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AFWAL-TR-81-1264

DECENTRALIZED RELATIVE NAVIGATION AND JTIDS/GPS/INS INTEGRATED NAVIGATION SYSTEMS

William S. Widnall
Giuseppe F. Gobbini
John F. Kelley

Department of Aeronautics and Astronautics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

January 1982

Final Report for Period August 1979 - October 1981

Approved for public release; distribution unlimited.

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James I. Barker
JAMES I. BARKER
AFWAL/AAAN-2, Project Engineer

Ronald L. Ringo
RONALD L. RINGO, Chief
Reference Systems Branch
System Avionics Division

FOR THE COMMANDER:

Richard H. Boivin
RICHARD H. BOIVIN, Colonel, USAF
Chief, System Avionics Division
Avionics Laboratory

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS
BEFORE COMPLETING FORM

1. REPORT NUMBER
AFWAL-TR-81-1264

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
Decentralized Relative Navigation and JTIDS/GPS/INS Integrated Navigation Systems

5. TYPE OF REPORT & PERIOD COVERED
Final Technical Report for period Aug 79 to Oct 81

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
William S. Widnall, Giuseppe F. Gobbini, and John F. Kelley

8. CONTRACT OR GRANT NUMBER(s)
F33615-79-C-1879

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Massachusetts Institute of Technology
Dept. of Aeronautics and Astronautics
Cambridge Massachusetts 02139

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
666A/07/01

11. CONTROLLING OFFICE NAME AND ADDRESS
Avionics Laboratory
Air Force Wright Aeronautical Laboratories (AFSC)
ATTN: AFWAL/AAAN-1
Wright-Patterson Air Force Base Ohio 45433

12. REPORT DATE
January 1982

13. NUMBER OF PAGES
213

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Decentralized Estimation. Relative Navigation. JTIDS (Joint Tactical Information Distribution System). GPS (Global Positioning System).

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

One contribution of this research is a deeper understanding of the stability and performance of community relative navigation, such as is currently implemented in the Joint Tactical Information Distribution System (JTIDS). Relative navigation in the JTIDS system is organized in a decentralized manner with the Kalman filter in each member estimating only the member's own navigation state, but based on measurements that rely in part on the reported positions of other members. Simulations have shown the interacting navigation solutions can be unstable. We have found an analytical proof of the stability of one decentralized organization—

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the fixed rank hierarchy. Next an alternate approach for the community navigation, based on measurement sharing, is presented. This alternate has no stability difficulties, has superior accuracy, but has some increase in communication and computer requirements. Another contribution of this research is an assessment of the benefits of and approaches to integrating the data from JTIDS, GPS, and inertial systems. Finally, an extensive Fortran simulation of the performance of JTIDS/GPS/INS integrated navigation in a multi member environment has been developed and is used to explore the performance of both JTIDS/INS navigation and JTIDS/GPS/INS navigation.

FOREWORD

The Massachusetts Institute of Technology has been under contract to the Avionics Laboratory of the Air Force Wright Aeronautical Laboratories at Wright-Patterson Air Force Base, Ohio, to conduct research into the fundamental issues and problems associated with decentralized relative navigation and JTIDS/GPS/INS integrated navigation systems. The contract F33615-79-C-1879 has been monitored for the Air Force by Mr. James I. Barker of the AFWAL. The principal investigator for M.I.T. has been Dr. William S. Widnall, Associate Professor, Department of Aeronautics and Astronautics. Research assistants have been Giuseppe F. Gobbini and John F. Kelley.

This final technical report summarizes the contract research results, from contract start in August 1979 to completion. The research on the stability of decentralized navigation and navigation based on measurement sharing was conducted by Mr. Gobbini and Prof. Widnall. The analysis of JTIDS/GPS/INS integration was provided by Prof. Widnall. The design and implementation of the JTIDS/GPS/INS simulator was conducted by Mr. Kelley and Prof. Widnall. Simulation results included in the report were generated by Mr. Gobbini and Mr. Kelley. The text of this report has been written by Prof. Widnall.

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CHAPTER 1

INTRODUCTION

1.1 Scope of Report

The joint services of the United States Department of Defense are developing a Joint Tactical Information Distribution System (JTIDS), a time division multiple access command control communication system. The JTIDS is to include a relative navigation (Relnav) capability whereby members of the tactical community can determine their position relative to other members of the community. Initial designs for the Relnav capability have been implemented in the prototype JTIDS terminals.

Future improvements to the navigation capability will depend in part on a deeper understanding of the fundamental concepts and issues associated with relative navigation and of the fundamental problems associated with optimizing the integration of JTIDS, Global Positioning System (GPS), and Inertial Navigation System (INS) data. The Massachusetts Institute of Technology has been under contract to the Avionics Laboratory of the Air Force Wright Aeronautical Laboratory to conduct research into such fundamental issues.

This contract final technical report summarizes the M.I.T. research results. Chapter 2 is a review of prior research and development as reported in both the applied literature on JTIDS Relnav and the theoretical literature on decentralized estimation. Chapter 3 treats the stability of decentralized navigation. Chapter 4 presents an alternate approach for community navigation based on measurement sharing. Chapter 5 discuses the integration of JTIDS, GPS, and INS navigation data. Chapter 6 describes the capabilities of the M.I.T. simulator developed during this research effort. Chapter 7 presents JTIDS/INS Relnav simulation results. Chapter 8 presents JTIDS/GPS/INS simulation results. Chapter 9 summarizes the research conclusions.

1.2 JTIDS Concepts

The Joint Tactical Information Distribution System (JTIDS) is a synchronous, time-division multiple-access (TDMA), spread spectrum communication system. The JTIDS will provide a secure antijam data link between military elements in a tactical environment The elements can include users in the air, on the sea, and on land. An introduction to the JTIDS system concept and a description of one communication terminal hardware implementaion is provided by Dell-Imagine of Hughes in Ref. 1.

The system architecture uses time division to allow multiple users to participate. Each terminal within as network is assigned a number of transmit time slots in which it can broadcast messages. Each terminal in the net can listen to all time slots in which it is not transmitting. Proper system operation requires all users to have precise knowledge of system time. Initial user clock synchronization is attained by listening for either a special net entry message or for regularly transmitted position messages. Reception of this message assures the terminal that it has time to within a fraction of a 7.8 millisecond time slot. Fine synchronization can then be achieved by one of two methods: round trip timing (RTT) or passive synchronization.

In the round trip timing method, the donor terminal receives an interrogation and includes the time of arrival of the interrogate message in the reply message. The interrogation terminal measures the time of arrival of the reply message and combines this measurement with the data in the reply message to determine its timing error. The RTT method is very accurate, but requires both terminals to break radio silence.

Passive synchronization, as its name implies, is achieved by a user without breaking radio silence. In its simplest form, assume a user has an independent means of determining its own position. The donor transmits a position message. The receiving terminal measures the time of arrival, computes the propagaion delay associated with the range between the donor and user, and finally calculates its timing error.

1.3 Relative Navigation Concepts

In addition to its primary communications capability, the JTIDS has the inherent capability of providing high accuracy relative navigation data with respect to other terminals within the network. This added capability follows from the fact that all JTIDS terminals perform the very high accuracy time-of-arrival measurements on the signals received from the other terminals in the network. A time-of-arrival measurement may be considered a pseudo-range measurement.

Implementation of a JTIDS relative navigation capability does not require hardware modification to the JTIDS terminals. However it does require appropriate coordinated additions to the computer software in the JTIDS terminals. It is necessary to establish a consistent set of navigation definitions, community navigation architecture, and rules for data interchange to assure successful operation of the relative navigation function.

In absolute navigation, as opposed to relative navigation, we are concerned with the determination of one's own position and orientation within an agreed upon fixed coordinate system, such as the geodetic latitude longitude altitude coordinate system. In relative navigation we are concerned with one's own position and orientation relative to other members of a network, without necessarily being concerned with the geodetic location of the members of the network. In some missions, accurate relative navigation is sufficient. For example, consider a tactical scenario in which one aircraft locates a target relative to itself, then a second aircraft determines its location relative to the first aircraft plus receives the target location data. The second aircraft now can deliver weapons to the target. All this can take place without accurate knowledge of the absolute geodetic location of the target or of either aircraft.

To attempt relative navigation one must define a relative navigation grid. Fig. 1.1 illustrates two communities having identical relative positions. Each community has three members.

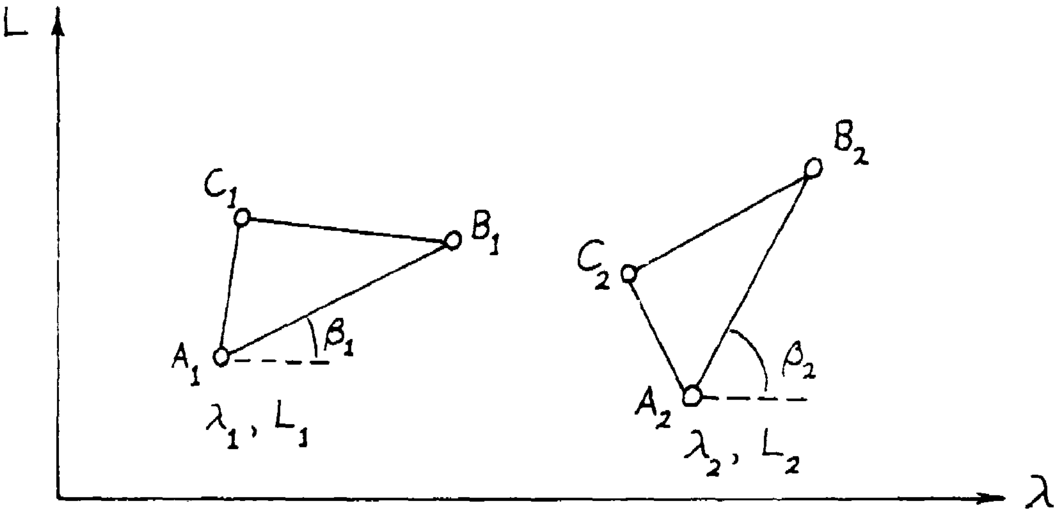
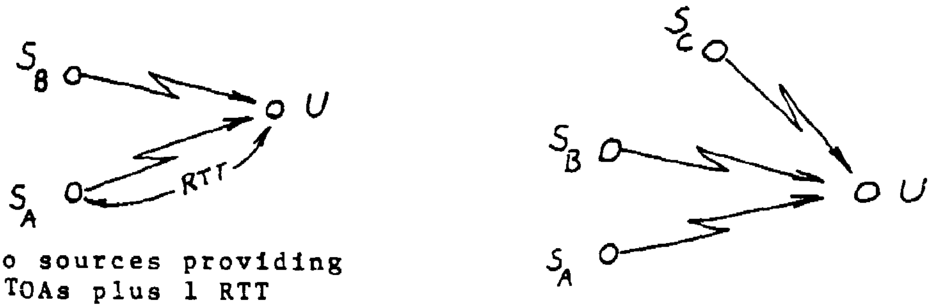
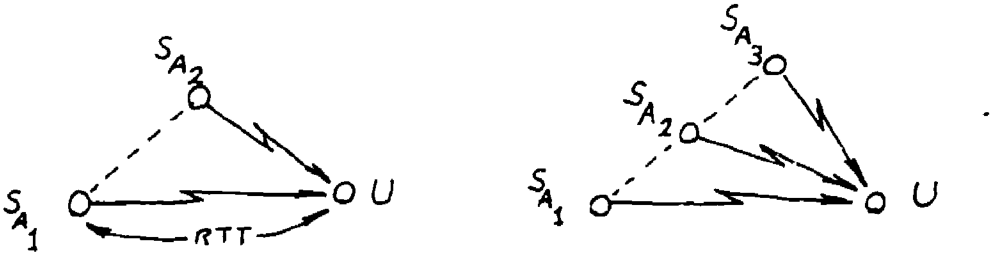


Fig. 1.1 Two Communities with Identical Relative Positions



Two sources providing
2 TOAs plus 1 RTT

Three sources providing 3 TOAs



Single moving source
providing 2 TOAs plus
1 RTT

Single moving source providing
3 TOAs

Fig. 1.2 Alternate Methods of Obtaining Three Independent Measurements

The distances between members in each community is the same, so the time of arrival measurements in each community could be the same (provided the clock errors were the same). The time of arrival measurements alone can not establish seven absolute navigation variables: the geodetic position of the origin of the relative grid (latitude, longitude, altitude), the geodetic orientation of the relative grid (three Euler angles), and the absolute time of the community.

In JTIDS relative navigation it is assumed that each member has an independent means of determining geodetic altitude and geodetic horizontal. The time of arrival measurements then are needed only to fix the horizontal position, the azimuth orientation, and the time. The relative navigation problem is reduced to a two dimensional navigation problem. It is convenient to set arbitrarily three of the unobservable seven variables relating the relative and absolute grids: the altitude of the origin of the relative grid is set at sea level and two axes of the relative grid are set tangent to the earth horizontal at the grid origin. This leaves unspecified the longitude and latitude of the origin of the relative grid, the azimuth orientation of the relative grid, and the grid time offset.

There are many ways of establishing the remaining four unobservable variables. The most convenient method of establishing the time offset is to choose one member to be the net time reference. All members attempt to synchronize their JTIDS clocks with that of the net time reference. The absolute error of the clock of the net time reference is the unobservable grid time offset. One method of establishing the horizontal position and azimuth of the relative grid is to use two fixed position references. One of the position references can establish the origin. The second establishes a baseline defining the orientation of the grid. A second method uses one fixed position reference and one aircraft with inertial navigation system (INS). A third method uses two aircraft with inertial systems. A fourth method uses a single rapidly moving aircraft with inertial system. This last method is the method currently implemented in the JTIDS software. This single grid setter is called the navigation controller.

In the current JTIDS navigation concept a user can obtain a fix of its own position using its own time of arrival measurements together with the reported positions of the sources. The user may also choose to use active round trip timing measurements. Assuming the reported positions of the sources have negligible error, three linearly independent measurements are necessary and sufficient for the user to obtain a fix of its own horizontal position (2 variables) plus its clock offset. Fig. 1.2 illustrates some of the methods of obtaining three independent measurements. Two sources can provide two independent TOA (time of arrival) measurements and one of them can provide a RTT (round trip timing) measurement. Or three sources can provide three TOA measurements permitting a passive fix. A single moving source can provide the necessary measurements sequentially. Two sequential TOA measurements plus one RTT measurement provides a fix. Or Three sequential measurements provide a passive fix.

If the reported positions of the sources were geodetic positions, the resulting fix is the geodetic position of the user. If the reported positions of the sources were positions in the relative navigation grid, the resulting fix gives the relative position of the user. The current JTIDS navigation concept permits dual grid operation, that is permits navigation fixing both in absolute geodetic coordinates and in relative grid coordinates.

Note the similarity of the three-source passive method of obtaining a horizontal navigation fix and the method utilized in GPS navigation to obtain a 3-D fix. In GPS navigation, time of arrival measurements from four of the available satellites combined with the reported geodetic positions of the satellites are sufficient to fix the three components of geodetic position and the clock error of the user.

However a fundamental difference between JTIDS navigation and GPS navigation is that in GPS navigation the reported positions of the satellites are highly accurate but in JTIDS the reported positions of the sources may be inaccurate. In fact the sources themselves may be trying to fix their own positions from time of arrival measurements from other members of the community. If closed loop information paths exist, the JTIDS navigation solution may be unstable. We will discuss the stability problem furthur in Chapter 3.

All active units transmit, perhaps every 12 sec, a position and status message (P message). Possible content of the P message includes: source terminal's horizontal position, speed, heading, altitude, as well as its position quality, time quality, and relative azimuth quality. Using these data and appropriate source selection logic, perhaps based on the source's and the user's own quality levels, the user terminal selects the desired sources, calculates the predicted pseudorange, and compares it to the measured pseudorange. By means of an appropriate recursive (Kalman) filter, these pseudorange differences are used to update the estimates of position, velocity, time bias, and time bias rate.

In most aircraft applications, dead reckoning data is available from an inertial system, a doppler system, or an air-data and heading reference system. The inertial or other data is used to extrapolate the aircraft navigation state between filter measurement incorporation time points. In such aircraft, an error state filter formulation would be implemented estimating the errors in the indicated position and velocity of the dead reckoning system. Additional states estimated would include the inertial alignment errors and/or other dead reckoning sources of error.

