Decentralized Reactive Power Compensation using Nash Bargaining Solution

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***Abstract*—We consider a distributed reactive power compensation problem in a distribution network, in which users locally generate reactive power using distributed generation units to contribute to the local voltage control. We model and analyze the interaction between one electric utility company and multiple users by using the Nash bargaining theory. On one hand, users determine the amount of active and reactive power generation for their distributed generation units. On the other hand, the electric utility company offers reimbursement for each user based on the amount of reactive power dispatched by that user. We first quantify the benefit for the electric utility company and users in the reactive power compensation problem. Then we derive the optimal solution for the active and reactive power generation as well as reimbursement for each user under two different bargaining protocols, namely sequential bargaining and concurrent bargaining. Numerical results show that both electric utility company and users benefit from the proposed decentralized reactive power compensation mechanism, and the overall system efficiency is improved.**

***Index Terms*—Reactive power compensation, demand side management, game theory, Nash bargaining solution.**

NOMENCLATURE

Indices/Sets:

n	Index of users/nodes.
g	Superscript for generation.
d	Superscript for demand.
$\mathcal{N}$	Set of users/nodes.

Parameters:

p_n^d	Active power demand of user.
q_n^d	Reactive power demand of user.
$r_n + jx_n$	Complex impedance of the link between node n and node $n+1$.
λ	Unit price of active power.
π	Cost parameter for compensating reactive power.
N	Number of nodes or users in the network.
s_n	Maximum apparent power that DG unit n can support.
$\hat{r}_n$	Cumulative resistance from the substation to node n.
$\hat{x}_n$	Cumulative reactance from the substation to node n.
$p^g_{n,max}$	Maximum available active power of the DG unit n.
p_n^{g0}	Amount of active power that user n generates when user n does not participate in reactive power compensation.
C_n^0	Cost for buying remaining active power from the electric company when user n does not participate in reactive power compensation.
C_n	Payment to the electric utility company if user n participates in reactive power compensation.
$\mathcal{V}_n$	User n's payoff.
$\mathcal{U}$	The electric utility company's payoff.

Variables:

p_n^g	Active power generation of user n.
q_n^g	Reactive power generation of user n.
z_n	Reimbursement that user n receives from the electric utility company.

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I. INTRODUCTION

Reactive power compensation is necessary in power systems in order to assure power quality and voltage support [1] [2] [3]. In traditional power systems, reactive power is provided by synchronous generators or shunt capacitor banks installed at specific locations of the distribution networks [4] [5] [6]. However, this centralized reactive power compensation can be costly and it can also increase the power loss on transmission and distribution lines [7] [8]. In addition, due to the increasing number of inductive residential appliances, such as microwaves, washing machines, air conditioners, and refrigerators, there is a need to explore new reactive power compensation options at distribution level [9] [10] [11] [12] [13].

With the introduction of distributed generation (DG), an alternative approach to compensate reactive power is to utilize the power electronics interfaces at DG units, c.f., [14] [15]. In [16], the optimal control schemes for reactive power dispatch to achieve the trade-off between distribution loss reduction and voltage variation minimization using distributed photovoltaic generators are proposed. A stochastic optimization model for real and reactive power management under the uncertainty of solar power generation has been proposed in [17] to maximize the sum utility of users while maintaining the voltage at every node at safe levels. In [18], a novel reactive power management strategy under stochastic parameters of the system has been proposed to allow system operators to detect the reactive power vulnerable part of the power grid. The work in [19] develops a convex optimization framework for reactive power compensation. The authors in [20] propose an online reactive power control scheme considering the stochastic nature of reactive demand and renewable generation.

Although end-user reactive power compensation via DG units has been advocated as a viable solution to achieve high system efficiency [10] [11] [21] [22], users will not actively participate in generating reactive power unless they have proper financial incentives from electric utility companies or distribution network operators. Recent research has applied game theory to propose incentive mechanisms to encourage users to participate in power management systems in the smart grid. For instance, the works in [23] and [24] investigate the demand side management problem as a noncooperative game and propose a smart pricing model to encourage users to participate in energy consumption scheduling program. In [25], the authors formulate the reactive power compensation as a Stackelberg game and derive a pricing scheme to encourage plug-in electric vehicles in generating and consuming reactive power.

In this paper, we consider the problem of controlling reactive power generation from DG units in a radial network, and focus on economic incentives that a utility company needs to provide for users to achieve high system efficiency. Specifically, each user individually controls its DG unit to determine the amount of active and reactive power generation to partially satisfy its own demand. Based on the amount of reactive power compensation, the electric utility company offers reimbursement to users as financial encouragement. The main contribution of this paper lies in the fact that we model and analyze the interaction between the electric utility company and users using the Nash bargaining theory [26]. We quantify the benefit for users and electric utility company in collaborative reactive power compensation. The closed-form optimal solutions for reactive and active power generation as well as the amount of reimbursement offered to users are derived under both sequential bargaining and concurrent bargaining. We also investigate the connections of the optimal solutions with the social welfare of the network.

The remainder of this paper is organized as follows. A model of decentralized reactive power compensation is formulated in Section II. We solve the reactive power compensation problem using the Nash bargaining theory under sequential and concurrent bargaining protocols in Section III and Section IV, respectively. The simulation results are provided in Section V. Section VI presents our conclusions. All analytical proofs are relegated to the Appendix.

II. SYSTEM MODEL

In this section, we describe the topology of the power distribution network considered for reactive power compensation and define the payoff functions for users and utility company.

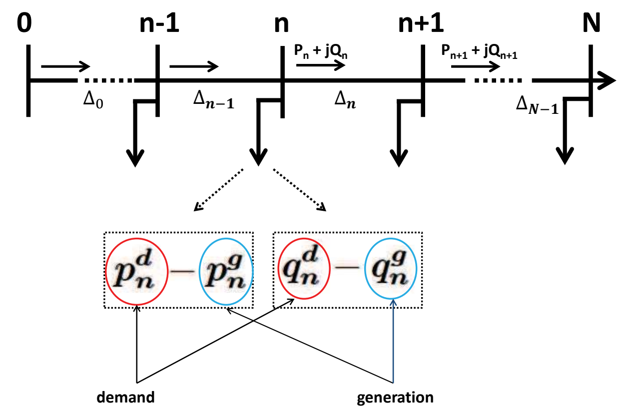

Fig. 1: A distribution network with local reactive power compensation.

