shock in the borrowing economy. This formulation is a special case of the discrete-time version of the Vasicek one-factor model of the term structure (see Vasicek (1977) and Backus et al. (1998)) and allows us to introduce risk premium in a tractable way. A more plausible alternative is one in which the lenders' valuation of future payments is not perfectly correlated with the endowment in the borrower's economy and in which the risk-free interest rate  $r$  is also subject to shocks. The advantage of our formulation is that it avoids introducing another state variable to the model.

As in recent quantitative studies of default risk, we assume that the government cannot commit to future default and borrowing decisions. Thus, one may interpret this environment as a game in which the government making the default and borrowing decisions in period  $t$  is a player who takes as given the default and borrowing strategies of other players (governments) who will decide after  $t$ . We focus on Markov Perfect Equilibrium. That is, we assume that in each period, the government's equilibrium default and borrowing strategies depend only on payoff relevant state variables. As discussed by Krusell and Smith (2003), there may be a problem of multiplicity of Markov perfect equilibria in infinite-horizon economies. In order to avoid this problem, we solve for the equilibrium of the finite-horizon version of our economy, and we increase the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium objects as the infinite-horizon-economy equilibrium objects.

## B. Recursive formulation of the baseline framework

Let  $b$  denote the number of outstanding coupon claims at the beginning of the current period, and  $b'$  denote the number of outstanding coupon claims at the beginning of next period. A negative value of  $b$  implies that the government was a net issuer of bonds in the past. Let  $d$  denote the current-period default decision. We assume that  $d$  is equal to 1 if the government defaulted in the current period and is equal to 0 if it did not. Let  $V(b, y)$  denote the government's value function at the beginning of a period, that is, before the default decision is made. Let  $\tilde{V}(d, b, y)$  denote its value function after the default decision has been made. Let  $F(y' | y)$  denote the conditional cumulative distribution function of the next-period endowment  $y'$ . For any bond price function  $q(b', y)$ , the function  $V(b, y)$  satisfies the following functional equation:

$$V(b, y) = \max_{d \in \{0,1\}} \{d\tilde{V}(1, b, y) + (1-d)\tilde{V}(0, b, y)\}, \quad (1)$$

where

$$\tilde{V}(d, b, y) = \max_{b' \leq 0} \left\{ u(c) + \beta \int V(b', y') F(dy' | y) \right\}, \quad (2)$$

and

$$c = y - d\phi(y) + (1-d)b - q(b', y) [b' - (1-d)(1-\delta)b]. \quad (3)$$

The bond price is given by the following functional equation:

$$q(b', y) = \frac{1}{1+r} \int M(y', y) [1 - h(b'')] F(dy' | y) \quad (4)$$

$$+ \frac{1-\delta}{1+r} \int M(y', y) [1 - h(b'')] q(g(h(b', y'), b', y'), y') F(dy' | y), \quad (5)$$

where  $h(b, y)$  and  $g(d, b, y)$  denote the future default and borrowing rules that lenders expect the government to follow. The default rule  $h(b, y)$  is equal to one if the government defaults, and is equal to zero otherwise. The function  $g(d, b, y)$  determines the number of coupons that will mature next period. The first term in the right-hand side of equation (5) equals the expected value of the next-period coupon payment promised in a bond. The second term in the right-hand side of equation (5) equals the expected value of all other future coupon payments, which is summarized by the expected price at which the bond could be sold next period.

Equations V-q illustrate that the government finds its optimal current default and borrowing decisions taking as given its future default and borrowing decision rules  $h(b, y)$  and  $g(d, b, y)$ . In equilibrium, the optimal default and borrowing rules that solve problems (1) and (2) must be equal to  $h(b, y)$  and  $g(d, b, y)$  for all possible values of the state variables.

**Definition 1** *A Markov Perfect Equilibrium is characterized by*

1. a set of value functions  $\tilde{V}(d, b, y)$  and  $V(b, y)$ ,
2. a default rule  $h(b, y)$  and a borrowing rule  $g(d, b, y)$ ,
3. a bond price function  $q(b', y)$ ,

such that:

- (a) given  $h(b, y)$  and  $g(d, b, y)$ ,  $V(b, y)$  and  $\tilde{V}(d, b, y)$  satisfy functional equations (1) and (2) when the government can trade bonds at  $q(b', y)$ ;
- (b) given  $h(b, y)$  and  $g(d, b, y)$ , the bond price function  $q(b', y)$  is given by equation (5); and
- (c) the default rule  $h(b, y)$  and borrowing rule  $g(d, b, y)$  solve the dynamic programming problem defined by equations (1) and (2) when the government can trade bonds at  $q(b', y)$ .

