Decentralized ellipsoidal state estimation for linear model predictive control of an irrigation canal

L. P. Rodriguez, J. M. Maestre, E. F. Camacho and M. C. Sánchez

ABSTRACT

A centralized linear MPC is used to stabilize an irrigation system whose operation is represented by an integrator-delay model. Since not all the state variables can be measured, a decentralized ellipsoidal estimation strategy is proposed. This approach keeps the quality of a centralized estimation and reduces significantly the computation time for the systems considered. An adaptation of Test Canal 1, developed by the ASCE Task Committee on Canal Automation Algorithms, is used as a case study to show the performance of the proposed methodology.

Key words | decentralized, ellipsoidal state estimation, irrigation canal model, predictive control

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INTRODUCTION

Irrigation is the application of controlled amounts of water to crops at established intervals. It is a necessary activity in dry areas and during periods of below-average rainfall. In these areas the irrigation water consumption competes directly with the municipal and industrial ones. Since the use of water for agriculture constitutes the largest consumer of fresh water, the modernization and automation of irrigation systems can largely improve the conservation of this resource. The water usually comes from a river, lagoon, or is pumped from underground aquifers and is distributed through a system of irrigation canals. In this context, the use of control strategies allows maintaining the water levels in their desired values in each section of the irrigation system, while using the smallest possible changes in the configuration of the structures, and using the least amount of energy for water pumping.

Model Predictive Control (MPC) presents important advantages over traditional control methods such as Proportional Integral Differential (PID) controllers (see Malaterre *et al.* (1998) for a survey on this topic). The MPC allows simultaneously dealing with multiple objectives, constraints, delay times and uncertainties in the variables (Camacho & Bordons 2004). Regarding irrigation systems, Van Overloop (2006) compared the performance of classic feedback and feedforward controllers and MPC, demonstrating the superiority of the latter. Within the framework of distributed control, a control strategy in two levels was developed (Núñez *et al.* 2013). The upper layer follows a risk management strategy to cope with unexpected changes in demand, failures and additional maintenance costs, and the lower layer optimizes the value of water flows using a distributed MPC. Likewise, Fele *et al.* (2013) presented a flexible hierarchical scheme of MPC which reorganizes the agent priorities for optimizing their control actions according to different operating conditions in a hydropower valley. Likewise, a distributed MPC strategy where local controllers exchange information aiming for a trade-off between global performance and cooperation costs was proposed for a system of irrigation canals (Fele *et al.* 2014). The so-called coalitional control technique used a linear Kalman filter to estimate the coupling dynamics between the different clusters of controllers.

Concerning centralized control strategies, an MPC technique was applied to control the water levels of irrigation systems and its performance was compared with that obtained using Gaussian linear quadratic regulators (LQR)

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(Van Overloop et al. 2010). To improve the operation of irrigation canals, an MPC strategy that maintains the water level of the canals within a target band between the maximum and minimum predefined water levels was proposed (Hashemy et al. 2013). Likewise, centralized MPC was applied for human-in-the-loop control of irrigation systems (Van Overloop et al. 2015).

Furthermore, Hashemy et al. (2015) presented a centralized control strategy to satisfy a fair distribution of water between users downstream and upstream of an irrigation canal when water is scarce. A new way of calculating the error associated with the difference in water level was introduced. The heart of the controller is a centralized MPC that accelerates the equal exchange of level errors between all sections of the canals. Recently, an MPC strategy that maximizes the net income derived from existing agricultural economic activities was developed (Hashemy et al. 2017). To this end, the authors used an economic model based on positive mathematical programming to determine the economic value of water for each extraction point along the irrigation canal. This information is compared with an operating model of the irrigated district, and the water deficit is proportionally divided along the canal, maximizing the district's economic gains. The application of MPC to irrigation systems with important fluctuations in the water supply was studied (Hashemy et al. 2016). The control system allows adjusting the regulators in such a way that the irrigators take water continuously and at a satisfactory flow during fluctuations. Water is stored during the excesses and distributed when there is a shortage.

Previous works showed that key operational problems of irrigation systems could be effectively solved applying MPC. In the same way for any control strategy, the availability of reliable knowledge of the system state is crucial for MPC because the measurement of all the state variables is not possible. As is well known, MPC uses models to perform its calculations of future states and inputs. While irrigation canal dynamics are highly nonlinear and may evolve with time, the use of very accurate models, such as the Saint–Venant equations which are widely used in literature for describing the behavior of canal reaches and simulating water levels and flows, may result in long computation times of the control actions, which are inappropriate for real-time applications. As a consequence, simplifying assumptions to make the solution method easier and more tractable are common, for example, a linear model with a quadratic cost function that leads to a convex optimization problem. In particular, we focus here on the linear and time-invariant Integrator-Delay (ID) model, which captures the delay time steps and the storage by which the backwater area moves up and down. Due to its simplicity, the ID model is very compact and fast for computations, thus being a popular choice for MPC. Nevertheless, the values of its parameters (delay time and storage area) are valid at given operation point and these may change when the flow changes (Van Overloop 2006). Moreover, the simplifications introduced also generate uncertainty in the model predictions. For these reasons, it is crucial to implement estimation methods that can explicitly handle uncertainty and several operation points, which, as will be seen later, are features of the method proposed in this article.

In the literature dealing with MPC for irrigation canals, stochastic observers, such as the Kalman filter (Kalman 1960), were used. These assume that the probability distribution of the disturbances and the noise of the measurements are known. In practice, these perturbations are not always known, and it is more natural that they belong to compact sets without assumptions about their distribution (Fogel & Huang 1982). These sets guarantee to contain the states that are consistent with the system model, the measurements, the noise of the measurements and the bounded perturbations.

Generally, the compact set that contains the states has a particular geometric shape. Alamo et al. (2005) and Le et al. (2013) developed estimation techniques based on zonotopes. Other types of widely used sets are the ellipsoids due to the simplicity of their formulation and the resulting stability of the estimates (Durieu et al. 2001; Polyak et al. 2004; Daryin et al. 2006; Daryin & Kurzhanski 2012; Chabane et al. 2014). Two different methodologies to reduce the size of the estimation set have been proposed. The first one uses a criterion based on the determinant of the shape matrix, while the second one minimizes the trace of this matrix. Durieu et al. (2001) showed that the computational complexity of these two methods is low at the expense of a loss of accuracy in comparison with the polytopic estimation. The radius of the ellipsoid estimation set is minimized at each time instant by solving a Linear Matrix Inequality (LMI) optimization problem (Chabane et al. 2014). The estimates precisions are similar or greater than the ones provided by the zonotopic estimation, but the computation time increases considerably with the size of the model.

