

Decentralized Anonymous Credentials

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Abstract—Anonymous credentials provide a powerful tool for making assertions about identity while maintaining privacy. However, a limitation of today’s anonymous credential systems is the need for a trusted credential issuer — which is both a single point of failure and a target for compromise. Furthermore, the need for such a trusted issuer can make it challenging to deploy credential systems in practice, particularly in the *ad hoc* network setting (e.g., anonymous peer-to-peer networks) where no single party can be trusted with this responsibility.

In this work we propose a novel anonymous credential scheme that eliminates the need for a trusted credential issuer. Our approach builds on recent results in the area of electronic cash that, given a public append-only ledger, do not need a trusted credential issuer. Furthermore, given a *distributed* public ledger, as in, e.g., Bitcoin, our system requires no credential issuer at all and hence is decentralized. Using such a public ledger and standard cryptographic primitives, we propose and provide a proof of security for a basic anonymous credential system that allows users to make flexible identity assertions with strong privacy guarantees without relying on trusted parties. Finally, we discuss a number of practical applications for our techniques, including resource management in *ad hoc* networks and prevention of Sybil attacks. We implement our scheme and measure its efficiency.

I. INTRODUCTION

Traditionally, making statements about identity on the Internet, whether actual assertions of identity (“I am Spartacus”) or about one’s identity (“I am a gladiator”) involves centralized providers who issue a credential attesting to that verification. These organizations, which include Certificate Authorities, DNS maintainers, or login providers like Google and Facebook, play a large role in securing internet infrastructure, email, and financial transactions. Our increasing reliance on these providers raises concerns about privacy and trust.

Anonymous credentials, introduced by Chaum [22] and developed in a line of subsequent works [4, 10, 14, 15, 17], represent a powerful solution to this privacy concern: they deprive even colluding credential issuers and verifiers of the ability to identify and track their users. Although credentials may involve direct assertions of identity, they may also be used for a large range of useful assertions, such as “my TPM says my computer is secure,” “I have a valid subscription for

content,” “I have a certain reputation,” or “I am eligible to vote.”

Indeed, anonymous credentials have already seen several practical applications. The most widely deployed example is the Direct Anonymous Attestation (DAA) portion of the Trusted Platform Module specification [2, 11]. DAA extends the standard attestation capabilities of the Trusted Platform Module to allow for anonymous attestations of TPM state and to admit *pseudonyms* that are cryptographically bound to the TPM’s internal identity certificate.

Unfortunately, current anonymous credential systems such as DAA have a fundamental limitation: while identity certification itself can be performed by a variety of centralized and decentralized processes, all existing anonymous credential systems employ blind signatures and thus require the appointment of a central, *trusted* party to issue the credentials. This issuer represents a single point of failure and its signing key an obvious target for compromise, either of which can seriously damage the reliability of the credential system. Moreover, compromise or issuer malfeasance can be particularly difficult to detect in an anonymous credential system. As a result, in distributed settings such as *ad hoc* or peer-to-peer networks, it may be challenging to identify parties who can be trusted to play this critical role or verify that the trust is well placed. The ability to remove this trusted party or even verify their continued good behavior is a distinct advantage.

These challenges raise two questions: 1) is it possible to build practical anonymous credential systems where the process of issuing credentials — if not the establishment of identity itself — no longer depends on a trusted party? And 2) is it possible to do so without the need for a central party?

Our contribution. In this paper we answer both questions in the affirmative, proposing a new technique for constructing anonymous credentials which does not rely on the continued integrity of signature keys. A consequence of this result is that our anonymous credential system can be instantiated on-demand and operated by an *ad hoc* group of mistrustful peers. We further show how to extend our credential scheme to create *updatable* (e.g., stateful) anonymous credentials in which users obtain new credentials based on changing properties of their identity.

As a basic ingredient, our protocols require the existence of a *public append-only ledger*. When the ledger is implemented using trusted hardware, or a central party who is audited by the rest of the network, we obtain a positive answer only to the first question. To answer both questions in the affirmative we require that 1) this ledger be maintained in a distributed manner that need not require a trusted party or parties and 2) the

identity claims we are issuing credentials on must be verifiable by everyone participating in the system. We refer to this new primitive as a *decentralized anonymous credential* system and elaborate on its properties herein. We note that one promising instantiation of a decentralized ledger is the “block chain” construction used by Bitcoin [39] to implement a decentralized digital currency. Not only can this technology be used to actually construct a separate distributed ledger for identities, but using existing techniques for embedding small amounts of data in the block chain [23] we can leverage Bitcoin’s existing ledger and protocol *without* modification to transform any reliable storage mechanism (whether a central server or a distributed mechanism like a DHT) into an append-only ledger.

We show that our techniques have several immediate applications. They include:

- **Decentralized Direct Anonymous Attestation.** We show how to decentralize the Direct Anonymous Attestation protocol [11], allowing individual collections of nodes in an ad hoc or distributed system to securely assert properties of their system state. We provide an exemplary description of our decentralized (dDAA) construction.
- **Anonymous resource management in ad hoc networks.** Peer-to-peer networks are vulnerable to impersonation attacks, where a single party simulates many different peers in order to gain advantage against the network [30]. We show that our credentials may be useful in mitigating these attacks. The basic approach is to construct an *anonymous subscription service* [13, 26, 36] where parties may establish unique or costly pseudonyms (for example by submitting a valid TPM credential or paying a sum of digital currency). They can then assert possession on their identity under a specific set of restrictions, e.g., a limit to the number of requests they can make in each time period.
- **Auditable credentials.** Our techniques may also be used to extend existing centralized credential systems by allowing for public audit of issued credentials. This helps to guard against compromised credential issuers and allows the network to easily detect and revoke inappropriate credential grants. For example, in Direct Anonymous Attestation (DAA) one might want to prevent a malicious DAA authority from covertly granting certificates to users who do not have a TPM or whose TPM did not attest.

Is decentralized credential issuance valuable? Before proceeding to describe our protocols, it is worth asking whether decentralizing the issuance of anonymous credentials is a useful goal at all. After all, identity credentialing is frequently a centralized process. One might ask: what do we gain by decentralizing the issuance of *anonymous* credentials?

A first response to this question is that most anonymous credential systems separate the process of *issuing* anonymous credentials from the process of *certifying* the underlying identity claims. Frequently, the claims being certified are publicly verifiable. For example, each TPM ships with an Endorsement Key (EK). Identity assertions using the EK could be publicly verifiable merely by checking the certificate chain on the EK

certificate and engaging in a challenge/response protocol to ensure the TPM can read nonces encrypted to the EK.¹ The problem is that transactions conducted using this certificate are linked to the particular TPM device.

DAA solves this issue by having a central party issue new anonymous credentials to a device. Organizations must configure a local server to validate identity certifications and issue the corresponding anonymous credential. All this server does is transform a publicly verifiable identity assertion into an anonymous one. This adds a cumbersome step to the anonymous attestation system and also introduces a point of failure. Indeed, this pattern of a trusted party transforming existing credentials into an *anonymous* credential repeats in many settings. Allowing for the distributed issue of anonymous credentials, even if they can only certify centrally validated assertions, removes this additional point of trust.

An obvious question is why, if the identity assertion is publicly verifiable, do we need any transformation mechanism at all? Why not present the information we used to convince the authority to issue the credential to everyone? The issue is that proving an identity statement may reveal far more information than the statement itself. For example, a driver’s license can prove to anyone that the bearer is over 21 but also reveals a whole host of other information that the statement that “some trusted mechanism says I am over 21” does not. Because anonymous credentials add a layer of indirection between certifying that an identity statement is true and actually showing that statement, they fix this issue and avoid linking any use of the credential to the information used to issue it.

A more interesting question is whether identity certification itself can be decentralized. At least for certain claims, this seems like a promising direction. For example, non-extended validation SSL certificates are simply an assertion that the bearer controls the specified domain.² Similarly, DNS names are generally an assertion that the owner was the first to register that name and wants it mapped to certain values (e.g., an IP address). In both cases, since these claims are publicly verifiable by simple criteria, a distributed set of entities can easily validate these claims for themselves.

In fact, a now largely unused fork of Bitcoin, Namecoin [40], shows that such modifications are readily achievable. Namecoin uses Bitcoin’s append-only ledger mechanism to maintain such first-come first-serve name-value mappings. Individuals register a name and an owning public key. Provided they are the first to register that name, they can make arbitrary updates to the associated value by signing them with the registered key. A DNS system built atop this — DotBIT — is already in experimental deployment. Namecoin can also be used to maintain mappings from names to public keys. One could imagine more complex semantics for allowing name registration — e.g., proofs of work, proofs of payment, TPM attestations, publicly verifiable proofs of storage and retrievability of files [49] — supporting

¹Conceptually the TPM’s EK can sign a statement and forgo any interactive issuing process. The TPM 1.1 spec places an arbitrary restriction against using the EK RSA key for signing.