### C. A model without debt dilution

In this section, we propose a modification to the model presented in Section A. that will allow us to study an economy without debt dilution and, in turn, measure the effects of debt dilution on the levels of borrowing and default risk. In the baseline model presented in Section A., the debt dilution problem arises as follows. An increase in the current borrowing level increases the probability of a default on previously issued debt and, thus, it decreases the market value of this debt—debt dilution occurs. Each period, the government borrows without internalizing the cost of diluting the value of debt issued in past periods. Lenders anticipate the effect of future borrowing on the probability of a default on the debt they buy and require to be compensated for future debt dilutions through a higher bond yield. Thus, the government could benefit from eliminating debt dilution in future periods because this would reduce the interest rate at which it can borrow in the current period.

We eliminate debt dilution by assuming that, each period, the government must pay to holders of debt issued in previous periods a compensation equal to the decline of the market value of their debt holdings implied by new issuances (but does not compensate bondholders for declines of the debt value that are the result of income shocks). Thus, when buying sovereign debt, investors anticipate that the future value of their investment is independent of future issuances and the dilution problem is eliminated.<sup>12</sup>

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<sup>12</sup>One could study alternative dilution rules or search for an optimal dilution rule. One could also assume that the government cannot commit to compensate lenders for their dilution losses but faces a penalty for breaking its compensation promises. Since the objective of the paper is to measure the effects of dilution, we simply focus on a rule that eliminates dilution, i.e., a rule that makes the value of debt holdings independent of future issuances.

## D. Recursive formulation of the framework without debt dilution

As before, let  $q(b', y)$  denote the price a sovereign bond. Let  $\tilde{b} \equiv (1 - d)(1 - \delta)b < 0$  denote the interim number of next-period coupon obligations. Suppose the government issues  $\tilde{b} - b' > 0$  bonds. The compensation the government must pay to holders of debt issued in previous periods is  $-\tilde{b}[q(\tilde{b}, y) - q(b', y)]$ . As in Section A., when the government wants to buy back its bonds, it does so at the secondary-market price  $q(b', y)$ .<sup>13</sup> Suppose the bond price is higher when the debt level is lower because the default probability is increasing with respect to the debt level (as is always the case in this paper and in previous quantitative studies of sovereign default). The equilibrium bond price is given by

$$\begin{aligned} q(b', y) = & \frac{1}{1+r} \int M(y', y) [1 - h(b', y')] F(dy' | y) \\ & + \frac{1-\delta}{1+r} \int M(y', y) [1 - h(b', y')] \max\{0, q(b'(1-\delta), y') - q(g(h(b', y'), b', y'), y')\} F(dy' | y) \\ & + \frac{1-\delta}{1+r} \int M(y', y) [1 - h(b', y')] q(g(h(b', y'), b', y'), y') F(dy' | y). \end{aligned}$$

The first term of the right-hand side of equation (8) represents the expected value of the next-period coupon payment. The second term represents the expected compensations bondholders would receive if the government issues new debt. This compensation implies that lenders price sovereign bonds anticipating that the value of their investment will not be diluted by future debt issuances. The third term represents the expected next-period value of a bond after the lender received the compensation for new issuances (if any). Note that the next-period value of a lender's investment may be affected by the income shock, a debt buyback, and a default, but will not be affected by new issuances.

The government's budget constraint reads

$$c = y - d\phi(y) + (1 - d)b + q(b', y)(\tilde{b} - b') + \tilde{b} \max\{0, q(\tilde{b}, y) - q(b', y)\}. \quad (9)$$

The last term of the right-hand side of equation (9) represents the government's compensation to existing bondholders for the issuance of new debt. Replacing equations (3) and (5) by equations (8) and (9) in the dynamic programming problem described in Section B. describes the problem without debt dilution.

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<sup>13</sup>Alternatively, we could have assumed that the government receives a compensation from lenders when it buys back debt in the same way it compensates lenders when it issues debt. That is, lenders could make transfers to the government when there is a debt buyback. This alternative assumption would be equivalent to assuming a voluntary renegotiation of sovereign debt. Even if such renegotiation could be ex-post mutually beneficial, the government may want to commit ex-ante to avoid it (for a similar discussion in the context of default negotiations see Benjamin and Wright (2008)). In order to measure the effects of debt dilution, we left the study of voluntary renegotiations for future research and only modify the baseline model to eliminate dilution.