1.4 JTIDS/GPS/INS Integration

The Global Positioning System (GPS) is a satellite navigation system capable of providing continuous worldwide navigation of unprecedented accuracy. When fully deployed, eighteen or more satellites in 12 hour orbits will provide the continuous coverage. The ground control segment tracks the satellites, determines their orbital ephemeris, and uploads the ephemeris to each satellite. The satellites rebroadcast the ephemeris data to the users of the system. Atomic clocks in the satellites maintain accurate time synchronization. The receivers of the users measure the times of arrival of the psuedo random coded signals. The crystal clock in a receiver has significant drift, so the time of arrival (or pseudo range) measurement has significant clock error. Four psuedo range measurements from four different satellites are sufficient to fix the navigation unknowns, the three components of position and the receiver clock error.

To integrate JTIDS, GPS, and INS data, a Kalman filter can be used to optimally combine the data. The use of both GPS navigation and JTIDS Relnav can provide significant performance benefits. We discuss these benefits in Chapter 5. Simulation results in Chapter 8 provide a quantitative demonstration of some of the benefits.

CHAPTER 2

REVIEW OF PRIOR RESEARCH AND DEVELOPMENT

2.1 PLRACTA and ITNS Navigation Literature

Bivin of the Naval Air Development Center in Ref. 2 provides a survey of prior programs that have contributed to the technology base for JTIDS and its relative navigation capability. A collision avoidance system was developed by McDonnel Douglas in 1965 for flight testing of high performance aircraft. This system was similar to JTIDS in that it used spread spectrum bi-phase modulation and used signal time of arrival to infer range. Interrogate-respond synchronization was used, all units transmitted in turn, and units received when not transmitting. Danger situations were determined based on range and range rate with altitude separation calculated from position reports.

Both the Air Force and the Navy sponsored programs to advance the technology. The Air Force Electronic Systems Division and the Mitre Corporation began the PLRACTA (Position Location, Reporting, and Control of Tactical Aircraft) project. The Naval Air Development Center and the Singer Kearfott Division began the ITNS (Integrated Tactical Navigation System) project.

Objectives of the PLRACTA project were to verify the feasibility of attaining and maintaining synchronization of remote airborne clocks based on inexpensive crystal oscillators by means of passive ranging, to use these clocks to support a time division multiple access data net, and to demonstrate the position location capability inherent in this process. A system of ground stations and two airborne units was built. Flight tests conducted in 1971-72 successfully demonstrated the basic concepts. One performance problem predicted in simulations and observed in the flight tests was the inherent instability of multi-user systems with completely passive synchronization. Further simulation work led to recommending a covariance based hierarchical community organization including limited round-trip timing for clock synchronization, as reported in Ref. 3. In PLRACTA the presence of surveyed ground sites was assumed, providing a geodetically referenced baseline for positioning.

In the ITNS project the Navy was pursuing a relative navigation approach designed for operation at sea with no fixed references or surveyed positions. The system developed by Singer had an active mode for round trip synchronization and a passive (receive only) mode. A community structure was established in which a master unit established a tactical navigation grid, all units would report their inertial position in grid coordinates, range would be measured and compared with expected range based on position reports, and inertial corrections applied. Flight tests were conducted with two airborne units and up to three ground units. Stable relative navigation was demonstrated. It was discovered that large initial errors could cause linearity and ambiguity problems at net entry. Techniques were developed to overcome this problem. It was also discovered that use of a single fixed ground station could cause a rotational instability.

The completely passive community used in PLRACTA was not attempted in ITNS.

Stow of Singer describes the system and relative navigation concept in Ref. 4. He provides simulation results for the case of two fixed ground units and one airborne unit, which is in the active mode. Danik of Singer in Ref. 5 also describes the system and the single aircraft simulation results. He also provides a qualitative discussion of a few other simple cases: one aircraft and one ground station, two aircraft (master and user), and helicopter and ship. No multiple aircraft simulations are presented.

Studies at Singer (Ref. 6), ITT (Ref. 7), Litton (Ref. 8), and Dynamics Research Corporation (Refs. 9,10) were conducted to investigate alternate approaches to relative navigation. The paper by Rome and Stambaugh (Ref. 10) summarizes the DRC study results. Many different community relative navigation organization concepts are considered. Some organizations have a designated master that relies on its own dead reckoning system and does not utilize measurements from other members. Other organizations have no master and the members attempt to arrive at a consistent definition of the relative grid in some democratic manner. Within both the with-master and master-free organizations further distinctions are identified, including the many-on-one, the baseline, the weak hierarchy, and the strong hierarchy organizations. A covariance analysis simulation is used to evaluated some of the alternate organizations. Scenarios include a rapid maneuver community having four aircraft and a community having two aircraft and two ships. The simulations of four organizations with masters are compared. The many-on-one organization consistently is less accurate than the others. Better accuracy is obtained when members use more than just the master as ranging sources. The end of baseline organization gives good accuracy. The covariance defined strong hierarchy and the weak hierarchy sometimes give good accuracy but sometimes are unstable. In these mechanizations each element uses reported position covariances to weight its range residuals in an attempt to model the effect of the position errors of the other members. This modeling of the correlated position errors as uncorrelated measurement noise gives unreliable results. After several observation cycles, the Kalman derived position covariance of all the members drop to a value where their range residuals are weighted equally with the master's. At this point there is no strong definition of the grid in the community and instabilities may result. Simulations of the master free organizations indicated that these are generally not as accurate as those with a master.

The studies showed that alternatives to inertial dead reckoning were practical. A potential for instability was discovered in communities with mixed dead-reckoning systems unless grid drift error states were modeled. The Rome and Stambaugh paper simulation results indicate that an all-Doppler community may have more accurate relative navigation than a mixed community having a Doppler master and some members with inertial systems.

The interest in a Doppler version of relative navigation for helicopter use led to additional flight tests involving the Army Electronics Command (Refs. 2,11). The tests in 1975 demonstrated that stable relative navigation is feasible using Doppler and, as predicted, the accuracy is somewhat less than with inertial dead reckoning. Mixed community operation was tried and measurement weighting using reported position covariances was attempted. The latter did not work. However there were software problems that permitted negative covariance diagonal elements, so the results should be considered inconclusive.

2.2 JTIDS Relative Navigation Literature

The efforts of the military services were merged in the JTIDS program. Two classes of terminals were defined, a Class 1 terminal for use on command and control platforms such as AWACS, and a Class 2 terminal for use on small tactical platforms. The Class 1 terminal is being developed by Hughes. The Class 2 terminal is being developed by Singer-Kearfott. The JTIDS Relnav Working Group was established to try to define a set of rules for data interchange and organization which would assure successful operation.

The 1976 paper by Fried (Ref. 12) is a good introduction to the JTIDS Relnav concepts and terminology. The paper has been also published in an archive journal (Ref. 13) with the addition of some references and slightly more Kalman filter information. A possible architecture for the relative navigation community is presented. The relationship of the members of the community is illustrated in Fig. 2.1. One member of the network, designated the time reference, establishes system time and is assigned the highest time quality. For relative grid operation, one member, designated the navigation controller, establishes the origin and orientation of the relative grid. The origin is at sea level and is assumed stationary. Actually the grid origin and orientation may be slowly moving as a result of the dead reckoning errors of the navigation controller. All net members attempt to determine their position with respect to these grid coordinates. The navigation controller is assigned the highest relative position quality. There may also be terminals possessing high accuracy absolute position information. These are designated position references and are assigned the highest absolute position quality. Below the position references and the navigation controller there could be two classes of users: primary and secondary users. The primary users are permitted to use round trip timing for clock synchronization at relatively frequent intervals, perhaps whenever their time quality (as estimated by the terminal's filter) falls below a certain level. These (relatively few) units, having excellent time quality, are used as primary navigation references. Secondary users do not perform round trip timing frequently and, in fact, must be capable of performing clock synchronization and relative navigation completely passively. Within the two classes of users, quality levels are established on the basis of accuracy estimated by each terminal's filter. Source selection logic perhaps can be developed based on these self-estimates of accuracy. For

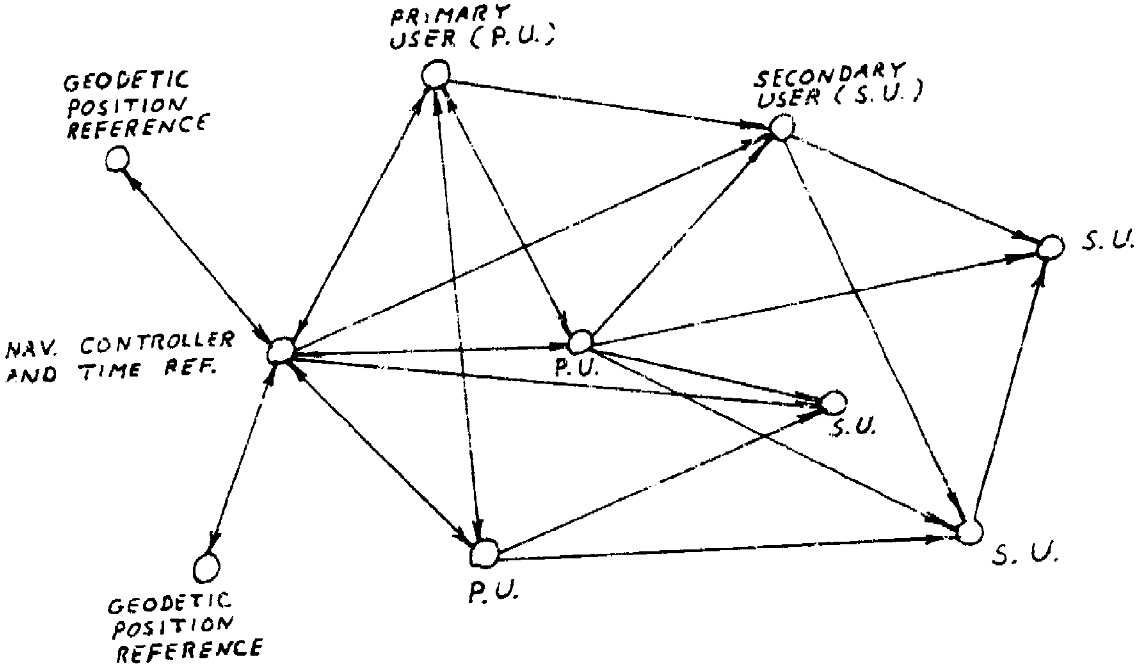


Fig. 2.1 Community Relative Navigation Architecture

example, one secondary user might use another secondary user as a source only if the latter has a higher position and time quality.

Fried (12,13) also presents some simulation results. The members of the community do not have a dead reckoning capability. All scenarios include four ground stations (position references), one of which is also the time reference. Two scenario types are investigated, a wide baseline geometry and a narrow baseline geometry. Airborne member measurement selection logic in some cases selects every P-message from all other airborne members an in other cases selects only to P-messages from the ground stations. Up to 11 members are included. The simulation results show that for certain poor geometry conditions, divergence occurs when airborne members are permitted to use every position message from all other units at all times. Fried concludes that it will be necessary to have some form of strong hierarchical source selection based on own and source's time and position quality. Recall however that Rome and Stambaugh (10) found that such a strong hierarchical organization was not sufficient to guarantee stability.

Steele and Schlenger of Singer in Ref. 14 discuss JTIDS relative navigation and its implementation in the Singer terminal. The paper discusses dual grid operation (relative and geographic) and the Kalman filter requirements to maintain independence of the grids. No simulation results are presented.

Greenberg and Rome in Ref. 15 summarize the more recent work at Dynamics Research Corp. The paper gives increased emphasis to the problem of stability. A covariance analysis simulation is used to evaluate alternate organizations. All organizations considered have a master member and all other members actively synchronize their clocks to the master. All members are aircraft and have inertial navigators. The organizations considered include a weak hierarchy, a position defined hierarchy, a heading defined hierarchy, and an end of baseline hierarchy. In the weak hierarchy, all members except the master use all received measurements. In the position defined hierarchy, a member uses a measurement only if the reported relative position quality is better than own computed relative position quality. In the heading defined hierarchy, a member uses a measurement only if the reported relative heading quality is better than own computed relative heading quality. In the end of baseline hierarchy, the end of baseline member uses measurements only from the master, and all other members use measurements only from the master and the end of baseline. Other criteria for establishing hierarchy are discussed but are not evaluated by simulation. The paper also discusses alternate choices for assumed measurement error variance in terms of the reported relative position quality. This influences the Kalman gains that are used to weigh the measurements. No mechanizations are discussed in which additional filter states rather than increased measurement noise are used to model the position and time errors of the source. The simulations are of a community of four aircraft. The simulation results indicate that the weak hierarchy has the best relative range accuracy at 30 min. of relative navigation, however at one hour the errors have increased. The authors conclude the weak hierarchy is marginally unstable. Furthermore if the reported position quality is not used to increase the assumed measurement variance, the relative navigation is completely unstable. The three strong hierarchies (position defined, heading defined, and end of baseline) all demonstrated stable behavior. The position defined hierarchy had better relative range accuracy and the heading defined hierarchy had the better pointing accuracy. The end of baseline organization had the worst relative range accuracy both at 30 min and at one hour. The position and heading defined organizations could be made unstable by reducing the assumed measurement error variance.

Companion papers presented at the 1979 National Aerospace Electronics Conference summarize progress at Hughes in implementing inertially aided JTIDS relative navigation. The paper by Fried (16) reviews JTIDS relative navigation architecture, error characteristics, and operational benefits. The paper by Fried of Hughes and Loeliger of Intermetrics (17) presents the Hughes terminal system configuration and the algorithm design for the inertial navigation processing, the source selection logic, the Kalman filter, and the process timing and sequencing. A combination of the two papers also has been published in an archive journal, Ref. 18. A Litton LN-31 has been interfaced with the Hughes JTIDS terminal. The relative navigation function is added to the JTIDS terminal computer. Velocity data from the INS is used to infer acceleration, which is integrated in a full inertial computational algorithm. The paper does not make it clear why it was necessary to duplicate the inertial navigation equations in the terminal computer.

The Kalman filter has 18 states. They have chosen to estimate the error in geodetic position and the error in relative grid origin geodetic position and the error in the relative grid azimuth. The error in relative position need not be explicitly estimated because it is linearly dependent on states which are estimated. This choice of states is said to minimize coordinate computations. However it will lead to a nearly singular covariance matrix when in relative navigation the geodetic position errors and the grid origin errors become highly correlated.

There is no assurance that the incoming pseudo range measurements will be conveniently spaced in time so that the filter can keep up with the measurements. On the contrary the time slots of the measurements are quite arbitrary and so groups of measurements can be bunched in time. Therefore buffering is required to save measurements and the simultaneous navigation states until the filter is ready to process a new set. The source selection logic screens the incoming position messages and selects the best set of data to store in the buffer. Up to eight measurements can be stored in the buffer. Five different criteria are used to select up to seven measurements. The criteria are comparisons of incoming reported accuracy with own computed accuracy for geodetic level position, grid level position, altitude, time, and grid azimuth. The geodetic and grid position criteria each can select a pair of measurements. The other three criteria each can select one measurement. There are four update types that can be constructed from the incoming P messages: grid pseudorange, geodetic pseudorange, offset along line of sight, and offset across line of sight. A round trip timing event is the eighth possible measurement in the buffer. A source selection cycle can be short or can include a full eight selected measurements, depending in part on the CPU availability. If message traffic allows, the filter, which has a lower priority, will complete a cycle very rapidly. The lower priority for Relnav and its filter was chosen so as to minimize the impact on the communication function of the JTIDS terminal.

Once per source selection cycle the inertial variables are reset according to the filter estimated errors. It appears that only the redundant variables are reset, not the LN-31 variables. Not discussed is the effect of allowing the LN-31 variables and the redundant variables to have different values. A possible acceleration error is introduced due to the unmodeled difference between the variables.

2.3 JTIDS/GPS/INS Integration Literature

The NAVSTAR Global Positioning System (GPS) and the Joint Tactical Information Distribution System (JTIDS) are both likely to be used by many military platforms. In addition most military aircraft carry inertial navigation systems (INS). Of concern is the effective integration of these and other avionic systems. A General Officers' Steering Committee for Communications, Navigation, and Position Integration (CNPI) was established by the Air Force in May 1977. An objective of this committee was to establish a feasible integration plan that reduced the overall equipment requirements for space, power, cooling, and weight. The C.S. Draper Laboratory and the ARINC Research Corporation as associate contractors performed a technical and life cycle cost analysis concerning GPS/JTIDS/INS integration. The results are summarized in Refs. 19,20,21. A conclusion was that it will be possible to install GPS and JTIDS user equipment sets on tactical fighter aircraft without integrating the hardware. To achieve this it will be necessary for the sets to be more compact than the early GPS and JTIDS development models, but this is considered to be within the state of the art of component technology, large scale integration, and packaging design.