²In practice, CA’s usually verify that the bearer controls some administrator email such as admin@domain or webmaster@domain.

more sophisticated functionality than simple DNS.

A. Overview of Our Construction

We now provide a brief overview for our construction, which is inspired by the electronic cash proposals of Sander and Ta-Shma [47] and Miers et al. [38].

Issuing and showing credentials. The ability to establish identities and bind them to a public key ensures that users can assert their identity in a non-anonymous fashion, simply by issuing signatures from the corresponding secret key. Unfortunately, this does not immediately show us how to construct anonymous credentials, since traditional anonymous credentials consist of a signature computed by a credential issuer. Since no central party exists to compute the credential signature, this approach does not seem feasible without elaborate (and inefficient) use of threshold cryptography.³

We instead take a different approach. To issue a new credential in our decentralized system, the user establishes an identity and related attributes as described above. She then attaches a vector commitment to her secret key sk_U along with the identity and attribute strings that are contained within her identity assertion. Finally, she includes a non-interactive proof that the credential is correctly constructed, i.e., that the attributes in the commitment correspond to those revealed in the identity assertion. The network will accept the identity assertion if and only if the assertion is considered correct and the attached proof is valid.

At a later point an individual can prove possession of such a credential by proving the following two statements in zero-knowledge:

- 1) She knows a commitment C_i in the set (C_1, \dots, C_N) of all credentials previously accepted to the block chain.
- 2) She knows the opening (randomness) for the commitment.

In addition to this proof, the user may simultaneously prove additional statements about the identity and attributes contained within the commitment C_i . The challenge in the above construction is to efficiently prove statements (1) and (2), i.e., without producing a proof that scales with N . Our solution, which adapts techniques from distributed e-cash systems [38], circumvents this problem by using an efficient publicly-verifiable accumulator [15] to gather the set of all previous commitments together. Using this accumulator in combination with an efficient membership proof due to Camenisch and Lysyanskaya [16], we are able to reduce the size of this proof to $O(\lambda)$ for security parameter λ , rather than the $O(N \cdot \lambda)$ proofs that would result from a naive OR proof.

Of course, merely applying these techniques does not lead to a *practical* credential system. A key contribution of this work is to supply a concrete instantiation of the above idea under well-studied assumptions and to prove that our construction provides for consistency of credentials (ensuring multiple users cannot

pool their credentials), the establishment of pseudonyms, and a long set of extensions built upon anonymous credentials. Last but not least, we need to formally define and prove the security of a distributed anonymous credential scheme and provide some model for the distributed ledger. Our instantiation requires a single trusted setup phase, after which the trusted party is no longer required.⁴

B. Outline of This Work

The remainder of this work is organized as follows. In the next section we discuss how to get a distributed bulletin board. In §III we discuss specific applications for decentralized anonymous credentials and argue that these systems can be used to solve a variety of problems in peer-to-peer networks. In §IV we define the notion of a *decentralized anonymous credential scheme* and provide an ideal-world security definition. In §V we describe the cryptographic building blocks of our construction, and in §VI we provide an overview of our basic construction as well as a specific instantiation based on the Discrete Logarithm and Strong RSA assumptions. In §VII we extend our basic construction to add a variety of useful features, including k -show credentials, stateful credentials, and credentials with hidden attributes. In §VIII we describe the implementation and performance of a prototype library realizing our credential system. Finally, in §IX, we show how to use our library to build a distributed version of anonymous attestation.

II. REAL-WORLD BULLETIN BOARDS AND DECENTRALIZED BULLETIN BOARDS

A core component of our system is an append-only bulletin board we can use to post issued credentials. The board must provide two strong security guarantees: (1) that credentials must not be tampered with once added to the board and (2) all parties will share a consistent view of the board. For the distributed instantiation we additionally require (3) no party can control the addition of credentials to the board. We detail ways to achieve both distributed and centralized versions of such a bulletin board here.

A. Bitcoin

Bitcoin is a distributed currency system [39], which has grown since 2009 to handle between \$2–\$5 million USD/day in transaction volume in a highly adversarial environment. The heart of Bitcoin is the block chain, which serves as an append-only bulletin board maintained in a distributed fashion by the Bitcoin peers. The block chain consists of a series of blocks connected in a hash chain.⁵ Every Bitcoin block memorializes a set of transactions (containing an amount of bitcoin, a sender, and a recipient) that are collected from the Bitcoin broadcast network. Thus the network maintains a consensus about what transactions have occurred and how much money each user has.

Bitcoin peers, who are free to enter and leave the network, compete to generate the next block by trying to calculate

³A possibility is to use ring signatures [46], which do not require a single trusted signer. Unfortunately, these signatures grow with the number of participating signers and require expensive communication to generate.

⁴In §VII we discuss techniques for removing this trusted setup requirement.

⁵For efficiency reasons, the hash chain is actually a Merkle Tree.

$\mathcal{H}(\text{block} \parallel \text{nonce}) < t$ where \mathcal{H} is a secure hash function and t is an adjustable parameter. This process is known as *mining*, and the difficulty level t is adjusted so that a block is created on average every 10 minutes. When a block is generated, it is broadcast to the network and, if valid, accepted as the next entry in the block chain. Bitcoin and related systems provide two incentives to miners: (1) mining a block (i.e., completing the proof of work) entitles them to a reward⁶ and (2) nodes can collect fees from every transaction in a block they mine.

While Bitcoin uses the hash chain for the specific purpose of implementing an electronic currency, the usefulness of the Bitcoin bulletin board has already been recognized by several related applications. One spinoff of the Bitcoin concept is Namecoin [40], a fork of Bitcoin that uses the block chain to maintain key–value mappings. Namecoin is currently being used to implement an experimental DNS replacement, dotBIT [29]. Users pay a small fee to register a key–value pair along with a controlling public key. They can then make updates to the pair provided (1) the updates are signed by that key and (2) if necessary, they pay a transaction fee.⁷ Due to this flexibility we use the Namecoin software in our implementations, but we stress that the same techniques can be used with nearly any hash chain based network, including mature deployments such as Bitcoin.

Because of the way Bitcoin’s block chain is constructed, recently added blocks maybe be removed, and, more importantly, it is possible to introduce short-term forks in the block chain that could be used to convince a single party that a poisoned-pill credential was issued and hence identify them (see §IV-C for more details). One solution, which is commonly used in Bitcoin, is to wait until a block has several blocks on top of it (known as confirmations) before using it. Typically, waiting six blocks, or roughly 60 minutes, is sufficient. Of course, peers are free to show credentials based off blocks sooner than that as doing so does not make the show less secure. However it comes at an increased privacy risk.

B. A Central Ledger

An alternative to using Bitcoin’s block chain technology is to simply use a central service to maintain an append-only ledger. This service must be trusted to give a consistent view of the credential ledger to all parties. The most effective way to do this is with trusted hardware (e.g., TPM attestations) that ensures that (1) the list is append only and (2) the same version of the list is shown to everyone for a given time period.

For lower security systems, it may be possible to simply run a service that signs the list and have users audit the system by periodically comparing the list they received. Similar mechanisms exist for auditing SSL authorities (e.g., Google’s Certificate Transparency project). Tampering would not only be readily apparent but, due to the signature on the list, provable. This, however, only acts as a deterrent to tampering as it would not be detected until the next such comparison. As such tampering can identify users when they authenticate, we only

⁶For Bitcoin this reward is set at 25 BTC but will eventually diminish and be eliminated.

⁷Currently, neither Namecoin nor Bitcoin require significant transaction fees.

recommend this approach when either the consequences of such a privacy breach are low or loss of reputation to an authority when its malfeasance is subsequently detected is prohibitively high.

C. A Hybrid Approach

A third approach is to use some reliable storage mechanism (e.g., a central server or a robust DHT) to store credential requests and insert checkpoints into Bitcoin’s actual block chain to ensure the ledger is append only. This can be done *without* any modification to Bitcoin. We can achieve this by periodically (e.g., every 10 minutes) inserting the digest of the ledger into the Bitcoin block chain. One way to accomplish this is by using CommitCoin [23] which encodes information into the keys used for transactions without destroying funds.⁸

Our one last technical challenge is to actually mark these transactions as checkpoints for anyone to see. To accomplish this we propose leveraging multi–sig transactions⁹ where one key encodes the checkpoint with CommitCoin’s techniques and another is a marker address that designates a checkpoint. For a distributed storage service, this requires that the network elect a node or set of nodes to hold the marker key and insert checkpoints and elect a new set of nodes with a fresh marker if the current set either fails to insert checkpoints or inserts too many (either case is a denial of service attack and will not compromise the integrity or anonymity of credentials).

