Decentralized Adaptive Control of Nonlinear Systems Using Radial Basis Neural Networks

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***Abstract*— Stable direct and indirect decentralized adaptive radial basis neural network controllers are presented for a class of interconnected nonlinear systems. The feedback and adaptation mechanisms for each subsystem depend only upon local measurements to provide asymptotic tracking of a reference trajectory. Due to the functional approximation capabilities of radial basis neural networks, the dynamics for each subsystem are not required to be linear in a set of unknown coefficients as is typically required in decentralized adaptive schemes. In addition, each subsystem is able to adaptively compensate for disturbances and interconnections with unknown bounds.**

I. Introduction

Decentralized control systems often arise from either the physical inability for subsystem information exchange or the lack of computing capabilities required for a single central controller. Furthermore, the difficulty of, and uncertainty in, measuring parameter values within a large-scale system may call for adaptive techniques. Since these restrictions encompass a large group of applications, a variety of decentralized adaptive techniques have been developed. Model reference adaptive control (MRAC) based designs for decentralized systems have been studied in [1]–[3] for the continuous time case and in [4] for the discrete time case. These approaches, however, are limited to decentralized systems with linear subsystems and possibly nonlinear interconnections. Decentralized adaptive controllers for robotic manipulators were presented in [5] and [6], while a scheme for nonlinear subsystems with a special class of interconnections was presented in [7]. It was shown in [8] that it is possible to provide stable tracking in decentralized systems which contain uncertainties which are bounded by polynomials with known order. These previous results consider subsystems which are linear in a set of unknown parameters, or consider the uncertainties to be contained within the dynamics describing the subsystem interconnections which are bounded.

On-line function approximation approaches have been successfully applied to a wide variety of control problems, in particular in the area of nonlinear adaptive control of SISO systems (see [9]–[13] for examples). On-line function approximation approaches adjust parameters within a universal approximator (such as a fuzzy system or neural network) to estimate unknown nonlinearities which may describe plant dynamics or a desired control law. Universal approximators possess the property that, given an appropriate approximator structure, it is possible to represent continuous nonlinearities over a compact space with arbitrary accuracy as described in [14] and [15]. This means that entire classes of nonlinearities may be represented with a single approximator structure using different parameter choices.

In this correspondence, we exploit the function approximation capabilities of radial basis functions to provide asymptotic tracking given a class of nonlinear subsystems with unknown interconnection strengths. Using radial basis neural networks to approximate unknown functions on-line allows us to extend the results in [16] to include the case of both parametric and dynamic uncertainty. A direct adaptive approach approximates unknown control laws required to stabilize each subsystem, while an indirect approach is provided which identifies the isolated subsystem dynamics to produce a stabilizing controller. Both approaches ensure asymptotic tracking using only local feedback signals.

This correspondence is organized as follows. In Section II, an overview of radial basis neural networks is given. In Section III, the details of the problem statement for the decentralized system are presented. The adaptive algorithms for each subsystem using only local information are presented and composite system stability is established in Sections IV and V for the direct and indirect cases, respectively. Simulation examples are given in Section VI, while concluding remarks are provided in Section VII.

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II. Radial Basis Neural Networks

A radial basis neural network (RBNN) is made up of a collection of parallel processing units called nodes. The output of the ith node is defined by a Gaussian function $z_i(x) = \exp(-|x - c_i|^2/\sigma_i^2)$, where $x \in \mathbb{R}^n$ is the input to the network, c_i is the center of the ith node, and σ_i is its size of influence. The output of a radial basis network, $y = \mathcal{F}(x, A)$, may simply be calculated by either a weighted sum so that

$$\mathcal{F}(x, A) = \sum_{i=1}^{p} a_i z_i(x) \tag{1}$$

or by a weighted average

$$\mathcal{F}(x, A) = \frac{\sum_{i=1}^{p} a_i z_i(x)}{\sum_{i=1}^{m} z_i(x)} \tag{2}$$

where $A = [a_1, \cdots, a_p]^\top$ is a vector of network weights. We notice that (1) and (2) may be rewritten as $\mathcal{F}(x, A) = A^\top \zeta(X)$, where $\zeta(X)$ is a set of radial basis functions defined by $\zeta^\top(x) = [z_1(x), \cdots, z_p(x)]$ for the weighted sum, (1), and $\zeta^\top(x) = [z_1(x), \cdots, z_p(x)]/\sum_{i=1}^{p} z_i(x)$, for the weighted average (2).

Given a single RBNN, it is possible to approximate a wide variety of functions simply by making different choices for A. In particular, if there are a sufficient number of nodes within the network, then there exists some A^* such that

$$\sup_{x \in S_x} |\mathcal{F}(x, A^*) - f(x)| < W$$

where S_x is a compact set, and $W > 0$ is a finite constant provided $f(x)$ is continuous [17]. This lets us express $f(x) = \mathcal{F}(x, A^*) + w(x)$ with $|w(x)| < W$ when $x \in S_x$. Notice that even though RBNN's are linear in a set of adjustable parameters, we may, e.g., approximate a function $f(x) = a + \cos(bx^\top x)$ which is not linear in an independent set of parameters $[a, b]^\top$. Thus we are using an approximator which is linear in the parameters to describe functions which are not necessarily linear in another set of parameters.

Even though we will be defining the control laws in terms of radial basis networks, it should be noted that any universal approximator which is linear in the adjustable parameters may be considered. Other examples are standard fuzzy systems with adjustable output membership centers [9], Takagi–Sugeno fuzzy systems [13], CMAC networks, among others.

III. A Class of Decentralized Systems

Here we consider each subsystem S_i to be SISO such that

$$\begin{aligned}\dot{x}_i &= f_i(x_1, \cdots, x_m) + g_i(x_i)u_i \\ y_i &= h_i(x_1, \cdots, x_m)\end{aligned} \tag{3}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state vector, $u_i \in \mathbb{R}$ is the input, and $y_i \in \mathbb{R}$ is the output of S_i. We assume that the functions $f_i(\cdot), g_i(\cdot) \in \mathbb{R}^{n_i}$ and $h_i(\cdot) \in \mathbb{R}$, $i = 1, \cdots, m$ are smooth. If each subsystem has strong relative degree d_i, then the output dynamics may be rewritten as

$$y_i^{(d_i)} = \alpha_i(x_i) + \beta_i(x_i)u_i + \Delta_i(t, x_1, \cdots, x_m) \tag{4}$$

where $y_i^{(d_i)}$ is the d_ith time derivative of y_i [16]. Here we are assuming that the influence of the other subsystems S_j, $j \neq i$, is represented by the $\Delta_i(t, x_1, \ldots, x_m)$ term in (4). We also assume that for some $\beta_{0_i} > 0$, we have $\beta_i(x_i) \geq \beta_{0_i}$ so that the control gain is bounded away from zero (for convenience we assume that $\beta_i(x_i) > 0$).

Assumption 1—Plant: The plant can be defined by (3) and transformed to (4) with input gain bounded by $0 < \beta_{0_i} \leq \beta_i(x_i) \leq \beta_{1_i}$. The zero dynamics for each subsystem are exponentially attractive [18]. The ith subsystem input gain rate of change is bounded by $|\dot{\beta}_i(x_i)| \leq B_i$ where $B_i \in \mathbb{R}$ is a finite constant defined later in Theorem 1.

The tracking error for S_i is defined by $e_i = r_i - y_i$. Our objective is to design an adaptive control system for each subsystem which will cause the output y_i of a relative degree d_i subsystem S_i to track a desired output trajectory r_i (i.e., $e_i \to 0$) in the presence of interconnections and unknown disturbances, using only local measurements. The desired output trajectory may be defined by a signal external to the control system so that the first d_i derivatives of the ith subsystem's reference signal r_i may be measured, or by a reference model, with relative degree greater than or equal to d_i which characterizes the desired performance. This requirement leads to the following assumption.

Assumption 2—Reference Model: The desired output trajectory and its derivatives $r_i, \cdots, r_i^{(d_i)}$ for the ith subsystem S_i are measurable and bounded.