The study also considered various levels of hardware integration. One ground rule was that the existing inertial navigators in the aircraft be used. Another ground rule was that any proposed integrated system had to be capable of being on board tactical aircraft and operational in the 1984 time frame. This meant that only current and near future technologies could be included. In the most integrated design, the GPS and JTIDS share a common oscillator, frequency synthesizer, power supply, computer, and RF and IF channels. A surprising conclusion was that this most integrated design only achieves a 8 or 9% reduction in required volume compared with the minimally integrated systems. The reduction in volume due to the sharing of common hardware was not enough to allow for a quantum jump to the next standard size line replaceable unit. The recommended design for the 1984 time frame has independent GPS and JTIDS hardware except for a single aircraft interface data unit and a common power supply. Data links are provided to interconnect the GPS, JTIDS, and INS sets.

Although little benefit was found from hardware integration, the study discussed the many benefits obtainable from data integration. These benefits will be discussed in the chapter on JTIDS/GPS/INS integration. Part of the Draper Laboratory study included the design of a Kalman filter for integrating GPS, JTIDS, and INS data, as summarized in the Kriegsman and Stonestreet paper (21). A covariance analysis simulation was used to compare the accuracy of the integrated JTIDS/GPS/INS system with that of a GPS/INS system and a JTIDS/INS system. No surprises were reported. The integrated system, as one would expect, was more accurate than either of the other systems. Only qualitative results are presented in the unclassified paper (21). The quantitative simulation results are in the classified Volume 3 of Ref. (19).

The MFBARS program at the Air Force Avionics Laboratory has been exploring the design of a Modular multiFunction multiBand Airborne Radio System (MFBARS). This program is driven by the same concerns that led to the Draper Lab CNPI study. However the MFBARS studies have not been constrained to look at only near term technology. Also the studies are considering more than JTIDS and GPS for possible integration. ITT and TRW have been the contractors on the Program. During Phase I a set of candidate architectures were developed and compared. During Phase II one of the architectures was selected as a preferred approach and became the focus of a more detailed analysis and design effort. In its Phase II report, Ref. 22, ITT summarizes its results. The 14 operationally required radio functions (JTIDS, GPS, Seek Talk, UHF communication, VORTAC navigation, ILS, Identification, etc.) can be accomplished by four radio sets, provided each is considerably more flexible than its single function predecessors. Two emerging technologies are used extensively in the recommended design: charge coupled devices and surface acoustic wave devices. These devices function as an analog memory for an incoming signal. This allows a time segment of signal to be stored and processed different ways at different times. Transversal filters are implemented to do the desired signal processing. An agile programmable transversal filter for RF signal processing is at the heart of the recommended design. It can change tuning faster than the reciprocal of the output bandwidth, which had not been possible with conventional filters. With this feature, high speed time sharing of the signal is possible with no loss in signal to noise ratio. The study found significant direct benefits to the highly integrated most advanced technology design in terms of weight savings, volume savings, production cost savings, and life cycle cost savings.

The MFBARS studies have concentrated on hardware considerations. The various computer algorithms needed to accomplish the digital signal processing and the data processing have been considered only to obtain an estimate of the computer thruput requirements and memory requirements. In particular no effort has been devoted to integrated software design including a possible universal Kalman filter for navigation data processing.

A paper by Rome, Reilly, and Ward (Ref. 23) further discusses the benefits of fully integrating JTIDS and GPS data. Their comments and recommendations will be reviewed in a later chapter.

2.4 Decentralized Estimation Literature

The JTIDS Relnav problem can be considered to be an example of decentralized estimation. A centralized solution to the Relnav problem could be implemented by radioing all measurements to a central computer whose Kalman filter would estimate the navigation state throughout the community. Such a centralized solution would require a large amount of data being sent to the central computer. Also it would require a powerful computer to implement the Kalman filter because the dimension of the state vector would be very large and the number of measurements to be processed would be large. The JTIDS approach is a decentralized solution. Responsibility for estimating the community navigation state is spread throughout the community. In particular, the navigation state of a particular member is estimated by that member. Furthermore the measurements used by a member are only the measurements obtained by its own receiver. In place of sharing the values of its measurements, a member broadcasts its own estimated position.

A complete and satisfactory theory of decentralized estimation does not exist. Kerr (Ref. 24) has reviewed some of the approaches to decentralized estimation. He has found two approaches to be applicable to the JTIDS Relnav problem. These are the Surely Locally Unbiased filter proposed by Sanders et al (Ref. 25) and the Sequentially Partitioned Algorithm proposed by Shah (Ref. 26). The application of both methods to the JTIDS problem would retain the partitioning and allocation of the community state as in the current JTIDS organization but would alter the weights associated with the accuracy of the measurements.

Speyer (Ref. 27) and Willsky et al (Ref. 28) propose decentralized organizations that reconstruct the centralized optimal estimate. Speyer's method requires all members to have a full sized filter modeling all the community navigation states. They must also have an auxiliary vector of the same size as the complete state vector. It is data dependent and must be updated on line. Each member knows only its own measurements, but it can reconstruct the optimal centralized estimate by linearly combining the estimates and the auxiliary vectors of all members. The members share their estimates and auxiliary vectors.

Willsky's method is an extension of Speyer's method. It allows every member to have an incomplete state vector (such as only its own state variables) provided certain conditions are satisfied. In particular the state vector must include any state variable that is needed to compute the expected value of that member's measurements. There is only one auxiliary vector, which has the same size as the complete state vector, is not data dependent, and must be updated by a central processor that must be supplied with the covariance matrices from all members. The central processor must also know all the local estimates. It obtains the centralized optimal estimate by linearly combining the local estimates and the auxiliary vector.

Neither Speyer's nor Willsky's method appear to be helpful in the JTIDS application. The Speyer method requires a large amount of computation since every member must have a full sized state vector and associated covariance matrix plus an equally large auxiliary vector and matrix. The Willsky method requires a central processor with large computational capability. Both methods require a large amount of information to be shared by the local estimators: all local estimates, all local covariance matrices, and in Speyer's method the auxiliary vectors as well.

In some applications the measurement geometries, the measurement schedules, and the state transition matrices are all known in advance. The required covariance matrices can be computed in advance and stored. But in the JTIDS application the mission trajectories are not known precisely in advance so the covariance matrices must be computed on line as a function of the geometry, the availability of measurements, and the dynamics.

The contributions of Speyer and Willsky have significance for the JTIDS application in that they inspire the hope that methods can be found that decentralize the computation burden and have acceptable communication requirements while preserving optimal estimation accuracy.

CHAPTER 3

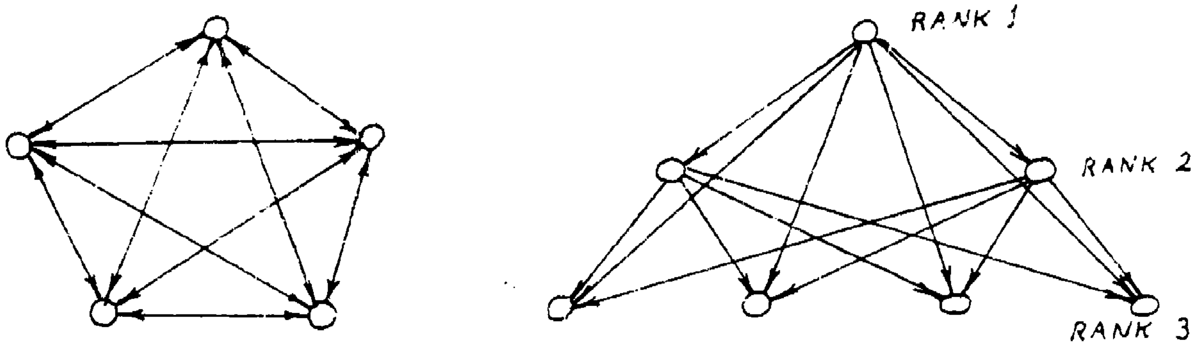
STABILITY OF DECENTRALIZED NAVIGATION

3.1 Ownstate Organization

The JTIDS Relnav concept currently implemented is an example of decentralized estimation of the community navigation state. Each member of the community is responsible for estimating its own navigation state. The member is to use only the time of arrival measurements and the round trip timing measurements that it has obtained with its own receiver. We call this the ownstate formulation of the decentralized estimation problem.

Within the broad category of ownstate formulations there are subcategories identified by their measurement selection logic. We will be discussing the democratic logic, the fixed rank hierarchy logic, and the covariance based hierarchy logic. Fig. 3.1 illustrates the difference between the democratic organization and the fixed rank hierarchy.

In the democratic organization each member uses all of the time of arrival measurements obtained by its receiver. Thus member A is using measurements of pseudorange from member B plus the reported position of member B to help solve for the position of member A. At the same time member B is using measurements of pseudorange from member A plus the reported position of member A to help solve for the position of member B. Simulations have shown that the democratic organization sometimes is unstable.



DEMOCRATIC ORGANIZATION

FIXED RANK HIERARCHY

Fig. 3.1 Democratic Organization Versus Fixed Rank Hierarchy

The instability is probably strongly related to the closed loop information patterns.

In the fixed rank hierarchy each member is assigned a certain rank. More than one member can be assigned an equal rank. Each member uses time of arrival measurements only from members with higher rank. This organization does not allow any closed loop information patterns. We shall prove that this organization is stable.

The covariance based hierarchy avoids the question of how to assign fixed ranks to the members. Instead the rankings are computed dynamically during a mission in terms of the navigation accuracies of the members. The community relies on the filter computed error variances provided by the ownstate filters. As long as the navigation error reported by member A is smaller than the navigation error reported by member B, then member B will use the time of arrival measurements from member A but member A will not use the measurements from member B. This dynamic assignment of ranking assures that information from the most accurate members can propagate to all members. It is hoped that this organization is stable. Note rank reversals can occur, so closed loop information patterns have not been eliminated, only retarded.

This chapter presents a simplified mathematical model of the ownstate method of decentralizing the community navigation problem. An outline of the proof of the stability of the fixed rank hierarchy is presented. Simulations with the simplified model demonstrate the stability issues. A more complete documentation of our stability results may be found in the doctoral thesis of Gobbini (Ref. 29).

3.2 Mathematical Model of the Ownstate Organization

A linear stochastic model for the navigation errors of one member of the community can be put into standard state equation form

$$x_n^i = \Phi_n^i \, x_{n-1}^i + w_n^i + u_n^i \qquad (3\text{-}1)$$

where x is the state vector, Φ is the state transition matrix, w is the zero mean state driving noise vector of covariance Q, and u is the reset vector. The subscript n is the time step variable. The superscript i identifies the ith member. Depending on the fidelity wanted, the number of state variables in the state vector can be small or large. We will be able to illustrate the basic stability characteristics of the ownstate formulations using only a few state variables. The minimum required are three: the two components of horizontal navigation error plus the clock error. These are the variables that directly enter into the measurement relationships.

The Kalman filter in each member extrapolates its own state vector estimate $\hat{x}$ and associated error covariance matrix P according to

$$\hat{x}_{n-}^i = \Phi_n^i \, \hat{x}_{n-1}^i + u_n^i \qquad (3\text{-}2)$$

$$P_{n-}^{i} = \Phi_n^i P_{n-1}^{i} \Phi_n^{i'} + Q_n^i \qquad (3\text{-}3)$$

The subscript n- denotes the estimate before incorporating any available measurement. It is assumed in this analysis that the filters have the correct ownstate transition matrices and correct driving noise statistics.

Let $\tilde{x}$ denote the estimation error vector. Let the sign convention be

$$\tilde{x} = x - \hat{x} \qquad (3\text{-}4)$$

The difference equation governing the time propagation of the estimation error is obtained by subtracting Eqs. (3-1) and (3-2).

$$\tilde{x}_{n-}^{i} = \Phi_n^i \tilde{x}_{n-1}^{i} + w_n^i \qquad (3\text{-}5)$$

At the nth time point there may be a broadcast event. Denote the broadcasting member as member j. At this time point member j is not taking a measurement, so its best estimate of its own navigation errors is the same before and after the event. The notation n- can be replaced by n in the case of member j. Member j broadcasts its own best estimate of position and its computed covariance matrix. In JTIDS Relnav the covariance matrix is not broadcast but instead quality words are broadcast. For simplicity in this analysis we are assuming the covariance matrix is broadcast.

All other members of the community listen to the broadcast of member j. They measure the time of arrival and they receive the position message. A time of arrival measurement is processed by a member by first subtracting the measurement and the predicted measurement. The predicted measurement is based on the member's own best estimate of position and the source's own best estimate of position, which was provided in the position message. It can be shown that this difference or innovation is linearly related to the estimation errors of the user i and the source j according to

$$\delta\rho^{i} = h^{i'} (\tilde{x}^{i}_{n-} - \tilde{x}^{j}_{n}) + v^{i} \qquad (3\text{-}6)$$

where v is the zero mean measurement noise of variance r in the receiver of member i and h is the measurement geometry vector. For the simplest formulation with the three element state vectors, the h vector is

$$h^{i'}_{TOA} = [c_x, c_y, -1] \qquad (3\text{-}7)$$

where cx and cy are the direction cosines of the horizontal line of sight from the source to the user. Note how the errors of the source and the user enter into the measurement difference by means of the same geometry vector but with opposite signs.

A round trip timing measurement has a measurement difference that is also linearly related to the estimation error of the user i and the source j according to Eq. (3-6). However the measurement geometry vector has a non zero entry only for the time error element of the state. For the simplest formulation with the three element state vectors, the RTT h vector is

$$h^{i'}_{RTT} = [0, \quad 0, \quad 1] \tag{3-8}$$

The variance r of the additive random error in a RTT measurement is half that of a passive time of arrival measurement.

The Kalman filter of the ith member uses a TOA or RTT measurement difference, the measurement geometry, and the source covariance to update its state vector estimate and associated covariance matrix according to

$$s^i = h^{i'} \; P^j_n \; h^i \tag{3-9}$$

$$k^i = P^i_{n-} \; h^i / (h^{i'} \; P^i_{n-} \; h^i + r^i + s^i) \tag{3-10}$$

$$\hat{x}^i_n = \hat{x}^i_{n-} + k^i \; \delta\rho^i \tag{3-11}$$

$$P^i_n = P^i_{n-} - k^i \; h^{i'} \; P^i_{n-} \tag{3-12}$$

These are the familiar Kalman measurement incorporation equations except for the addition of Eq. (3-9) and its use in Eq. (3-10). The variable s is the variance of the contribution of source error to the measurement residual. The denominator in Eq. (3-10) would be the correct expression for the measurement innovation variance if there were no correlation between the errors of the source and the user and if the covariance matrics computed by the source and user were correct.

The change in estimation error at a measurement incorporation is obtained combining Eqs. (3-4), (3-6), and (3-11)

$$\tilde{x}_n^i = (I - A^i)\, \tilde{x}_{n-}^i + A^i\, \tilde{x}_n^j - k^i v^i \tag{3-13}$$

$$A^i = k^i h^{i'} \tag{3-14}$$

The community navigation error equations (3-5) and (3-13) (with i running from 1 to M, the number of community members) are a set of coupled linear difference equations with time varying coefficients. The solution to the set of equations is a function of the initial error vectors $\tilde{x}_0^i$, the state noise sequences w_n^i, and the measurement noise sequences v_n^i.

3.3 Proof of Stability of Fixed Rank Hierarchy

It is highly desirable that the community navigation solution be exponentially stable. By definition the community navigation solution is exponentially stable if every perturbation of the initial estimation errors produces a solution that approaches within an exponential bound the unperturbed (or reference) solution. The definition assumes the same noise sequences w and v are driving both the reference solution and the perturbed solution. Fig. 3.2 illustrates the convergence of a perturbed solution to the reference solution and Fig. 3.3 illustrates the convergence occurs exponentially fast.