III. APPLICATIONS

In this section we discuss several of the applications facilitated by decentralized anonymous credentials. While we believe that these credential systems may have applications in a variety of environments, we focus specifically on settings where trusting a central credential issuer is not an option or where issued credentials must be publicly audited.

Mitigating Sybil attacks in ad hoc networks. Impersonation attacks can have grave consequences for both the security and resource allocation capabilities of ad hoc networks. A variety of solutions have been proposed to address this problem. One common approach is to require that clients solve computational puzzles [7]. For example, for a challenge c and a difficulty target t , find a nonce n such that $\mathcal{H}(c \parallel n) < t$. Solving such a puzzle takes a meaningful amount of effort — thus deterring Sybil attacks — and, as anyone can hash n and c , is publicly verifiable. For a centralized service, this proof can be done once per client on registration. In a peer-to-peer system, however, far more complex mechanisms are needed to avoid having to provide a proof of work per each pair of interacting peers [7]. We stress that the issue with distributed approaches is not the lack of publicly verifiable puzzles but the number of puzzles and who they are sent to. This is even more difficult if we require the system to be anonymous.

⁸The naive approach replaces the public key specifying the recipient with the hash of the data, making it impossible to retrieve the funds. CommitCoin fixes this.

⁹Transactions that require signatures from multiple parties to redeem.

Our solution to this problem is to use k -show anonymous credentials. In this approach, peers establish a single credential by solving a proof of work (similar to using a central service). This allows the peer to obtain a credential that can be used a limited number of times or a limited number of times within a given time period. When a peer exceeds the k -use threshold (e.g., by cloning the credential for a Sybil attack), the credential can be identified and revoked. We note that this proposal is a distributed variant of the *anonymous subscription service* concept, which was first explored by Damgård et al. [26] and Camenisch et al. [13].

Managing resource usage. In networks where peers both contribute and consume resources, ensuring fair resource utilization can be challenging. For example, a storage network might wish to ensure peers provide as much storage as they consume [42] or ensure that peers fairly use network bandwidth [43]. This can be problematic in networks that provide anonymity services (e.g., Tor), where peers may be reluctant to identify which traffic they originated. An anonymous credential system allows peers to identify their contributions to routing traffic in exchange for a credential which they can then use to originate traffic. Of course, we are restricted to issuing credentials on metrics which peers can publicly establish. Thankfully this is a fairly expressive set. Eigenspeed [50] allows peer-to-peer networks to form accurate bandwidth estimates for all peers even in the presence of active attackers. Similarly, there exist publicly verifiable proofs of retrievability that can be used to verify storage of a file [49]. Both of these are effective metrics for resource management.

IV. DECENTRALIZED ANONYMOUS CREDENTIALS

A traditional anonymous credential system has two types of participants: users and organizations. Users, who each have a secret key sk_U , are known by pseudonyms both to each other and organizations. Nym_A^O , for example, is the pseudonym of user A to organization O . Decentralized anonymous credentials have no single party representing the organization. Instead, this party is replaced with a quorum of users who enforce a specific *credential issuing policy* and collaboratively maintain a list of credentials thus far issued. For consistency with prior work, we retain the term “organization” for this group.

A distributed anonymous credential system consists of a global transaction ledger, a set of transaction semantics, as well as the following (possibly probabilistic) algorithms:

- $\text{Setup}(1^\lambda) \rightarrow params$. Generates the system parameters.
- $\text{KeyGen}(params) \rightarrow sk_U$. Run by a user to generate her secret key.
- $\text{FormNym}(params, U, E, sk_U) \rightarrow (Nym_U^E, sk_{Nym_U^E})$. Run by a user to generate a pseudonym Nym_U^E and an authentication key $sk_{Nym_U^E}$ between a user U and some entity (either a user or an organization) E .
- $\text{MintCred}(params, sk_U, Nym_U^O, sk_{Nym_U^O}, attrs, aux) \rightarrow (c, sk_c, \pi_M)$. Run by a user to generate a request for a credential from organization O . The request consists of a candidate credential c containing public attributes $attrs$; the user’s key sk_U ; auxiliary data aux justifying

the granting of the credential; and a proof π_M that (1) Nym_U^O was issued to the same sk_U and (2) the credential embeds $attrs$.

- $\text{MintVerify}(params, c, Nym_U^O, aux, \pi_M) \rightarrow \{0, 1\}$. Run by nodes in the organization to validate a credential. Returns 1 if π_M is valid, 0 otherwise.
- $\text{Show}(params, sk_U, Nym_U^V, sk_{Nym_U^V}, c, sk_c, C_O) \rightarrow \pi_S$. Run by a user to non-interactively prove that a given set of attributes are in a credential c in the set of issued credentials C_O and that c was issued to the same person who owns Nym_U^V . Generates and returns a proof π_S .
- $\text{ShowVerify}(params, Nym_U^V, \pi_S, C_O) \rightarrow \{0, 1\}$. Run by a verifier to validate a shown credential. Return 1 if π_S is valid for Nym_U^V , 0 otherwise.

We now describe how these algorithms are used in the context of an anonymous credential system.

A. Overview of the Protocol Semantics

To realize the full anonymous credential system, we integrate the above algorithms with a decentralized hash chain based bulletin board as follows. We assume a bulletin board such as Namecoin that provides a means for storing arbitrary key–value pairs.¹⁰ We provide a concrete realization of our protocols in §VI and §VIII.

Formulating a pseudonym. Prior to requesting a new credential, the user executes the KeyGen algorithm to obtain sk_U and then runs the FormNym algorithm to obtain a pseudonym for use with this organization. This requires no interaction with the bulletin board, hence the user can perform these actions offline.

Obtaining a credential. To obtain a credential, the user places the organization name and some public identity assertion — for example, a TPM attestation and AIK certificate chain — into the auxiliary data field aux , then executes the MintCred routine to obtain a credential and a signature of knowledge on that information. She then formulates a transaction including both the resulting credential and the auxiliary data and broadcasts it into the hash chain network, along with (optionally) some sum of digital currency to pay for the transaction fees. She retains the secret portion of the credential.

Once received by the network, all parties can verify the correctness of the credential and the identity assertion using the MintVerify routine and whatever external procedures are needed to verify the auxiliary data. This process can be conducted directly by the network nodes, or it can be validated after the fact by individual credential verifiers.

Showing a credential. When a user wishes to show a credential to some Verifier, she first scans through the bulletin board to obtain a set C_O consisting of all candidate credentials belonging to a specific organization. She next verifies each credential using the MintVerify routine (if she has not already done so) and validates the auxiliary identity certification information. She

¹⁰While this functionality is supported by default in Namecoin, it is also possible to store arbitrary data in existing block chains such as the Bitcoin chain.

- $RegNym(Nym_U^O, U, O)$: U logs into T_P with sk_U to register a nym with organization O . If she does not have an account, she first creates one. She gives T_P a unique random string Nym_U^O for use as her nym with O . T_P checks that the string is indeed unique and if so stores (Nym_U^O, U, O) and informs U .
- $MintCred(Nym_U^O, O, attrs, aux)$: U logs into T_P authenticating with sk_U . If Nym_U^O is not U 's nym with O or sk_U is wrong, reject. Otherwise, T_P checks that aux justifies issuing a credential under O 's issuing policy and if so generates a unique random id ID and stores $(Nym_U^O, U, ID, attrs)$. It then adds ID to its public list of issued credentials for O .
- $ShowOnNym(Nym_U^O, Nym_V^V, O, V, attrs, C)$: U logs into T_P with sk_U . If Nym_U^O is not U 's nym with O or Nym_V^V is not U 's nym with V , reject. Else, T_P checks if the tuple (Nym_U^O, U) exists, if ID associated with that tuple is in the set of credentials C that U provided, and if the given attributes $attrs$ match the attributes associated with that tuple. If all conditions hold, T_P informs V that Nym_V^V has a credential from O in the set C . V then retrieves the set of credentials C_O issued by O from T_P and accepts T_P 's assertion if and only if $C \subseteq C_O$ and O 's issuing policy is valid $\forall c' \in C_O$.
- $GetCredList(O)$: T_P retrieves the list of credentials for organization O and returns it.

Fig. 1: Ideal Functionality. Security of a basic distributed anonymous credential system.

then runs the Show algorithm to generate a credential, which she transmits directly to the Verifier. The Verifier also collects the set of credentials in C_O and validates the credential using the ShowVerify routine. She accepts the credential certification if this routine outputs 1.