Let the scalars $\gamma_{i,j}$ quantify the "strength" of the interconnections and the output error vector for the ith subsystem be defined by $\boldsymbol{e}_i = [e_i, \cdots, e_i^{(d_i-1)}]^\top$. It is assumed that the interconnections satisfy $\Delta_i(x_1, \cdots, x_m) = \theta_i(t) + \delta_i(x_1, \cdots, x_m)$, where $|\delta_i(x_1, \cdots, x_m)| \leq \sum_{j=1}^m \gamma_{i,j}|\boldsymbol{e}_j|_2$ and $\theta_i(t) \in \mathcal{L}_\infty$. Let $\tilde{\Theta}_i^* = (\sup(\theta_i(t)) - \inf(\theta_i(t)))/2$ be a measure of the variation of $\theta_i(t)$ and $\overline{\theta}_i = (\sup[\theta_i(t)] + \inf(\theta_i(t)))/2$ be a measure of the center position of $\theta_i(t)$. We may thus let $\theta_i(t) = \overline{\theta}_i + \tilde{\theta}_i(t)$ for some $\tilde{\theta}_i(t)$, where $|\tilde{\theta}_i| \leq \tilde{\Theta}_i^*$, with $\tilde{\Theta}_i^*$ assumed to be bounded. Also, if $\overline{\theta}_i$ is nonzero, we may absorb $\overline{\theta}_i$ into $\alpha_i(x_i)$ within (4) as an unknown bias since it is not dependent upon the other subsystem states. The assumptions for the interconnections are summarized as follows.

Assumption 3—Interconnections: The interconnections satisfy

$$\Delta_i(x_1, \cdots, x_m) = \theta_i(t) + \delta_i(x_1, \cdots, x_m)$$

where

$$|\delta_i(x_1, \cdots, x_m)| \leq \sum_{j=1}^m \gamma_{i,j}|\boldsymbol{e}_j|_2 \quad \text{and} \quad \theta_i(t) \in \mathcal{L}_\infty.$$

This assumption on the interconnections can be satisfied by a variety of decentralized nonlinear systems. For instance, in [16] it is shown to be satisfied for an intervehicle spacing regulation problem in a platoon of an automated highway system. In this correspondence, we show that it is satisfied for the control of two inverted pendulums connected by a spring. It should be noted that if the interconnections satisfy $|\delta_i(\cdot)| \leq \sum_{j=1}^m \gamma_{i,j}|\boldsymbol{y}_j|_2$ (which is the case for many mechanical systems), then $|\delta_i(\cdot)| \leq \sum_{j=1}^m \gamma_{i,j}(|\boldsymbol{e}_j|_2 + |\boldsymbol{r}_j|_2)$ where $\boldsymbol{y}_i = [y_i, \cdots, y_i^{(d_i)}]^\top$ and $\boldsymbol{r}_i = [r_i, \cdots, r_i^{(d_i)}]^\top$ which satisfies Assumption 3 provided each $\boldsymbol{r}_i$ is bounded.

IV. Direct Adaptive Control

Because state information about the ith subsystem is only available for the ith controller, a standard feedback linearizing control law may not be defined for the composite system, even if the plant dynamics are known. Ideally we may, however, define a controller which compensates for the dynamics of each isolated subsystem. For the ith isolated subsystem, a feedback linearizing controller is defined by

$$u_i^*(x_i, \nu_i) = \frac{-\alpha_i(x_i) + \nu_i(t)}{\beta_i(x_i)} = u_{u_i}(x_i, \nu_i) + u_{k_i} \tag{5}$$

where the signal ν_i will be defined below, $u_{u_i}(x_i, \nu_i)$ is the unknown portion of the control law that is smooth in its arguments, and $u_{k_i}(x_i)$ is a known part of the control which is assumed to be well defined *a priori*. The term u_{k_i} is included to allow *a priori* control knowledge into the decentralized controller design. The ideal decentralized control function (5) may be represented by an RBNN (or other approximation structure), $\mathcal{F}_{u_i}$, such that

$$u_i^* = \mathcal{F}_{u_i}\big(x_i, \nu_i, A_{u_i}^*\big) + u_{k_i}(x_i) + w_{u_i}(x_i, \nu_i) \tag{6}$$

where the vector of ideal control parameters is defined as

$$A_{u_i}^* = \arg\min_{A_{u_i} \in \Omega_{u_i}} \left[\sup_{x_i \in S_{x_i}, \nu_i \in S_{\nu_i}} |\mathcal{F}_{u_i}(x_i, \nu, A_{u_i}) - u_{u_i}(x_i, \nu_i)|\right] \tag{7}$$

so that $w_{u_i}(x_i, \nu_i)$ is the representation error which arises when $u_{u_i}(x_i, \nu_i)$ is represented by an RBNN of finite size. From the universal approximation property, we know that for a given approximator structure, there exists $A_{u_i}^*$ such that $|w_{u_i}| \leq W_{u_i}$ for some finite $W_{u_i} > 0$. The subspaces S_{x_i} and S_{ν_i} are defined as the compact sets through which the state trajectories for the ith subsystem and ν_i may travel. The subspace Ω_{u_i} is the convex compact set which contains feasible parameter sets for $A_{u_i}^*$. The stability proof to follow will establish bounds for S_{x_i} and S_{ν_i}. The following assumption summarizes the controller requirements.

Assumption 4—Control: If $x_i \in \mathcal{L}_\infty^{n_i}$, then $u_{k_i} \in \mathcal{L}_\infty$. Also, assume that the representation error $w_{u_i}(x_i, \nu_i)$ is bounded by some $W_{u_i} > 0$, i.e., $|w_{u_i}(x_i, \nu_i)| \leq W_{u_i}$.

An adaptive algorithm will be defined to estimate $A_{u_i}^*$ with A_{u_i}. These estimates are then used to define the control law as

$$u_i = \mathcal{F}_{u_i}(x_i, \nu_i, A_{u_i}) + u_{k_i}(x_i) \tag{8}$$

where $\mathcal{F}_{u_i}(x_i, \nu_i, A_{u_i})$ is the RBNN used to approximate an ideal controller for the ith subsystem. A parameter error vector is defined as $\phi_{u_i} = A_{u_i} - A_{u_i}^*$ for each subsystem.

It is desired that the output error of the ith subsystem follow $e_i^{(d_i)} + k_{i,d_i-1}e_i^{(d_i-1)} + \cdots + k_{i,0}e_i = 0$, where the coefficients are picked so that each $\hat{L}_i(s) = s^{d_i} + k_{i,d_i-1}s^{d_i-1} + \cdots + k_{i,0}$ has its roots in the open left-half plane (is Hurwitz). The error dynamics for the ith subsystem may be expressed as

$$e_i^{(d_i)} = r_i^{(d_i)} - \alpha_i(x_i) - \beta_i(x_i)u_i - \Delta_i(x_1, \cdots, x_m).$$

Adding and subtracting $\beta_i(x_i)u_i^*$ and using the definition of u_i^* in (5), we obtain

$$e_i^{(d_i)} = r_i^{(d_i)} - \beta_i(x_i)[u_i - u_i^*] - \nu_i(t) - \Delta_i(x_1, \cdots, x_m). \tag{9}$$

Let $a_{\theta_i}(t)$ and $a_{\kappa_i}(t)$ be scalar time functions and

$$\nu_i = r_i^{(d_i)} + k_{i,\,d_i-1}e_i^{(d_i-1)} + \cdots + k_{i,\,0}e_i + a_{\theta_i}(t) \cdot \operatorname{sgn}\left(\boldsymbol{e}_i^\top P_i b_i\right) + a_{\kappa_i}(t)\boldsymbol{e}_i^\top P_i b_i/2.$$

where $P_i \in \mathbb{R}^{d_i \times d_i}$ is a positive definite matrix defined by a Lyapunov matrix equation and $b_i \in \mathbb{R}^{d_i}$ is a vector. These will both be defined shortly. Using the definition of ν_i, the error dynamics may be expressed as

$$\dot{\boldsymbol{e}}_i = \Lambda_i \boldsymbol{e}_i + b_i\Big(-\beta_i(x_i)[u_i - u_i^*] - a_{\theta_i}\operatorname{sgn}\left(\boldsymbol{e}_i^\top P_i b_i\right) - a_{\kappa_i}\boldsymbol{e}_i^\top P_i b_i/2 - \Delta_i(x_1, \cdots, x_m)\Big) \tag{10}$$

where

$$\Lambda_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_{i,\,0} & -k_{i,\,1} & -k_{i,\,2} & \cdots & -k_{i,\,d_i-1} \end{bmatrix}$$

and $b_i = [0, 0, \cdots, 0, 1]^\top \in \mathbb{R}^{d_i}$.