A. System Description

Without loss of generality, consider a linear distribution network as in Fig. 1. The set of nodes is denoted by $\mathcal{N} = \{1, \ldots, N\}$. The reference node is connected to the substation, denoted by node 0. Let P_n and Q_n represent active and reactive power flowing down the network from node n to node $n+1$. At each node n, the complex power demand is denoted by $p_n^d + jq_n^d$, where p_n^d and q_n^d are active and reactive power demand, respectively. The exact values of p_n^d and q_n^d depend on the demand condition of each node over time. However, since our focus in this paper is on per-time-slot analysis, we assume that the demand of each node remains unchanged during the period of study. The length of each time slot is a design parameter. In general, a shorter time slot may improve the accuracy in predicting demand; however, such improvement may also come at the expense of increasing computational complexity due to the need for solving the optimal reactive power compensation problem more frequently. In this paper, the length of each time slot is assumed to be equal to the length of each time slot of power price so that power price does not change during the period of study. However, our analysis can apply to any particular choice for the length of time slots.

We further assume that each node $n \in \mathcal{N}$ has a DG unit, e.g., a solar panel or wind turbine, which is capable of generating p_n^g active and q_n^g reactive power respectively. The amount of power that a DG unit can generate must satisfy the following constraint:

$$(p_n^g)^2 + (q_n^g)^2 \leq s_n^2, \tag{1}$$

where s_n is the maximum apparent power that the DG unit can support. Note that, our model can be applied to systems that have only a subset of nodes have DG units by setting $s_n = 0$ for any node which is not equipped with a DG unit. For the distribution network illustrated in Fig. 1, the power flow and voltage for each link between nodes n and $n+1$

satisfies the following equations [7] [8]

$$P_{n+1} = P_n - r_n \frac{P_n^2 + Q_n^2}{V_n^2} - p_{n+1}^d + p_{n+1}^g, \quad (2)$$

$$Q_{n+1} = Q_n - x_n \frac{P_n^2 + Q_n^2}{V_n^2} - q_{n+1}^d + q_{n+1}^g, \quad (3)$$

$$V_{n+1}^2 = V_n^2 - 2(r_n P_n + x_n Q_n) + (r_n^2 + x_n^2)\frac{P_n^2 + Q_n^2}{V_n^2}, \quad (4)$$

where $r_n + jx_n$ is the complex impedance of the link between node n and node $n+1$, V_n is the voltage at node n. Since the quadratic terms in (2), (3), and (4) are relatively small [7] [8], we can approximate (2), (3), and (4) as linear equations as

$$P_{n+1} = P_n - p_{n+1}^d + p_{n+1}^g, \quad (5)$$

$$Q_{n+1} = Q_n - q_{n+1}^d + q_{n+1}^g, \quad (6)$$

$$V_{n+1} = V_n - (r_n P_n + x_n Q_n)/V_0. \quad (7)$$

Since V_0 is constant, we can absorb it into the voltage at each node and define the voltage variation ΔV_n between node n and node $n+1$ as

$$\Delta V_n = V_{n+1} - V_n = -(r_n P_n + x_n Q_n). \quad (8)$$

Then, the total voltage deviation of the system with respect to the reference bus can be calculated as

$$\begin{aligned}\sum_{n=0}^{N-1} |\Delta_n| &= \sum_{n=0}^{N-1} (r_n P_n + x_n Q_n) \\ &= \sum_{n=1}^{N} \big[(p_n^d - p_n^g)(r_0 + \ldots + r_{n-1}) \\ &\qquad + (q_n^d - q_n^g)(x_0 + \ldots + x_{n-1})\big] \\ &= \sum_{n=1}^{N} \big[(p_n^d - p_n^g)\hat{r}_n + (q_n^d - q_n^g)\hat{x}_n\big] \quad (9)\end{aligned}$$

where $\hat{r}_n = \sum_{k=0}^{n-1} r_k$ and $\hat{x}_n = \sum_{k=0}^{n-1} x_k$, which are the cumulative resistance and reactance from the substation to node n.

B. User's Payoff Modeling

Each user has a DG unit that can generate both active and reactive power to satisfy its demand. However, it can decide not to generate reactive power if there is no incentive from electric utility company for reactive power compensation. Next, we quantify the benefit that each user can receive if it decides to participate in reactive power dispatch. We focus on the benefit for each user from reactive power dispatch due to the reduction of payment to the electric utility company.

We first calculate the payment of each user $n \in \mathcal{N}$ if it decides *not* to participate in reactive power compensation. In this case, user n only generates active power from its DG unit to meet its own load demand p_n^d. Let $p_{n,max}^g$ be the maximum available active power of the DG unit n, which depends on solar irradiance and temperature for solar panels or wind speed for wind turbines. Based on the amount of available capacity $p_{n,max}^g$, user n will generate p_n^{g0} amount of active power to serve its own active power demand. The amount of p_n^{g0} is determined by user n as follow

$$p_n^{g0} \triangleq \min\{p_n^d, p_{n,max}^g\}. \quad (10)$$

Note that, in (10), user n can predict its power demand p_n^d and maximum available active power generation $p_{n,max}^g$ at the current period of study. Therefore, p_n^{g0} is a fixed parameter and not a control variable in this problem. Moreover, equation (10) means that user n only generates active power to satisfy its demand and does not inject surplus active power back to grid even if there is active power available. The cost for user n to buy the remaining active power from the electric utility company is obtained as

$$\mathcal{C}_n^0 = \lambda(p_n^d - p_n^{g0}), \quad (11)$$

where λ is the unit price of active power at the current period of study. The unit price of active power may vary during the day. However, we assume that λ will be unchanged during each decision making time slot. Given the price information from the electric utility company, each user n can calculate its payment to the electric utility company since p_n^d and p_n^{g0} do not change during the period of study.