## E. Discussion of the framework without debt dilution

There are different mechanisms that would implement the government's transfer to bondholders we use to eliminate dilution. For instance, one could assume that before issuing debt, the government must receive the consent of existing bondholders. In exchange of this consent, the government offer to pay a compensation to bondholders. Note that if the government offers to pay the compensation we propose, existing bondholders would be better off accepting the offer and allowing the government to issue new debt. Implementing this mechanism may not be as difficult as one may first think. The implementation could be facilitated by the intermediation of a bondholder representative like the ones Bolton and Jeanne (2009) or G-10 (2002) suggest for post-default sovereign debt restructuring processes. Majority clauses in sovereign bonds could also be used.

In addition, the government could give bondholders the right to redeem their debt claims at face value if the government issues debt without their consent. This would be enough to prevent such government's behavior. Note that another way of thinking about our model without dilution is to assume that before issuing debt the government must buy back all previous issuances at the market price that would be observed if no new debt had been issued in the current period. Suppose that this is the case and that the government wants to issue debt (i.e.,  $b' < \tilde{b}$  and, therefore,  $q(b', y) < q(\tilde{b}, y)$ ). Then, the government's budget constraint

$$c = y - d\phi(y) + (1 - d)b + \tilde{b}q(\tilde{b}, y) - b'q(b', y) \quad (10)$$

is equivalent to the one in equation (9).

It should also be noticed that the resources the government obtains from borrowing in our model without dilution are the same resources it would obtain when dealing with an *exclusive* lender if the government could make take-it-or-leave-it offers to this lender. Suppose all government debt is held by a lender who is the only one who can buy bonds from the government. If this lender chooses not to buy more debt from the government, the end-of-period value of its debt holdings would be  $-\tilde{b}q(\tilde{b}, y)$ . If he buys  $\tilde{b} - b'$  bonds from the government, the end-of-period value of his debt holdings would be  $-b'q(b', y)$ . Thus, the exclusive lender is willing to buy  $\tilde{b} - b'$  bonds from the government for  $\tilde{b}q(\tilde{b}, y) - b'q(b', y)$ , which is equal to the amount the government obtains when issuing bonds while compensating existing bondholders:  $(\tilde{b} - b')q(b', y) + \tilde{b}[q(\tilde{b}, y) - q(b', y)]$ . This illustrates how one can think about debt dilution as a nonexclusivity problem.

## III. Results

In this section we compare the predictions of the models with and without debt dilution and as in most previous quantitative studies on sovereign default, we use Argentina as a case study. Following Hatchondo et al. (2010), we solve the models numerically using

Borrower's risk aversion	$\sigma$	2
Interest rate	$r$	1%
Output autocorrelation coefficient	$\rho$	0.9
Standard deviation of innovations	$\sigma_\epsilon$	2.7%
Mean log output	$\mu$	$(-1/2)\sigma_\epsilon^2$
Duration	$\delta$	0.0341
Discount factor	$\beta$	0.969
Default cost	$d_0$	-0.69
Default cost	$d_1$	1.01
Risk premium	$\alpha$	4

Table 1: Parameter values.

value function iteration and interpolation.<sup>14</sup>

## A. Calibration

Table 1 presents the calibration. We assume that the representative agent in the sovereign economy has a coefficient of relative risk aversion of 2, which is within the range of accepted values in studies of real business cycles. A period in the model refers to a quarter. The risk-free interest rate is set equal to 1%. The parameter values that govern the endowment process are chosen so as to mimic the behavior of GDP in Argentina from the fourth quarter of 1993 to the third quarter of 2001, following Hatchondo et al. (2009). The parameterization of the output process is similar to the parameterization used in other studies that consider a longer sample period (see, for instance, Aguiar and Gopinath (2006)).

With  $\delta = 3.41\%$ , bonds have an average duration of 4.19 years in the simulations of the baseline model. Cruces et al. (2002) report that the average duration of Argentinean bonds included in the EMBI index was 4.13 years in 2000. This duration is not significantly different from what is observed in other emerging economies. Using a sample of 27 emerging economies, Cruces et al. (2002) find an average duration of 4.77 years with a standard deviation of 1.52.

We calibrate the discount factor, the output cost (two parameter values), and the risk premium parameter to target four moments: a mean spread of 7.4, a standard deviation of the spread of 2.5, a mean debt level of 28% of the mean quarterly output in the pre-default samples of our simulations (the exact definition of these samples is presented in Section 3.2), and a default frequency of three defaults per one hundred years. The targets for the spread distribution are taken from the spread behavior in Argentina before

<sup>14</sup>We use linear interpolation for endowment levels and spline interpolation for asset positions. The algorithm finds two value functions,  $\tilde{V}(1, b, y)$  and  $\tilde{V}(0, b, y)$ . Convergence in the equilibrium price function  $q(b', y)$  is also assured.