In centralized optimal estimation theory, sufficient

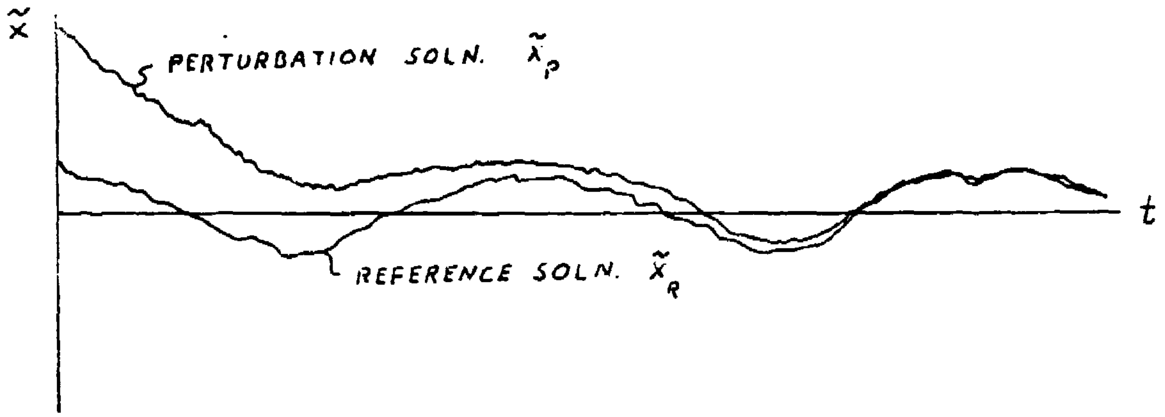


Fig. 3.2 Convergence of a Perturbed Solution to a Reference Solution

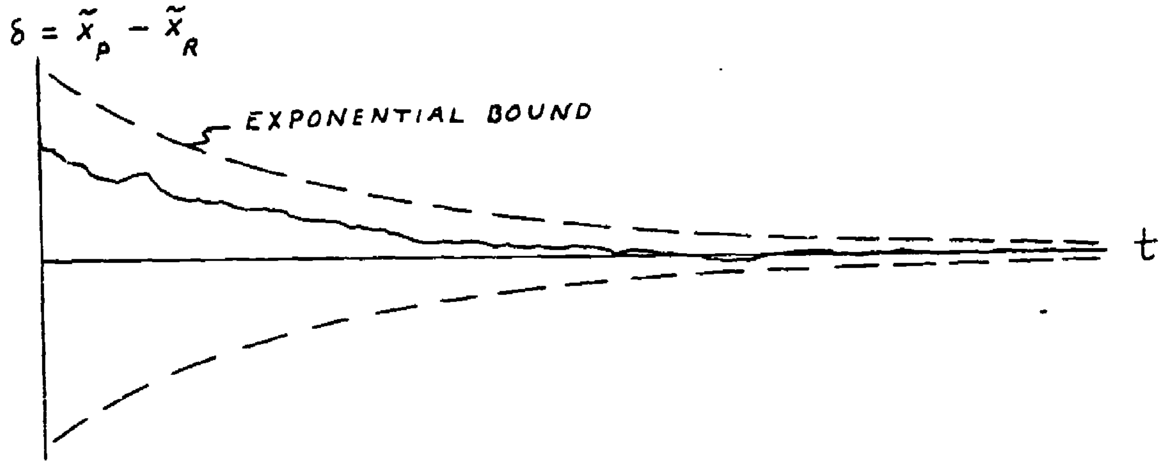


Fig. 3.3 Convergence Within an Exponential Bound

conditions guaranteeing the stability of a Kalman filter have been found. The Deyst-Price theorem (Ref. 30) states that a Kalman filter is exponentially stable if there is present observability and controllability by the assumed driving noises. Observability is a property of the measurement geometries and the state transition matrix. Controllability by the assumed driving noises in the model used for filter synthesis assures that the Kalman gains do not go to zero.

Suppose now that the ownstate filters of the navigation community by themselves satisfy the sufficient conditions of the Deyst-Price theorem and therefore are stable if there were no error interaction. What is the response of such a self stable filter when there is an unmodeled error in the measurement due to the source error? A key theorem proven by Gobbini (Ref. 29) is that if there are exponentially bounded unmodeled perturbations in the measurements used by an exponentially stable Kalman filter, there will be exponentially bounded perturbations in the filter estimates. This theorem is like the theorem in linear system theory that a bounded input into a stable system produces a bounded output. Gobbini has extended the familiar theorem to treat exponential bounding and time varying systems with multiple inputs, of which a Kalman filter is an example.

With Gobbini's theorem, one can now deduce the stability of a fixed rank hierarchy. Suppose we have a single navigation controller who is also the net time reference. Assign rank 1 to this and only this member. This rank 1 member accepts measurements from no other member. Its indicated position and time are perfect by definition. There is no stability issue with respect to these variables.

One or more members may be assigned rank 2. These rank 2 members accept measurements only from the rank 1 member. Since there are no unmodeled errors in the measurements, the ownstate filters are optimal. If the controllability and observability conditions are met, the rank 2 solutions are exponentially stable by the Deyst-Price theorem.

One or more members may be assigned rank 3. These rank 3 members accept measurements only from the rank 1 and rank 2 members. There are no unmodeled errors in the measurements from the rank 1 member, but there are unmodeled errors in the measurements from the rank 2 members. Considering the effect of perturbations to the initial estimation errors of the community, the perturbations in the unmodeled errors will be exponentially bounded. By the Gobbini theorem, if the controllability and observability conditions are met, the perturbations to the rank 3 estimation error vectors will be exponentially bounded.

The proof continues by induction. At every rank level if the controllability and observability conditions are met, the estimates are exponentially stable. Fig. 3.4 illustrates the stability of the several ranks.

The same inductive reasoning can be applied to prove the stability of other fixed rank hierarchy organizations. Suppose for example the navigation grid is established by a two-member method. The primary navigation controller and net time reference is assigned rank 1. The secondary navigation controller, who

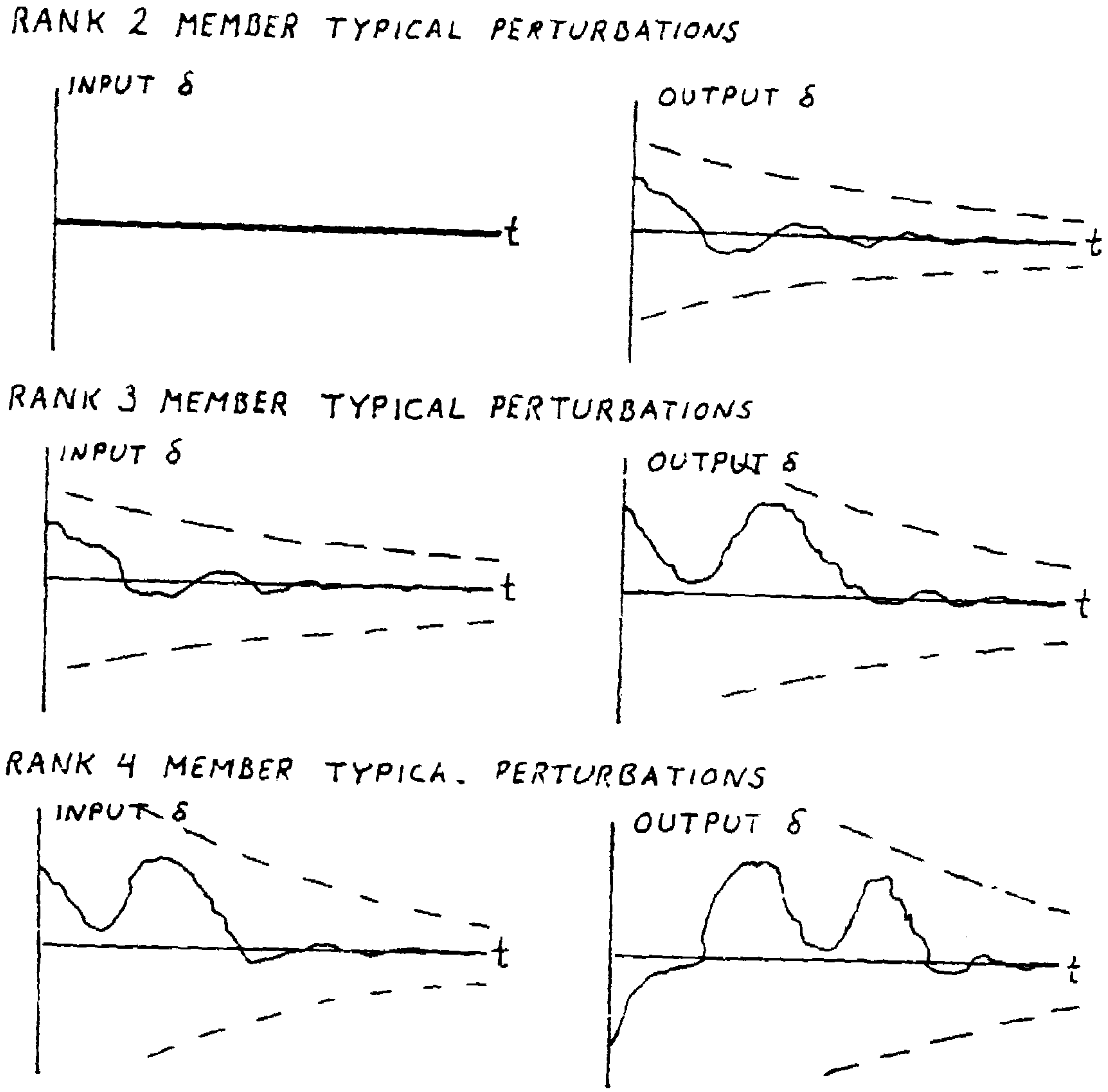


Fig. 3.4 Stability of a Fixed Rank Hierarchy

establishes the end of the grid baseline, is assigned rank 2. This rank 2 member uses measurements from the rank 1 member to estimate its distance and time with respect to the rank 1 member. Again there are no unmodeled errors in the measurements used by the rank 2 member, so given controllability and observability the rank 2 member solution is stable. The stability of rank 3, rank 4, etc. members is proven as in the single navigation controller organization.

Note the integral part that the controllability and observability conditions play in the proof. The filter designer can assure that the controllability condition is met by injecting driving noise at appropriate points in the mathematical model used for filter synthesis. Observability is partially under the control of the filter designer through the measurement selection logic. However there may not be available sufficiently diverse measurements to provide observability. This is beyond the control of the filter designer and is determined by each actual mission scenario. Some of the situations that provide positional and time observability were discussed in Chapter 1 and illustrated in Fig. 1.2.

3.4 Time Domain Covariance and Stability Analyses

One method of evaluating the performance of a proposed community organization is to compute the time variation of the covariances of the actual estimation error vectors. This section presents the actual error covariance equations for the ownstate organization whose mathematical model was presented in Section

3.2. The stability of the organization can also be examined in the time domain. Equations for exhibiting the stability are also presented in this section.

The linear equations governing the estimation error vectors were given earlier. Eq. (3-5) governs the time propagation and Eq. (3-13) governs the change at a measurement incorporation. The state driving noises and the measurement noises have zero means. If the initial estimation errors have zero means, then the estimation errors have zero mean for all time. Calculation of the error covariance matrices is then the same as the calculation of the mean squared error matrices. The cross-covariance of the estimation error of member i and member k is

$$U^{ik} = E\,[\tilde{x}^{i}\ \tilde{x}^{k'}] \qquad (3\text{-}15)$$

We include the possibility that i is the same as k. Using Eq. (3-5) in Eq. (3-15) one obtains the equation governing the time propagation of the cross-covariances

$$U^{ik}_{n-} = \Phi^{i}_{n}\ U^{ik}_{n-1}\ \Phi^{k'}_{n} + Q^{i}_{n}\ \delta_{ik} \qquad (3\text{-}16)$$

It has been assumed that the state driving noise vectors are independent of the prior estimation error vectors and are independent of each other.

At time n there may be a broadcast event. Assume member j is the broadcast source. Using Eq. (3-13) in Eq. (3-15) one obtains the equation governing the changes in the cross-covariances as a result of the measurement incorporations

$$U_n^{ik} = (I-A^i)U_{n-}^{ik}(I-A^k)' + (I-A^i)U_{n-}^{ij}A^{k'} + A^iU_{n-}^{jk}(I-A^k)' + A^iU_n^{jj}A^{k'} + k^ir^ik^{i'}\delta_{ik} \tag{3-17}$$

If either i or k does not incorporate a measurement because it refuses the measurement or because it happens to be the broadcaster j, then Eq. (3-17) is still correct provided the appropriate Kalman gains k and matrices A are set to zero.

These equations governing the actual error covariance have been included in the simulations of the ownstate communities discussed in the next section.

In general the actual error covariances of the members do not go to zero because of the presence of state driving noises and measurement noises. This makes it difficult to make statements about stability based on observations of the error covariance. It is desireable to have some time domain method of assessing stability that is more closely related to the definition of stability, namely the convergence to zero of all perturbations.

The equations governing perturbations to the community estimation error vectors can be obtained by subtracting the equations governing a reference solution from the equations governing a perturbed solution. The perturbations are found to be governed by

$$\delta\tilde{x}_{n-}^i = \Phi_n^i\ \delta\tilde{x}_{n-1}^i \tag{3-18}$$

$$\delta\tilde{x}_n^i = (I - A^i)\,\delta\tilde{x}_{n-}^i + A^i\,\delta\tilde{x}_n^j \qquad (3\text{-}19)$$

Note since the same noise sequences were driving the reference and perturbed solutions and since the system is linear, the driving noises and measurement noises do not force the perturbation equations.

The definition of exponential stability requires that every perturbation converge to zero within an exponential bound. It is not practical to simulate the response to every possible initial perturbation. Fortunately this is not necessary in the case of a linear system. Let L be the dimension of the complete community navigation error vector. Choose a set of L linearly independent vectors to span the L-dimensional vector space. Every initial perturbation vector can be expressed as a linear combination of these basis vectors. Because the system is linear, the time variation of the perturbation can be expressed as the same linear combination of the system responses to the individual basis vectors. Therefore it is sufficient to examine the convergence of the L basis vector responses. If all L elements of all L response vectors go to zero exponentially, then the system is exponentially stable.

The simulation to be discussed in the next section can be used for such a stability analysis. The responses to the L independent basis vectors are summarized by providing the sum of the squares of all L elements of all L response vectors. If this sum converges to zero, the system is stable.

3.5 Simulations of Ownstate Communities

To demonstrate the stability issues associated with the ownstate organizations we have utilized a simple simulation that incorporates the mathematical model of the ownstate organization presented in Sect. 3.2. The simulation also incorporates the covariance analysis equations of the previous section. This makes it possible to compare a single case error trace, the filter computed error standard deviation, and the actual error standard deviation. This section describes the simulation and presents simulation results.

The navigation problem is a two dimensional flat earth navigation problem. There are four members in the community located as shown in Fig. 3.5. They are initially located at the corners of a 20 km square. Members 1,3,and4 are not moving. Member 2 moves rapidly on the closed course shown, with a period of 300 sec.

The relative navigation grid is established by choosing member 1 to be the navigation controller and time master and by choosing member 2 to be an end of baseline subcontroller. In particular member 1 has by definition perfect knowledge of its own x,y position and the time t. Member 2 has by definition perfect knowledge of its own y position.

There are eight navigation error variables in the community: the member 2 x and t errors, the member 3 x, y, and t errors, and the member 4 x,y, and t errors. The truth models for these errors are simple random walks with variance parameters of $(10\text{ m})^2$

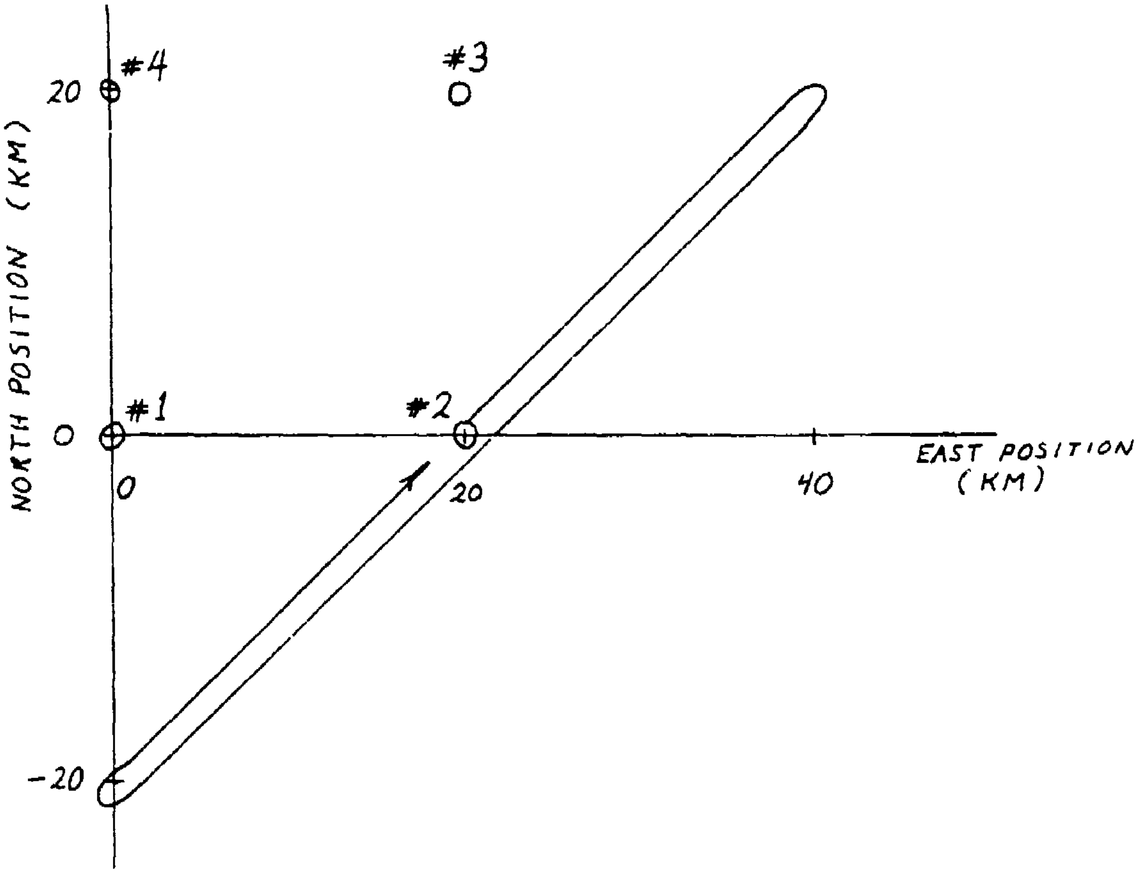


Fig. 3.5 Location and Flight Paths of Community Members

MEMBER	0	3	6	9	12	15	18	21	24	. . .
	BROADCAST TIME (SEC)									
# 1		●				●				
# 2			●				●			
# 3				●				●		
# 4					●				●	

Fig. 3.6 Broadcast Assignments

/sec. Initial values of these errors were set to plus or minus 1 km or 2 km. Note we present clock errors in range equivalent units. These random walk models are the most simple possible models for the growth of the inertial navigation errors and the clock errors. We have assumed that the inertial system errors and the clock errors are of comparable significance.