B. Security

We define our system in terms of an ideal functionality implemented by a trusted party T_P that plays the role that our cryptographic constructions play in the real system. All communication takes place through this ideal trusted party. Security and correctness for our system comes from a proof that this ideal model is indistinguishable from the real model provided the cryptographic assumptions hold. Our ideal functionality is outlined in Figure 1.

It consists of organizations who issue credentials and users who both prove that they have these credentials and verify such proofs. Organizations have only two things: 1) an efficient and publicly evaluable policy, $policy_O$, for granting credentials and 2) an append-only list of credentials meeting that policy maintained by the trusted party.

C. Trusting the Ledger

An obvious question is whether the append-only transaction ledger is necessary at all. Indeed, if the list of valid credentials can be evaluated by a set of untrusted nodes, then it seems that a user (Prover) could simply maintain a credential list compiled from network broadcasts and provide this list to the Verifier during a credential show. However, this approach can enable sophisticated attacks where a malicious Verifier manipulates the Prover's view of the network to include a poisoned-pill credential that — although valid by the issuing heuristic — was not broadcast to anyone else. When the Prover authenticates, she has completely identified herself.

The distributed transaction ledgers employed by networks such as Bitcoin and Namecoin provide a solution to this problem, as their primary purpose is to ensure a shared view among a large number of nodes in an adversarial network. In practice this is accomplished by maintaining a high degree of network connectivity and employing computational *proofs of work* to compute a hash chain.

For an attacker to execute the poisoned credential attack against such a ledger, she would need to both generate and maintain a false view of the network to delude the Prover. This entails both simulating the Prover's view of the rest of the network complete with *all* its computational power and forging any assurances the Prover might expect from known peers about the present state of the network. If the Prover has a reasonable estimate of the actual network's power (e.g., she assumes it monotonically increases), then an attacker must actually have equivalent computational power to the entirety of the network to mount such an attack. For the purposes of this paper we assume such active attacks are impossible even if the attacker controls a simple majority of the computational power. Attackers are still free to attempt any and all methods of retroactively identifying a user and mount any other active attacks.

V. PRELIMINARIES

We make use of the following complexity assumptions and cryptographic building blocks to construct our scheme.

A. Complexity Assumptions

The security of our scheme relies on the following two complexity assumptions:

Strong RSA Assumption [3, 32]. Given a randomly generated RSA modulus n and a random element $y \in \mathbb{Z}_n^*$, it is hard to compute $x \in \mathbb{Z}_n^*$ and integer exponent $e > 1$ such that $x^e \equiv y \pmod n$. We can restrict the RSA modulus to those of the form pq , where $p = 2p' + 1$ and $q = 2q' + 1$ are safe primes.

Discrete Logarithm (DL) Assumption [27]. Let G be a cyclic group with generator g . Given $h \in G$, it is hard to compute x such that $h = g^x$.

B. Cryptographic Building Blocks

Zero-knowledge proofs. In a zero-knowledge protocol [34] a user (the prover) proves a statement to another party (the verifier) without revealing anything about the statement other than that it is true. Our constructions use zero-knowledge proofs

that can be instantiated using the technique of Schnorr [48], with extensions due to, e.g., [9, 18, 20, 25]. We convert these into *non-interactive* proofs by applying the Fiat-Shamir heuristic [31]. When we use these proofs to authenticate auxiliary data, we refer to the resulting non-interactive proofs as *signatures of knowledge* as defined in [21].

When referring to these proofs we will use the notation of Camenisch and Stadler [19]. For instance, $\text{NIZKPoK}\{(x, y) : h = g^x \wedge c = g^y\}$ denotes a non-interactive zero-knowledge proof of knowledge of the elements x and y that satisfy both $h = g^x$ and $c = g^y$. All values not enclosed in $()$'s are assumed to be known to the verifier. Similarly, the extension $\text{ZKSoK}[m]\{(x, y) : h = g^x \wedge c = g^y\}$ indicates a *signature of knowledge* on message m .

Accumulators [38]. An accumulator allows us to combine many values into one smaller value (the accumulator). We then have a single element, called the witness, that allows us to attest to the fact that a given value is actually part of the accumulator. Our constructions use an accumulator based on the Strong RSA assumption. The accumulator we use was first proposed by Benaloh and de Mare [6] and later improved by Baric and Pfitzmann [3] and Camenisch and Lysyanskaya [15]. We describe the accumulator using the following algorithms:

- $\text{AccumSetup}(\lambda) \rightarrow \text{params}$. On input a security parameter, sample primes p, q (with polynomial dependence on the security parameter), compute $N = pq$, and sample a seed value $u \in QR_N, u \neq 1$. Output (N, u) as *params*.
- $\text{Accumulate}(\text{params}, \mathbf{C}) \rightarrow A$. On input *params* (N, u) and a set of prime numbers $\mathbf{C} = \{c_1, \dots, c_i \mid c \in [\mathcal{A}, \mathcal{B}]\}$,¹¹ compute the accumulator A as $u^{c_1 c_2 \dots c_n} \bmod N$.
- $\text{GenWitness}(\text{params}, v, \mathbf{C}) \rightarrow \omega$. On input *params* (N, u) , a set of prime numbers \mathbf{C} as described above, and a value $v \in \mathbf{C}$, the witness ω is the accumulation of all the values in \mathbf{C} besides v , i.e., $\omega = \text{Accumulate}(\text{params}, \mathbf{C} \setminus \{v\})$.
- $\text{AccVerify}(\text{params}, A, v, \omega) \rightarrow \{0, 1\}$. On input *params* (N, u) , an element v , and witness ω , compute $A' \equiv \omega^v \bmod N$ and output 1 if and only if $A' = A$, v is prime, and $v \in [\mathcal{A}, \mathcal{B}]$ as defined previously.

For simplicity, the description above uses the full calculation of A . Camenisch and Lysyanskaya [15] observe that the accumulator may also be *incrementally* updated, i.e., given an existing accumulator A_n it is possible to add an element x and produce a new accumulator value A_{n+1} by computing $A_{n+1} = A_n^x \bmod N$.¹²

Camenisch and Lysyanskaya [15] show that the accumulator satisfies a strong *collision-resistance* property if the Strong RSA assumption is hard. Informally, this ensures that no *p.p.t.* adversary can produce a pair (v, ω) such that $v \notin \mathbf{C}$ and yet

¹¹“Where \mathcal{A} and \mathcal{B} can be chosen with arbitrary polynomial dependence on the security parameter, as long as $2 < \mathcal{A}$ and $\mathcal{B} < \mathcal{A}^2$.” [16] For a full description, see [16, §3.2 and §3.3].

¹²This allows the network to maintain a running value of the accumulator and prevents individual nodes from having to recompute it [38].

AccVerify is satisfied. Additionally, they describe an efficient zero-knowledge proof of knowledge that a committed value is in an accumulator. We convert this into a non-interactive proof using the Fiat-Shamir transform and refer to the resulting proof using the following notation:

$$\text{NIZKPoK}\{(v, \omega) : \text{AccVerify}((N, u), A, v, \omega) = 1\}.$$

Verifiable Random Functions. A pseudorandom function (PRF) [33] is an efficiently computable function whose output cannot be distinguished (with non-negligible advantage) from random by a computationally bounded adversary. We denote the pseudorandom function as $f_k(\cdot)$, where k is a randomly chosen key. A number of PRFs possess efficient proofs that a value is the output of a PRF on a set of related public parameters. Two examples of this are the Dodis-Yampolskiy (DY) PRF [28] and the Naor-Reingold PRF [41].

Pedersen Commitments. A commitment scheme allows a user to bind herself to a chosen value without revealing that value to the recipient of the commitment. This commitment to the value ensures that the user cannot change her choice (i.e., *binding*), while simultaneously ensuring that the recipient of the commitment does not learn anything about the value it contains (i.e., *hiding*) [24]. In Pedersen commitments [45], the public parameters are a group G of prime order q , and generators (g_0, \dots, g_m) . In order to commit to the values $(v_1, \dots, v_m) \in \mathbb{Z}_q^m$, pick a random $r \in \mathbb{Z}_q$ and set $C = \text{PedCom}(v_1, \dots, v_m; r) = g_0^r \prod_{i=1}^m g_i^{v_i}$.

VI. A CONCRETE INSTANTIATION

We now provide a concrete instantiation of our construction and prove the security of our construction under the Discrete Logarithm and Strong RSA assumptions.