its 2001 default (see Table 2). Even though it is not clear which data values for the mean debt level and the default frequency one should target, we choose to target these statistics because they have received attention in the literature, they are clearly influenced by the parameter values we are calibrating, and they will influence the welfare gains from eliminating dilution. For the period we studied, Chatterjee and Eyigungor (2009) target a mean level of unsecured sovereign debt of 70% of quarterly output. Since our model is a model of external debt and Sturzenegger and Zettelmeyer (2006) estimates that 60% of the debt Argentina defaulted on was held by residents, we choose to target a mean debt level that is 40% of the value targeted by Chatterjee and Eyigungor (2009). We target a frequency of three defaults per 100 years because that is the value used as reference in previous quantitative studies (see, for example, Arellano (2008) or Aguiar and Gopinath (2006)). The discount factor value we obtain is relatively low but higher than the ones assumed in previous studies (for instance, Aguiar and Gopinath (2006) assume  $\beta = 0.8$ ). Low discount factors may be a result of political polarization in emerging economies (see Amador (2003) and Cuadra and Sapriza (2008)).

## B. Simulation results

This section discusses quantitative effects of debt dilution. In order to do so, it presents simulation results from the models with and without debt dilution. To facilitate the comparison of our results with the ones in previous studies, we report results for pre-default simulation samples, as these studies do. We simulate the model for a number of periods that allows us to extract 500 samples of 32 consecutive periods before a default. Except for the computation of default frequencies, which are computed using all the simulation data, we focus on samples of 32 periods because we compare the artificial data generated by the model with Argentine data from the fourth quarter of 1993 to the third quarter of 2001.<sup>15</sup> In order to facilitate the comparison of simulation results with the data, we only consider simulation sample paths in which the last default was declared at least two periods before the beginning of each sample.

Table 2 reports moments in the data and in our simulations.<sup>16</sup> The moments reported in the table are chosen so as to illustrate the ability of the model to replicate distinctive business cycle properties of emerging economies. These economies feature a high, volatile and countercyclical interest rate, and high consumption volatility. The interest rate spread ( $R_s$ ) is expressed in annual terms.<sup>17</sup> The logarithm of income and consumption are

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<sup>15</sup>The qualitative features of this data are also observed in other sample periods and in other emerging markets (see, for example, Aguiar and Gopinath (2007), Alvarez et al. (2009), Boz et al. (2008), Neumeyer and Perri (2005), and Uribe and Yue (2006)). The only exception is that in the data we consider, the volatility of consumption is slightly lower than the volatility of income, while emerging market economies tend to display a higher volatility of consumption relative to income.

<sup>16</sup>The data for output and consumption were obtained from the Argentinean Finance Ministry. The spread before the first quarter of 1998 is taken from Neumeyer and Perri (2005), and from the EMBI Global after that. For the debt level and the default frequency, we report the targeted values discussed in Section 3.1.

<sup>17</sup>Let  $r^* = \frac{1}{q(b', y)} - \delta$  denote the per-period constant yield implied by a bond price  $q(b', y)$ . The annualized spread is given by  $R_s = \left(\frac{1+r^*}{1+r}\right)^4 - 1$ .



denoted by  $y$  and  $c$ , respectively. The standard deviation of  $x$  is denoted by  $\sigma(x)$  and is reported in percentage terms. The coefficient of correlation between  $x$  and  $z$  is denoted by  $\rho(x, z)$ . Moments are computed using detrended series. Trends are computed using the Hodrick-Prescott filter with a smoothing parameter of 1,600. Table 2 also reports the mean debt market value (computed as the mean  $b$  divided by  $\delta + r^*$ , where  $r^*$  is the mean equilibrium interest rate) and the mean debt face value (computed as the mean  $b$  divided by  $\delta + r$ ).

	Data	With debt dilution	Without debt dilution
Defaults per 100 years	3.00	3.10	0.42
Mean debt market value		0.20	0.18
Mean debt face value	0.28	0.28	0.18
$E(R_s)$	7.44	7.38	0.57
$\sigma(R_s)$	2.51	2.45	0.72
$\sigma(y)$	3.17	3.03	3.36
$\sigma(c)/\sigma(y)$	0.94	1.04	1.21
$\rho(c, y)$	0.97	1.00	0.99
$\rho(R_s, y)$	-0.65	-0.80	-0.63

Table 2: Business cycle statistics. The second column is computed using data from Argentina from 1993 to 2001. Other columns report the mean of the value of each moment in 500 simulation samples.