To incentivize reactive power compensation, the electric utility company offers a reimbursement z_n to user n for its amount of q_n^g reactive power dispatch. Therefore, the user n's payment to the electric utility company if participating in reactive power compensation can be calculated as the cost of purchasing remaining active power minus reimbursement:

$$\mathcal{C}_n = \lambda(p_n^d - p_n^g) - z_n. \quad (12)$$

In (12), the remaining amount of active power that user n purchases from the electric utility company is $(p_n^d - p_n^g)$. Also, by generating q_n^g reactive power, user n may potentially reduce the amount of active power p_n^g, constrained by (1). Moreover, the amount of active power generation p_n^g cannot be greater than the amount of active power generation in case the user does not participate in reactive power compensation:

$$p_n^g \leq p_n^{g0}. \quad (13)$$

User n controls the amount of active and reactive power generation $\{p_n^g, q_n^g\}$ so that it can reduce payment to the electric utility company.

Then we define the user n's payoff as the payment reduction when compensating reactive power for the electric utility company, denoted by $\mathcal{V}_n$

$$\mathcal{V}_n(p_n^g, q_n^g, z_n) = \mathcal{C}_n^0 - \mathcal{C}_n = z_n - \lambda(p_n^{g0} - p_n^g). \quad (14)$$

From (14), we realize that when user n does not participate in reactive power compensation, its payoff is $\mathcal{V}_n^0 = 0$.

C. Electric Utility Company's Payoff Modeling

By offering financial incentive for users to locally generate reactive power, the utility company can reduce the amount of remaining reactive power it has to provide, and thus reduce the cost for reactive power compensation. Let $Q_{inj}^{nocomp} = \sum_{n=1}^{N} q_n^d$ and $Q_{inj} = \sum_{n=1}^{N}(q_n^d - q_n^g)$ be the total amount of reactive power that node 0 has to inject into the distribution feeder to satisfy all reactive power demand of the overall system, without and with reactive power compensation from users, respectively. We further define $\boldsymbol{f}(Q) = \pi Q$ as the cost for the electric utility company to compensate Q units of reactive power at node 0, where π is a constant parameter [20]. Then the saving cost for reactive power compensation can be calculated as

$$\Delta f_{cost} \triangleq \boldsymbol{f}(Q_{inj}^{nocomp}) - \boldsymbol{f}(Q_{inj}) = \pi \sum_{n=1}^{N} q_n^g. \tag{15}$$

Moreover, by locally compensating for reactive power, the total voltage deviation along the network can be reduced. Based on (9), we first determine the total voltage deviation of the network in case users do not participate in local reactive power compensation as

$$\sum_{n=1}^{N} |\Delta_n^o| = \sum_{n=1}^{N} [\hat{r}_n (p_n^d - p_n^{g0}) + \hat{x}_n q_n^d]. \tag{16}$$

From (9) and (16), the reduction of voltage deviation by locally compensating for reactive power can be computed as

$$\Delta f_{vol} \triangleq \sum_{n=1}^{N} |\Delta_n^o| - \sum_{n=1}^{N} |\Delta_n| = \sum_{n=1}^{N} [\hat{r}_n (p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]. \tag{17}$$

Then the electric utility company's payoff can be defined as the saving cost for reactive power compensation and the reduction of voltage deviation along the distribution network

$$\begin{aligned} \mathcal{U}(\boldsymbol{q}^g, \boldsymbol{p}^g, \boldsymbol{z}) &= \left(\Delta f_{cost} - \sum_{n=1}^{N} z_n \right) + \alpha \Delta f_{vol} \\ &= \pi \sum_{n=1}^{N} q_n^g - \sum_{n=1}^{N} z_n + \alpha \sum_{n=1}^{N} [\hat{r}_n (p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]. \end{aligned} \tag{18}$$

where α is a positive weighted parameter to capture the trade-off between saving cost and voltage deviation.

D. Network Social Welfare Maximization

We define the social welfare as the aggregate payoff of electric utility company and users in the network

$$\begin{aligned} &\Psi(\boldsymbol{p}^g, \boldsymbol{q}^g, \boldsymbol{z}) = \mathcal{U}(\boldsymbol{q}^g, \boldsymbol{z}) + \sum_{n=1}^{N} \mathcal{V}_n(p_n^g, q_n^g, z_n) \\ &= \pi \sum_{n=1}^{N} q_n^g - \lambda \sum_{n=1}^{N} (p_n^{g0} - p_n^g) + \alpha \sum_{n=1}^{N} [\hat{r}_n (p_n^g - p_n^{g0}) + \hat{x}_n q_n^g] \\ &\triangleq \Psi(\boldsymbol{p}^g, \boldsymbol{q}^g). \end{aligned} \tag{19}$$

Then, the social welfare maximization problem can be formulated as

$$\begin{aligned} \max \quad & \Psi(\boldsymbol{p}^g, \boldsymbol{q}^g) \\ \text{s.t.} \quad & \{q_n^g, p_n^g\} \in \mathcal{X}_n, \forall n \in \mathcal{N}, \end{aligned} \tag{20}$$

where $\mathcal{X}_n$ is the set of feasible $\{q_n^g, p_n^g\}$ of user n, which is defined as

$$\mathcal{X}_n \triangleq \{q_n^g, p_n^g | q_n^g \in [0, q_n^d], p_n^g \in [0, p_n^{g0}], \text{constraint (1)}\}.$$

Given the complete knowledge and centralized control of the network, we can solve the network social welfare maximization problem in (20) to obtain the optimal active and reactive power generation. However, this requirement is difficult to fulfill in practice due to the distributed nature of the network topology. Moreover, solving (20) is not able to determine the amount of reimbursement for users. Therefore, in the next section, we use the Nash bargaining theory to determine the optimal solutions for power generation and reimbursement in a distributed fashion.

III. Sequential Bargaining

In this section, we first analyze the Nash bargaining solution (NBS) of the decentralized reactive power compensation under sequential bargaining protocol for a simple network consisting of one electric utility company and one user. Then we use this result to generalize the solution for multi-user network.