Likewise, note that there are other alternatives available in the literature. For example, there is a growing interest in data assimilation procedures that do not require assumptions related to probability distribution of noise measurements, and they can be also used in combination with deterministic models (Sun *et al.* 2010; Wang *et al.* 2016), for example, these techniques have been applied for real-time correction of deviations between the simulation model forecast and the observed variables. The same holds for optimization-based control methods. Besides the previously mentioned LQR and MPC, other approaches can be found in the literature, for example, genetic algorithms, which provide an optimization framework that is suitable for complex problems with variables of continuous and discrete nature, for example Tian *et al.* (2019) used it to find Pareto optimal solutions for a multi-scenario operational water resources management problem and Li & Lian (2007) for PID parameter tuning. Arauz *et al.* (2020) optimized the coefficients of PI controllers for irrigation canals using LMI constraints to guarantee the stability of the overall system and minimize undesired mutual interactions. Also, more recently, data-based methods have also appeared in this context. For example, in Salvador *et al.* (2019), previous trajectories of the system were optimized to calculate control actions for a water system, and Barreiro-Gomez *et al.* (2017) used them in combination with evolutionary games, which are a valuable tool for resource allocation optimization.

As can be seen, optimization and control strategies may be essential in many areas of the water domain, from the mitigation of severe droughts in many regions of the planet up to control of urban floods, which are increasing on a worldwide scale. Also, model-based decision-making methods are sensitive to model simplifications and inaccuracies, for example, in Diogo & do Carmo (2019), the influence of boundary conditions in peak flows is assessed via numerical integration of the Saint–Venant equations. To deal with these issues, we develop a decentralized ellipsoidal estimation methodology suitable for linear models of irrigation canals. This is relevant for control methods that require an estimate of the state such as MPC, where the use of this type of model is common to increase the computation speed and guarantee the convexity of the resulting optimization problem, for example, the ID model (Schuurmans 1997; Van Overloop *et al.* 2010; ZafraCabeza *et al.* 2011; Fele *et al.* 2013, 2014; Hashemy *et al.* 2013, 2015, 2016, 2017; Maestre *et al.* 2014). In general, this type of model does not intend to be accurate from a hydraulic viewpoint and clearly its simplicity generates mismatches with the real dynamics of the canal. Indeed, it is a Markov-type model, i.e. it is a stochastic model where future states depend only on the current state and not on previous events. The same holds for the exogenous inputs considered in the model update, which are simplified and treated as random disturbances, thus ignoring issues such as the water demand can be highly affected by the water-availability and the precipitation-drought behavior, which exhibit a long-term persistent behavior – the so-called Hurst phenomenon or Hurst–Kolmogorov behavior. These phenomena can be detected by performing auto-correlation analysis and may be relevant to avoid failures in extreme events such as severe droughts (Koutsoyiannis 2003; Dimitriadis & Koutsoyiannis 2018; Tyralis *et al.* 2018). Despite all the simplifications performed, the proposed framework has been shown to provide good performance for canal regulation purposes and can benefit from a significant advantage of the method proposed in the article, which is that the estimation process can handle explicitly mismatches and disturbances as long as they are bounded.

Indeed, throughout the article we apply the decentralized estimator in combination with MPC to show its applicability. The proposed approach has several appealing features: (i) it deals explicitly with uncertainty in the system evolution, thus accounting for possible model misspecifications and errors, which are typical issues when simple models are used to capture nonlinear systems dynamics and/or different operation points; (ii) the computation is decentralized so that it can be implemented in real world problems where local controllers and observers may exist for different part of the canal; (iii) the exploitation of the structure of irrigation canals allows significant speeding up of the computations in comparison with that of Chabane *et al.* (2014). To this end, the developed procedure takes advantage of the structure of the matrices that model the system evolution, the measurements, the weight of the disturbances, and the weight of the measurements noise, which can be arranged in block diagonal form. Each block may represent one or more sections of the irrigation canal, and sub-systems are connected through the input-to-state matrix. The nominal state of the system is estimated using a Luenberger observer, which is calculated by means of an LMI optimization problem solved in parallel for each subsystem. The resulting local observer gains are then gathered into a single global gain matrix. After accomplishing the estimation step, the system is regulated using a centralized MPC controller. An adaption of the Test Canal 1, developed by the ASCE Task Committee on Canal Automation Algorithms, is used as a case study.

The rest of the paper is organized as follows. Next, the formulation of the ad-hoc decentralized ellipsoidal state estimation and the centralized MPC used to stabilize the irrigation system are presented. The performance of the proposed methodology for different simulated scenarios of the ASCE Test Canal 1 is discussed in the following section. Finally, conclusions and hints about future works are discussed.

PROBLEM FORMULATION

System model

We consider canals that transport water for irrigated agriculture. In particular, a canal is composed of several sections separated by gates, which can be moved to control the water volume stored in each section of the canal. From a mathematical viewpoint, we are interested in the more simple internal models used by many control methods such as LQR and MPC, so we consider an irrigation canal represented by the following discrete-time linear time-invariant (LTI) system:

$$x_{k+1} = Ax_k + Bu_k + Ew_k \tag{1}$$

$$y_k = Cx_k + Fw_k \tag{2}$$

where $x_k \in \mathbb{R}^{n_x}$ is the discrete state vector of the system, $u_k \in \mathbb{R}^{n_u}$ is the input vector, $y_k \in \mathbb{R}^{n_y}$ is the measured output vector, $w_k \in \mathbb{R}^{n_w} (n_w = n_x + n_y)$ contains the state and measurements perturbations (noise, offset, etc.) that are considered bounded by unitary boxes, and $A \in \mathbb{R}^{n_x \cdot n_x}$, $B \in \mathbb{R}^{n_x \cdot n_u}$, $C \in \mathbb{R}^{n_y \cdot n_x}$, $E \in \mathbb{R}^{n_x \cdot n_w}$, $F \in \mathbb{R}^{n_y \cdot n_w}$, being the pair (*A*,*C*) detectable and the pair (*A*,*B*) stabilizable.