In the analysis to follow, we will use the fact that $u_i - u_i^* = \phi_{u_i}^\top \zeta_{u_i} - w_{u_i}$, where

$$\zeta_{u_i}^\top = \frac{\partial \mathcal{F}_{u_i}(x_i, \nu_i, A_{u_i})}{\partial A_{u_i}}$$

with $|w_{u_i}| \le W_{u_i}$. Consider the following update laws:

$$\dot{A}_{u_i} = \eta_{u_i}\zeta_{u_i}\boldsymbol{e}_i^\top P_i b_i \tag{11}$$

$$\dot{a}_{\theta_i} = \eta_{\theta_i}\left|\boldsymbol{e}_i^\top P_i b_i\right| \tag{12}$$

$$\dot{a}_{\kappa_i} = \eta_{\kappa_i}\left(\boldsymbol{e}_i^\top P_i b_i\right)^2 \tag{13}$$

where $\eta_{u_i} > 0$, $\eta_{\theta_i} > 0$, and $\eta_{\kappa_i} > 0$, $i = 1, \cdots, m$ are adaptation gains. The update law (11) is used to estimate the dynamics of the subsystem under control, while the update laws (12) and (13) are used to stabilize the subsystem by estimating the effects of the interconnections. Both (12) and (13) increase monotonically and we require that $a_{\theta_i}(0), a_{\kappa_i}(0) \ge 0$ so a projection algorithm may be required to ensure that they do not become unnecessarily large.

Theorem 1: Given the decentralized system with reference models satisfying Assumption 2, subsystems satisfying Assumption 1, interconnections satisfying Assumption 3, and controllers satisfying Assumption 4, then the control law (8) with adaptation laws (11)–(13) will ensure that for $i = 1, \cdots, m$ if $B_i < (\beta_{0_i}\lambda_{\min}(R_i)/\lambda_{\max}(P_i))$ then:

1) the subsystem outputs and their derivatives, $y_i, \cdots, y_i^{(d_i-1)}$, are bounded;
2) each control signal is bounded, i.e., $u_i + u_{k_i} \in \mathcal{L}_\infty$;
3) the magnitude of each output error, $|e_i|$, decreases asymptotically to zero, i.e., $\lim_{t\to\infty}|e_i| = 0$;
4) $\lim_{t\to\infty}|\dot{A}_{u_i}| = 0$, $\lim_{t\to\infty}|\dot{a}_{\theta_i}| = 0$, and $\lim_{t\to\infty}|\dot{a}_{\kappa_i}| = 0$;

where R_i is defined below.

Proof: Consider the following Lyapunov-type function for the ith subsystem:

$$v_i = \frac{\boldsymbol{e}_i^\top P_i \boldsymbol{e}_i}{\beta_i(x_i)} + \frac{1}{\eta_{u_i}}\phi_{u_i}^\top\phi_{u_i} + \frac{1}{\beta_{1_i}\eta_{\theta_i}}\phi_{\theta_i}^2 + \frac{1}{2\beta_{1_i}\eta_{\kappa_i}}\phi_{\kappa_i}^2 \tag{14}$$

where $\phi_{\theta_i} = a_{\theta_i} - \Theta_i^*$, $\phi_{\kappa_i} = a_{\kappa_i} - \kappa_i^*$, and each $P_i \in \mathbb{R}^{d_i \times d_i}$ is a positive definite and symmetric matrix (Θ_i^* and κ_i^* will be defined shortly). Taking the time derivative of v_i yields

$$\begin{aligned}\dot{v}_i = {} & \frac{\boldsymbol{e}_i^\top\left(P_i\Lambda_i + \Lambda_i^\top P_i\right)\boldsymbol{e}_i}{\beta_i(x_i)} - \frac{2\boldsymbol{e}_i^\top P_i b_i}{\beta_i(x_i)}\Delta_i(x_1, \cdots, x_m) \\ & - \frac{\boldsymbol{e}_i^\top P_i \boldsymbol{e}_i \dot{\beta}(x_i)}{\beta_i^2(x_i)} + \frac{2}{\eta_{u_i}}\phi_{u_i}^\top\dot{\phi}_{u_i} + \frac{2\boldsymbol{e}_i^\top P_i b_i}{\beta_i(x_i)} \\ & \cdot\Big(-\beta_i(x_i)[u_i - u_i^*] - a_{\theta_i}\operatorname{sgn}\left(\boldsymbol{e}_i^\top P_i b_i\right) - a_{\kappa_i}\boldsymbol{e}_i^\top P_i b_i/2\Big) \\ & + \frac{2}{\beta_{1_i}\eta_{\theta_i}}\phi_{\theta_i}\dot{\phi}_{\theta_i} + \frac{1}{\beta_{1_i}\eta_{\kappa_i}}\phi_{\kappa_i}\dot{\phi}_{\kappa_i}.\end{aligned} \tag{15}$$

Since each Λ_i is negative definite, given some positive definite R_i, there exits a unique symmetric positive definite P_i satisfying $P_i\Lambda_i + \Lambda_i^\top P_i = -R_i$, a Lyapunov matrix equation.

Since $\dot{\phi}_{u_i} = \dot{A}_{u_i}$, $\dot{\phi}_{\theta_i} = \dot{a}_{\theta_i}$, and $\dot{\phi}_{\kappa_i} = \dot{a}_{\kappa_i}$, using the definition of the adaptive laws (15) may be written as

$$\begin{aligned}\dot{v}_i = {} & \frac{-\boldsymbol{e}_i^\top R_i \boldsymbol{e}_i}{\beta_i(x_i)} - \frac{2\boldsymbol{e}_i^\top P_i b_i}{\beta_i(x_i)}\Delta_i(x_1, \cdots, x_m) \\ & - \frac{\boldsymbol{e}_i^\top P_i \boldsymbol{e}_i \dot{\beta}(x_i)}{\beta_i^2(x_i)} + \frac{2}{\beta_{1_i}}\phi_{\theta_i}\left|\boldsymbol{e}_i^\top P_i b_i\right| + \frac{2\boldsymbol{e}_i^\top P_i b_i}{\beta_i(x_i)} \\ & \cdot\Big(\beta_i(x_i)w_{u_i} - a_{\theta_i}\operatorname{sgn}\left(\boldsymbol{e}_i^\top P_i b_i\right) - a_{\kappa_i}\boldsymbol{e}_i^\top P_i b_i/2\Big) \\ & + \frac{1}{\beta_{1_i}}\phi_{\kappa_i}\left(\boldsymbol{e}_i^\top P_i b_i\right)^2.\end{aligned} \tag{16}$$

Since $1/\beta_{1_i} \le 1/\beta_i(x_i)$, and $a_{\theta_i} \ge 0$, $a_{\kappa_i} \ge 0$ we have

$$\begin{aligned}\dot{v}_i \le {} & \frac{-\boldsymbol{e}_i^\top R_i \boldsymbol{e}_i}{\beta_i(x_i)} - \frac{2\boldsymbol{e}_i^\top P_i b_i}{\beta_i(x_i)}\Delta_i(x_1, \cdots, x_m) - \frac{\boldsymbol{e}_i^\top P_i \boldsymbol{e}_i \dot{\beta}(x_i)}{\beta_i^2(x_i)} \\ & + 2\boldsymbol{e}_i^\top P_i b_i w_{u_i} - \frac{2\Theta_i^*}{\beta_{1_i}}\left|\boldsymbol{e}_i^\top P_i b_i\right| - \frac{\kappa_i^*}{\beta_{1_i}}\left(\boldsymbol{e}_i^\top P_i b_i\right)^2.\end{aligned} \tag{17}$$