A. One-To-One Nash Bargaining Solution

In this subsection, we determine the NBS for a simple two-person bargaining, one electric utility company and one user. Let $\mathcal{Z}_n$ be the sets of feasible z_n

$$\mathcal{Z}_n \triangleq \{z_n \mid z_n \in [0, +\infty)\}. \tag{21}$$

Then the NBS is the solution of the following optimization problem

$$\begin{aligned} \max \quad & \left[\mathcal{U}(q_n^g, p_n^g, z_n) - \mathcal{U}^0\right] \cdot \left[\mathcal{V}_n(q_n^g, p_n^g, z_n) - \mathcal{V}_n^0\right] \\ \text{s.t.} \quad & \{q_n^g, p_n^g\} \in \mathcal{X}_n, \; z_n \in \mathcal{Z}_n, \end{aligned} \tag{22}$$

where $\mathcal{U}^0$ and $\mathcal{V}_n^0$ are the disagreement points of the electric utility company and user n, respectively. From (18), we can calculate the disagreement point of the electric utility company, which is the electric utility company's payoff without reactive power compensation $\mathcal{U}^0 = \mathcal{U}(0, 0, 0) = 0$. Then we can explicitly express the optimization problem (22) as

$$\begin{aligned} \max \quad & \left[\pi q_n^g - z_n + \alpha \hat{r}_n (p_n^g - p_n^{g0}) + \alpha \hat{x}_n q_n^g\right] \\ & \cdot \left[z_n - \lambda (p_n^{g0} - p_n^g)\right] \\ \text{s.t.} \quad & 0 \le q_n^g \le q_n^d, \\ & 0 \le p_n^g \le p_n^{g0}, \\ & (p_n^g)^2 + (q_n^g)^2 \le s_n^2, \\ & z_n \ge 0. \end{aligned} \tag{23}$$

By solving the optimization problem (23), we obtain the NBS for the two-person bargaining as the following theorem.

Theorem 1: The NBS $(q_n^{g*}, p_n^{g*}, z_n^*)$ for the one-to-one bargaining is

- If $(p_n^{g0})^2 + (q_n^d)^2 \le s_n^2$

$$q_n^{g*} = q_n^d, \tag{24}$$

$$p_n^{g*} = p_n^{g0}, \tag{25}$$

$$z_n^* = \frac{1}{2} \pi q_n^d + \frac{\alpha}{2} \hat{x}_n q_n^d. \tag{26}$$

- If $(p_n^{g0})^2 + (q_n^d)^2 > s_n^2$

$$q_n^{g*} = \min\{\frac{(\pi + \alpha\hat{x}_n)s_n}{\sqrt{(\lambda+\alpha\hat{r}_n)^2 + (\pi+\alpha\hat{x}_n)^2}}, q_n^d\}, \quad (27)$$

$$p_n^{g*} = \min\{\frac{(\lambda + \alpha\hat{r}_n)s_n}{\sqrt{(\lambda+\alpha\hat{r}_n)^2 + (\pi+\alpha\hat{x}_n)^2}}, p_n^{g0}\}, \quad (28)$$

$$z_n^* = \lambda(p_n^{g0} - p_n^{g*}) + \frac{1}{2}[\pi q_n^{g*} - \lambda(p_n^{g0} - p_n^{g*}) + \alpha[\hat{r}_n(p_n^{g*} - p_n^{g0}) + \hat{x}_n q_n^{g*}]]. \quad (29)$$

Proof: See Appendix A. ■

From the result in Theorem 1, we realize that the reimbursement covers the cost incurred by reducing the active power generation $\lambda(p_n^{g0} - p_n^{g*})$ and a half of its portion of social welfare contributed to the system, i.e., $\frac{1}{2}[\pi q_n^{g*} - \lambda(p_n^{g0} - p_n^{g*}) + \alpha[\hat{r}_n(p_n^{g*} - p_n^{g0}) + \hat{x}_n q_n^{g*}]]$.

B. Generalized Sequential Bargaining for Multiple Users

In this subsection, we find the NBS for a general model of reactive power compensation with multiple users under sequential bargaining protocol. The electric utility company will bargain with each user $n \in \mathcal{N}$ sequentially to determine (q_n^g, p_n^g, z_n). Without loss of generality, we assume that the electric utility company will bargain with users in the order of $1, 2, \ldots, N$ to obtain the NBS.

We first assume that at the current bargaining stage, the electric utility company already finished bargaining with prior users $1, 2, \ldots, n-1$, and starts bargaining with user n. Then the NBS $(q_n^{g*}, p_n^{g*}, z_n^*)$ between the electric utility company and user n is obtained via solving the following optimization problem

$$\begin{aligned} \max \quad & \left[\mathcal{U}_{[n]} - \mathcal{U}_{[n]}^0\right] \cdot \left[\mathcal{V}_n(q_n^g, p_n^g, z_n) - \mathcal{V}_n^0\right] \quad (30) \\ \text{s.t.} \quad & \{q_n^g, p_n^g\} \in \mathcal{X}_n,\ z_n \in \mathcal{Z}_n. \end{aligned}$$

Note that in (30), we use the subscript $[n]$ to denote the bargaining stage index. Moreover, the disagreement point of the electric utility company at the current bargaining stage is $\mathcal{U}_{[n]}^0$ rather than $\mathcal{U}^0$, which is calculated as the payoff that electric utility company achieved after bargaining with prior users $1, 2, \ldots, n-1$. From (18), we can determine $\mathcal{U}_{[n]}^0$ as the following equation

$$\mathcal{U}_{[n]}^0 = \pi \sum_{i=1}^{n-1} q_i^{g*} - \sum_{i=1}^{n-1} z_i^* + \alpha \sum_{i=1}^{n-1} [\hat{r}_i(p_i^{g*} - p_i^{g0}) + \hat{x}_i q_i^{g*}]. \quad (31)$$

We further calculate the payoff of the electric utility company at the current bargaining stage $[n]$ as from (18)

$$\begin{aligned} \mathcal{U}_{[n]} = \pi \sum_{i=1}^{n-1} q_i^{g*} - \sum_{i=1}^{n-1} z_i^* + \alpha \sum_{i=1}^{n-1} [\hat{r}_i(p_i^{g*} - p_i^{g0}) + \hat{x}_i q_i^{g*}] \\ + \pi q_n^g - z_n + \alpha[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]. \quad (32) \end{aligned}$$

Therefore, from (31) and (32) the payoff gain $\mathcal{U}_{[n]} - \mathcal{U}_{[n]}^0$ that the electric utility company receives if bargaining with user n is

$$\mathcal{U}_{[n]} - \mathcal{U}_{[n]}^0 = \pi q_n^g - z_n + \alpha[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]. \quad (33)$$

By substituting (33) into (30), we obtain

$$\begin{aligned} \max \quad & \left[\pi q_n^g - z_n + \alpha[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]\right] \\ & \cdot \left[z_n - \lambda(p_n^{g0} - p_n^g)\right] \quad (34) \\ \text{s.t.} \quad & \{q_n^g, p_n^g\} \in \mathcal{X}_n,\ z_n \in \mathcal{Z}_n. \end{aligned}$$

It is readily to realize that the optimization problem (34) at the bargaining stage $[n]$ is identical in case of one-to-one bargaining in (23). Therefore, the optimal solution for $\{q_n^{g*}, p_n^{g*}, z_n^*\}$ is similar to Theorem 1. Furthermore, we analyze the connection between the bargaining result and social welfare problem as the following theorem.