A. Overview of the Construction

Alice’s pseudonym with a given organization/user is an arbitrary identity that she claims in a transaction. She tags this value with a Pedersen commitment to her secret key sk and *signs* the resulting transaction using a signature of knowledge that she knows the secret key. There is no separate process for registering a pseudonym: instead they are simply used in issue and show to allow operations to be linked if necessary. Alice’s credential c is a vector Pedersen commitment to both sk and a set of public attributes $\text{attrs} = a_0, \dots, a_m$, which Alice also includes in her credential. To issue a credential, Alice provides the network with a credential, a pseudonym, her attributes, optionally some auxiliary data justifying the credential issue (e.g., a proof of work that Alice is not a Sybil), and a proof that (1) the commitment and the pseudonym contain the same secret key and (2) the attributes are in some allowed set. If all of this validates, the entry is added to the ledger. Alice shows the credential under a different pseudonym by proving in zero-knowledge that (1) she knows a credential on the ledger from the organization, (2) the credential opens to the same sk as her pseudonym, and (3) it has some attributes.

- $\text{Setup}(1^\lambda) \rightarrow \text{params}$. On input a security parameter λ , run $\text{AccumSetup}(1^\lambda)$ to obtain the values (N, u) . Next generate primes p, q such that $p = 2^w q + 1$ for $w \geq 1$. Let \mathbb{G} be an order- q subgroup of \mathbb{Z}_p^* , and select random generators g_0, \dots, g_n such that $\mathbb{G} = \langle g_0 \rangle = \dots = \langle g_n \rangle$. Output $\text{params} = (N, u, p, q, g_0, \dots, g_n)$.
- $\text{KeyGen}(\text{params}) \rightarrow sk$. On input a set of parameters params , select and output a random master secret $sk \in \mathbb{Z}_q$.
- $\text{FormNym}(\text{params}, sk) \rightarrow (Nym, sk_{Nym})$. Given a user's master secret sk , select a random $r \in \mathbb{Z}_q$ and compute $Nym = g_0^r g_1^{sk}$. Set $sk_{Nym} = r$ and output (Nym, sk_{Nym}) .
- $\text{MintCred}(\text{params}, sk, Nym_U^O, sk_{Nym_U^O}, \text{attrs}, \text{aux}) \rightarrow (c, sk_c, \pi_M)$. Given a nym Nym_U^O and its secret key $sk_{Nym_U^O}$; attributes $\text{attrs} = (a_0, \dots, a_m) \in \mathbb{Z}_q$; and auxiliary data aux , select a random $r' \in \mathbb{Z}_q$ and compute $c = g_0^{r'} g_1^{sk} \prod_{i=0}^m g_{i+2}^{a_i}$ such that $\{c \text{ prime} \mid c \in [\mathcal{A}, \mathcal{B}]\}$.¹³ Set $sk_c = r'$ and output (c, sk_c, π_M) where π_M is a signature of knowledge on aux that the nym and the credential both belong to the same master secret sk , i.e.:

$$\pi_M = \text{ZKSoK}[\text{aux}]\{(sk, r', r) : \\ c = g_0^{r'} g_1^{sk} \prod_{i=0}^m g_{i+2}^{a_i} \wedge Nym_U^O = g_0^r g_1^{sk}\}$$

Finally, submit the resulting values $(c, \pi_M, \text{attrs}, Nym_U^O, \text{aux})$ to the public transaction ledger.

- $\text{MintVerify}(\text{params}, c, \text{attrs}, Nym_U^O, \text{aux}, \pi_M) \rightarrow \{0, 1\}$. Given a credential c , attributes attrs , a nym Nym_U^O , and proof π_M , verify that π_M is the signature of knowledge on aux . If the proof verifies successfully, output 1, otherwise output 0. The organization nodes should accept the credential to the ledger if and only if this algorithm returns 1.
- $\text{Show}(\text{params}, sk, Nym_U^V, sk_{Nym_U^V}, c, \text{attrs}, sk_c, \mathbf{C}_O) \rightarrow \pi_S$. Given a user's master secret sk ; a nym Nym_U^V between the user and the verifier and its secret key $sk_{Nym_U^V}$; a credential c and its secret key sk_c ; the attributes (a_0, \dots, a_m) used in the credential; and a set of credentials \mathbf{C} , compute $A = \text{Accumulate}(\text{params}, \mathbf{C}_O)$ and $\omega = \text{GenWitness}(\text{params}, c, \mathbf{C}_O)$ and output the following proof of knowledge:

$$\pi_S = \text{NIZKPoK}\{(sk, \omega, r', c, r, Nym_U^V) : \\ \text{AccVerify}(\text{params}, A, c, \omega) = 1 \wedge c = g_0^{r'} g_1^{sk} \prod_{i=0}^m g_{i+2}^{a_i} \wedge Nym_U^V = g_0^r g_1^{sk}\}$$

- $\text{ShowVerify}(\text{params}, Nym_U^V, \pi_S, \mathbf{C}_O) \rightarrow \{0, 1\}$. Given a nym Nym_U^V , proof of possession of a credential π_S , and the set of credentials issued by organization O \mathbf{C}_O , first compute $A = \text{Accumulate}(\text{params}, \mathbf{C}_O)$. Then verify that π_S is the aforementioned proof of knowledge on c , \mathbf{C}_O , and Nym_U^V using the known public values. If the proof verifies successfully, output 1, otherwise output 0.

Fig. 2: Our basic decentralized anonymous credential scheme.

B. The Construction

The full construction is provided in Figure 2. We use Pedersen commitments and a Strong RSA based accumulator to instantiate the core of the protocol. The proofs of knowledge in the Show algorithm are conducted using Schnorr-style proofs modified using the Fiat-Shamir heuristic as in previous work [15, 48]. The implementation of the proofs are similar to those used by Miers et al. in [38].

Theorem 6.1: The basic distributed anonymous credential system described in Figure 2 is secure in the random oracle model under the Strong RSA and the Discrete Logarithm assumptions.

We provide a sketch of the proof of Theorem 6.1 in Appendix A.

VII. EXTENSIONS

We consider extending the basic system in several ways.

A. k -show Credentials

Damgård et al. [26] first suggested a credential system where users could only authenticate once per time period. Camenisch et al. [13] independently proposed a significantly more efficient construction that allows for up to k authentications per time period, with the ability to revoke all cloned credentials if a credential was used beyond this limit. Camenisch et al. suggested that these techniques might be used to build anonymous subscription services, allowing users to access a resource (such as a website) within reasonable bounds. We briefly show that these same techniques can be applied to our basic credential system.

In the system of [13] an authority issues a credential on a user's secret seed s . To show a credential for the i^{th} time in validity period t , the user generates a serial number S using a verifiable random function (VRF) as $S = f_s(0 || t || i)$. She also includes a non-interactive zero-knowledge proof that this serial number is correctly structured.¹⁴ This technique can be applied

¹³“Where \mathcal{A} and \mathcal{B} can be chosen with arbitrary polynomial dependence on the security parameter, as long as $2 < \mathcal{A}$ and $\mathcal{B} < \mathcal{A}^2$.” [16]. For a full description, see [16, §3.2 and §3.3].

¹⁴The re-use of a credential would result in a repeated serial number, and yet the nature of the VRF's output (for an honest user) ensures that attackers cannot link individual shows.

- $\text{Update}(params, sk, c, sk_c, \mathbf{C}_O, \text{update_relation}, state') \rightarrow (c', sk'_c, \pi_u)$. Given a credential c and associated secret key sk_c , a set of credentials \mathbf{C}_O , an updated state $state' = (s'_0, \dots, s'_m) \in \mathbb{Z}_q$, and an update relation update_relation , generate a fresh random serial number $S' \in \mathbb{Z}_q$ and random value $r' \in \mathbb{Z}_q$ to form a new credential $c' = g_0^{r'} g_1^{sk} g_2^{S'} \prod_{i=0}^m g_{i+3}^{s'_i}$ with the aforementioned restrictions. Compute $A = \text{Accumulate}(params, \mathbf{C}_O)$ and $\omega = \text{GenWitness}(params, c, \mathbf{C}_O)$. Output (c', sk'_c, π_u) where $sk'_c = (S', state', r')$ and

$$\begin{aligned} \pi_u &= \text{NIZKPoK}\{(sk, \omega, c, state, r, c', S', state', r') : \\ &\text{AccVerify}(params, A, c, \omega) = 1 \\ &\wedge c = g_0^{r'} g_1^{sk} g_2^S \prod_{i=0}^m g_{i+3}^{s_i} \wedge c' = g_0^{r'} g_1^{sk} g_2^{S'} \prod_{i=0}^m g_{i+3}^{s'_i} \\ &\wedge \text{update_relation}(state, state') = 1\} \end{aligned}$$

- $\text{UpdateVerify}(params, c, \mathbf{C}_O, \pi_u) \rightarrow \{0, 1\}$. Given a stateful credential c , a credential set \mathbf{C}_O , and proof π_u , output 1 if π_u is correct, the proved state transition is a legal one, and the serial number S was not previously used. Otherwise 0.