Table 2 shows that the baseline model with dilution matches the data reasonably well. As in the data, in the simulations of the baseline model, consumption and income are highly correlated, and the consumption volatility is higher than the income volatility. The model also matches well the moments we choose to target in order to impose discipline to our measurement exercise (the default frequency, the mean debt level, and the mean and standard deviation of the spread). With this in mind, we concentrate on the main question this paper intends to answer: What are the quantitative effects of the debt dilution problem?

Table 2 shows that the number of default per 100 years decreases from 3.10 in the baseline to 0.42 in the model without debt dilution. That is, we find that debt dilution accounts for 86% of the default risk in the simulations of the baseline model. Eliminating dilution decreases the mean spread in the simulations from 7.38% to 0.57% (recall that reducing default risk allows the sovereign to pay a lower risk premium). That is, debt dilution accounts for 92% of the spread paid by the sovereign. The standard deviation of the spread decreases from 2.45 with debt dilution to 0.72 without debt dilution. The mean face value of outstanding bonds decreases by 36%. But most of this decline is explained by the lower interest rate in the simulations of the model without debt dilution: The mean market value of outstanding bonds decreases only by 11%.

In order to shed light on how the debt dilution problem influences equilibrium allocations, Figure 1 presents the implied spread demanded by lenders as a function of the face value of next-period debt—defined as the present value of future payment obligations discounted

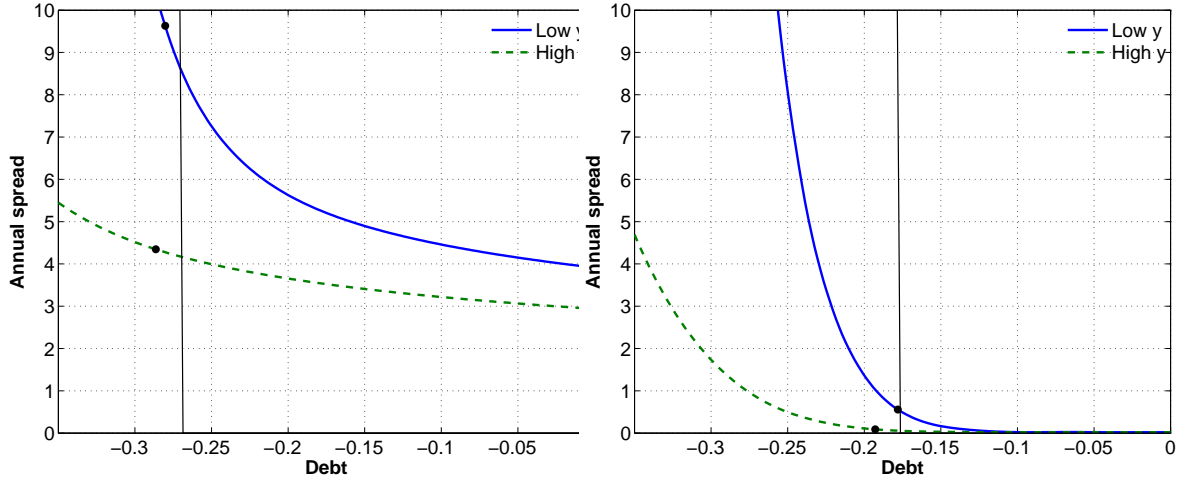


Figure 1: Menu of combinations of spreads and next-period debt levels ( $\frac{b'}{\delta+r}$ ) from which the government can choose. The left panel corresponds to the baseline case. The right panel corresponds to the case without debt dilution. In each of these two cases, solid dots illustrate the optimal decision of a government that inherits a debt level equal to the average debt observed in our simulations for that case. Vertical lines mark the government’s debt level before its issuance decision. The low (high) value of  $y$  corresponds to an endowment realization that is one standard deviation below (above) the unconditional mean.

at the risk-free rate,  $\frac{b'}{\delta+r}$ . This function defines the set of combinations of spreads and next-period debt levels that the government can choose from. The figure also presents the combination of spread levels and next-period debt chosen by the government when its initial debt level is the average level in the simulations of each case.

Figure 1 shows that eliminating dilution implies a significant shift in the government’s set of borrowing opportunities (combinations of debt levels and spreads). For the baseline model with dilution, the left panel of Figure 1 illustrates how the government cannot borrow paying spreads close to zero. In contrast, the right panel of Figure 1 illustrates how eliminating debt dilution gives the government the opportunity to borrow paying spreads close to zero. In particular, the equilibrium combinations of debt and spread levels without dilution are not part of the governments choice set with dilution. For the equilibrium debt levels without dilution, equilibrium spread levels would be about 400 basis points higher in the economy with dilution. Thus, Figure 1 shows that the shift in the government’s choice set plays an important role in accounting for the reduction in spreads (and default risk) implied by the elimination of debt dilution.