The time of arrival measurements have an additive random error of 10 m standard deviation. The broadcast time slots are as shown in Fig. 3.6. One full cycle of broadcasts has a period of 12 sec. No round trip timing measurements are included.

The ownstate Kalman filters are matched to the truth model. The filters of members 2, 3, and 4 have 2, 3, and 3 state variables respectively. They have the correct statistics for the driving noises and the measurement noise. The initial covariance matrices of the filters are diagonal with all diagonal elements set at $(1\ \mathrm{km})^2$.

Exploring nonlinear difficulties was not an objective of the stability simulations. Accordingly the Kalman filters were allowed to use the actual measurement geometry vectors in place of the estimated geometry vectors.

The simulation results for the democratic organization are shown in Figs. 3.7, 3.8, and 3.9. In these plots the trace labeled e is the single case error trace. The σ_C plot is the filter computed one sigma value of the error. The σ_A plot is the actual error one sigma value. The performance is clearly unacceptable. The actual one sigma value of many of the errors after 500 sec of navigation is as large as the initial 1 km one sigma error. The filter computed uncertainties are completely unaware of these large oscillatory actual errors.

The time domain stability test was applied to the democratic organization. The eight initial basis vectors were the orthogonal unit vectors. The sum of the squares of all elements of all responses is plotted in Fig. 3.10. The sum is not converging to zero so the system is unstable.

A fixed rank hierarchy has been simulated. Member 1 has been assigned rank 1, member 2 has been assigned rank 2, and members 3 and 4 have both been assigned rank 3. At each rank the members have the necessary observability. Member 2 using measurements only from member 1 has observability because of the relative motion. Members 3 and 4 have observability provided by member 1 static measurements and member 2 measurements that have relative motion. Simulation results for the fixed rank hierarchy are shown in Figs. 3.11, 3.12, and 3.13. Here the performance is good. The rank 2 member (member 2) has filter computed uncertainty in perfect agreement with the actual error uncertainty. After an initial transient, the rank 3 members (members 3 and 4) have good agreement between the filter computed uncertainties and the actual uncertainties.

The time domain stability result for the fixed rank hierarchy is shown in Fig. 3.14. The sum is converging to zero, showing that the fixed rank hierarchy is stable. The rankings in this stability test are not quite the same as in the previous run. Here member 4 is assigned rank 4.

A covariance based hierarchy has been simulated in which the ranking is computed dynamically as a function of the filter computed covariance matrices. If the trace (the sum of the diagonal elements) of a member's own covariance matrix is larger than the trace of the source's covariance matrix, then the member uses the time of arrival measurement from the source. This is not precisely the logic used in JTIDS Relnav, but it captures the essence of the JTIDS logic. Simulation results for this covariance based hierarchy are shown in Figs. 3.15, 3.16, and 3.17. The performance is acceptable but not quite as good as the fixed rank hierarchy performance. Initial transient errors are quite large. Eventually the errors die down but the filter computed uncertainties are optimistically small.

The stability test result for the covariance based hierarchy is shown in Fig. 3.18. After a large initial transient, the sum is converging toward zero. Apparently this covariance based hierarchy with this mission scenario is stable.

Other runs with this simulator may be found in the Gobbini thesis (Ref. 29). For example one run shows the fixed rank hierarchy not performing satisfactorily when there is no motion in the community. This illustrates the importance of the observability condition.

3.6 Stability Conclusions

It is clear that the stability of ownstate organizations of the decentralized navigation problem is critically dependent on the source selection logic.