Fig. 3: Extensions for a stateful anonymous credential system. $\text{update_relation}(\dots) = 1$ denotes that the update encodes some arbitrary state transition (e.g. $\forall i s'_i = s_i + 1$).

to our construction provided we can securely store a seed for the VRF. This is easy: the user simply generates a random seed s and includes this value in the commitment she stores in the transaction ledger. We note that for the trivial case of one-time show credentials, we can simply reveal the seed. For k -show, the user provably evaluates the VRF on the seed plus a secret counter.¹⁵

B. Credentials with Hidden Attributes

In our basic construction of §VI, users provide a full list of attributes when requesting and showing credentials. While this is sufficient for many applications, there exist cases where a user might wish to conceal the attributes requested or shown, opting instead to prove statements about them, e.g., proving knowledge of a secret key or proving that an attribute is within a certain range. There are two simple ways to do this. First, we can simply use multi-message commitments where each message is an attribute. This increases the size of our zero-knowledge proofs (they are linear in the number of messages in a commitment) but does not change our schemes. A more efficient construction is to encode the attributes in one single value and then prove statements about that committed value rather than reveal it. For example, one could prove that a given bit corresponding to a certain attribute was set. One could also use the first x bits for attribute one, the next x bits for attribute two, etc. and use range proofs [8, 12, 35, 37] to reveal only those attributes we want to display.

C. Stateful Credentials

A stateful anonymous credential system [24] is a variant of an anonymous credential system where credential attributes encode some state that can be updated by issuing new credentials. This credential issuance is typically conditioned on the user showing a previous credential and offering proof

that the new credential should be updated as a function of the original.

Intuitively, we can already have this capability quite easily due to the fact that our credentials are non-interactively issued. We can make stateful credentials simply by changing the policy by which we issue credentials: to issue a credential in a new state s_1 , we require a user to demonstrate that they had a credential in state s_0 and discard it by revealing its single use serial number.

We construct a “single show” credential c embedding some state $state$ in the attributes and a serial number S . Users are free to show c as many times as they like without revealing the serial number. However, to update the state of the credential, they must author a transaction that shows the original credential and reveals the serial number S and “mint” a new candidate credential c' containing the updated state $state'$ (hidden inside of a commitment) and a proof that there exists a valid relationship between the state encoded in c and the new state in c' (for example, that the attributes have been incremented).

This requires only minor extensions to our basic scheme composing the existing secure functionality. In this case we add an Update algorithm that operates similarly to MintCred but includes the earlier credential and a proof of its construction. A valid proof of the existing credential now becomes a condition for the organization accepting the updated credential into the ledger. We provide a description of this new algorithm in Figure 3.

VIII. INTEGRATING WITH PROOF-OF-WORK BULLETIN BOARDS

We provide a basic implementation of our credential scheme as a library and construct a basic example using Namecoin as the bulletin board. Our prototype system allows users to prove they have a (fresh) commitment to some attributes in an issued credential. For our purposes it is sufficient to merely reveal the content of that commitment (the attributes) in its entirety

¹⁵Camenisch et al. [13] describe a further extension that reveals the user’s identity in the event of a credential double-show. We omit the details here for space reasons but observe that the same technique can be applied to our construction.

during a show. However, selectively disclosable attributes are trivially realizable, see §VII-B.

A. Integration

Namecoin integration is straightforward. Namecoin provides a built in mechanism for storing key-value pairs which, by convention, have a namespace as a prefix. It also provides a basic functionality to scan the list of existing names. Thus we can scan for credentials, validate them, and then accumulate them. It is then simply matter of generating and verifying proofs against that computed accumulator value.

For Alice to obtain a credential, she:

- 1) Pays a very small fee (currently 0.0064 USD) to purchase some name in the system's namespace by registering a public key as the owner of the name. This corresponds to a transaction looking like:

```
1 665a... OP_2DROP
OP_DUP OP_HASH160 6c1abe34
OP_EQUALVERIFY OP_CHECKSIG
```

- 2) Prepares a fresh credential with some attributes and any supporting documentation necessary for her identity claim and stores the private portion of the credential.
- 3) Updates, using the public key from step 1, her registered name to contain a credential and its supporting documentation.

```
2 642f7... 7b...
OP_2DROP OP_2DROP
OP_DUP OP_HASH160
14d... OP_EQUALVERIFY OP_CHECKSIG
```

Once this update is confirmed, Alice has a fully formed credential.

To show the credential to Bob, Alice:

- 1) Scans through the list of added names and retrieves all candidate credentials.
- 2) Checks the supporting documentation for each candidate and puts valid ones in *C*.
- 3) Runs Show with the public parameters, the private portion of her credentials, and *C* and sends the result to Bob.
- 4) Bob does steps 1 and 2 and computes *C* himself.
- 5) Bob runs ShowVerify on Alice's supplied credential and *C* to verify it.

Alice has now proved she has a credential to Bob.

What the supporting documentation is and how it is verified is an application specific problem. For some applications, merely having paid the tiny registration fee may be sufficient and no verification is necessary. For others, some digital signature may need to be verified or some assertion about resource management (e.g., a proof of storage/retrievability) may need to be verified. Without modifications to Namecoin/Bitcoin, any assertion must be verifiable by all participants.¹⁶ We consider one such application in the next section.

¹⁶With modifications, identity assertions can be validated as part of the consensus protocol, abrogating relying parties from validating credential issue and allowing the use of ephemeral supporting documentation.

B. Operating Cost

Namecoin is not free to use as purchasing a name costs a small (less than 0.10 USD as of 12/1/2013) amount of money. This fee is necessary both to prevent mass name hoarding and to provide an economy to pay the miners who maintain the block chain. This cost must minimally be paid by users when creating a credential. For certain applications (e.g., *k*-anonymous credentials), relying parties must also post data on the block chain (e.g., double spend tags and serial numbers). This, again, costs a small fee. As such, there are monetary costs to using such an identity scheme.

C. Latency

A third consideration for the limited show credentials is the latency of inserting items into the block chain. Because completely meaningful proofs of work take time, some time must elapse in any such system. Namecoin and Bitcoin both aim to create blocks every 10 minutes. Thus, the naive wait time from a block is about 5 minutes. Propagation delays in the network and transaction volume, however, skew this distribution. While historical data for Namecoin is not available, for Bitcoin it takes slightly less than 9 minutes for a transaction to first be confirmed. In practice, it then takes multiple confirmations to solidify the transaction's place in the block chain. Variants of Bitcoin operate with faster confirmation times (e.g., Feathercoin, which aims to get a block every 2.5 minutes), though it is not yet clear if the more aggressive of these are entirely stable.

Given these latency constraints, our system, at least built on top of proof of work based bulletin boards, is not suitable for applications that require fast credential issue or quick detection of multi-spends across mutually distrusting parties.¹⁷ A side effect of this is that double spend prevention mechanisms for fast transactions need to rely on detection and punishment (e.g., forfeiture of an escrowed value), not prevention.

D. Performance

We now examine the performance of our anonymous credential system. There are four underlying operations: minting a credential, verifying that the mint is correct, showing a credential, and verifying that show. Showing and verifying credentials also entail computing the accumulation of all or all but one of the current credentials. However, both the accumulator and the witnesses can be computed incrementally as credentials are added: for each added credential, the nodes must update both the accumulator and the witness for each credential they intend to show. Because this cost is both amortized for any individual credential show or verify, it does not come into play. Hence, we measure the accumulation cost separately and run our other benchmarks with a precomputed witness and accumulator. We also give measurements for our performance with different security parameters. See Figure 4.

All experiments were conducted on a 2010 MacPro with 16GB of RAM and two 2.4GHz quad core Xeon E5620 processors running OSX 10.8.3. Experiments were measured

¹⁷Obviously, parties could cooperate and maintain a faster store of double spend tags, alleviating this problem.