Why does debt dilution shift the set of government’s opportunities? Suppose the government wants to issue an amount of debt for which, without future issuances, the expected recovery rate—i.e., the fraction of the loan lenders expect to recover—is close to one (i.e., without future issuances, the default probability and the spread are close to zero). Consider first the model without debt dilution. The government could issue such amount of debt paying a spread close to zero because lenders anticipate that even if the government were to increase the default probability with future issuances, they would not

be affected by these issuances. As illustrated in Figure 1, in equilibrium the government does not choose to increase the default probability significantly with its issuances. Recall that doing so would be expensive for the government because it would have to compensate holders of debt issued in previous periods.<sup>18</sup> In contrast, in the baseline model with dilution, even for issuances levels such that the expected recovery rate would be close to one without future issuances, lenders would ask for a significant spread. As illustrated in Figure 1, this is true even for debt levels arbitrary close to zero. This is the case because lenders anticipate that the government will decrease the expected recovery rate with its future issuances.

Table 2 also shows that consumption volatility is higher in the economy without dilution. This is the case because, for low income levels, consumption is more sensitive to changes in income in that economy. As illustrated in Figure 1, when income is low, issuance levels tend to be lower in the economy without dilution than in the benchmark with dilution (in the figure, issuance levels are represented by the horizontal distance between the dark dots and the vertical solid line).<sup>19</sup> Thus, the government is more effective in mitigating the effects of low income realizations on consumption in the benchmark with dilution.

### C. Welfare costs of debt-dilution

Figure 2 shows the ex-ante welfare gain that domestic agents would experience after moving from an economy with dilution to one without dilution. The solid line represents the welfare gain when the government enters the period with zero debt. The dashed line represents the welfare gain when the government enters the period with the the mean debt level observed in the simulations of the economy with dilution (note that for low endowment levels the government defaults or is likely to default in the future). Figure 2 shows that domestic consumers are better off in the economy without dilution. Welfare gains from eliminating dilution tend to be lower when the debt level is higher in part because of the adjustment cost of transiting to an ergodic distribution with lower debt levels.<sup>20</sup>

It should be mentioned that one may want to take our measure of the welfare gain with a grain of salt. In particular, one could argue that our measure is too low. First, our measure of the gain from reducing the risk premium is likely to be too low. This gain is increasing in the level of debt for which the risk premium is paid. We chose to calibrate our model to a low debt level (28% of quarterly output) to resemble the level of sovereign defaultable debt held by foreigners. However, eliminating debt dilution is also likely to

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<sup>18</sup>Note that the default probability (and the spread) increases when income is low even without debt issuances but the government does not compensate lenders for this increase. In the simulations, typically the government does not issue debt when income is very low.

<sup>19</sup>Note that, for the same debt level, the spread curves in Figure 1 are steeper when income is lower. This implies that in the economy without dilution, for the same issuance level, the compensation the government would have to pay to existing bondholders would be larger when income is lower.

<sup>20</sup>In a default period, the government writes off all debt liabilities but has a stronger incentive to borrow than in case in which it enters the period with zero debt—because it wants to smooth out the default cost. This may explain why the better borrowing terms faced in the economy without dilution may be more valuable in default periods than when the government enters the period without debt.

reduce the risk premium paid for other government debt, and even private debt (sovereign spreads are likely to influence private spreads, see, for example, Mendoza and Yue (2008)). Furthermore, several studies find evidence of a significant effect of interest rates on productivity (through the allocation of factors of production), and of a significant role of interest rate fluctuations in the amplification of shocks (see, for example, Mendoza and Yue (2008), Neumeyer and Perri (2005), and Uribe and Yue (2006)). Since there is no production in our setup, we cannot capture productivity gains from reducing the level and volatility of interest rates.