Theorem 2: The NBS $\{q_n^{g*}, p_n^{g*}\}_{n=1,2,\ldots,N}$ under the sequential bargaining maximizes the social welfare problem (20).

Proof: See Appendix B. ■

IV. Concurrent Bargaining

In this section, we find the NBS for the decentralize reactive power compensation under the concurrent bargaining protocol, where the electric utility company bargains with users concurrently. We also analyze the connection between the NBS and the network social welfare problem.

The generalized NBS under concurrent bargaining is the solution of the following optimization problem

$$\begin{aligned} \max \quad & \left[\mathcal{U}(q_n^g, p_n^g, z_n) - \mathcal{U}^0\right] \cdot \prod_{n=1}^{N} \left[\mathcal{V}_n(q_n^g, p_n^g, z_n) - \mathcal{V}_n^0\right] \\ & \quad (35) \\ \text{s.t.} \quad & \{q_n^g, p_n^g\} \in \mathcal{X}_n,\ z_n \in \mathcal{Z}_n. \end{aligned}$$

By solving the optimization problem (35), we obtain the following result.

Theorem 3: The NBS under concurrent bargaining $\{q_n^{g*}, p_n^{g*}\}_{n=1,2,\ldots,N}$ also maximizes the social welfare problem (20) and is identical to NBS under sequential bargaining. However, the reimbursement for each user is given by

$$\begin{aligned} z_n = \lambda(p_n^{g0} - p_n^g) \\ + \frac{1}{N+1} \Bigg[\pi \sum_{n=1}^{N} q_n^g - \lambda \sum_{n=1}^{N} (p_n^{g0} - p_n^g) \\ + \alpha \sum_{n=1}^{N} [\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]\Bigg]. \quad (36) \end{aligned}$$

Proof: See Appendix C. ■

Remark: From the above results, we conclude that the NBS $\{q_n^{g*}, p_n^{g*}\}_{n=1,2,\ldots,N}$ in both sequential bargaining and concurrent bargaining will maximize the social welfare problem (20). The reimbursement for user n covers the cost incurred by reducing the active power generation $\lambda(p_n^{g0} - p_n^g)$ and a half of its portion of social benefit contributed to the system, under sequential bargaining. While in concurrent bargaining, the social welfare of the system is equally divided among all users and the electric utility company.

Based on Theorems 2 and 3, the NBS for the decentralized reactive power compensation can be implemented by two steps as follows. First, each user individually determines the amount of active and reactive power generation, which maximizes the social welfare problem. Then, in the second step, depending on the amount of power generation from users and which bargaining protocol is selected, the electric utility company determines the amount of reimbursement offered to each user. Due to the distributed topology of power distribution networks as well as lacking coordination among users, the sequential bargaining is more practical to be deployed in realistic applications. For the concurrent bargaining, it can be applied in the scenario in which a group of users located in the same geographical area acts as a single entity and negotiates with the electric utility company in the reactive power compensation problem.

V. Simulation Results

In this section, we use numerical simulations to demonstrate the effectiveness of decentralized reactive power compensation. We test a distribution network with $N = 250$ users/nodes. The voltage at the node 0 is $V_0 = 7.2kV$. The line impedance is $(0.33 + j0.38)\Omega/km$, and the distances between neighboring nodes are drawn from a uniform distribution from 0.2km to 0.3km. Each node has the active power demand uniformly generated in the range $[1kW, 3kW]$, and the corresponding reactive power demand is generated in the range of $[0kVAR, 1.8kVAR]$. The number of users equipped with DG units is selected randomly and accounts for 50% of the total users in all simulations, unless otherwise stated, while the other users do not have DG units to participate in the reactive power compensation. The maximum apparent capacity for all DG units is $s_n = 2.2kVA$. The amount of available active power generation using renewable resources is generated randomly from a uniform distribution with lower and upper limits $[0.75s_n, s_n]$. The active power price is $\lambda = ¢6.6/kWh$ [20] and the constant parameter $\pi = 0.25 * \lambda/kVARh$. We set $\alpha = 1$.

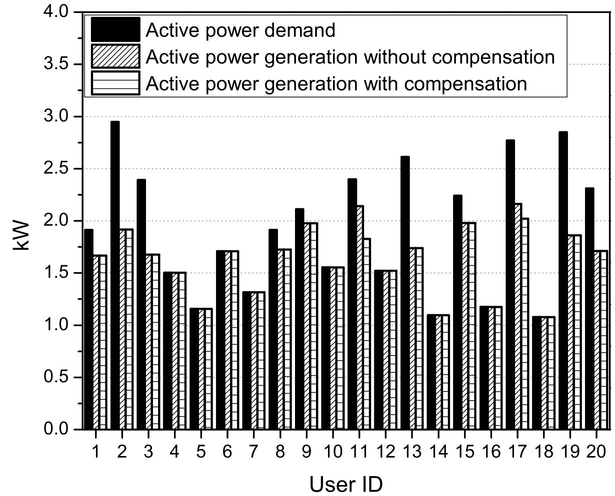

Fig. 2: The active power demand and generation profiles.

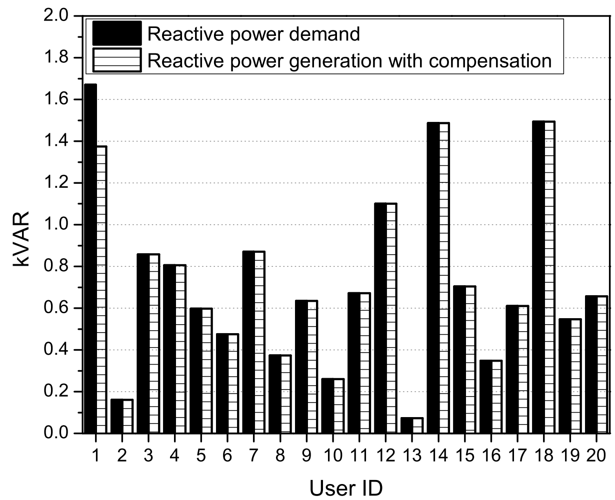

Fig. 3: The reactive power demand and generation profiles.