If we choose $\Theta_i^* = \beta_{1_i}\tilde{\Theta}_i^*/\beta_{0_i} + \beta_{1_i}W_{u_i}$, then the inequality

$$-\frac{\boldsymbol{e}_i^\top P_i b_i}{\beta_i(x_i)}\tilde{\theta}_i(t) + \boldsymbol{e}_i^\top P_i b_i w_{u_i} \le \frac{\Theta_i^*}{\beta_{1_i}}\left|\boldsymbol{e}_i^\top P_i b_i\right| \tag{18}$$

holds. Substituting (18) in (17) yields

$$\begin{aligned}\dot{v}_i \le {} & \frac{-\boldsymbol{e}_i^\top R_i \boldsymbol{e}_i}{\beta_i(x_i)} - \frac{\kappa_i^*}{\beta_{1_i}}\left(\boldsymbol{e}_i^\top P_i b_i\right)^2 - \frac{2\boldsymbol{e}_i^\top P_i b_i \delta_i(x_1, \cdots, x_m)}{\beta_i(x_i)} \\ & - \frac{\boldsymbol{e}_i^\top P_i \boldsymbol{e}_i \dot{\beta}(x_i)}{\beta_i^2(x_i)}.\end{aligned} \tag{19}$$

It is possible to set $\kappa_i^* = 0$ and use M-matrix techniques to determine sufficient conditions for system stability [19]. Due to the conservativeness of the M-matrix techniques, however, the resulting composite system stability results are very restrictive for systems with relative degree $d_i > 1$. Here, we complete the squares (in an analogous manner to [2]) to obtain

$$\dot{v}_i \le \frac{-\boldsymbol{e}_i^\top R_i \boldsymbol{e}_i}{\beta_i(x_i)} + \frac{\beta_{1_i}\delta_i^2(x_1, \cdots, x_m)}{\kappa_i^*\beta_i^2(x_i)} + \frac{\lambda_{\max}(P_i)B_i\boldsymbol{e}_i^\top\boldsymbol{e}_i}{\beta_i^2(x_i)} \tag{20}$$

where each $\kappa_i^* > 0$ and $\lambda_{\max}(P_i)$ denotes the maximum eigenvalue of P_i.

Now consider the composite system Lyapunov candidate $V = \sum_{i=1}^m c_i v_i$, where each $c_i > 0$. Taking the derivative of V and using (20) and Assumption 3 gives

$$\dot{V} \le \sum_{i=1}^m c_i\left[-\frac{\boldsymbol{e}_i^\top R_i \boldsymbol{e}_i}{\beta_i(x_i)} + \frac{\beta_{1_i}}{\kappa_i^*\beta_i^2(x_i)}\left(\sum_{j=1}^m \gamma_{i,j}|\boldsymbol{e}_j|_2\right)^2 + \frac{\lambda_{\max}(P_i)B_i\boldsymbol{e}_i^\top\boldsymbol{e}_i}{\beta_i^2(x_i)}\right]. \tag{21}$$

Since $\sum_{j=1}^{m} \gamma_{i,j} |\boldsymbol{e}_j|_2 = \chi^\top \Gamma_i$, where $\chi = [|\boldsymbol{e}_1|_2, \cdots, |\boldsymbol{e}_m|_2]^\top$ and $\Gamma_i = [\gamma_{i,1}, \cdots, \gamma_{i,m}]^\top$, (21) may be written as

$$\dot{V} \le \sum_{i=1}^{m} c_i \left[\left(-\lambda_{\min}(R_i) + \frac{\lambda_{\max}(P_i) B_i}{\beta_i(x_i)} \right) \frac{|\boldsymbol{e}_i|_2^2}{\beta_i(x_i)} + \frac{\beta_{1_i}}{\kappa_i^* \beta_i^2(x_i)} \chi^\top \Gamma_i \Gamma_i^\top \chi \right] \tag{22}$$

where $\lambda_{\min}(R_i)$ is the real part of the eigenvalue of R_i with the minimum magnitude. Require that $B_i < \beta_{0_i} \lambda_{\min}(R_i)/\lambda_{\max}(P_i)$ so that $-\lambda_{\min}(R_i) + \lambda_{\max}(P_i) B_i/\beta_{0_i} < -\epsilon_i$, where each $\epsilon_i > 0$ is some finite constant. This gives us

$$\dot{V} \le \sum_{i=1}^{m} c_i \left[\frac{-\epsilon_i |\boldsymbol{e}_i|_2^2}{\beta_i(x_i)} + \frac{\beta_{1_i}}{\kappa_i^* \beta_i^2(x_i)} \chi^\top \Gamma_i \Gamma_i^\top \chi \right]. \tag{23}$$

Define $K^* = [\kappa_1^*, \cdots, \kappa_m^*]$. Let $\kappa_i^* = \kappa^*$, $i = 1, \cdots, m$ for some $0 < \kappa^*$, define

$$D = \operatorname{diag}\{c_1 \epsilon_1/\beta_{1_1}, \cdots, c_m \epsilon_m/\beta_{1_m}\}$$

and $M = \sum_{i=1}^{m} c_i \beta_{1_i} \Gamma_i \Gamma_i^\top / \beta_{0_i}^2$, so that $\dot{V} \le -\chi^\top A \chi$, where $A = D - (1/\kappa^*) M$. Then for some sufficiently large $\kappa^* > 0$, the matrix A is positive definite. This diagonal dominance property may be established using Gershgorin's Theorem [20]. Now define $K^* = [\kappa^*, \cdots, \kappa^*]^\top \in \mathbb{R}^m$ as

$$K^* = \arg \min_{\substack{K^* \in \mathbb{R}^m \\ 0 < \kappa^*}} \left\{ K^{*\top} K^* \colon A = D - \frac{1}{\kappa^*} M \text{ is positive definite} \right\}. \tag{24}$$

There exists sufficiently large κ^* such that A, defined by (24), is positive definite, which implies that $V \in \mathcal{L}_\infty$, and thus $|\chi|_2 \in \mathcal{L}_\infty$. Given bounded reference signals, Part 1 is established. With exponentially attractive zero dynamics, the states for each subsystem are bounded. Boundedness of the Lyapunov function thus ensures that $u_i + u_{k_i} \in \mathcal{L}_\infty$ so Part 2 holds. Also

$$\int_0^\infty \chi^\top A \chi \, dt \le -\int_0^\infty \dot{V} \, dt + \text{const} \tag{25}$$

so that $|\chi|_2 \in \mathcal{L}_2$. Since all of the signals are well defined, we also have $\dot{\boldsymbol{e}}_i \in \mathcal{L}_\infty^{d_i}$ so that $d/dt |\boldsymbol{e}_i|_2 = \boldsymbol{e}_i^\top \dot{\boldsymbol{e}}_i / |\boldsymbol{e}_i|_2 \le |\dot{\boldsymbol{e}}|_2 \in \mathcal{L}_\infty$. Using Barbalat's Lemma, we thus establish that $\lim_{t\to\infty} |\chi|_2 = 0$, thus we are guaranteed asymptotically stable tracking for each of the subsystems so Part 3 holds. In addition, since each of the plant and control signals is bounded and $\lim_{t\to\infty} |e_i| = 0$, convergence of the update law derivatives to zero is established by their definitions. ☐

Remark 1: The stability results are semiglobal in the sense that they hold for $x_i \in S_{x_i}$ where S_{x_i} is dependent upon the span of the RBNN. In the case that the RBNN's may approximate the feedback linearizing controller (5) globally, S_{x_i} may be defined as $\mathbb{R}^{n_i}$ and S_{ν_i} as $\mathbb{R}$ (rather than compact sets) so the results hold globally.

Remark 2: The bound for the representation error, W_{u_i}, does not need to be known for our choice of adaptation laws (all we needed to know earlier was that it existed and we are guaranteed this). In addition, the magnitude of the interconnections are estimated on-line to produce stable tracking.