As can be seen, the starting LTI model is very general. Without loss of generality, we will work in this article with the particular case of the previously mentioned ID model, a simple AR(1) model that provides us with a low frequency approximation of canal dynamics introduced by Schuurmans (1997), which has become a popular choice as a control model for MPC controllers that regulate average water levels in irrigation canals. Since estimators are needed for state space MPC controllers, the choice of this simple approximation was straightforward, but note that more sophisticated linear models can be used, for example, higher-order Markov models and linearized versions of Saint–Venant equations. In the chosen model, the state vector x_k contains water levels (or errors with respect to the operation point) and delayed flows, which are provided by the input vector u_k. More information in this regard is provided below under 'Case study', where the model is explained for the considered case study.

The methodology proposed in this paper is based on a decentralized ellipsoidal state estimation technique, which assumes that the noise of the disturbances and the measurements are unknown but bounded. Moreover, a centralized MPC is used to stabilize the whole system.

Decentralized ellipsoidal state estimation

Recently, a centralized ellipsoidal state estimation strategy for LTI discrete-time systems based on the minimization of the ellipsoidal estimation set radius at each time instant k was developed (Chabane *et al.* 2014). The estimation is computed by solving an LMI optimization problem. Although the strategy shows good performance for small systems, the computation time increases considerably with the size of the model. Here, we demonstrate how the efficiency of the estimation technique can be significantly improved.

It should be noticed that matrices A, C, E, F of the irrigation canal have a block diagonal structure, and each block represents the ith section of the irrigation canal. Each section may represent one or more reaches, and all sections are coupled by the matrix B. Therefore, the system state vector x_k can be represented by $x_k = \{ x_k^1 \quad \dots \quad x_k^i \quad \dots \quad x_k^p \}$, where $x_k^i \in R^{n_x^i}$ is the state vector corresponding to the ith section of the irrigation canal, and p is the total number of sections. In the same way, the measurement vector y_k is $y_k = \{ y_k^1 \quad \dots \quad y_k^i \quad \dots \quad y_k^p \}$, ($y_k^i \in \mathbb{R}^{n_y^i}$), and the disturbance vector is $w_k = \{ w_k^1 \quad \dots \quad w^i \quad \dots \quad w_k^p \}$, ($w_k^i \in \mathbb{R}^{n_w^i}$). Moreover the state transition matrix, the measurements matrix, the process noise matrix and the measurements noise matrix of the ith section of the irrigation canal are, respectively, $A^i \in \mathbb{R}^{n_{x^i} \cdot n_{x^i}}$, $C^i \in \mathbb{R}^{n_{y^i} \cdot n_{x^i}}$, $E^i \in \mathbb{R}^{n_{x^i} \cdot n_{w^i}}$, $F^i \in \mathbb{R}^{n_{y^i} \cdot n_{w^i}}$.

In this work the following assumptions are considered:

Assumption 1. Bounded disturbances: vector w_k is bounded by the unitary interval $M^{n_x + n_y}$.

Assumption 2. Bounded Initial State: the initial state vector x_0^i, $i = 1, \dots p$ is bounded by the ellipsoid $\varepsilon(P^i,\ \bar{x}_o^i,\ \rho_0^i) = \{x^i \in \mathbb{R}^{n_x^i} : (x^i - \bar{x}_o^i)^T P^i (x^i - \bar{x}_o^i) \le \rho_0^i\}$, where $\bar{x}_o^i$ is the initial nominal state of the ith section, P^i is the shape matrix of the ith ellipsoid, and ρ_0^i represents its initial radius (recall that in its simplest form the quadratic form of an ellipsoid can be expressed in matrix form as $x^T P x < \rho$, with P being a positive definite matrix).

Given an $\varepsilon(P^i,\ \bar{x}_k^i,\ \rho_k^i)$ for x_k^i, where $\bar{x}_k^i$ is its center and ρ_k^i is its radius at time k and $k > 0$, the ellipsoid $\varepsilon(P^i,\ \bar{x}_{k+1}^i,\ \rho_{k+1}^i)$ can be calculated for x_{k+1}^i using an ellipsoidal state estimation (Chabane et al. 2014) in such a way that:

$$\rho_{k+1}^i - \beta^i \rho_k^i > 0 \tag{3}$$

where $\beta^i \in (0, 1)$. In particular, ρ_{k+1}^i and β^i are minimized at $k+1$ to guarantee a non-increasing ellipsoidal radius.

The decentralized ellipsoidal state estimation technique developed in this work is based on the following theorem.

Theorem 1. Consider that at time k, the state vector of the ith section of the irrigation canal x_k^i belongs to the ellipsoid $\varepsilon(P^i,\ \bar{x}_k^i,\ \rho_k^i)$, with a symmetric positive definite matrix $P^i = P^{iT} \succ 0$. If there exists a matrix $\Xi_k^i \in \mathbb{R}^{n_x i \cdot n_y i}$, a scalar $\beta^i \in (0, 1)$ and a radius ρ_{k+1}^i satisfying the following LMI optimization problem for all $w_k^i \in M^{n_x i + n_y i}$:

$$\min_{\Xi_k^i,\, \beta^i,\, \rho_{k+1}^i} \rho_{k+1}^i \tag{4}$$

s.t.

$$\begin{bmatrix} \beta^i P^i & A^{iT} P^i - C^i \Xi_k^i & 0 \\ P^i A^i - \Xi_k^i C^i & P^i & P^i E^i - \Xi_k^i F^i \\ 0 & E^{iT} P^i - F^{iT} \Xi_k^i & \Phi^i \end{bmatrix} \succ 0$$

$$\rho_{k+1}^i - \beta^i \rho_k^i > 0$$

Then, the system state x_{k+1}^i at time $k+1$ belongs to $\varepsilon(P^i,\ \bar{x}_{k+1}^i,\ \rho_{k+1}^i)$ for all $w_k^i \in M^{n_x i + n_y i}$. The proof of Theorem 1 is beyond the scope of this work, and can be found in Chabane et al. (2014).