We have proven that a fixed rank hierarchy is stable,

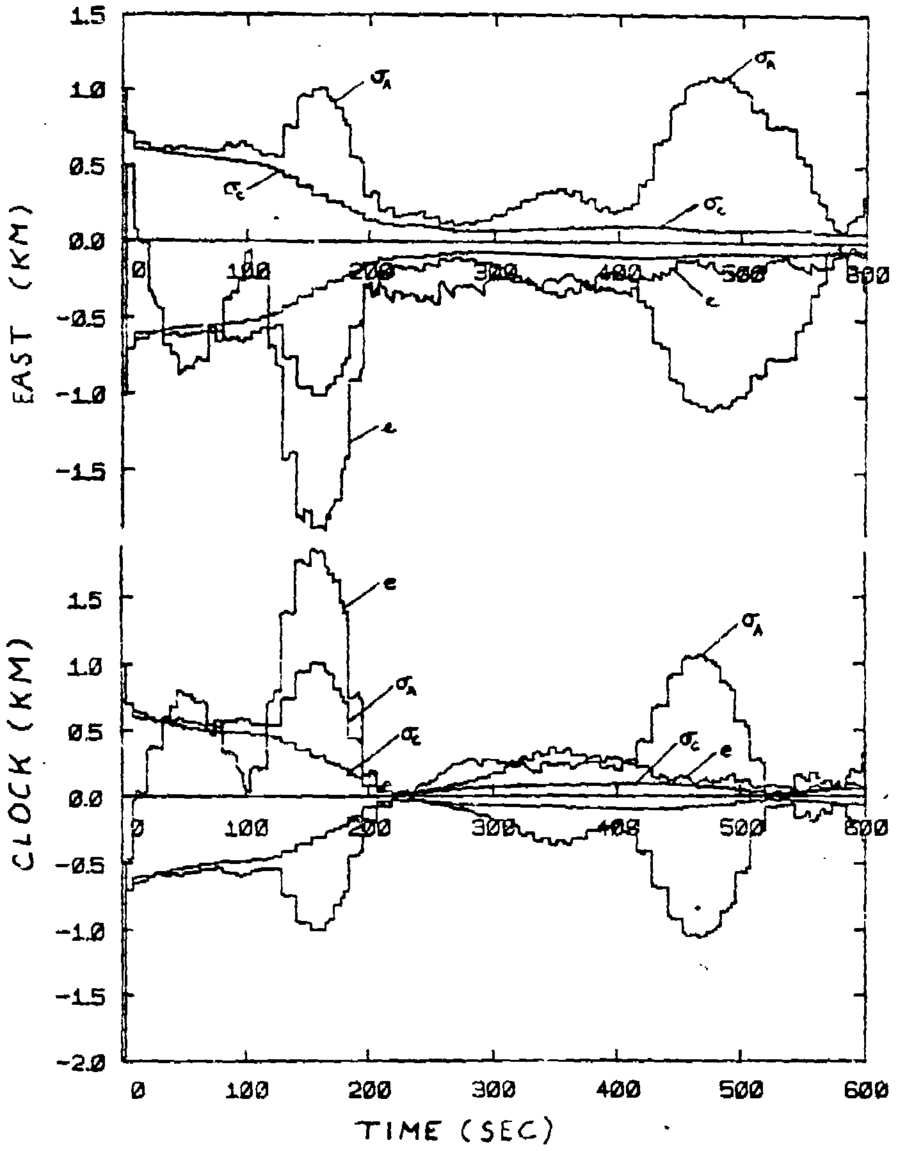


Fig. 3.7 Democratic Organization, Member 2 Errors

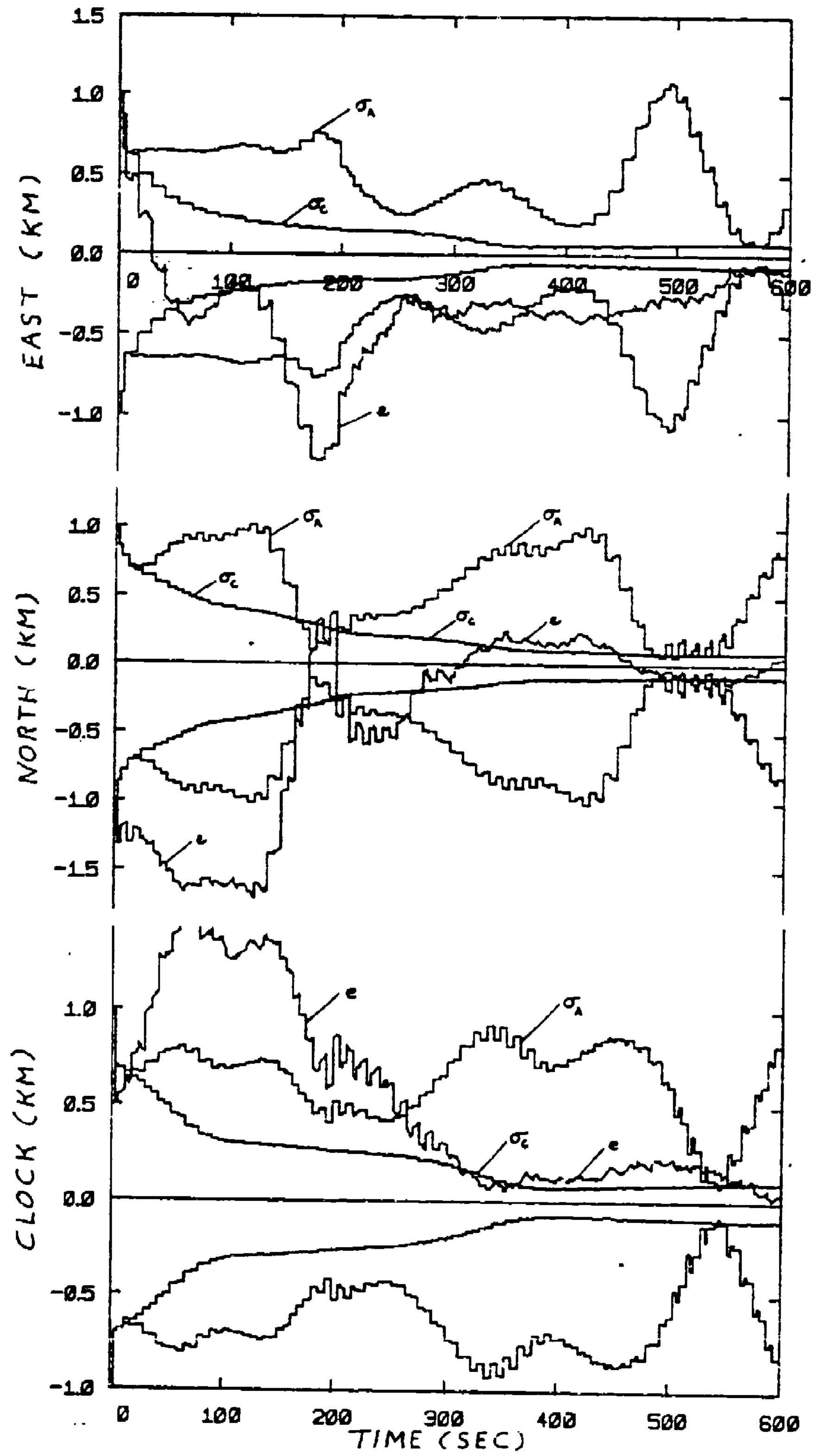


Fig. 3.8 Democratic Organization, Member 3 Errors

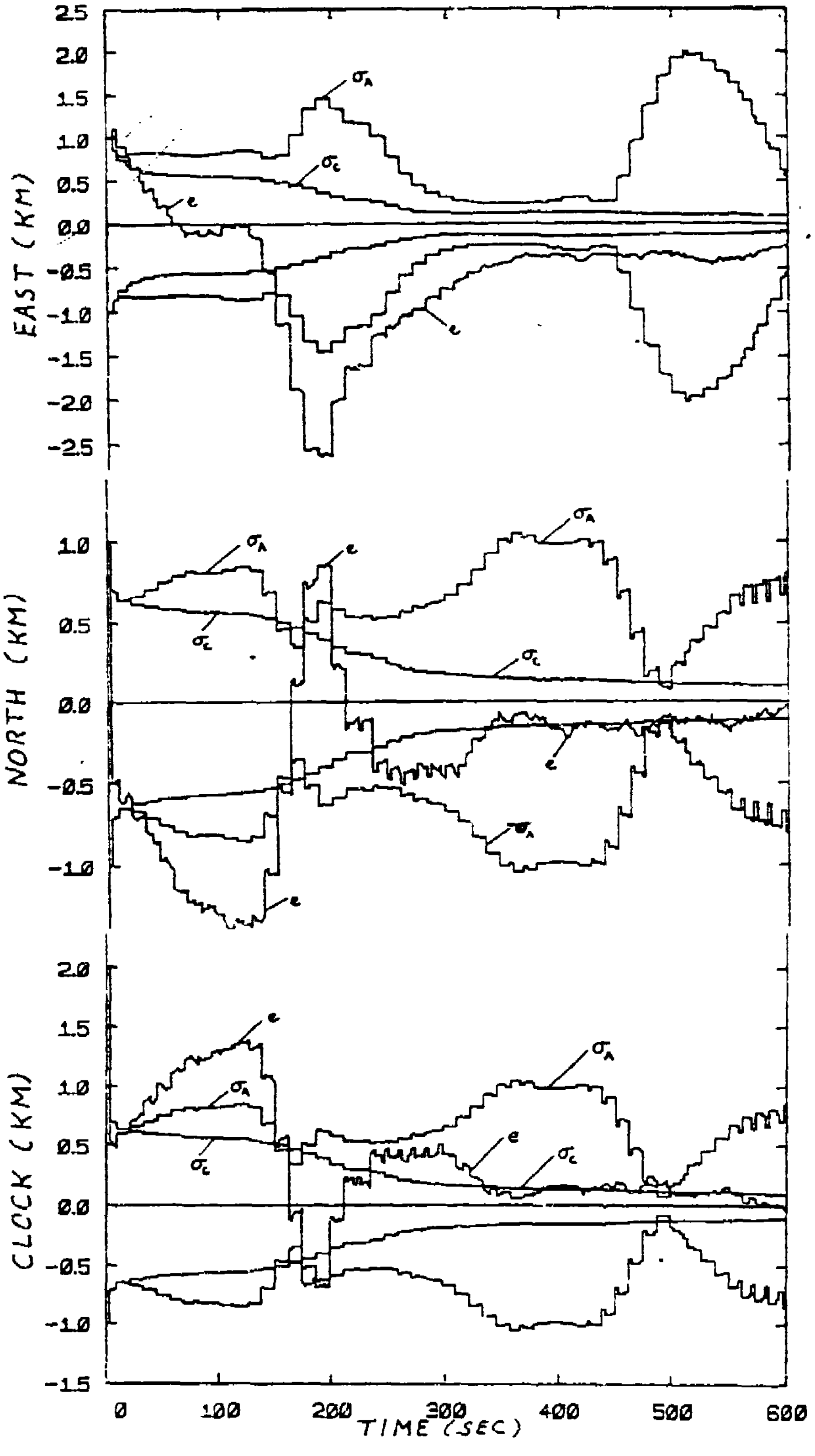


Fig. 3.9 Democratic Organization, Member 4 Errors

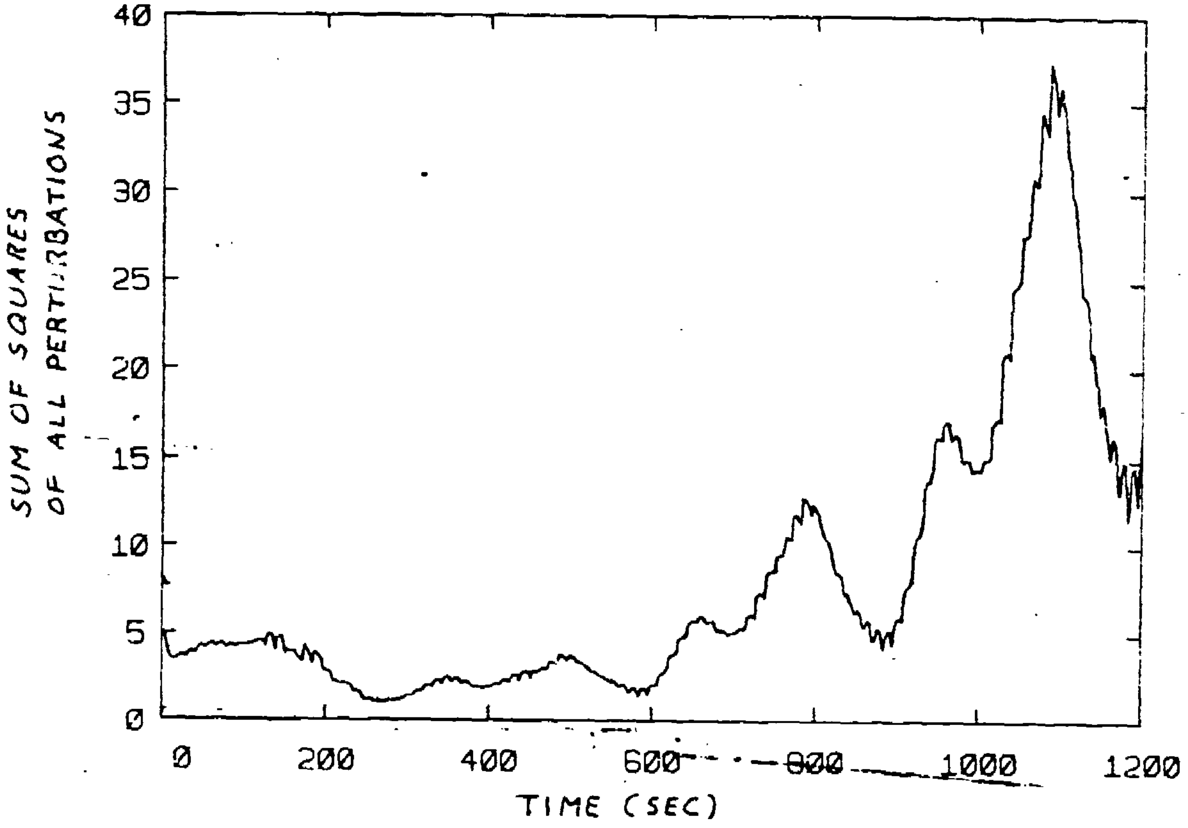


Fig. 3.10 Democratic Organization Stability Test

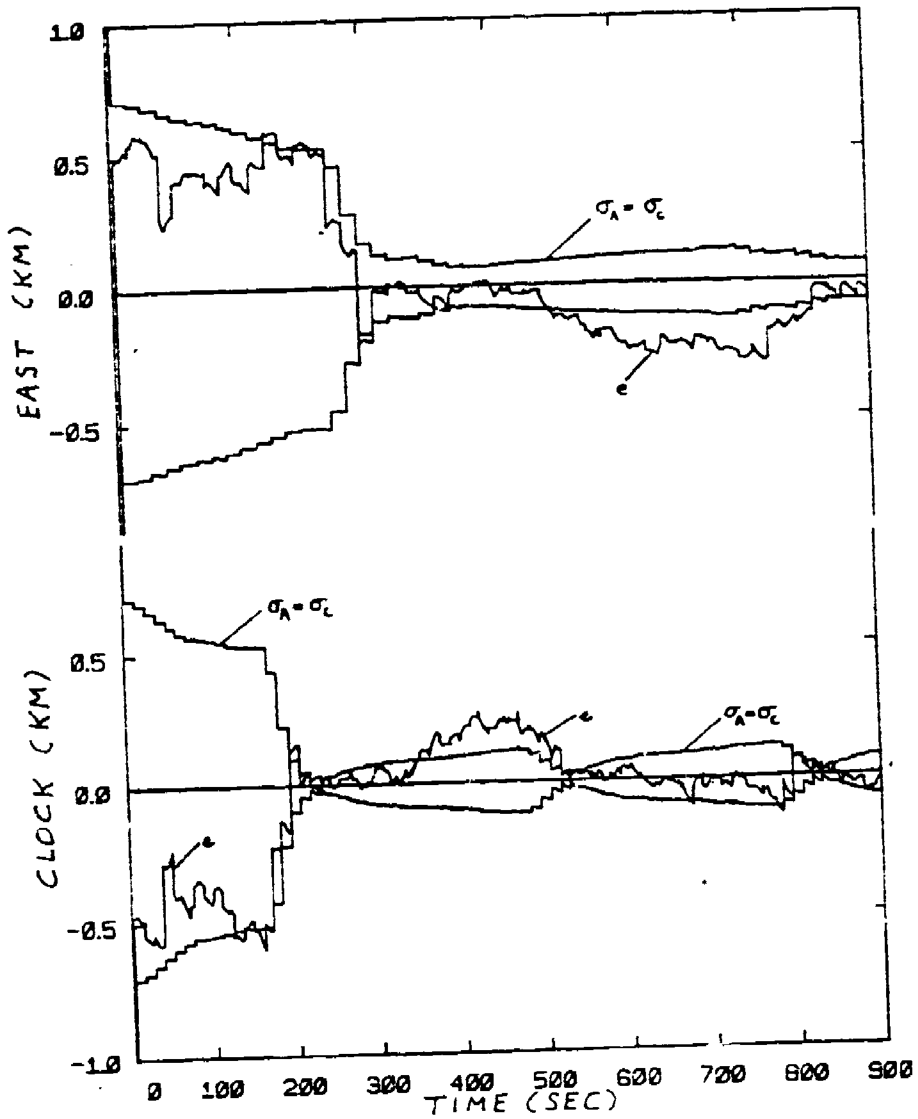


Fig. 3.11 Fixed Rank Hierarcny, Member 2 Errors

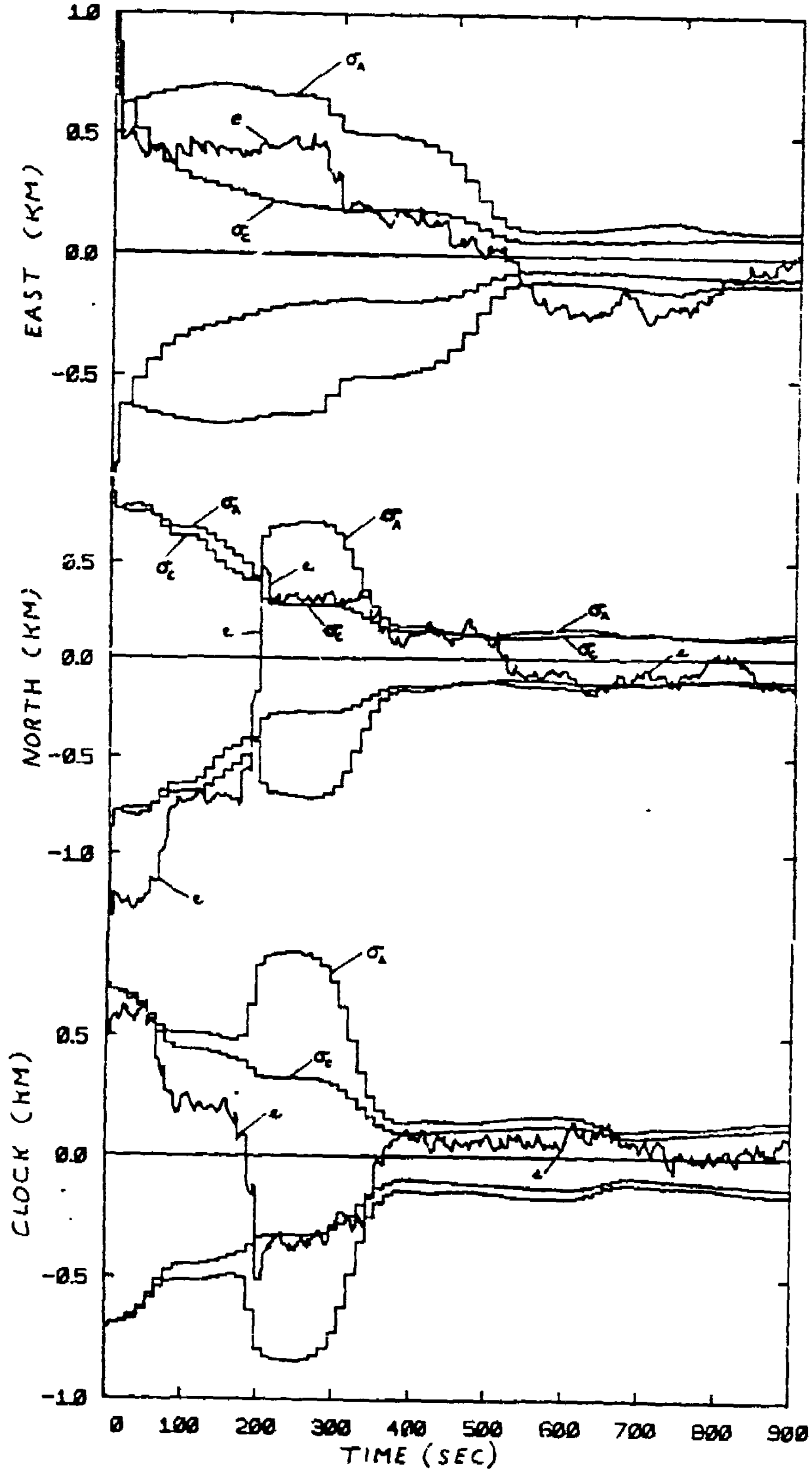


Fig. 3.12 Fixed Rank Hierarchy, Member 3 Errors

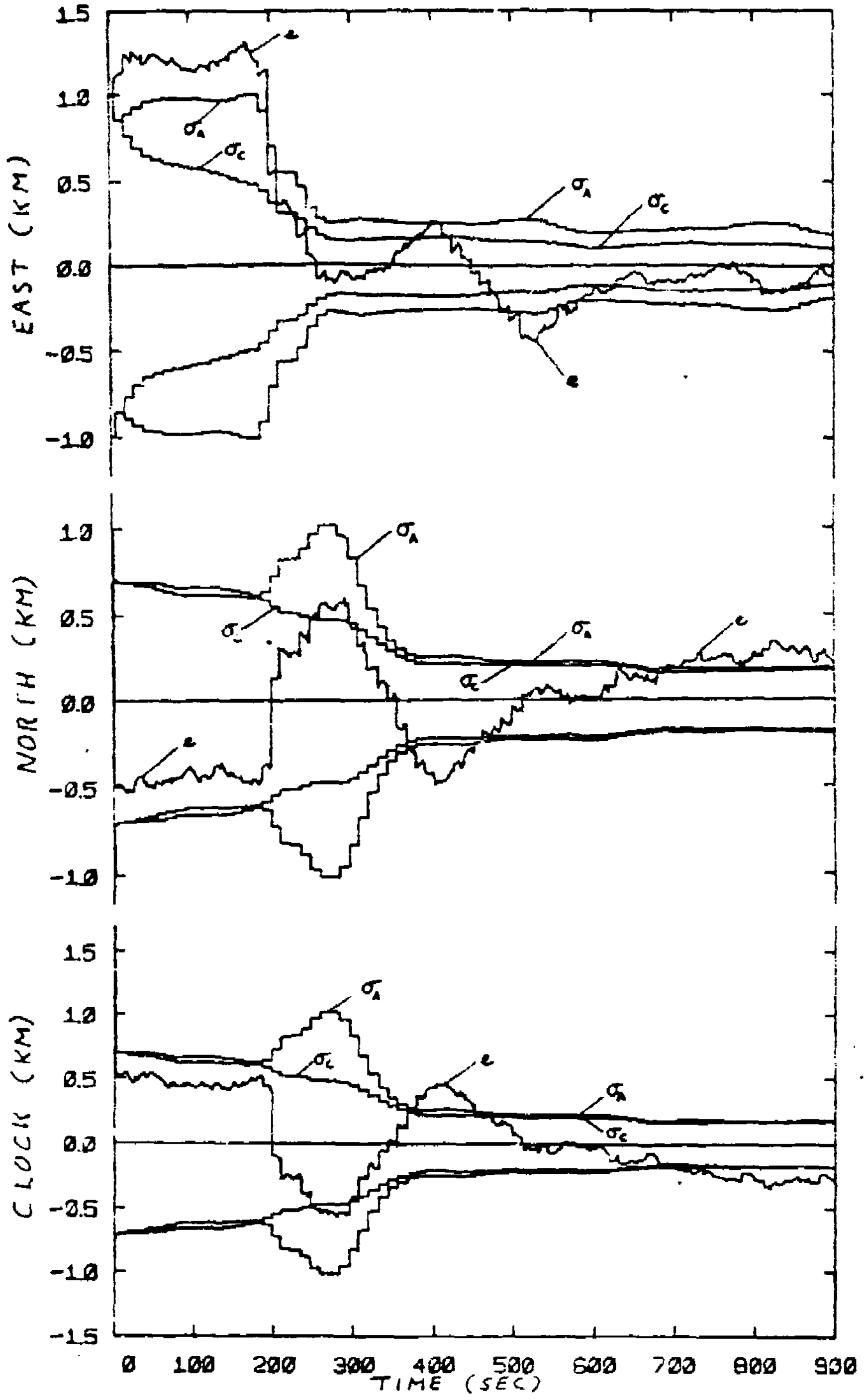


Fig. 3.13 Fixed Rank Hierarchy, Member 4 Errors

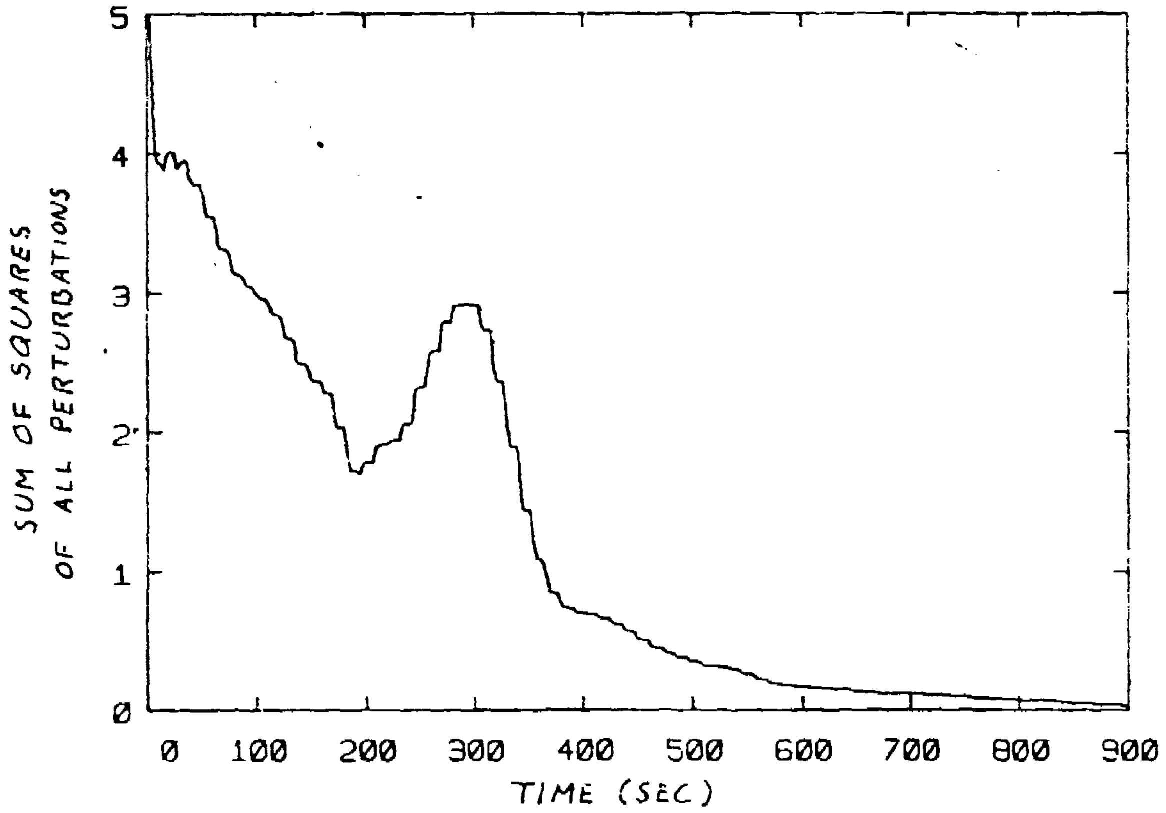


Fig. 3.14 Fixed Rank Hierarchy Stability Test

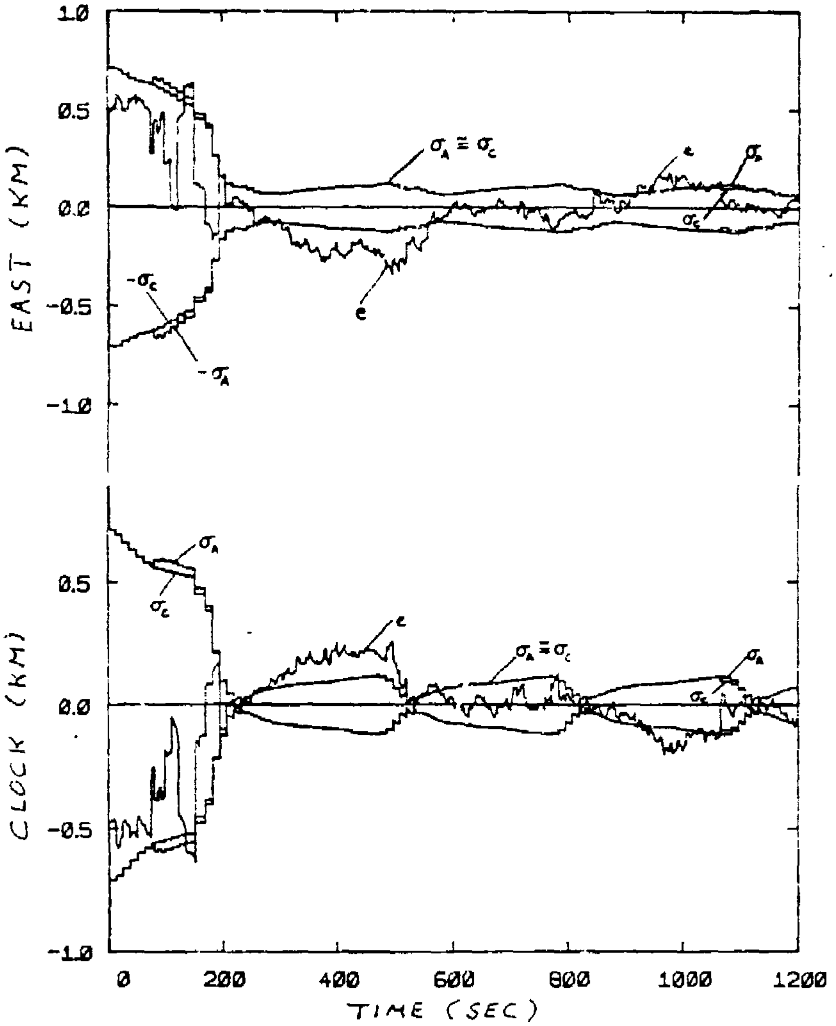


Fig. 3.15 Covariance Based Hierarchy, Member 2 Errors

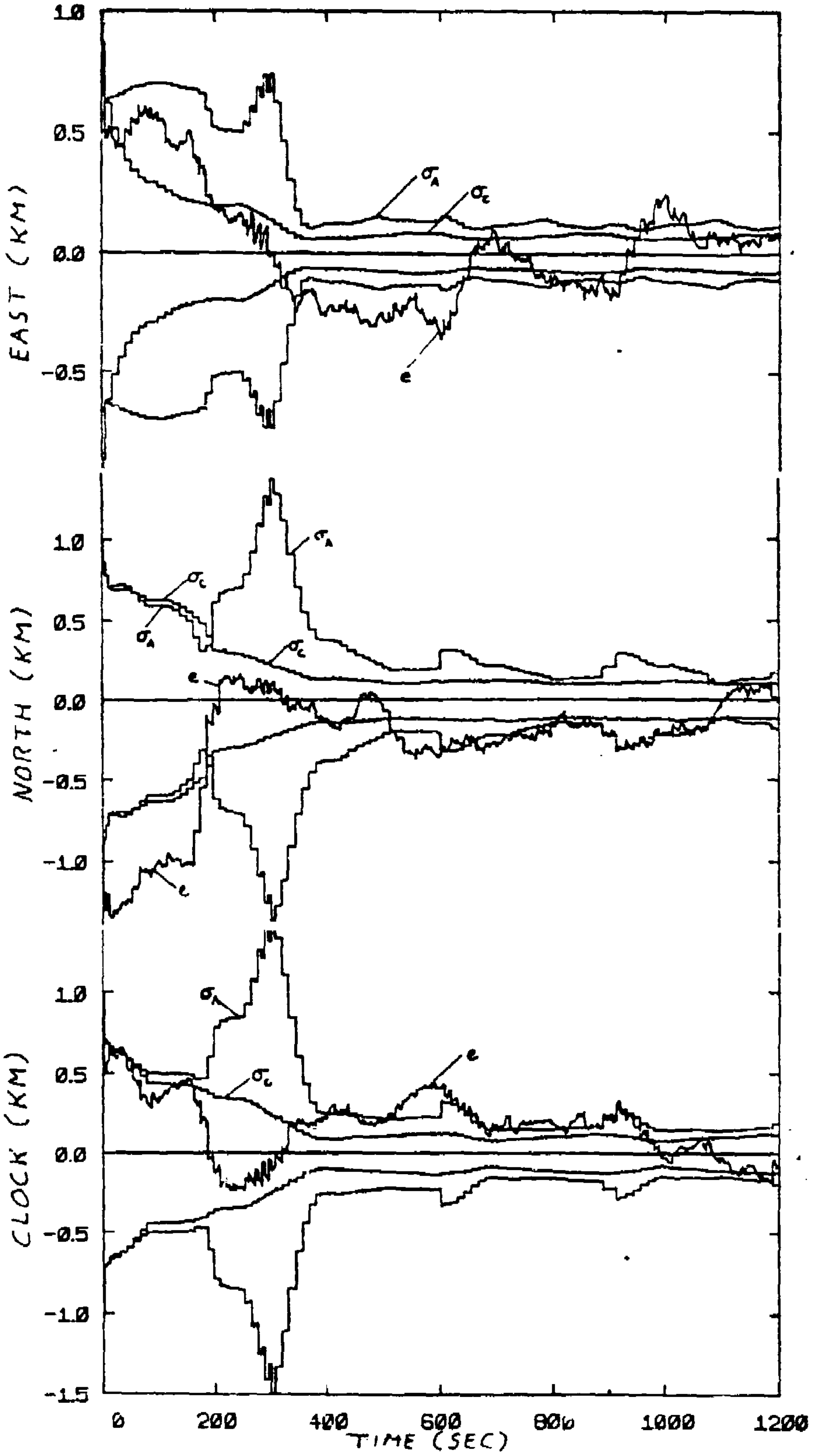


Fig. 3.16 Covariance Based Hierarchy, Member 3 Errors

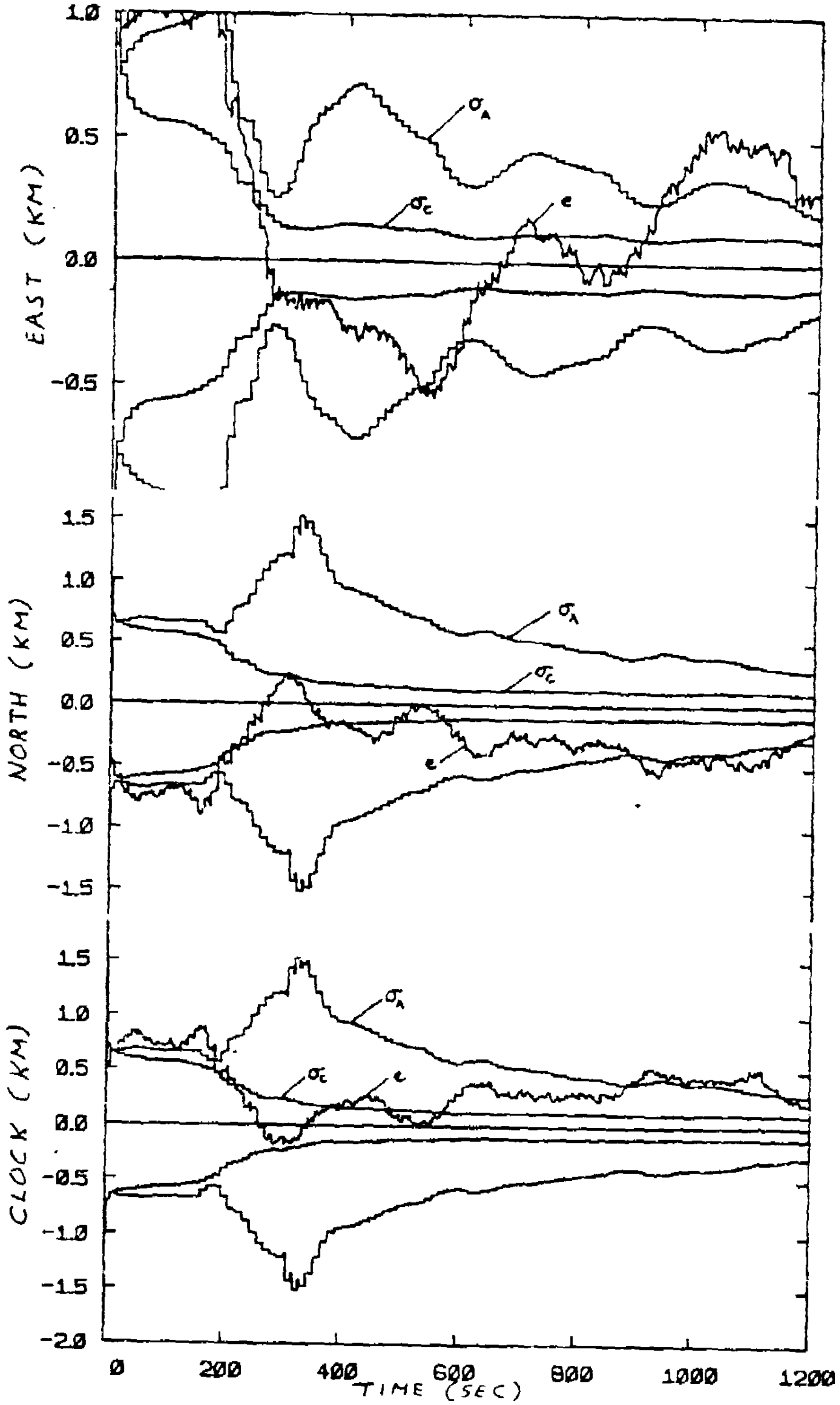


Fig. 3.17 Covariance Based Hierarchy, Member 4 Errors

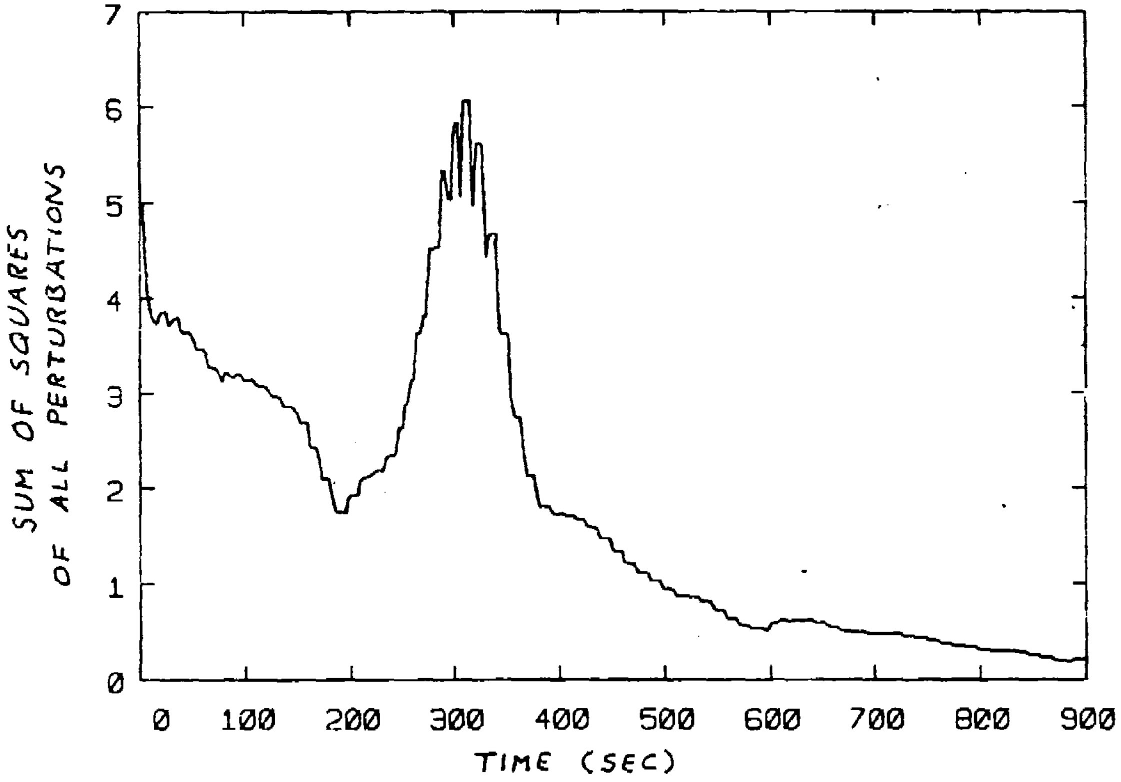


Fig. 3.18 Covariance Based Hierarchy Stability Test

provided the controllability and observability conditions are satisfied for each member. Controllability is built in by the filter designer. Observability depends on the mission geometry. For some members it may depend on there being motion or being allowed to use round trip timing.

The disadvantage of the fixed rank hierarchy is that there is no way of assigning fixed ranks that will be good for all missions. Suppose for example in our simulation we had assigned rank 1 to member 1, rank 2 to member 4, and rank 3 to members 2 and 3. As a result, members 4 and 3 fail to have the required observability. Member 4 at a fixed relative position with respect to member 1 and using measurements only from member 1 is unable to solve for its own errors. Member 3 with static measurements from members 1 and 4 can not solve for its own errors. Only member 2 with its motion is able to solve for its own errors, but its estimates will be corrupted by the poor measurements from member 4.

The democratic organization is totally unreliable. In the simulation presented the community navigation was clearly unstable.

The covariance based hierarchy overcomes the disadvantage of the fixed rank hierarchy, assuring information from members with accurate navigation will propagate throughout the community. However it is not clear that this organization can be proven stable. Rank reversals can occur because after processing many measurements from supposedly more accurate sources, the member will believe it now has better accuracy