Remark 3: The direct adaptive control scheme presented here does require that $B_i < \beta_{0_i} \lambda_{\min}(R_i)/\lambda_{\max}(P_i)$. That is, the rate at which the control gain changes may influence the design of the update and control laws through the choice of P_i given some Λ_i. For subsystems with $\beta_i(x_i)$ a constant, then this requirement is always satisfied since $B_i = 0$ is a valid choice. As the rate of change of the control gain increases, however, this bound becomes more restrictive for the control design.

V. Indirect Adaptive Control

The direct adaptive decentralized control law was defined using an RBNN with adjustable parameters to approximate u_i^*. For the indirect case, however, an identifier will be used to approximate the isolated system dynamics so that a feedback linearizing controller may be defined based on the certainty equivalence principle. We will first represent the isolated system dynamics (4) as $\alpha_i(x_i) = \alpha_{u_i} + \alpha_{k_i}(x_i)$ and $\beta_i(x_i) = \beta_{u_i} + \beta_{k_i}(x_i)$, where α_{u_i} and β_{u_i} represent the unknown dynamics, while α_{k_i} and β_{k_i} represent the known dynamics (which can be equal to zero and all the results to follow still hold). Representing the unknown dynamics with an RBNN, we see

$$\alpha_i(x_i) = \mathcal{F}_{\alpha_i}\left(x_i, A_{\alpha_i}^*\right) + \alpha_{k_i} + w_{\alpha_i} \tag{26}$$

$$\beta_i(x_i) = \mathcal{F}_{\beta_i}\left(x_i, A_{\beta_i}^*\right) + \beta_{k_i} + w_{\beta_i}. \tag{27}$$

The parameters for $\mathcal{F}_{\alpha_i}(x_i, A_{\alpha_i}^*)$ are defined by

$$A_{\alpha_i}^* = \arg \min_{A_{\alpha_i} \in \Omega_{\alpha_i}} \left[\sup_{x_i \in S_{x_i}} |\mathcal{F}_{\alpha_i}(x_i, A_{\alpha_i}) - \alpha_{u_i}| \right] \tag{28}$$

while the parameters for $\mathcal{F}_\beta(x_i, A_{\beta_i}^*)$ are defined by

$$A_{\beta_i}^* = \arg \min_{A_{\beta_i} \in \Omega_{\beta_i}} \left[\sup_{x_i \in S_{x_i}} |\mathcal{F}_{\beta_i}(x_i, A_{\beta_i}) - \beta_{u_i}| \right] \tag{29}$$

where the representation errors for α_{u_i} and β_{u_i} are defined by w_{α_i} and w_{β_i}, respectively. The bounds on the representation errors are given by $|w_{\alpha_i}| \le W_{\alpha_i}$ and $|w_{\beta_i}| \le W_{\beta_i}$ for some $W_{\alpha_i} > 0$ and $W_{\beta_i} > 0$. The subspace S_{x_i} is a compact set through which the state trajectory for the ith subsystem may travel. The subspaces Ω_{α_i} and Ω_{β_i} are compact convex sets which contain the feasible parameter sets for $A_{\alpha_i}^*$ and $A_{\beta_i}^*$, respectively. The components of the isolated subsystem dynamics are approximated by RBNN's so that

$$\hat{\alpha}_i(x_i) = \mathcal{F}_{\alpha_i}(x_i, A_{\alpha_i}(t)) + \alpha_{k_i}(x_i) \tag{30}$$

$$\hat{\beta}_i(x_i) = \mathcal{F}_{\beta_i}(x_i, A_{\beta_i}(t)) + \beta_{k_i}(x_i) \tag{31}$$

where $A_{\alpha_i}(t)$ and $A_{\beta_i}(t)$ will be updated on-line in an attempt to identify the isolated system dynamics.

An adaptive algorithm is used to estimate $A_{\alpha_i}^*$ and $A_{\beta_i}^*$ with A_{α_i} and A_{β_i}, respectively. Parameter error vectors are defined as $\phi_{\alpha_i} = A_{\alpha_i} - A_{\alpha_i}^*$ and $\phi_{\beta_i} = A_{\beta_i} - A_{\beta_i}^*$. Using the current estimate for the ith subsystem with no interconnections, a certainty equivalence control term for the ith subsystem is defined as

$$u_i = -\hat{\alpha}_i(x_i) + \nu_i / \hat{\beta}_i(x_i) \tag{32}$$

assuming that $\hat{\beta}_i(x_i)$ is bounded away from zero (this may be ensured using a projection algorithm). Let

$$\nu_i = r_i^{(d_i)} + k_{i,d_i-1} e_i^{(d_i-1)} + \cdots + k_{i,0} e_i + a_{\theta_i}(t) \operatorname{sgn}\left(\boldsymbol{e}_i^\top P_i b_i\right) + a_{\kappa_i}(t) \boldsymbol{e}_i^\top P_i b_i / 2 + a_{w\beta_i}(t) \operatorname{sgn}\left(\boldsymbol{e}_i^\top P_i b_i u_i\right) \tag{33}$$

where the adaptive parameters $a_{\theta_i}(t)$, $a_{\kappa_i}(t)$, and $a_{w\beta_i}(t)$ are yet to be defined. The term $a_{\theta_i} \operatorname{sgn}(\boldsymbol{e}_i^\top P_i b_i)$ is used to reject unknown disturbances, while the term $a_{\kappa_i} \boldsymbol{e}_i^\top P_i b_i / 2$ is used to compensate for unknown effects from the interconnections. In addition, $a_{\beta_i} \operatorname{sgn}(\boldsymbol{e}_i^\top P_i b_i u_i)$ has been included to compensate for the representation error w_{β_i}. The control assumptions for the indirect adaptive controller are summarized as follows.

Assumption 5—Control: If $x_i \in \mathcal{L}_\infty^{n_i}$, then $\alpha_{k_i}, \beta_{k_i} \in \mathcal{L}_\infty$. The representation errors w_{α_i} and w_{β_i} are bounded, i.e., $|w_{\alpha_i}| \le W_{\alpha_i}$ and $|w_{\beta_i}| \le W_{\beta_i}$ for some $W_{\alpha_i} > 0$ and $W_{\beta_i} > 0$. A projection algorithm is used to ensure that $\hat{\beta}_i(x_i) > \beta_{0_i}$.

Proceeding in a similar manner to the direct case, we have

$$\begin{aligned} e_i^{(d_i)} &= (\hat{\alpha}_i(x_i) - \alpha_i(x_i)) + (\hat{\beta}_i(x_i) - \beta_i(x_i))u_i \\ &\quad - \Delta_i(x_1, \cdots, x_m) - k_{i,d_i-1}e_i^{(d_i-1)} \\ &\quad - \cdots - k_{i,0}e_i - a_{\theta_i}\operatorname{sgn}\left(\boldsymbol{e}_i^\top P_i b_i\right) \\ &\quad - a_{\kappa_i}\boldsymbol{e}_i^\top P_i b_i/2 - a_{w\beta_i}\operatorname{sgn}\left(\boldsymbol{e}_i^\top P_i b_i u_i\right) \end{aligned} \tag{34}$$

and

$$\begin{aligned} \dot{\boldsymbol{e}}_i = \Lambda_i \boldsymbol{e}_i + b_i\Big[&(\hat{\alpha}_i(x_i) - \alpha_i(x_i)) + (\hat{\beta}_i(x_i) - \beta_i(x_i))u_i \\ &- \Delta_i(x_1, \cdots, x_m) - a_{\theta_i}\operatorname{sgn}\left(\boldsymbol{e}_i^\top P_i b_i\right) \\ &- a_{\kappa_i}\boldsymbol{e}_i^\top P_i b_i/2 - a_{w\beta_i}\operatorname{sgn}\left(\boldsymbol{e}_i^\top P_i b_i u_i\right)\Big] \end{aligned}$$

where Λ_i and b_i are as defined for the direct adaptive case.