In contrast to the centralized estimation technique proposed by Chabane et al. (2014), which updates the radius ρ_k of the ellipsoid ε that contains the state vector x_k and the gain of estimator G_k at each time instant k for the whole irrigation canal system, the decentralized estimation updates the radius and the gain for each canal section individually. In this way, the state vector x_{k+1}^i corresponding to the ith section of the irrigation canal belongs to the ellipsoid $\varepsilon^i(P^i,\ \bar{x}_{k+1}^i,\ \rho_{k+1}^i)$ for all $w_k^i \in M^{n_x^i + n_y^i}$ and the gain of ith section, G_k^i, can be calculated from:

$$P^i G_k^i = \Xi_k^i \tag{5}$$

In particular, Equations (4) and (5) are used to compute ρ_{k+1}^i and G_k^i for each section of the irrigation system. After that, the radius and gain for the overall system, ρ_{k+1} and G_k, are computed as follows:

$$G_k = \begin{bmatrix} G_k^1 & 0 \cdots 0 & 0 & 0 \\ \vdots & \vdots \cdots \vdots & \vdots & \vdots \\ 0 & 0 \cdots G_k^i & 0 & 0 \\ \vdots & \vdots\ \vdots & \vdots & \vdots \\ 0 & 0 \cdots 0 & 0 & G_k^p \end{bmatrix} \tag{6}$$

$$\rho_{k+1} = max(\rho_k^1; \quad \dots \quad ; \ \rho_k^i; \quad \cdots \quad ; \rho_k^p) \tag{7}$$

where $G_k \in \mathbb{R}^{n_x \cdot n_y}$, is a block diagonal matrix and ρ_{k+1} is the largest ellipsoid radius.

Finally, the nominal state vector of the system $\bar{x}_{k+1} \in \mathbb{R}^{n_x}$ at time instant $k+1$ is calculated by using the following equation:

$$\bar{x}_{k+1} = A\bar{x}_k + Bu_k + G_k(y_k - C\bar{x}_k - Du_k) \tag{8}$$

In summary, the steps for updating the state vector are:

- Solve the LMI Optimization Problem (4) for each section by obtaining ρ_{k+1}^i and the matrix Ξ_k^i for each ith section of the canal.

- Obtain G_k^i for each section of the canal by solving Equation (5).
- Obtain G_k for the whole system through the block diagonal matrix (6).
- Calculate the radius of the ellipse that contains the states of the system, ρ_{k+1}, by means of Equation (7).
- Update the states of the whole system, $\bar{x}_{k+1}$, through Equation (8).

Remarkably, this procedure can be performed in a decentralized fashion which allows parallelizing the computation of the solution. In this regard, the structure of irrigation canals and their relatively low coupling between different subsystems is key in distributing the computation of the ellipsoids. As a consequence, the resulting problems to solve are smaller, also providing additional gains in the time required for the computation. As will be shown later, all these elements provide a significant reduction of computation time with respect to the original method proposed in Chabane *et al.* (2014).

After updating $\bar{x}_{k+1}$, the control action is calculated in a centralized way by solving a centralized MPC regardless of the configuration chosen to estimate the states of the system.

Centralized model predictive control

Management of irrigation canals systems can be described by a set of logical and mathematical rules within a controller. These can be classified according to the property of the irrigation system that is controlled as flow control, water level control and volume control. Also, rules can be classified according to the relative position of the controlled water level in the reach as downstream control and upstream control. Finally, the controller may be classified based on general control theory feedback control, feedforward control, optimal control and heuristic control (Van Overloop 2006), the MPC technique proposed in this work has elements of the feedback control, feedforward and optimal control.

The objective of the proposed controller is to maintain a constant level at offtakes, which are located at the downstream end of each canal reach. A centralized MPC is developed to stabilize the system by maintaining state variables and inputs within specified ranges.

The control signal u_k is calculated by minimizing a quadratic criterion subject to a set of constraints. The optimization problem is stated as follows (Camacho & Bordons 2004; Van Overloop 2006):

$$\underset{u_{k|k}}{\arg\min} \sum_{l=0}^{L-1} x_{k+l+1|k}^T \, Q \, x_{k+l+1|k} + u_{k+l+1|k}^T R \, u_{k+l+1|k} \tag{9}$$

s.t.

$$x_{min} \leq x_{k+l|k} \leq x_{max}$$
$$u_{min} \leq u_{k+l|k} \leq u_{max}$$

where $x_{k+l|k}$ and $u_{k+l|k}$ are the states and inputs predicted at time $k+l$ $(l{:}1 \ldots L)$, L is the prediction horizon, $\mathrm{Q} \in \mathbb{R}^{n_x \cdot l}$ and $\mathrm{R} \in \mathbb{R}^{n_u \cdot l}$ are the weighting matrices, (x_{min}, x_{max}) and (u_{min}, u_{max}) are the limits on the state variables and the control actions, respectively, chosen taking into account safety and performance specifications. The particular specifications of the case study will be discussed below under 'Case study'.

The control signal u_k, applied to the system, is composed by the first n_u elements of the solution vector $\boldsymbol{u}_{k|k} \in \mathbb{R}^{n_u \cdot L}$ corresponding to the current time. For $k+1$, u_{k+1} is obtained by solving the MPC with the updated process information again.

CASE STUDY

The case study used in this paper is an adaption of the Test Canal 1 of the ASCE Task Committee on Canal Automation Algorithms, based on lateral canal WM within the Maricopa–Stanfield Irrigation and Drainage District in central Arizona. Additional details about the Test Canal 1 can be found elsewhere (Clemmens *et al.* 1994, 1998, 2004). The length of the original canal is 9.5 km, the maximum capacity at the head gate is 2.8 m^3/s and the irrigation canal supplies water to a number of large farms. The canal is characterized by high Froude numbers and little storage. Many pools have an initially very steep slope, followed by a mild slope, which was simplified to a single slope. It is assumed that each canal reach has an unsubmerged vertical sluice gate and the regulation time step is equal to 5 minutes.

While the original system consists of eight pools, here we use 24 pools by concatenating three times the original system to have a larger system for our tests (see Figure 1). The irrigation canal is fed by a constant water level reservoir at its head and the flow released to the canal is controlled by a gate. The canal reaches are separated by control structures and each one consists of an adjustable undershot control gate in parallel with weirs. A fixed crest level at both sides of the undershot gate is assumed. To the extreme of each reach (5 m from the downstream end) are located the offtake undershot gates and the offtake flows are conveyed to the secondary canals by a culvert. The canal has almost no flow at the downstream end. To model the canal, some assumptions were made: sections with supercritical slopes were modeled with a drop followed by a subcritical slope. These approximations result in slower canal response to upstream flow changes than the prototype canal.