than one or more of its previous sources. The role of source and user then reverses. Thus closed loop information paths do exist. It seems reasonable to predict that if the rank reversals occur infrequently relative to the settling times of the individual filters, then the community will be stable. But if the rank reversals are frequent and the settling times of the filters are large, then the community might be unstable.

Furthur research is needed to establish the conditions under which the covariance based hierarchy can be guaranteed stable. It may be necessary to enforce some rules concerning the allowable rate of rank reversals.

CHAPTER 4

NAVIGATION BASED ON MEASUREMENT SHARING

4.1 Measurement Sharing Versus Estimate Sharing

The current JTIDS Relnav implementation is an example of decentralized estimation. Individual members estimate their own state using only their own measurements. However incorporation of a time of arrival measurement requires knowledge of the position of the source, so members are required to share their best estimates of position.

A conceptually different approach would be based on measurement sharing rather than estimate sharing. If one knows the values of all measurements taken throughout the community (together with necessary supporting data), one can in theory estimate the navigation state of the entire community. Such an estimator would be the theoretical optimal estimator.

In a large community it would be impractical to implement the theoretical optimal estimator. The number of state variables in the filter is proportional to the number of members. To model the significant errors of a single inertially equiped member typically requires 12 or more state variables. If the filter must model 10 such members in the community the total state vector has dimension 120. Another difficulty is that the number of measurements obtained in the community each cycle is proportional to the square of the number of members. Each of the

M members takes M-1 measurements each cycle. At some sufficiently large M the communication demands on the JTIDS network would become excessive and/or the Kalman filtering processing requirements would exceed the computer capacity.

The JTIDS literature does not provide an evaluation of the possibility of implementing measurement sharing. The working assumption is that the communication requirements would be too great and the computer capacity required would be too large. The JTIDS development effort has concentrated on the ownstate formulation, which has the advantage that the filter state size is independent of the number of members and position message traffic grows only linearly with the number of active members.