The identifier errors may be expressed as

$$\hat{\alpha}_i(x_i) - \alpha_i(x_i) = \phi_{\alpha_i}^\top \zeta_{\alpha_i} - w_{\alpha_i} \tag{35}$$

$$\hat{\beta}_i(x_i) - \beta_i(x_i) = \phi_{\beta_i}^\top \zeta_{\beta_i} - w_{\beta_i} \tag{36}$$

where $\overline{w}_{\alpha_i}$ and $\underline{w}_{\beta_i}$ are representation errors. At this point, we also define the total parameter identification error for the isolated subsystem dynamics as $\phi_{\alpha\beta_i} = [\phi_{\alpha_i}^\top, \phi_{\beta_i}^\top]^\top$ with the total regressor as $\zeta_{\alpha\beta_i} = [\zeta_{\alpha_i}^\top, \zeta_{\beta_i}^\top u_i]^\top$.

The following update laws are now defined for the decentralized indirect adaptive controller:

$$\dot{A}_i = -\eta_{\alpha\beta_i}\zeta_{\alpha\beta_i}\boldsymbol{e}_i^\top P_i b_i \tag{37}$$

$$\dot{a}_{w\beta_i} = \eta_{w\beta_i}\left|\boldsymbol{e}_i^\top P_i b_i u_i\right| \tag{38}$$

$$\dot{a}_{\theta_i} = \eta_{\theta_i}\left|\boldsymbol{e}_i^\top P_i b_i\right| \tag{39}$$

$$\dot{a}_{\kappa_i} = \eta_{\kappa_i}\left(\boldsymbol{e}_i^\top P_i b_i\right)^2 \tag{40}$$

where $\eta_{\alpha\beta_i}$, $\eta_{w\beta_i}$, η_{θ_i}, and η_{κ_i} are fixed adaptive gains. The parameter update law for the isolated system identifier (37) is used to estimate the dynamics of the subsystem under control. The update law (38) is designed to compensate for the effects of the representation error w_{β_i}, while the update laws (39) and (40) are used to stabilize the subsystem by estimating the effects of the interconnections and w_{α_i}. The parameter error associated with the representation error w_{β_i} is defined as $\phi_{w\beta_i} = a_{w\beta_i} - W_{\beta_i}$, while ϕ_{θ_i} and ϕ_{κ_i} are as defined for the direct case. The adaptive parameters are initialized such that $a_{w\beta_i}(0) \ge 0$, $a_{\theta_i}(0) \ge 0$, and $a_{\kappa_i}(0) \ge 0$ so that $a_{w\beta_i}(t)$, $a_{\theta_i}(t)$, $a_{\kappa_i}(t)$ remain positive.

Theorem 2: Given the decentralized system with subsystems satisfying Assumption 1, interconnections satisfying Assumption 3, and controllers satisfying Assumption 5, then the control law (32) with adaptation laws (37)–(40) will ensure that, for $i = 1, \cdots, m$:

1) the subsystem output and its derivatives, $y_i, \cdots, y_i^{(d_i-1)}$, are bounded;
2) each control signal is bounded, i.e., $u_i \in \mathcal{L}_\infty$;
3) the magnitude of each output error, $|e_i|$, decreases asymptotically to zero, i.e., $\lim_{t\to\infty}|e_i| = 0$;
4) $\lim_{t\to\infty}|\dot{A}_{\alpha\beta_i}| = 0$, $\lim_{t\to\infty}|\dot{a}_{w\beta_i}| = 0$, $\lim_{t\to\infty}|\dot{a}_{\theta_i}| = 0$, and $\lim_{t\to\infty}|\dot{a}_{\kappa_i}| = 0$.

Proof: Consider the following Lyapunov-type function for the ith subsystem:

$$v_i = \boldsymbol{e}_i^\top P_i \boldsymbol{e}_i + \frac{1}{\eta_{\alpha\beta_i}}\phi_{\alpha\beta_i}^\top\phi_{\alpha\beta_i} + \frac{1}{\eta_{\theta_i}}\phi_{\theta_i}^2 + \frac{1}{2\eta_{\kappa_i}}\phi_{\kappa_i}^2 + \frac{1}{\eta_{w\beta_i}}\phi_{w\beta_i}^2 \tag{41}$$

where each $P_i \in \mathbb{R}^{d_i \times d_i}$ is a positive definite and symmetric matrix. From the definition of ϕ_{θ_i}, ϕ_{κ_i}, and $\phi_{w\beta_i}$, we use a similar approach to the direct case to obtain

$$\begin{aligned} \dot{v}_i = &-\boldsymbol{e}_i^\top R_i \boldsymbol{e}_i + 2\boldsymbol{e}_i^\top P_i b_i[-w_{\alpha_i} - w_{\beta_i}u_i - \Delta_i(x_1, \cdots, x_m)] \\ &- 2\Theta_i^*\left|\boldsymbol{e}_i^\top P_i b_i\right| - \kappa_i^*\left(e_i^\top P_i b_i\right)^2 - 2W_{\beta_i}\left|\boldsymbol{e}_i^\top P_i b_i u_i\right|. \end{aligned} \tag{42}$$

Choosing $\Theta_i^* = W_{\alpha_i} + \tilde{\Theta}_i^*$ ensures that $\boldsymbol{e}_i^\top P_i b_i[w_{\alpha_i} + \tilde{\theta}_i(t)] \le \Theta_i^*|\boldsymbol{e}_i^\top P_i b_i|$. Also $\boldsymbol{e}_i^\top P_i b_i w_{\beta_i} u_i \le W_{\beta_i}|\boldsymbol{e}_i^\top P_i b_i u_i|$ so that

$$\dot{v}_i \le -\boldsymbol{e}_i^\top R_i \boldsymbol{e}_i - \kappa_i^*\left(\boldsymbol{e}_i^\top P_i b_i\right)^2 - 2\boldsymbol{e}_i^\top P_i b_i \delta_i(x_1, \cdots, x_m). \tag{43}$$

We require each $\kappa_i^* > 0$, then complete the squares to obtain $\dot{v}_i \le -\boldsymbol{e}_i^\top R_i \boldsymbol{e}_i + (1/\kappa_i^*)\delta_i^2(x_1, \cdots, x_m)$.

Now consider the composite system Lyapunov candidate $V = \sum_{i=1}^m c_i v_i$, where each $c_i > 0$. Taking the derivate of V gives

$$\dot{V} \le \sum_{i=1}^m c_i\left[-\lambda_{\min}(R_i)|e_i|_2^2 + \frac{1}{\kappa_i^*}\chi^\top\Gamma_i\Gamma_i^\top\chi\right] \tag{44}$$

where $\lambda_{\min}(R_i)$ is the real part of the eigenvalue of R_i with the minimum magnitude. Define $K^* = [\kappa^*, \cdots, \kappa^*] \in \mathbb{R}^m$. Let $D = \operatorname{diag}\{c_1\lambda_{\min}(R_1), \cdots, c_m\lambda_{\min}(R_m)\}$ and $M = \sum_{i=1}^m c_i\Gamma_i\Gamma_i^\top$, so that $\dot{V} \le -\chi^\top A\chi$, where $A = D - \frac{1}{\kappa^*}M$. Then for some sufficiently large $\kappa^* > 0$, the matrix A is positive definite. The remainder of the theorem follows as for the direct adaptive case beginning with (24).□

Remark 4: The results obtained here are again semiglobal due to the definition of the RBNN's with global results obtained in the case where $S_{x_i} = \mathbb{R}^{n_i}$ for $i = 1, \cdots, m$.

Remark 5: Notice that for the indirect adaptive controller, we have u_i a function of ν_i and ν_i a function of $\operatorname{sgn}(u_i)$ due to the $a_{w\beta_i}(t)\operatorname{sgn}(e_i^\top P_i b_i u_i)$ term in (33). For implementation purposes, it is possible to bias the controller output so that u_i only takes on positive values so that ν_i and thus u_i may be easily calculated. This is analogous to considering the controller output to be positive values passed out of an analog-to-digital board, while the scaling and biasing of the actuator are considered to be part of the plant dynamics. If, for a particular application and choice of the RBNN's, we may set $W_{\beta_i} = 0$, then it is possible to let $a_{w\beta_i}(t) = 0$ so that $\nu_i = r_i^{(d_i)} + k_{i,d_i-1}e_i^{(d_i-1)} + \cdots + k_{i,0}e_i + a_{\theta_i}(t)\operatorname{sgn}(\boldsymbol{e}_i^\top P_i b_i) + a_{\kappa_i}(t)\boldsymbol{e}_i^\top P_i b_i/2$, thus eliminating any dependence upon u_i.