The mains characteristics of the case study were taken from Clemmens *et al.* (1998) and are shown in Tables 1–3. Nevertheless, note that the level of detail of the real canal, including its multiple reaches between pools, culverts, and supercritical flow sections, is not necessary for a general evaluation of canal control algorithms via simulation (Clemmens *et al.* 1994). Thus, single canal reaches and subcritical flow were considered for simplicity.

Table 1 | Main characteristics of the case study

Pool #	Pool length (m)	Bottom width (m)	Canal depth (m)	Capacity flow (m^3/s)	Design flow (m^3/s)
1, 9, 17	100	1.0	1.1	2.0	0.8
2, 10, 18	1,200	1.0	1.1	2.0	0.7
3, 11, 19	400	1.0	1.0	2.0	0.6
4, 12, 20	800	0.8	1.1	1.6	0.5
5, 13, 21	2,000	0.8	1.1	1.6	0.4
6, 14, 22	1,700	0.8	1.0	1.6	0.3
7, 15, 23	1,600	0.6	1.0	1.3	0.2
8, 16, 24	1,700	0.6	1.0	1.1	0.1

Also, the bottom slope is 0.002, the Manning coefficient is 0.014, the side slope is 1.5, and the drop at the gate expressed in meters is 1.0 for all the pools.

The original test was adapted to consider that the elements of w_k are bounded in the range [1,–1]. Also, the case study was simulated using (1) and (2) for $k = [0; 1; \ldots; 287; 288]$. The measurements trajectories were obtained from simulations. A linear response model in open irrigation canals can be obtained by linearizing the Saint–Venant equations over a bounded range of operating conditions. The model is linearized around stationary flow

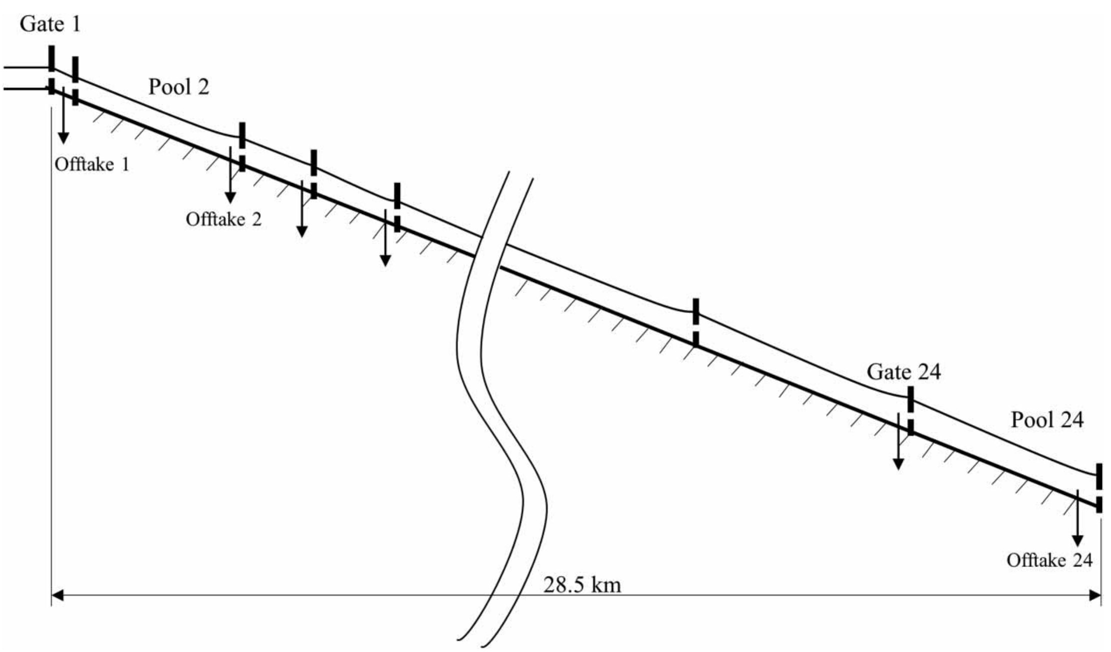

Figure 1 | Profiles of case study.

Table 2 | Check structures properties

Pool #	Gate width (m)	Gate height (m)	Weir height (m)	Weir width (m)
Inlet	1.5	1.0	No weir	No weir
1, 9, 17	1.5	1.0	0.99	4.5
2, 10, 18	1.5	1.0	0.99	4.5
3, 11, 19	1.5	0.9	0.91	4.2
4, 12, 20	1.2	1.0	1.07	4.4
5, 13, 21	1.2	1.0	0.91	3.9
6, 14, 22	1.2	0.9	0.90	3.7
7, 15, 23	1.0	0.9	0.91	3.3
8, 16, 24	–	–	0.95	4.0

Table 3 | Offtake discharges

Pool #	Discharges (m^3/s) initial	Discharges (m^3/s) final	Target level (m)	Offset (m)	Head discharge exponent
1, 9, 17	0.16	0.16	0.90	0.3	0.6
2, 10, 18	0.0	0.0	0.90	–	–
3, 11, 19	0.10	0.10	0.80	0.2	0.5
4, 12, 20	0.21	0.21	0.90	0.36	0.6
5, 13, 21	0.0	0.0	0.90	–	–
6, 14, 22	0.19	0.19	0.80	0.24	0.6
7, 15, 23	0.0	0.14	0.80	0.0	0.2
8, 16, 24	0.19	0.19	0.80	0.0	0.5

conditions, while the flow regimen considered along the canal and along the time is not stationary. A linear ID model assumes that each canal reach is formed by two different parts: the integrator and reservoir sections (Schuurmans 1997), as illustrated in Figure 2. The first one acts as a water reservoir without delay time (τ). The second section is represented by its delay time, which is the time necessary to go from one steady-state flow to another one. Note that this model is not accurate from a hydrological viewpoint and simply captures the transport and storage of water for average water level regulation purposes. As was said before, linear models are convenient for the sake of computation and the proposed estimation method can handle this loss of information as long as it is bounded, as is the case.