In Figs. 2 and 3, we plot the demand and generation profiles for active power and reactive power, respectively, of 20 randomly selected users from the set of users equipped with DG units. We compare active power generation when users do participate and when users do not participate in reactive power compensation. For users who have the generation capacity greater than demand, they generate as much reactive power and active power as possible to satisfy their power demand. Moreover, some users reduce the amount of active power generation to increase the amount of reactive power generation. For instance, users 11 and 17 even decrease the active power generation when they participate in reactive power compensation since at the current period, compensating for reactive power brings higher reimbursement than generating active power.

We further compare the reimbursement of 20 randomly selected users under sequential bargaining and concurrent bargaining protocols in Figs. 4 and 5. Specifically, the reimbursement that each user received covers the cost incurred due to the reduction of active power generation and its net payoff. For instance, users 11, 17 reduce active power generation to reserve capacity for reactive compensation. Therefore, the reimbursements that they received cover these reduction costs. Specifically, under the sequential bargaining protocol, the net payoff that each user received will be determined from a half portion of social welfare that the user contributed to the system, as shown in (29). Thus, each user has a different payoff. However, under the concurrent bargaining protocol, the net payoff each user received is equally divided from the total social welfare of the system for all users and the electric utility company, and hence the same for all users.

We study the effect of DG unit penetration level on the system reliability and efficiency. Note that, for all simulations so far, we assume that the number of users equipped with DG units accounted for 50% of the total number of users. In Fig. 6, we plot the percentage of voltage deviation of the system when the DG unit penetration level varies from 10% to 80%. Furthermore, three types of weather conditions are considered, namely, *sunny*, *partly cloudy*, and *cloudy*. For each of these weather types, $p^g_{n,max}$ is generated from a uniform distribution within the ranges $[0.75s_n, s_n]$ $[0.5s_n, 0.75s_n]$

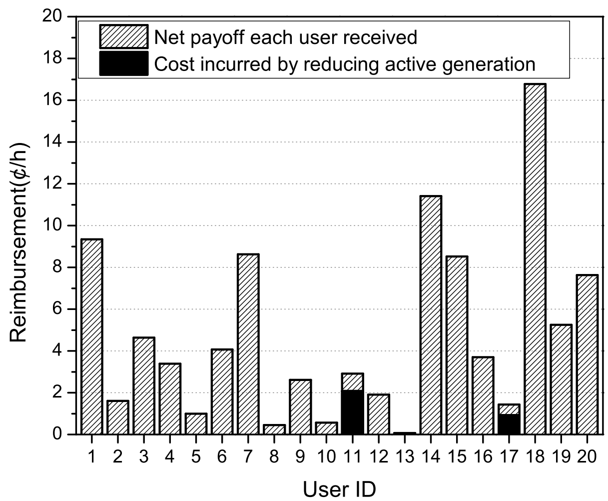

Fig. 4: The reimbursement of users in sequential bargaining.

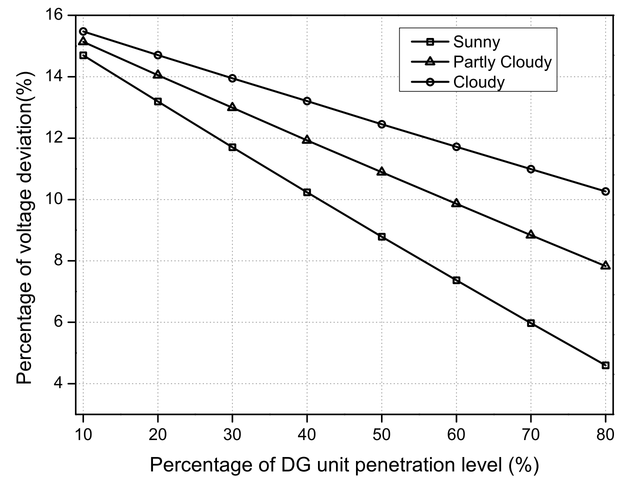

Fig. 6: The effect of DG unit penetration level on voltage deviation.

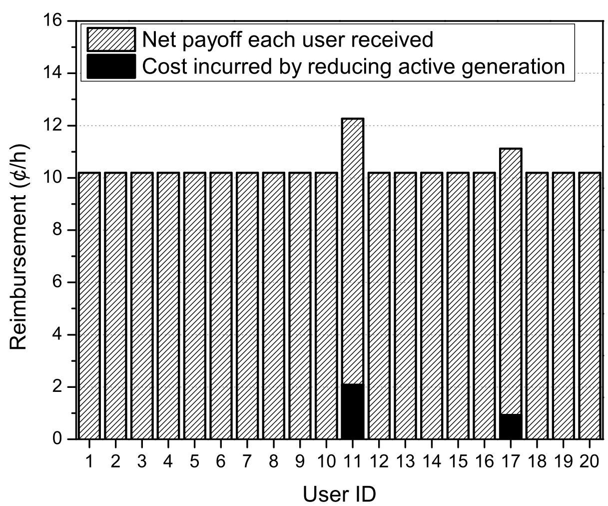

Fig. 5: The reimbursement of users in concurrent bargaining.

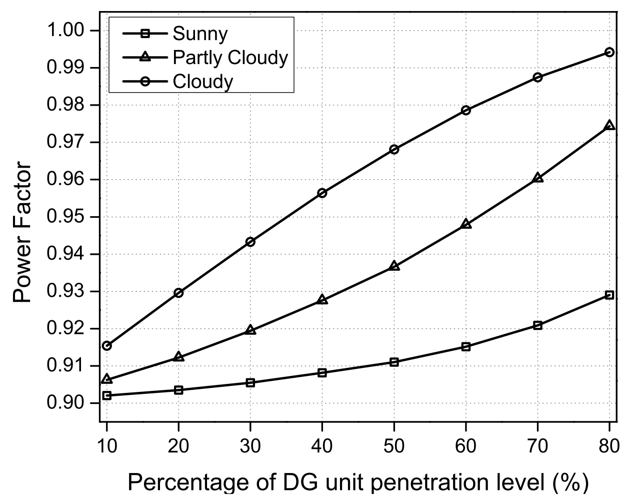

Fig. 7: The effect of DG unit penetration level on power factor.