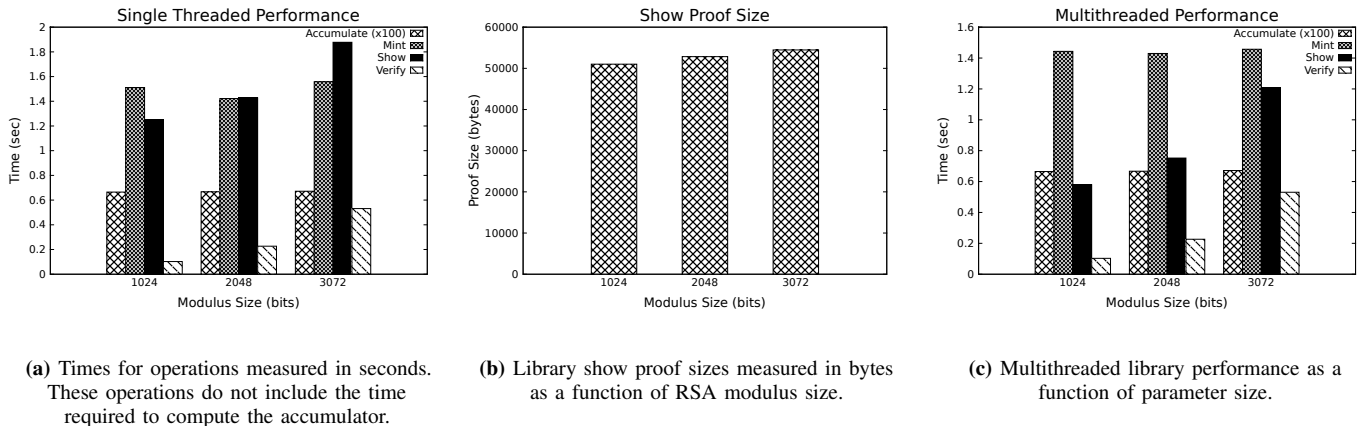


Fig. 4: Library performance as a function of parameter size.

in seconds via wall clock run time and were repeated for 500 iterations. Because of the speed of accumulating elements, we measure accumulator time in seconds per 100 accumulations.

The primary performance bottleneck for our library is the complexity of the proof of knowledge generated during the credential show. Because this double discrete logarithm proof uses cut-and-choose techniques, we need to perform between 80 and 128 iterations depending on the security parameter. This entails approximately 800-1000 exponentiations. Luckily, the same cryptographic requirements that force these iterations also mandate that such computations are independent and hence they can easily be parallelized. To exploit this, we make use of OpenMP to parallelize proof generation and verification. As shown in Figure 4c, this offers significant performance benefits.

Unfortunately, OpenSSL, which we use for the computations underpinning our system, is not fully parallelizable due to the fact that its PRNG is synchronous. The resulting locks around RNG usage prevent us from effectively parallelizing portions of our code for showing a credential. It also causes problems when minting a credential. The resource intensive portion of credential mint is creating commitments and then testing if they are prime. This requires random numbers both directly for commitment generation and indirectly for primality testing which uses randomized Miller-Rabin. We believe further performance gains could be realized by using a parallelizable RNG (e.g., Intel’s RDRand instruction).

IX. EXAMPLE APPLICATION: DISTRIBUTED DIRECT ANONYMOUS ATTESTATION (DDAA)

In the original TPM 1.1b specification [1], attestations are signed by a TPM’s Attestation Identity Key (AIK). Each TPM device can generate many AIKs, and prior to use each AIK public key is signed by a trusted third party called a Privacy CA, creating an AIK certification. The purpose of this awkward mechanism is to provide a layer of indirection between attestations and the manufacturer-specified keys programmed into the device, such as the permanent Endorsement Key (EK). By introducing a third party, it becomes possible to create many

identities and thus remove the risk that a given device could be linked to all of its endorsements.

Direct Anonymous Attestation (DAA) [11], replaced the privacy CA with a cryptographically sound group signature scheme. Instead of signing attestations with an AIK, a TPM signs attestations with a private key for a group signature scheme that preserves the signer’s anonymity. The TPM obtains the group signing key from a DAA authority by authenticating non-anonymously to the authority with the AIK. Because the group signature key is used anonymously, the authority can never link its use to the AIK key that caused its issue. Unfortunately, the integrity of this process depends fundamentally on the integrity of the software running in the DAA authority. This makes deploying a DAA instance somewhat problematic: each organization is responsible for deploying and securing this DAA authority, and any compromise of this server opens the door for credential theft or denial of service. Given the critical role envisioned for TPM attestations, this may inhibit the deployment of DAA systems.

We propose a new TPM design that retains the privacy advantages of DAA without the need to run a separate DAA server for each deployment. The advantage of our approach is that organizations may still maintain separate trust environments for private assertions by TPM devices but without the need to run a vulnerable separate server. Our solution requires one modification to the existing TPM infrastructure, namely that the TPM be updated to include a (non-anonymous) signing key, with a permission level similar to that of the Endorsement Key (EK). We will refer to this key as the Endorsement Signing Key, or ESK, and assume that each new TPM will ship with an Endorsement Signing Key Certificate signed by the manufacturer.¹⁸ Given this modification we show how to use our anonymous credential scheme as a replacement for DAA.

To obtain a credential in the new scheme, the TPM runs the MintCred routine, securely store the resulting sk and

¹⁸The TPM Endorsement Key and Endorsement Key Certificate would be sufficient for this role. However this key is limited by the specification to performing a decryption-only role, largely for privacy reasons. Our sole modification is to allow signing with this key, or a second key of a similar nature.

transmitting the resulting credential up to the block chain along with a signature under the TPM’s Endorsement Signing Key (ESK).¹⁹ This signature authenticates the credential as having been generated by a valid TPM. Once the credential and signature are validated, they can be accumulated by verifiers. The TPM can later attest to a particular configuration by running a modified version of Show that ensures π_S is a signature of knowledge on the attestation values (i.e., the program configuration registers (PCRs) and an optional nonce). Running Show/Verify with the appropriate modifications for checking the signature of knowledge validates the attestation. We stress that even though the TPM’s ESK (or AIK) is on the ledger with the issued credential, showing a credential never identifies which issued credential was used and hence does not identify the ESK (or AIK).

X. RELATED WORK

Anonymous credentials. Introduced by Chaum [22] and developed in a line of subsequent works (e.g., [10, 14, 17]), anonymous credentials allow a user to prove that she has a credential issued by some organization, without revealing anything about herself other than that she has the credential. Under standard security definitions, even if the verifier and credential issuer collude, they cannot determine when the credential was issued, who it was issued to, or when it was or will be used. A common construction involves issuing a credential by obtaining a signature from an organization on a committed value (e.g., using the signature scheme of [15]) then proving in zero-knowledge that one has a signature under the organization’s public key on that value. The contents of the commitment may be revealed outright or various properties can be proved on the committed values (e.g., Alice can prove she is over 21 years old). Extensions to this work describe credentials that can only be shown anonymously a limited number of times [13] or delegated to others [4]. All of these schemes require issuing organizations to maintain a secret key.

Bitcoin and append-only ledgers. Our construction relies on the existence of a *distributed append-only transaction ledger*, a technology that makes up the core component of the Bitcoin distributed currency: the log of all currency transactions called the block chain [39]. These ledgers are maintained by an ad hoc group of network nodes who are free to enter and leave the network (there is no key provisioning necessary for them to join). A typical transaction ledger consists of a sequence of blocks of data that are widely replicated among the participating nodes, with each block connected to the previous block using a hash chain. Nodes compete for the opportunity to add new blocks of transactions to the ledger by producing a partial hash collision over the new data and the hash of the last block in the chain. The hash collision serves two purposes: first, it is a computationally-difficult-to-forge authenticator of the ledger and second, since finding a partial hash collision involves substantial computational effort, the peer who finds it is chosen “at random” with a probability proportional to the rate at which

he can compute such partial collisions. As a result, an ad hoc group of mutually distrusting and potentially dishonest peers can correctly manage such a ledger provided that a majority of their computational power is held by honest parties. Recent experience with Bitcoin and Namecoin provides evidence that this assumption holds in practice.

Namecoin. Namecoin [40] is a decentralized identity system that uses the same block chain technology as Bitcoin. Namecoin’s primary purpose is to associate names with arbitrary data. A user can claim a name provided (1) they pay the price in NMC for it and (2) it is unclaimed. At that point, an entry is inserted into the block chain mapping the name to a public key and some arbitrary data. The public key allows the owner to update the data by signing a new record. The data allows for various uses. If it is an IP address, then one has a distributed DNS system (such a system, .bit, is already deployed). On the other hand, if it is a public key, the result is a basic PKI. The first-come first-served nature of Namecoin seems somewhat anachronistic, however it replicates in miniature the way normal DNS names are generally assigned, where the first person to claim the name gets it. Similarly, standard (non-extended validation) SSL certificates for a domain are typically issued to anyone who can demonstrate control of a domain (usually via an email to admin@domain).