The ID model for each canal reach can be described by the following equation:

$$\frac{de_i}{dt} = \frac{1}{A_{b_i}}[q_{in,i}(k-\tau) - q_{out,i}(k) - q_{offtake,i}(k)] \tag{10}$$

For the ith canal reach, e_i is the deviation of the water level i with respect to the set one in the reservoir section, $q_{in,i}$ and $q_{out,i}$ are the deviations of inflow and outflow from their initial values, respectively, and A_{b_i} is the surface of the reservoir section (see Figure 2). Both τ and A_{b_i} can be determined by unsteady flow simulation, (10) can be formulated in state-space form (1) and (2) (Van Overloop

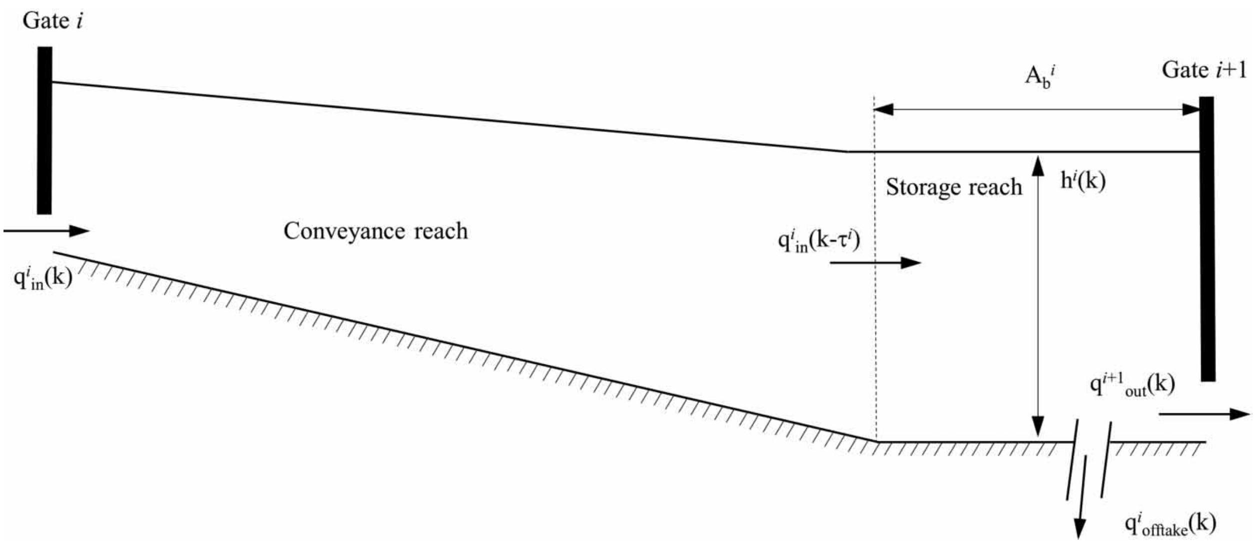

Figure 2 | ID model of the ith canal reach.

2006). For the purposes of this work, the state variables considered in each canal reach are the deviations in water level and inflow, i.e. e_i and $q_{in}^i(k-\tau_i)$:

$$\mathrm{x}_\mathrm{k}^\mathrm{i} = \{e_{i,k};\ q_{in}^i(k);\ q_{in}^i(k-\tau_1);\ q_{in}^i(k-2\tau_1);\ q_{in}^i(k-3\tau_1);\ q_{in}^i(k-4\tau_1)\} \tag{11}$$

Then, the state vector of the irrigation canal is represented by $\mathbf{x_k} = \{\mathrm{x}_\mathrm{k}^\mathrm{i}\}^T$ with $i = \{1;\ \ldots;\ 24\}$. Also, the vector of inputs at time k calculated by the MPC controller is formed by the deviation in inflow $q_{in}^i(k)$ at each canal reach:

$$u_k = \{q_{in}^1(k);\ q_{in}^2(k);\ q_{in}^3(k);\ \ldots;\ q_{in}^{24}(k)\}^T \tag{12}$$

Moreover, the output at sample time k, y_k, are the deviations in water level in each canal reach:

$$y_k = \{e_{1,k};\ e_{2,k};\ e_{3,k};\ e_{4,k};\ \ldots;\ e_{23,k};\ e_{24,k}\}^T \tag{13}$$

Also, matrix A contains information on the temporal evolution of water and flow levels, whereas matrix B injects changes in the flows in certain states variables. In addition, modeling errors may be incorporated through matrix E.

The original test was adapted to consider that the elements of w_k are bounded in the range [1,–1]. Also, the case study was simulated using (1) and (2) for $k = [0;\ 1;\ \ldots;\ 287;\ 288]$. The measurements trajectories were obtained from simulations.

At first, the control algorithm presented in Figure 3 was executed considering the centralized model (a) of the irrigation canal. The states of the system were updated by solving the optimization problem (4) using an LMI solver, whose parameters were adjusted by trial and error. The values of w_k are generated randomly, being $\|w_k\| \le 1$, and $x_0 \in \varepsilon(I^{144.144},\ 0^{144.1},\ 1)$. The control signal was calculated by solving the MPC problem (8) using a QP optimization code. No hard constraints on x_k and hard constraints on $u_k(100,\ -100)$ are considered, also $R = 0.01 \cdot I^{25x25}$ and Q is a diagonal matrix of dimension (144,144). The diagonal elements of this matrix are 1 in positions corresponding to e^i and 0 for those related to q_{in}^i. The controller parameters are the control and prediction horizons, which were adjusted by trial and error and set at 15 and 43 time instants, respectively.