$[0, 0.25s_n]$, corresponding to sunny, partly cloudy, and cloudy. From the figure, we realize that when the percentage of DG unit penetration level increases, the voltage variation of the system will decrease consequentially. This improvement happens due to local compensate for reactive demand, which renders the voltage variation to decrease. In addition, in Fig. 7, we plot the power factor of the system, which is calculated by $PF = P_0/\sqrt{P_0^2 + Q_0^2}$, when the DG unit penetration level varies from 10% to 80% for three different types of weather as well. The figure reveals that the power factor increases correspondingly to the increase of the penetration level.

Finally, we compare the performance of our proposed method with the centralized reactive power compensation control. The results are shown in Table I. To obtain the centralized control solution, we assume that the electric utility company has the ability to fully control the amount of reactive and active power generation of all DG units of all users and to solve the optimization problem in (20). Since the DG units are controlled by the electric utility company, the amount of surplus active power and reactive power can be injected back to the power grid in centralized control solution. We compare the amount of reactive power reduction and the percentage of voltage deviation along the distribution network between the centralized control and our proposed model for the case with 50% users equipped with DG units. From the results in Table I, the centralized solution can help the electric utility company reduce 71.5% amount of reactive power generation. Similarly, a higher power quality in terms of voltage deviation is shown for the centralized solution, where the total voltage deviation is only 6.8% while it is 8.5% in our proposed decentralized solution. Such performance improvement for centralized control is obtained due to the fact that the surplus reactive power and active power from users equipped with DG units can be utilized to supply demand to neighbor users who do not have DG units.

TABLE I: Performance comparison with centralized control

	Reactive power reduction(%)	Voltage deviation(%)
Centralized	71.5	6.8
Our approach	45.5	8.5

VI. Conclusions

In this paper, the decentralized reactive power compensation problem in a distribution network has been studied. Each user independently determines the amount of active and reactive power generation for its DG unit to locally compensate for reactive power. Based on the amount of power dispatch, users will receive reimbursement from the electric utility company. We investigate the economic interaction between users and electric utility company using the Nash bargaining theory. Optimal solutions of power generation and reimbursement are derived under both sequential bargaining and concurrent bargaining protocols. Numerical results in a distribution network with 250 nodes/users are conducted to illustrate the effectiveness of the decentralized reactive power compensation in enhancing system reliability.

Appendix A
Proof of Theorem 1

We can rewrite the optimization problem (23) as an equivalent optimization problem by taking $\ln$ of the objective function

$$\begin{aligned} \max \quad & \ln[\pi q_n^g - z_n + \alpha \hat{r}_n (p_n^g - p_n^{g0}) + \alpha \hat{x}_n q_n^g)] \\ & + \ln\left[z_n - \lambda(p_n^{g0} - p_n^g)\right] \qquad (37) \\ \text{s.t.} \quad & 0 \le q_n^g \le q_n^d, \\ & 0 \le p_n^g \le p_n^{g0}, \\ & (p_n^g)^2 + (q_n^g)^2 \le s_n^2, \\ & z_n \ge 0. \end{aligned}$$

The optimization problem (37) can be solved by decomposing into the following two steps. First, for fixed q_n^g, p_n^g, solve for optimal z_n by setting the first derivative of the objective function (37) to zero, we obtain

$$z_n = \frac{1}{2}\pi q_n^g + \frac{\lambda}{2}(p_n^{g0} - p_n^g) + \frac{\alpha}{2}[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]. \qquad (38)$$

By substituting (38) into the problem (37), we obtain the following subproblem for decision variables $\{q_n^g, p_n^g\}$

$$\begin{aligned} \max_{q_n^g, p_n^g} \quad & 2\ln \frac{\pi q_n^g - \lambda(p_n^{g0} - p_n^g) + \alpha[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]}{2} \qquad (39) \\ \text{s.t.} \quad & 0 \le p_n^g \le p_n^{g0}, \\ & 0 \le q_n^g \le q_n^d, \\ & (p_n^g)^2 + (q_n^g)^2 \le s_n^2. \end{aligned}$$

We now solve (39) to find the optimal $\{q_n^{g*}, p_n^{g*}\}$. Let consider two cases:

- If $(p_n^{g0})^2 + (q_n^d)^2 \le s_n^2$ (total demand of active power and reactive power is less than generation capacity), the user n can generate both active and reactive power to fully satisfy its own demand. Therefore, the optimal reactive power generation is

$$q_n^{g*} = q_n^d, \qquad (40)$$
$$p_n^{g*} = p_n^{g0}. \qquad (41)$$

Then we can easily obtain the optimal value for reimbursement

$$z_n^* = \frac{1}{2}\pi q_n^d + \frac{\alpha}{2}\hat{x}_n q_n^d. \qquad (42)$$

- If $(p_n^{g0})^2 + (q_n^d)^2 > s_n^2$, we realize that the objective function (39) is an increasing function of p_n^g and q_n^g. Therefore, the constraint $(p_n^g)^2 + (q_n^d)^2 = s_n^2$ must be hold at the optimality. Then we can express p_n^g as a function of decision variable q_n^g as

$$p_n^g = \sqrt{s_n^2 - (q_n^g)^2}. \qquad (43)$$

By substituting (43) to the objective function (39) and taking the first derivative of the objective function to zero, we can find the optimal solution for q_n^g and p_n^g as

$$q_n^{g*} = \frac{(\pi + \alpha\hat{x}_n)s_n}{\sqrt{(\lambda + \alpha\hat{r}_n)^2 + (\pi + \alpha\hat{x}_n)^2}}, \qquad (44)$$
$$p_n^{g*} = \frac{(\lambda + \alpha\hat{r}_n)s_n}{\sqrt{(\lambda + \alpha\hat{r}_n)^2 + (\pi + \alpha\hat{x}_n)^2}}. \qquad (45)$$