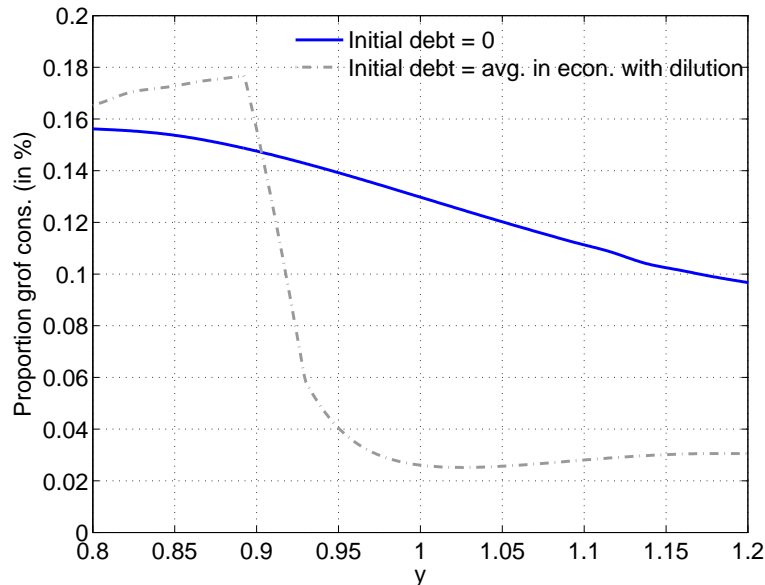


Figure 2: Consumption compensation (in percentage terms) that makes domestic agents indifferent between living in an economy with or without dilution. The figure was constructed assuming that the initial debt level is equal to zero or to the mean debt level in the simulations of the economy with dilution. A positive number means that agents prefer the economy without dilution.

#### D. One-period bonds vs. long-duration bonds without debt dilution

An alternative model without intertemporal debt dilution is the commonly used one-period-bond model. With one-period bonds, when the government decides its borrowing level, there are no previous issuances that can be diluted (either because the government honored its debt obligations at the beginning of the period or because it defaulted on them). In this subsection, we compare the predictions of our model without dilution with the ones of the one-period-bond version of our baseline model. It should be emphasized that we do not intend to use the model to discuss optimal maturity. The model is missing key ingredients for that discussion (e.g., issuance costs and interest rate risk).

Table 3 presents simulation results obtained with the baseline model assuming one-period bonds ( $\delta = 1$ ). In order to facilitate comparisons, we report again the statistics obtained with our model with long-duration bonds and without debt dilution ( $\delta = 0.0341$ ). The table shows that simulation results obtained with one-period bonds differ from those obtained with our model without debt dilution. One-period bonds and long-duration bonds without dilution are different assets.

	$\delta = 0.0341$	$\delta = 1$
Defaults per 100 years	0.42	0.23
Mean debt (market value)	0.18	0.19
Mean debt (face value)	0.18	0.19
$E(R_s)$	0.57	0.45
$\sigma(R_s)$	0.72	0.45
$\sigma(y)$	3.36	3.11
$\sigma(c)/\sigma(y)$	1.21	1.23
$\rho(c, y)$	0.99	0.97
$\rho(R_s, y)$	-0.63	-0.73

Table 3: Business cycle statistics without debt dilution.

In order to shed more light on the different allocations observed in the two economies without dilution, Figure 3 presents the no-arbitrage spread curves and the government optimal choices for these two economies. The figure shows that the response of the equilibrium spread to a negative income shock is different in the two economies. While for the “very low” income level (2 standard deviations below the mean) the equilibrium spread is higher in the economy with long bonds, for the “low” income level (one standard deviation below the mean) the equilibrium spread is higher in the economy with one-period bonds.

For the “low” income level, the government chooses to pay a lower spread with long bonds because the government’s financial needs are weaker with these bonds. As discussed in Section 2.2.2, one may think about the economy with long bonds and without dilution as an economy in which, before issuing debt, the government must buy back all previous issuances at the market price that would be observed without new issuances. In that economy, the government’s financial needs are lower when its income is lower because the market price of debt is lower when income is lower. In contrast, with one-period bonds, government’s obligations do not depend on the income level.

In the economy with long bonds, for the “very low” income level, it does not pay off to issue new debt because the revenue raised by new debt issuances would not be enough to compensate existing bondholders.<sup>21</sup> Thus, the high spread observed when income is “very low” corresponds to previous-period issuances and it is not paid by the government for

<sup>21</sup>For sufficiently low income levels, initial debt levels in the simulations tend to be on the decreasing part of the sovereign-debt-market-value function  $-q(b', y)b'$ . Thus, if the government chooses  $b' < b(1 - \delta)$ , its issuance revenue would be  $-q(b', y)[b' - b(1 - \delta)]$ , which is lower than the amount it would have to pay existing bondholders,  $-[q(b(1 - \delta), y) - q(b', y)]b(1 - \delta)$ .