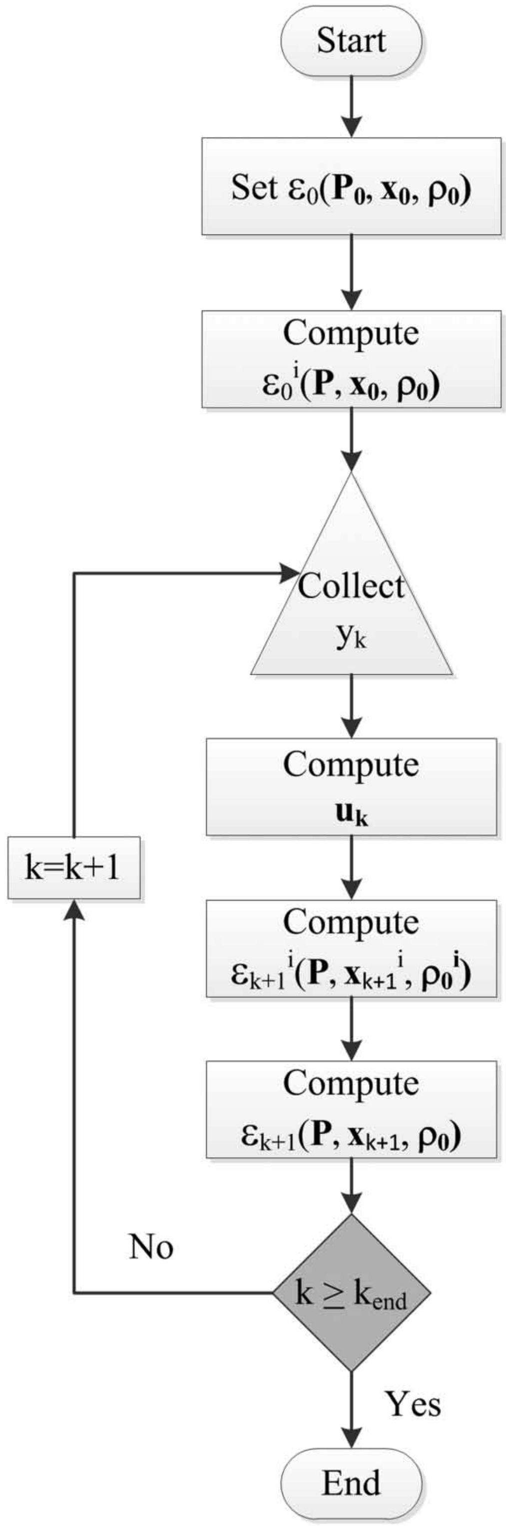

Figure 3 | Control algorithm.

Then, the algorithm was executed considering seven different configurations of the irrigation system for the ellipsoidal state estimation, using the parameters adjusted for the centralized configuration (a). These configurations are the following:

- two sections of 12 pools each one (b);
- three sections of eight pools each one (c);
- four sections of six pools each one (d);
- six sections of four pools each one (e);
- eight sections of three pools each one (f);
- 12 sections of two pools each one (g);
- 24 sections of one pool each one (h).

Also, the MPC is always solved in a centralized way regardless of the proposed state estimation configuration. The mean computing time (MCT) of the eight configurations can be observed in Table 4.

From Table 4 it can be observed that the MCT for the centralized scenario is 1,487 s. If distributed configurations (b), (c), (d), (e), (f) and (g) are chosen, the computation time decreases by a factor of 73, 585, 870, 2,609, 5,312 and 8,263, respectively. If the state estimation is totally decentralized (configuration h), the reduction factor of MCT is 11,441.

Also, Mean Absolute Error (MAE), the Integrated Absolute Error (IAE), the Integrated Quadratic Error (IQE) and the Accumulated Cost (AC) were used as performance indexes (Dimitriadis & Koutsoyiannis 2018). These were evaluated using the following equations:

$$MAE = \frac{\sum_{k=0}^{K} |e_k^T|}{K} \tag{14}$$

Table 4 | Mean computing time

Configuration	MCT (s)
(a) Centralized	1,487.40
(b) Distributed	20.36
(c) Distributed	2.54
(d) Distributed	1.71
(e) Distributed	0.57
(f) Distributed	0.28
(g) Distributed	0.18
(h) Decentralized	0.13

Table 5 | Performance indices MAE, IAE, IQE

Configuration		MAE	IAE	IQE
a	mean	0.0038	1.1026	0.0081
	max	0.0059	1.7232	0.0182
b	mean	0.0038	1.1026	0.0081
	max	0.0059	1.7232	0.0182
c	mean	0.0038	1.1026	0.0081
	max	0.0059	1.7232	0.0182
d	mean	0.0038	1.1026	0.0081
	max	0.0059	1.7232	0.0182
e	mean	0.0038	1.1026	0.0081
	max	0.0059	1.7232	0.0182
f	mean	0.0038	1.1026	0.0081
	max	0.0059	1.7232	0.0182
g	mean	0.0038	1.1026	0.0081
	max	0.0059	1.7232	0.0182
h	mean	0.0038	1.1026	0.0081
	max	0.0059	1.7232	0.0182

$$IAE = \sum_{k=0}^{K} |e_k^T| \tag{15}$$

$$IQE = \sum_{k=0}^{K} (e_k^T)^2 \tag{16}$$

$$AC = \sum_{k=0}^{K} x_k^T Q x_k + u_k^T \, R \, u_k \tag{17}$$

where K is the time period analyzed, and e_k^T is a vector that contains the nominal values of the state variables which represent the error in water levels. These performance indices have been widely used in MPC papers and, in particular,

Table 6 | Performance index AC

Configuration	AC
a	0.4349
b	0.4349
c	0.4349
d	0.4349
e	0.4349
f	0.4349
g	0.4349
h	0.4349

for the ASCE Test Canal 1 (Clemmens *et al.* 1998, 2004). The values of the performance indices calculated for each configuration are shown in Tables 5 and 6.

The analysis of previous results indicates that the mean and max MAE, IAE, IQE performance indexes are independent of the chosen configuration. Also, it is worth mentioning that as the value of the power superscripts increases, the performance indicators become more sensitive to the extreme values, which may bias decisions taken in that regard towards the extremes.

Figures 4–6 show the trajectories of the state variables 1, 49 and 97, obtained by simulating the proposed case study. These variables represent the deviation of the water level of the 1st, 9th and 17th pools of the irrigation canal, respectively, which correspond to the first pools of the first, second and third concatenated sections, respectively.