Since the reactive power and active power that user n generated cannot exceed its own demand, therefore we have

$$q_n^{g*} = \min\{\frac{(\pi + \alpha\hat{x}_n)s_n}{\sqrt{(\lambda + \alpha\hat{r}_n)^2 + (\pi + \alpha\hat{x}_n)^2}}, q_n^d\}, \qquad (46)$$
$$p_n^{g*} = \min\{\frac{(\lambda + \alpha\hat{r}_n)s_n}{\sqrt{(\lambda + \alpha\hat{r}_n)^2 + (\pi + \alpha\hat{x}_n)^2}}, p_n^{g0}\}. \qquad (47)$$

Moreover, we rewrite the (38) for purpose of analysis as

$$\begin{aligned} z_n &= \lambda(p_n^{g0} - p_n^{g*}) \\ &+ \frac{1}{2}[\pi q_n^{g*} + \alpha[\hat{r}_n(p_n^{g*} - p_n^{g0}) + \hat{x}_n q_n^{g*}] - \lambda(p_n^{g0} - p_n^{g*})]. \end{aligned} \qquad (48)$$

Appendix B
Proof of Theorem 2

From (39) in subproblem 2, we realize that the NBS in sequential bargaining is the optimal solution of the following optimization problem

$$\begin{aligned} \{q_n^{g*}, p_n^{g*}\} = \underset{(q_n^g, p_n^g) \in \mathcal{X}_n}{\arg\max} \; & [\pi q_n^g + \alpha[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g] \\ & - \lambda(p_n^{g0} - p_n^g)], \forall n \in \mathcal{N}. \end{aligned} \qquad (49)$$

We need to show that $\{q_n^{g*}, p_n^{g*}\}_{n=1,2,\ldots,N}$ also maximize the social welfare optimization problem (20). First, we decouple the objective function of the social welfare maximization problem (20) into

$$\begin{aligned} \Psi(\boldsymbol{p}^g, \boldsymbol{q}^g) &= \pi \sum_{n=1}^N q_n^g + \alpha \sum_{n=1}^N [\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g] \\ &\quad - \lambda \sum_{n=1}^N (p_n^{g0} - p_n^g) \\ &= \sum_{n=1}^N [\pi q_n^g + \alpha\hat{r}_n(p_n^g - p_n^{g0}) + \alpha\hat{x}_n q_n^g - \lambda(p_n^{g0} - p_n^g)] \\ &= \sum_{n=1}^N \Psi_n(p_n^g, q_n^g), \end{aligned}$$

where $\Psi_n(p_n^g, q_n^g) = [\pi q_n^g + \alpha \hat{r}_n(p_n^g - p_n^{g0}) + \alpha \hat{x}_n q_n^g - \lambda(p_n^{g0} - p_n^g)]$. Then we can rewrite (49) as

$$\{q_n^{g*}, p_n^{g*}\} = \arg\max_{(q_n^g, p_n^g)\in\mathcal{X}_n} \quad \Psi_n(p_n^g, q_n^g), \forall n \in \mathcal{N}. \tag{50}$$

From (50), for any $\{q_n^{g*}, p_n^{g*}\} \neq \{q_n^g, p_n^g\}$, we have

$$\sum_{n=1}^{N} \Psi_n(p_n^{g*}, q_n^{g*}) \triangleq \Psi(\boldsymbol{p}^{g*}, \boldsymbol{q}^{g*}) \geq \sum_{n=1}^{N} \Psi_n(p_n^g, q_n^g) \triangleq \Psi(\boldsymbol{p}^g, \boldsymbol{q}^g). \tag{51}$$

Therefore, the NBS in sequential bargaining maximizes the social welfare.

Appendix C
Proof of Theorem 3

Since the disagreement point $\mathcal{U}^0 = 0, \mathcal{V}_n^0 = 0, \forall n$, and by taking $\ln$ of the objective function in (35), we obtain

$$\begin{aligned} \max \quad & \ln\left[\pi\sum_{n=1}^{N} q_n^g - \sum_{n=1}^{N} z_n + \alpha\sum_{n=1}^{N}[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]\right] \\ & + \sum_{n=1}^{N}\ln\left[z_n - \lambda(p_n^{g0} - p_n^g)\right] \qquad (52)\\ \text{s.t.} \quad & \{q_n^g, p_n^g\} \in \mathcal{X}_n,\ z_n \in \mathcal{Z}_n, \forall n. \end{aligned}$$

We can solve the optimization problem (52) using similar method in Appendix A. Given the fixed q_n^g, p_n^g, the optimal solution z_n can be obtained by setting the first derivative of the objective function (52) with respect to z_n to zero

$$\frac{-1}{\pi\sum_{n=1}^{N} q_n^g - \sum_{n=1}^{N} z_n + \alpha\sum_{n=1}^{N}[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]} + \frac{1}{z_n - \lambda(p_n^{g0} - p_n^g)} = 0, \forall n. \tag{53}$$

Solving the set of N equations (53), we can obtain the expression of z_n as

$$\begin{aligned} z_n = \lambda(p_n^{g0} - p_n^g) + \frac{1}{N+1}\Bigg[\pi\sum_{n=1}^{N} q_n^g - \lambda\sum_{n=1}^{N}(p_n^{g0} - p_n^g) \\ + \alpha\sum_{n=1}^{N}[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]\Bigg]. \qquad (54) \end{aligned}$$

By substituting (54) into the objective function of (52), we obtain

$$\begin{aligned} \max (N+1)\ln\Bigg[\frac{1}{N+1}\Bigg(\pi\sum_{n=1}^{N} q_n^g - \lambda\sum_{n=1}^{N}(p_n^{g0} - p_n^g) \\ + \sum_{n=1}^{N}\alpha[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]\Bigg)\Bigg]. \qquad (55) \end{aligned}$$

From (55), the optimal solution $\{p_n^{g*}, q_n^{g*}\}_{n=1,2,\ldots,N}$ maximizes the inner term $\pi\sum_{n=1}^{N} q_n^g - \lambda\sum_{n=1}^{N}(p_n^{g0} - p_n^g) + \sum_{n=1}^{N}\alpha[\hat{r}_n(p_n^g - p_n^{g0}) + \hat{x}_n q_n^g]$, which is identical to the objective function of the social welfare problem (20). Therefore, the optimal solutions are as in Theorem 1. And the reimbursement is given in (54).

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