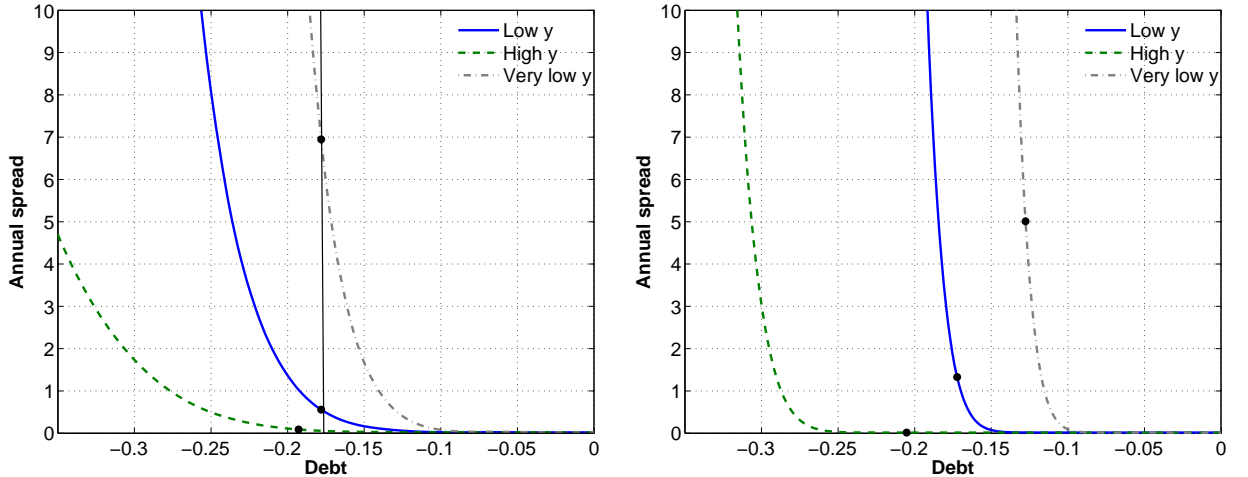


Figure 3: Menu of combinations of spreads and next-period debt levels ( $\frac{b'}{\delta+r}$ ) from which the government can choose. The left panel corresponds to the case of long-duration bonds without debt dilution. The right panel corresponds to the case of one-period bonds. In each of these two cases, solid dots illustrate the optimal decision of a government that inherits a debt level equal to the average debt observed in our simulations for that case. Vertical lines mark the government’s debt level before its issuance decision. The low (high) value of  $y$  corresponds to an endowment realization that is one standard deviation below (above) the unconditional mean. The very low value of  $y$  corresponds to an endowment realization that is two standard deviations below the unconditional mean.

new issuances. In the economy with one-period bonds, when income is “very low” the government still issues debt because it has to pay all the debt it issued in the previous period (with long bonds it only has to pay  $\delta = 3.41\%$  of its debt). But the government chooses to pay a spread lower than the high spread implied by previous-period issuances in the long-bond economy (recall that the government also chooses not to pay this very high spread in the latter economy).

#### IV. Conclusions

We proposed an extension of a baseline sovereign default framework à la Eaton and Gersovitz (1981) that allowed us to study the case in which the sovereign eliminates debt dilution. We found that debt dilution accounts for 86% of the default risk, 92% of the mean spread, and 71% of the spread volatility in the simulations of the baseline model with dilution. That is, even without commitment to future repayment policies and without contingency of sovereign debt, if the sovereign could eliminate debt dilution, it would reduce significantly the level and volatility of default risk. We showed that the reduction in default risk implied by the elimination of dilution occurs in part because of a sizable reduction in debt levels (36% of the mean face value and 11% of the mean market value), but that the most important effect on default risk results from an shift in the government’s set of borrowing opportunities.

Analyzing debt dilution in a production economy would allow for a better quantification of the welfare gains from eliminating dilution. Several studies find evidence of a significant effect of interest rates on productivity (through the allocation of factors of production), and of a significant role of interest rate fluctuations in the amplification of shocks (see, for example, Mendoza and Yue (2008), Neumeyer and Perri (2005), and Uribe and Yue (2006)). In the light of these findings, our results indicate that the welfare cost of debt dilution may be large. The developing of a sovereign default framework that accommodates effects of interest rates on factors allocation is the subject of ongoing research (see, for example, Mendoza and Yue (2008) and Sosa Padilla (2010)). An interesting extension of our analysis would be to study the implications of the debt dilution problem in such a framework.

There are also many other directions in which our analysis can be extended. For instance, one could allow the sovereign to change the debt duration, one could allow the risk premium to be affected by variables other than the domestic income, and one could introduce other shocks to the economy. However, these extensions would require including additional state variables and, the computation cost of doing so is significant.

One could also investigate how to exploit the gains from eliminating dilution. For instance one could explore the benefits to committing to a future borrowing behavior through fiscal rules, which several countries are introducing (see IMF (2009) and the references therein). Our results indicate that eliminating debt dilution should be an important motivation for the implementation of fiscal rules that could allow to reduce significantly the risk of debt crises and the mean and volatility of interest rates.

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