The control algorithm was executed for each proposed estimation configuration and the bounds of the state variables were displayed. As can be seen in Figure 4, the values of the bounds for the state variable 1 are independent

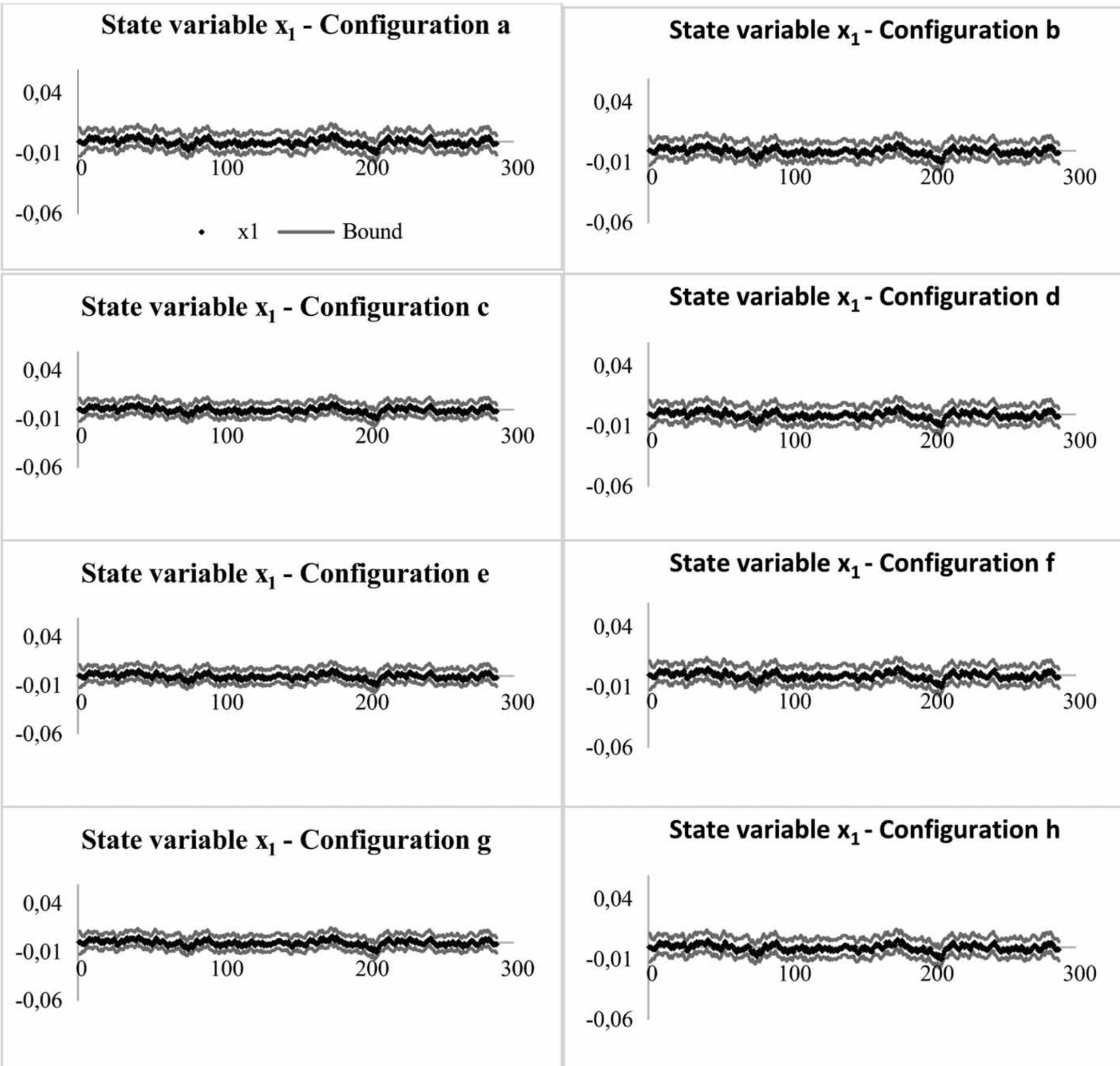

Figure 4 | Trajectory of state variable x_1 (estimator configuration a–h).

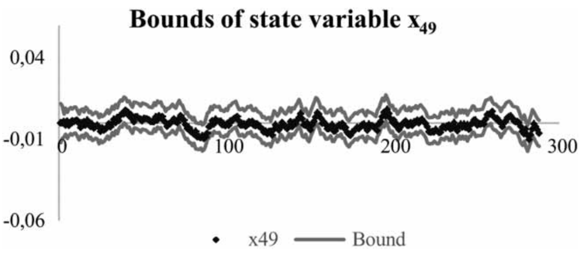

Figure 5 | Trajectory of state variable x_{49}.

of the estimation configuration. Also, it can be observed that trajectories are defined within the ellipsoid limits calculated from the nominal state estimates and the radius (Figures 4–6).

The same analysis was performed for the rest of the state variables that take part in all configurations. It was observed that: $x_k \leq \bar{x}_k \pm \rho_k$ for $k = 0; 1; \ldots; 288$.

CONCLUSIONS

A control algorithm devoted to regulating an irrigation system whose operation is represented by a ID model is proposed in this paper. An ad-hoc decentralized ellipsoidal estimation technique is used to compute the states of the system, and the centralized MPC allows stabilization of the water levels in the pools by adjusting the control action on the gates. The performance of the proposed strategy was tested for different estimator configurations, using the MAE, IAE, IQE, AC and MCT indicators. The computational time is significantly reduced, keeping the quality of the estimates with respect to the centralized estimation. Therefore, the decentralized ellipsoidal estimation method can deal with larger irrigation systems in comparison with centralized strategies. Regarding future work, we will study the application of both decentralized state estimation and MPC to a real large-scale system whose model is based on a more sophisticated linearized version of the Saint–Venant equations. The use of more sophisticated linear models than the ID model is interesting because it can provide additional performance but it may affect the way the system is decomposed to perform the local parallel estimations. Nevertheless, this should not be a significant issue given the naturally distributed nature of irrigation canal problems.

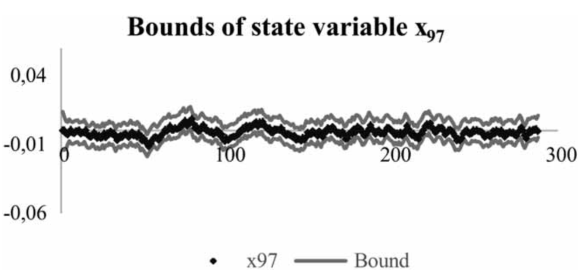

Figure 6 | Trajectory of state variable x_{97}.

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