Remark 6: The indirect adaptive controller does require a projection algorithm to ensure that the control signal is well defined for all time. This may be easily achieved for RBNN's with adjustable output weights using a weighted average radial basis calculation since the output of the RBNN is then no less than the value of the smallest weight.

Remark 7: The indirect adaptive control routine does not make any requirements upon the rate of change of the input gain for each subsystem. In addition, we did not need to know the interconnection strengths, representation errors, or bounds on $\theta_i(t)$. Because of the functional approximation properties of RBNN's, the functional form of the subsystem dynamics does not need to be known.

VI. Simulations

Within this section, we will present illustrative examples for both the direct and indirect approaches. While the approach could be applied to intervehicle spacing regulation in a platoon of an

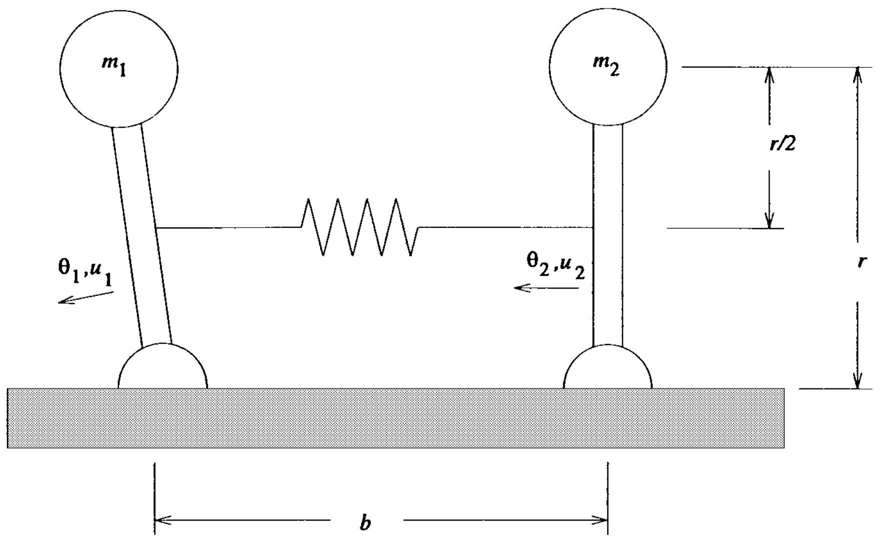

Fig. 1. Two inverted pendulums connected by a spring.

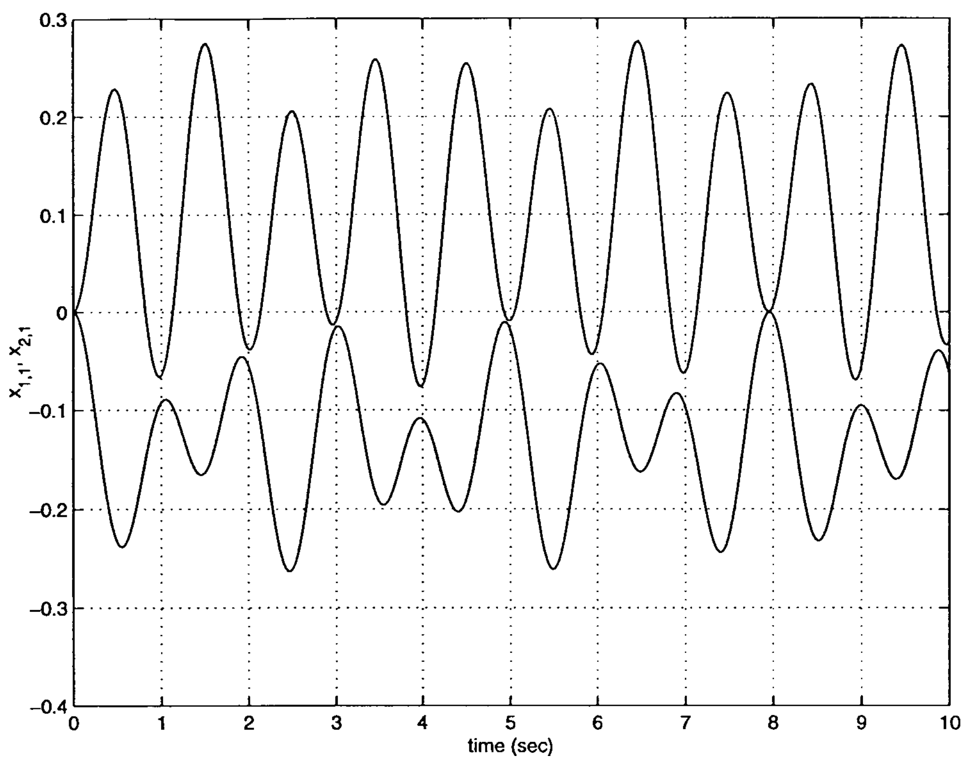

Fig. 2. Control of the pendulums using the a proportional feedback controller (using u_{k_i}).

automated highway system, since that system fits the assumptions of our framework [16], instead we study the control of two inverted pendulums connected by a spring as shown in Fig. 1. Each pendulum may be positioned by a torque input u_i applied by a servomotor at its base. It is assumed that both θ_i and $\dot{\theta}_i$ (angular position and rate) are available to the ith controller for $i = 1, 2$.

The equations which describe the motion of the pendulums are defined by

$$\dot{x}_{1,1} = x_{1,2} \tag{45}$$

$$\dot{x}_{1,2} = \left(\frac{m_1 g r}{J_1} - \frac{k r^2}{4 J_1}\right) \sin(x_{1,1}) + \frac{k r}{2 J_1}(l - b) + \frac{u_1}{J_1} + \frac{k r^2}{4 J_1} \sin(x_{2,1}) \tag{46}$$

$$\dot{x}_{2,1} = x_{2,2} \tag{47}$$

$$\dot{x}_{2,2} = \left(\frac{m_2 g r}{J_2} - \frac{k r^2}{4 J_2}\right) \sin(x_{2,1}) - \frac{k r}{2 J_2}(l - b) + \frac{u_2}{J_2} + \frac{k r^2}{4 J_2} \sin(x_{1,2}) \tag{48}$$

where $\theta_1 = x_{1,1}$ and $\theta_2 = x_{2,1}$ are the angular displacements of the pendulums from vertical. The parameters $m_1 = 2$ kg and $m_2 = 2.5$ kg are the pendulum end masses, $J_1 = 0.5$ kg and $J_2 = 0.625$ kg are the moments of inertia, $k = 100$ N/m is the spring constant of the connecting spring, $r = 0.5$ m is the pendulum height, $l = 0.5$ m is the natural length of the spring, and $g = 9.81$ m/s^2 is gravitational acceleration. The distance between the pendulum hinges is defined as $b = 0.4$ m, where, in this example, $b < l$ so that the pendulums