XI. CONCLUSION

In this work we constructed a distributed anonymous credential system and several extensions. Our constructions are secure in the random oracle model under standard cryptographic assumptions provided there exists a trustworthy global append-only ledger. To realize such a ledger we propose using the block chain system already in real world use with the distributed cryptographic currency Bitcoin. Although we are limited in the class of identity assertions we can certify, we argue that several basic assertions are of particular use in peer-to-peer systems, as they can be used to mitigate Sybil attacks, ensure fair resource usage, and protect users’ anonymity while verifying their computer’s correctness.

Future work. We leave two open problems for future work. First, the proofs in this work assumed the security of a transaction ledger. We leave a precise formal model of the ledger, which attacks are allowable, and what bounds may be placed on their consequence as an open problem. Second, the efficiency of our construction can be improved. Although all of our algorithms are efficient (in that they do not scale with the size of the ledger), the need for double-discrete logarithm proofs leads to somewhat large proof sizes when showing a credential (roughly 50KB for modest parameters). Our construction may be optimized for certain applications that do not require the full flexibility of our construction. For example, schemes not requiring selective disclosure of credentials require about half that proof size. At the same time, we hope that advances in bilinear accumulators, mercurial commitments, or lattice based techniques may provide a more efficient construction. We are particularly hopeful that generic work in verifiable computation [5, 44] will offer drastically smaller proof sizes without resorting to bespoke proofs and protocols.

¹⁹In principle this ESK signature can be replaced with an AIK signature with no changes to the TPM mechanism, but AIKs will have to be issued by a trusted third party.

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APPENDIX A

PROOF SKETCH OF SECURITY FOR OUR BASIC SYSTEM

We now provide a sketch of the proof of security for our basic distributed anonymous credentials system.

Our basic approach is to show that for every real-world adversary \mathcal{A} against the credential system, we can construct an ideal-world adversary \mathcal{S} against the ideal-world system such that the transcript of \mathcal{A} interacting with the real system is computationally indistinguishable from the transcript produced by \mathcal{A} interacting with \mathcal{S} . We assume a static corruption model in which the adversary controls some set of users and leave a proof in the adaptive corruption model for future work. For this sketch we also assume that our zero-knowledge signatures of knowledge include an efficient extractor and simulator and that the *params* are created using a trusted setup process. Note that in the random oracle model this assumption holds for the Fiat-Shamir proofs we employ, provided we conduct the proofs sequentially.

Our proof assumes the existence of a global, trusted transaction ledger, which we use as a black box. We leave a complete proof that considers this construction and models it to future work.

We begin by sketching the simulator \mathcal{S} for our system.

A. Description of the Simulator

Minting a credential. When a user controlled by the adversary with nym Nym_U^O wants a credential, the user first generates $(c, \pi_M, attrs)$. When the simulator receives notification of this, it first verifies that the credential and proof are valid and meet the organization’s policy. If so it employs the knowledge extractor for the signature of knowledge on π_M to obtain (sk, aux) .

The simulator then checks if it has a record of (U, sk, Nym_U^O) on its list of users. If the user with key sk and nym Nym_U^O exists, then \mathcal{S} retrieves sk_U associated with

(U, sk, Nym_U^O) and proceeds. If it is not on the list, the simulator checks if it has previously seen a user with key sk . If the user with key sk is not present, then the simulator creates a user U and runs $RegNym(Nym_U^O, U, O)$ to register Nym_U^O and obtain sk_U for further interactions with T_P . \mathcal{S} then stores (U, sk, sk_U, Nym_U^O) in its list of users controlled by the adversary. If a user U with key sk exists, then it runs $RegNym(Nym_U^O, U, O)$ to register Nym_U^O and adds Nym_U^O to U ’s record.

Once the simulator has registered the nym or verified it already exists, it runs $MintCred(Nym_U^O, O, attrs, aux)$. The simulator then transmits the credential information to the trusted store and acknowledges the credential’s issuance. \mathcal{S} stores $(sk, Nym_U^O, attrs, aux, c, \pi_M)$ in its list of granted credentials.

When an honest user, through T_P , wants to establish a credential, the simulator creates a credential c (using the publicly available *attrs*) and uses the simulator for the signature of knowledge π_M to simulate the associated proof. It then transmits the credential information $(c, \pi_M, attrs)$ to the trusted store.

Showing a credential. When a user controlled by the adversary wants to show a credential from organization O to verifier V with which it has nym Nym_U^O and Nym_U^V respectively, the user first generates π_S . When the simulator receives notification of this, it verifies the proof as in the real protocol (rejecting if it is invalid). If the show verifies, it runs the knowledge extractor for the proof of knowledge on π_S to get sk .

The simulator then checks if it has a record of $(U, sk, Nym_U^O, Nym_U^V)$ on its list of users. If the user with key sk and nym Nym_U^O and Nym_U^V exists, then \mathcal{S} retrieves sk_U associated with (U, sk, Nym_U^O) and proceeds. If the record does not exist, either in part or in full, the simulator checks if it has previously seen a user with key sk . If the user with key sk is not present, then the simulator creates a user U and runs $RegNym(Nym_U^O, U, O)$ and $RegNym(Nym_U^V, U, V)$ to register Nym_U^O and Nym_U^V and obtain sk_U for further interactions with T_P . \mathcal{S} then stores $(U, sk, sk_U, Nym_U^O, Nym_U^V)$ in its list of users controlled by the adversary. If a user U with key sk exists, then it runs $RegNym(Nym_U^O, U, O)$ (resp. $RegNym(Nym_U^V, U, V)$) to register Nym_U^O (resp. Nym_U^V) and adds Nym_U^O (resp. Nym_U^V) to U ’s record.

Now, the simulator \mathcal{S} runs $ShowOnNym(Nym_U^O, Nym_U^V, O, V, \mathbf{C})$ where \mathbf{C} is obtained by the simulator through a call to $GetCredList(O)$.

When an honest user (through T_P) wants to show a credential to a verifier V controlled by the adversary, the simulator generates a random prime commitment and runs the zero-knowledge simulator for π_S to simulate a proof that it then sends to V .

1) *Proof (sketch) of a Successful Simulation:* Our simulation is computationally indistinguishable from the real protocol if the Strong RSA and the Discrete Logarithm assumptions hold. While we do not provide a full proof here due to space reasons, we provide an overview of the argument for security.

We first begin by discussing the signatures/proofs π_M and π_S . Under the Discrete Logarithm assumption, π_M is a computational zero-knowledge signature of knowledge on aux of the values sk , r , and r' such that the nym Nym_U^O and the credential c both belong to the same master secret sk . The proof is constructed using standard techniques in the random oracle model [48], and the resulting proofs are (at least) computationally zero knowledge. An attacker who forges this proof to spend a new coin would violate the soundness guarantee of the proof system. Alternatively, an attacker might forge this message by identifying a collision on the commitments, which occurs with negligible probability under the Discrete Logarithm assumption [45]. In the event that this occurs, we can use the extractor for the zero knowledge proof to obtain the collision with all but negligible probability.

Under the Strong RSA and Discrete Logarithm assumptions, π_S is a statistical non-interactive zero-knowledge proof of knowledge of the values sk , ω , c , Nym_U^V , r , and r' such that ω is a witness that c is in the accumulator A and nym Nym_U^V and the credential c both belong to the same master secret sk . This proof is again constructed using standard techniques [15, 48] similar to the proofs used by Miers et al. in [38]. In order to forge such a proof, the adversary would need to either find a collision on the commitments or forge an accumulator membership proof. We previously discussed how the first case occurs with negligible probability. The second case occurs with negligible probability under the Strong RSA assumption due to [15]. See the full version of the paper for a formal treatment/reduction of these statements.

Intuitively, we can now see that the simulator will fail with at most negligible probability because it deals solely with zero-knowledge signatures of knowledge and zero-knowledge proofs of knowledge, which have efficient extractors and simulators. Our proofs π_M and π_S have knowledge extractors that succeed with probability $1 - \nu(\lambda)$ for some negligible function $\nu(\cdot)$. Since signatures and proofs are the sole point of failure for our simulator described above, it fails with negligible probability. Because the adversary only sees the simulated zero-knowledge proofs and signatures, and the simulated signatures and proofs are computationally indistinguishable from legitimate ones, the adversary cannot distinguish a simulated transcript from the real protocol except with negligible advantage. Hence the adversary cannot distinguish between an interaction with the simulator and the real protocol.

We note that the Pedersen commitments we use are non-standard in that we output only commitments that are prime. We stress that these commitments remain information theoretically hiding and computationally binding under the assumption that the Discrete Logarithm assumption holds in $\langle g \rangle$.