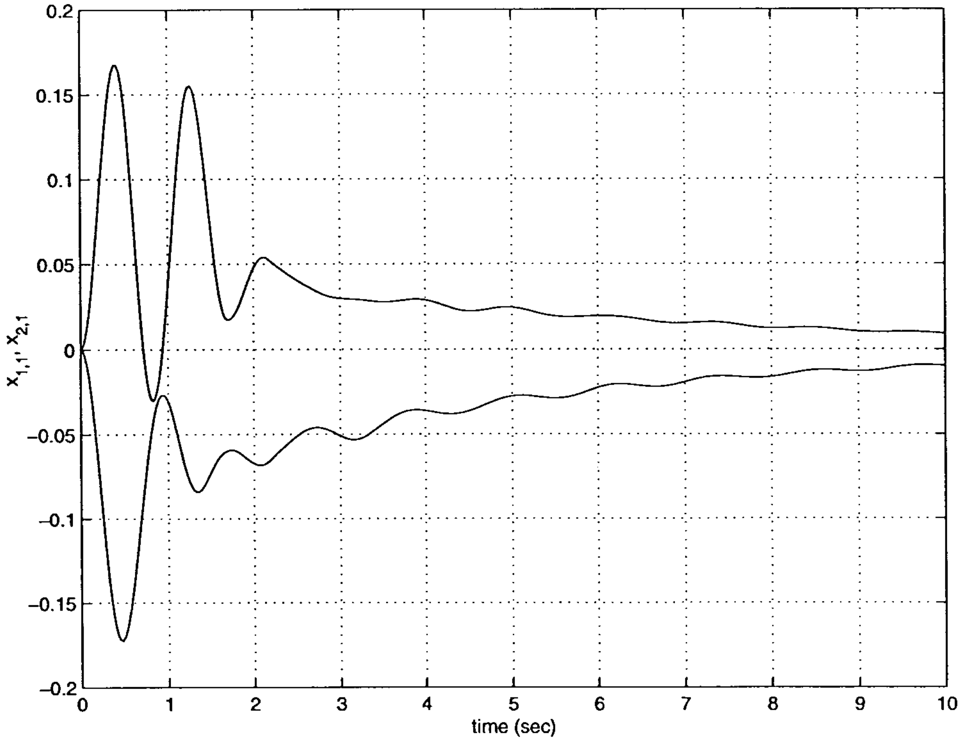

Fig. 3. Control of the pendulums using the proposed direct adaptive decentralized technique.

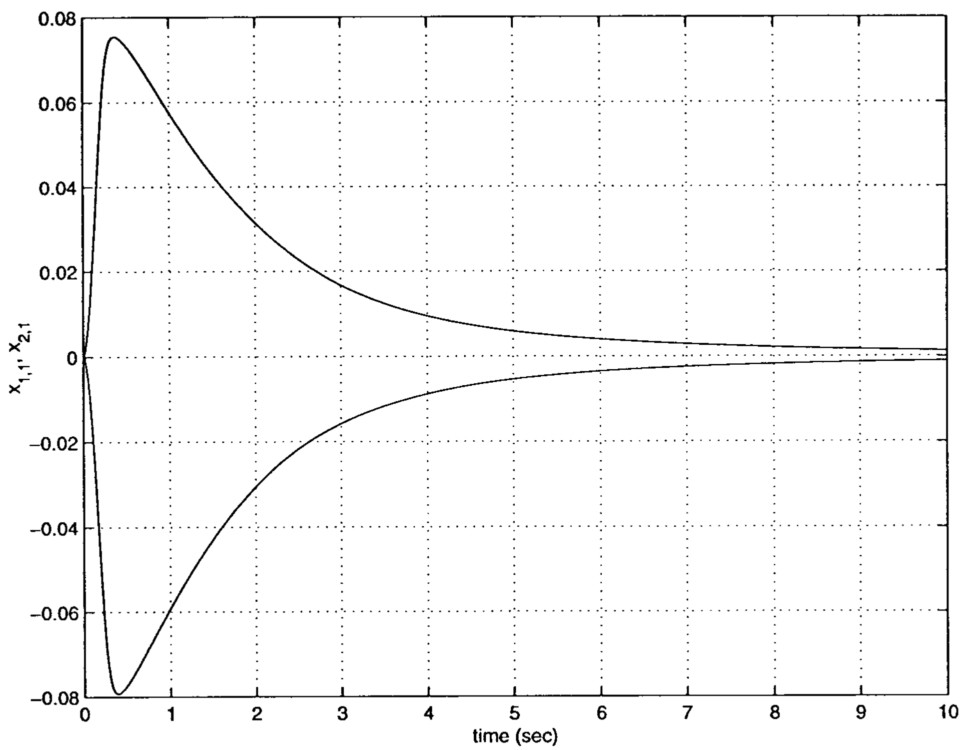

Fig. 4. Control of the pendulums using the proposed indirect adaptive decentralized technique.

repel one another when both are in the upright position. It is easy to see that the pendulum equations of motion fit (4).

Here we will attempt to drive the angular positions to zero, so that $e_i = -\theta_i$ [i.e., $r_i(t) = 0$]. We will first demonstrate that simple decentralized proportional feedback controllers appear to stabilize the system. Choosing $u_i = 20e_i$ for $i = 1, 2$, we find that the pendulums appear to be stabilized, but exhibit relatively large oscillatory behavior due to the lack of damping as shown in Fig. 2. Selecting $u_{k_i} = 20e_i$, it is possible to augment the proportional controllers with the decentralized direct adaptive controllers to help regulate the system. Here we choose to input θ_i/π and $\nu_i/5$ to the ith RBNN controller with centers evenly spaced between [−1, 1] for the $\nu_i/5$ input and [−1, 1] for the θ_i/π input with $\sigma_i = 1$. The inputs were normalized because the RBNN definition considers σ

as a constant. Thus scaling the inputs ensures that the RBNN basis function's size of influence will be appropriate for each input. Using 10 centers for each input dimension yields a total of 100 adjustable parameters for each RBNN with a weighted average formulation. We additionally chose $\eta_i = 2000$, $\eta_{\theta_i} = 10$, $\eta_{\kappa_i} = 10$ and each Λ_i so that $\hat{L}(s) = s^2 + 4s + 4$ has roots at $(-2, -2)$. The performance of direct adaptive controller is shown in Fig. 3.

Next, we apply the indirect adaptive scheme using weighted average RBNN's for the inverted pendulum example. Rather than choosing some u_{k_i}, we may now choose α_{k_i} and β_{k_i} if desired. Here we let $\alpha_{k_i} = 0$ and $\beta_{k_i} = 1$. Since the control gains are simply constants for this example (i.e., the control inputs are multiplied by constants $1/J_1$ and $1/J_2$ for u_1 and u_2, respectively) we let $W_{\beta_i} = 0$ for $i = 1, 2$. This choice is valid since an RBNN may exactly approximate a constant. This implies that we may set $a_{w\beta_i}(t) = 0$ to simplify the control law. In addition, a projection algorithm was used to ensure that $\mathcal{F}_{\beta_i} + \beta_{k_i} \geq 0.1$. This is done using a projection algorithm such that each weight for $\mathcal{F}_{\beta_i}$ remains greater than -0.9 so that $\mathcal{F}_{\beta_i} > -0.9$ when using a weighted average RBNN. We chose θ_i/π as the input for $\mathcal{F}_{\alpha_i}$ and $\mathcal{F}_{\beta_i}$ with 20 centers evenly spaced between $[-1, 1]$. With $\eta_i = 200$, $\eta_{\theta_i} = 2$, and $\eta_{\kappa_i} = 2$, the output trajectories for the inverted pendulums are shown in Fig. 4.

VII. Concluding Remarks

Within this paper, we presented direct and indirect adaptive control schemes appropriate for a class of interconnected nonlinear systems using radial basis neural networks. Using an on-line approximation approach, we have been able to relax the linear in the parameter requirements of traditional nonlinear decentralized adaptive control without considering the dynamic uncertainty as part of the interconnections or disturbances. Semiglobal asymptotic stability results were obtained with global results achieved by placing additional assumptions upon the RBNN's.

Although the adaptive schemes presented here relax the linear in the parameter requirements for subsystems, there are distinct disadvantages associated with the on-line approximation approach to decentralized control. 1) The schemes presented here do not necessarily identify physically meaningful parameters, which schemes for linear in the parameter subsystems might do. Often, however, the parameters identified using traditional approaches are a combination of a number of physical parameters so that it may be difficult to extract useful information about the subsystems. 2) Even though we have shown asymptotic stability for the tracking errors, we have made no guarantee about convergence of the controller parameters to their ideal values (e.g., A_{u_i} may not converge to $A^*_{u_i}$). This may not be a concern in cases where stable control is the primary objective as it is here. 3) The approaches here may require a large number of adjustable parameters for each RBNN due to the curse of dimensionality associated with RBNN's which are linear in the parameters. If using a decentralized approach was decided based upon computational overhead, then there may be circumstances for which other less computationally intensive approaches would be more appropriate.

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