However after considerable effort has been devoted to implementing the ownstate organization concept, it has become clear that the ownstate formulations have some performance shortcomings. The navigation accuracy at best falls short of the theoretical optimal, in part because a member's filter uses only its own subset of the total measurements taken in the community. At worst some ownstate formulations in some mission scenarios are unstable. The community convergence can be noticeably slower than the optimal because unmodeled error transients of a source excite error transients in a user. It would be highly desireable to find alternate community organizations that approach more closely the theoretical optimal performance.

In this chapter we explore the possibility of implementing JTIDS navigation based on measurement sharing. The goal is to propose an organization that approaches the theoretical optimal in performance but meets the communication and computation constraints of the JTIDS system.

4.2 Practical Measurement Sharing Organization

It may be that navigation estimates close to the theoretical optimal can be obtained using a subset of the total measurements in the community. It should be possible to develop a primary member concept in which certain members are identified as being of particular significance in establishing the navigation grid and providing good measurement geometry for all members. The number of primary members might be of the order of five, but certainly not more than ten. It would be these primary members that would be responsible for sharing the values of their measurements. By limiting the number of members that do measurement sharing, the amount of message data no longer grows as the square of the total number of members but rather linearly with the total number of members.

Gobbini (Ref. 29) has estimated the number of bits that must be broadcast in the navigation messages. In addition to the values of the measurements obtained during the last cycle, the broadcast must include the inertial indicated position at each measurement time. It may also be necessary to include INS resetting data and clock resetting data. Consider a nine member community, all designated primary members. If data supporting eight measurements is included in each navigation message, Gobbini estimates that 685 bits would be required. The nine member broadcasts per cycle would total 6165 bits. According to

Ref. 1, every 12 sec cycle of JTIDS broadcasts contains 1536 time slots, whose capacities are 545 bits each, for a total of 837,120 bits per cycle. Thus the required navigation communication for nine members would be less than 1% of the total capacity.

Optimal processing of the available measurement data by an individual member would require a filter state vector that included the navigation errors of the member plus the navigation errors of the primary sources. If the member were a secondary member and there were nine primary members, this could require 12 states for each of the 10 members or 120 state variables. Such a filter is beyond the capability of current flight computers.

It is possible that a suboptimal filter with far fewer states can meet the performance goals of an individual member. A member is primarily interested in having accurate estimates of its own navigation errors. It is interested in estimating the errors of the others only if these errors are strongly correlated with its own errors. This suggests that a reduced state filter could be implemented in each member. The suboptimal filter would model all the ownstate variables but would delete most of the other member variables except those that directly influence the measurements. The retained variables are the horizontal position errors and the clock offset errors of each primary source.

In the nine primary member example, such a suboptimal filter would have the member's own 12 states plus 9 times 3 or 27 of the primary member states for a total of 39 state variables. This is still a large real time filter by current standards, but it could be implemented.

4.3 Simulation of Optimal and Suboptimal Filters

To provide some estimates of the potential performance of the optimal filter and the reduced state suboptimal filter, we have implemented low order versions of both filters in the 2-D simulator.

Again four members are simulated. This time there is no motion. The members are at the corners of the 20 km square of Fig. 3.5.

Again the navigation grid is established by choosing member 1 to be the navigation controller and time master and by choosing member 2 to be an end of baseline subcontroller.

The truth model for the navigation errors has been augmented to include rate states. There are now 16 navigation error variables in the community. Member 2 has four errors: x position and velocity errors, and clock phase and clock frequency errors. Members 3 and 4 each have six errors: x and y position and velocity errors, and clock phase and clock frequency errors. The truth models for the rate states are simple random walks. The positional and phase states are integrals of the related rates. Inital values of the positional and clock phase errors are set randomly to 1 km standard deviation. Initial values of the velocity and clock frequency errors are set randomly to 0.5 m/sec standard deviation. These initial values are representative of INS positional and velocity errors. The clock errors have again been assumed to be of the same level of significance as the inertial errors. The random walk rate states all have variance parameter of $(0.017 \text{ m/sec})^2$/sec. This level of noise provides a crude model of the effect of the Schuler oscillations on the velocity errors of the inertial systems. Clock frequency noise is at the same level of significance.

The time of arrival measurements again have an additive random error of 10 m standard deviation. The broadcast time slots are as before as shown in Fig. 3.6.

All four members are considered primary members, so all measurements taken by all members will be shared. The simulation neglects any lag between taking a measurement and broadcasting the measurement.

The optimal filter implementation has 16 state variables that match the state variables in the truth model. The initial covariance matrix is diagonal with positional or clock phase variances set to $(1 \text{ km})^2$ and velocity or clock frequency variances set to $(0.5 \text{ m/sec})^2$. The filter has correct values for the driving noise and measurement noise statistics.

The performance of the optimal filter is shown in Figs. 4.1, 4.2, 4.3. The estimation error is rapidly reduced to very small levels. By the end of the first cycle of measurements at t=12 all positional variables have been estimated to an accuracy of the order of the noise in the time of arrival measurements (10 m). By the end of the second cycle some progress has been made in reducing the velocity errors.

Each suboptimal filter implementation deletes the rate states of the other members. The suboptimal filter of member 2 has 10 states (x2,vx2,t2,f2; x3,y3,t3; x4,y4,t4). The suboptimal

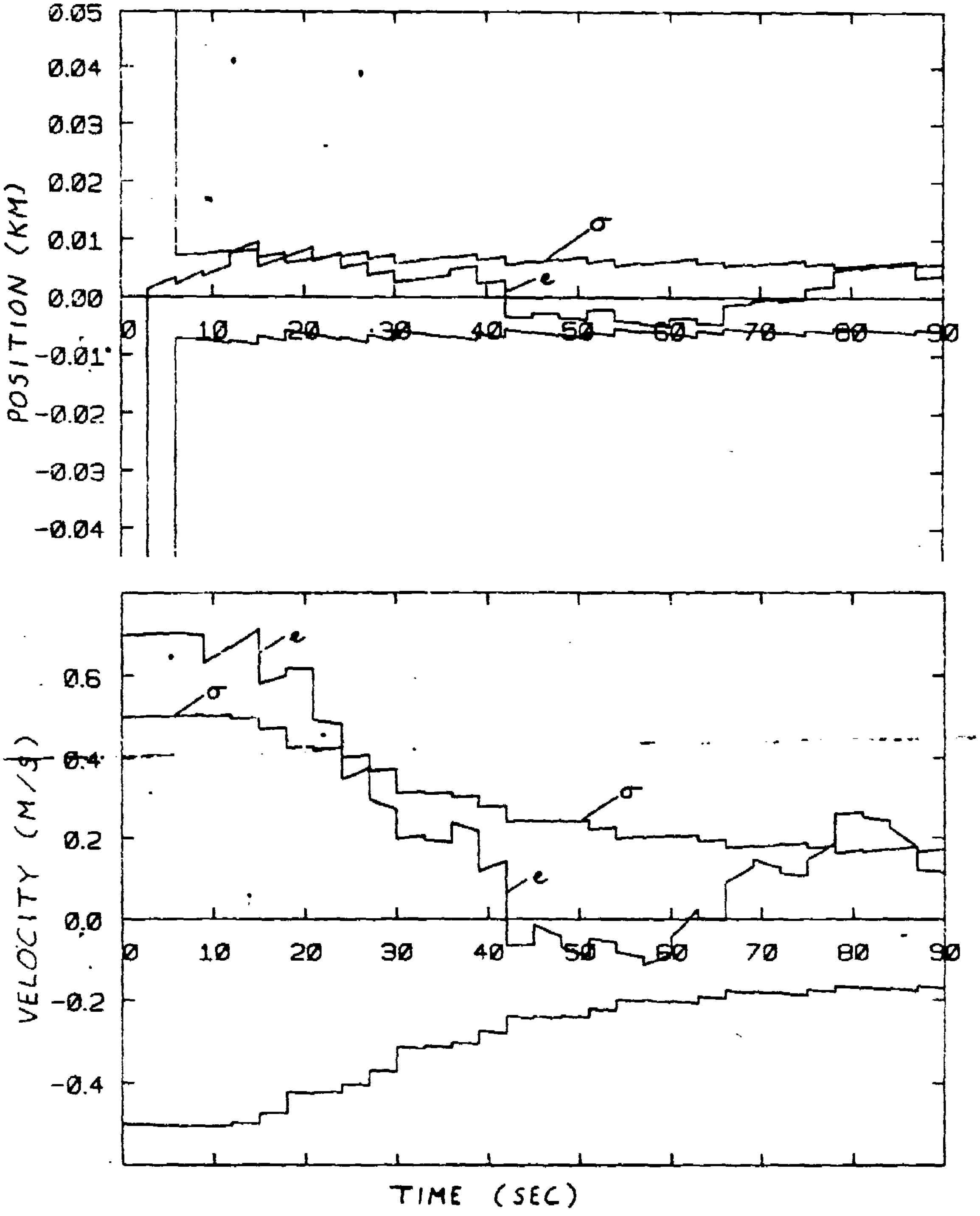


Fig. 4.1 Optimal Estimate of Member 2 East Errors

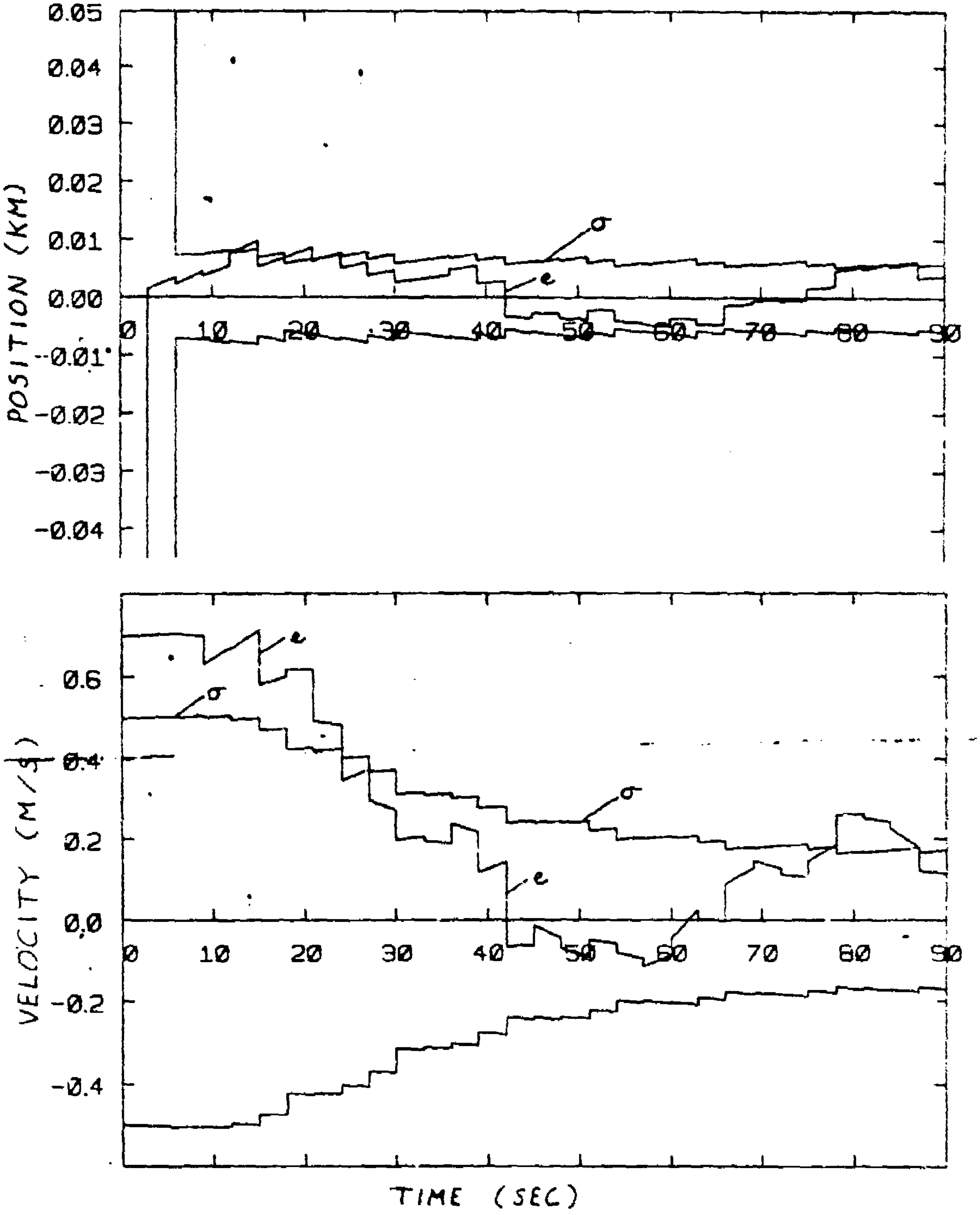


Fig. 4.1 Optimal Estimate of Member 2 East Errors

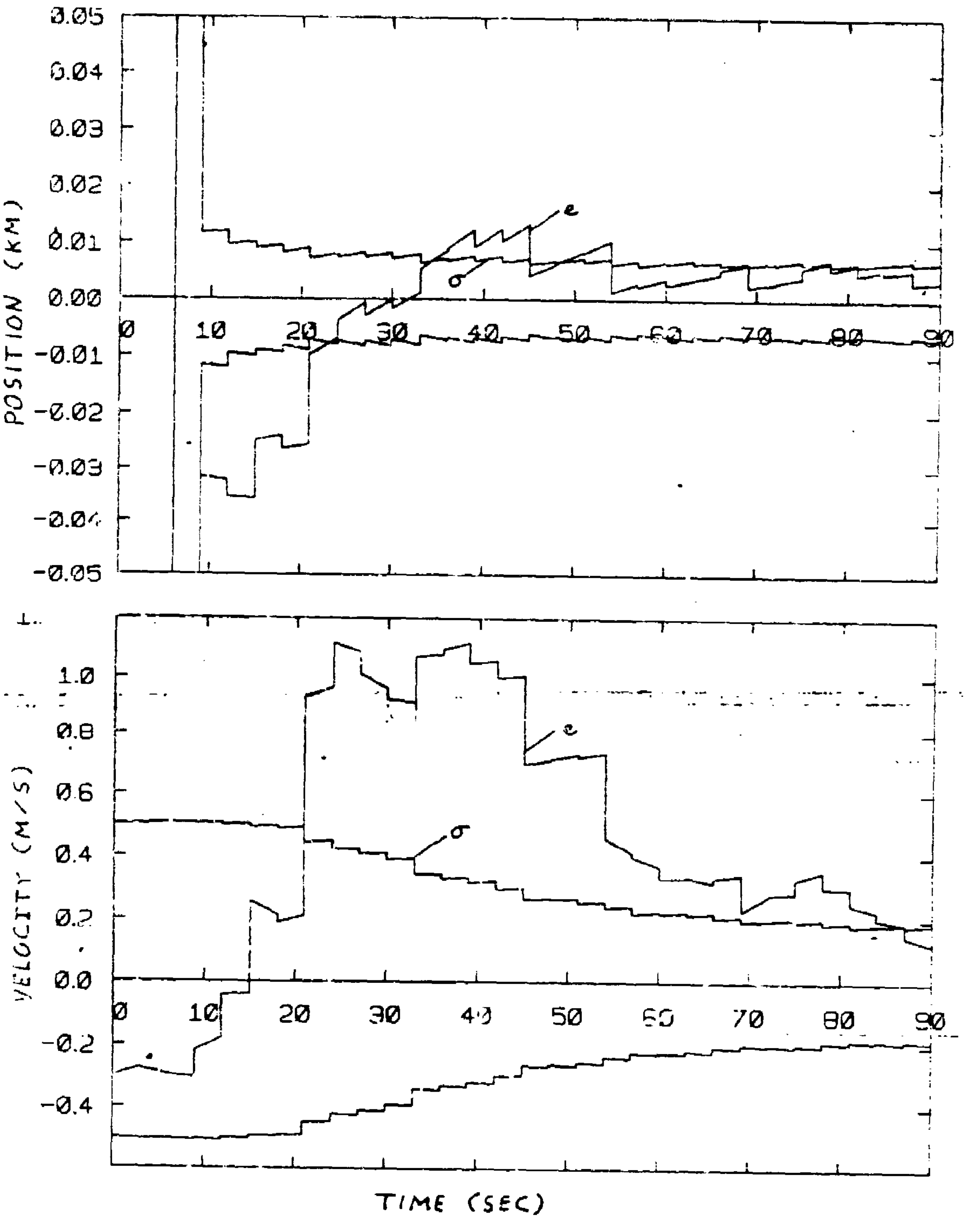


Fig. 4.2 Optimal Estimate of Member 3 East Errors

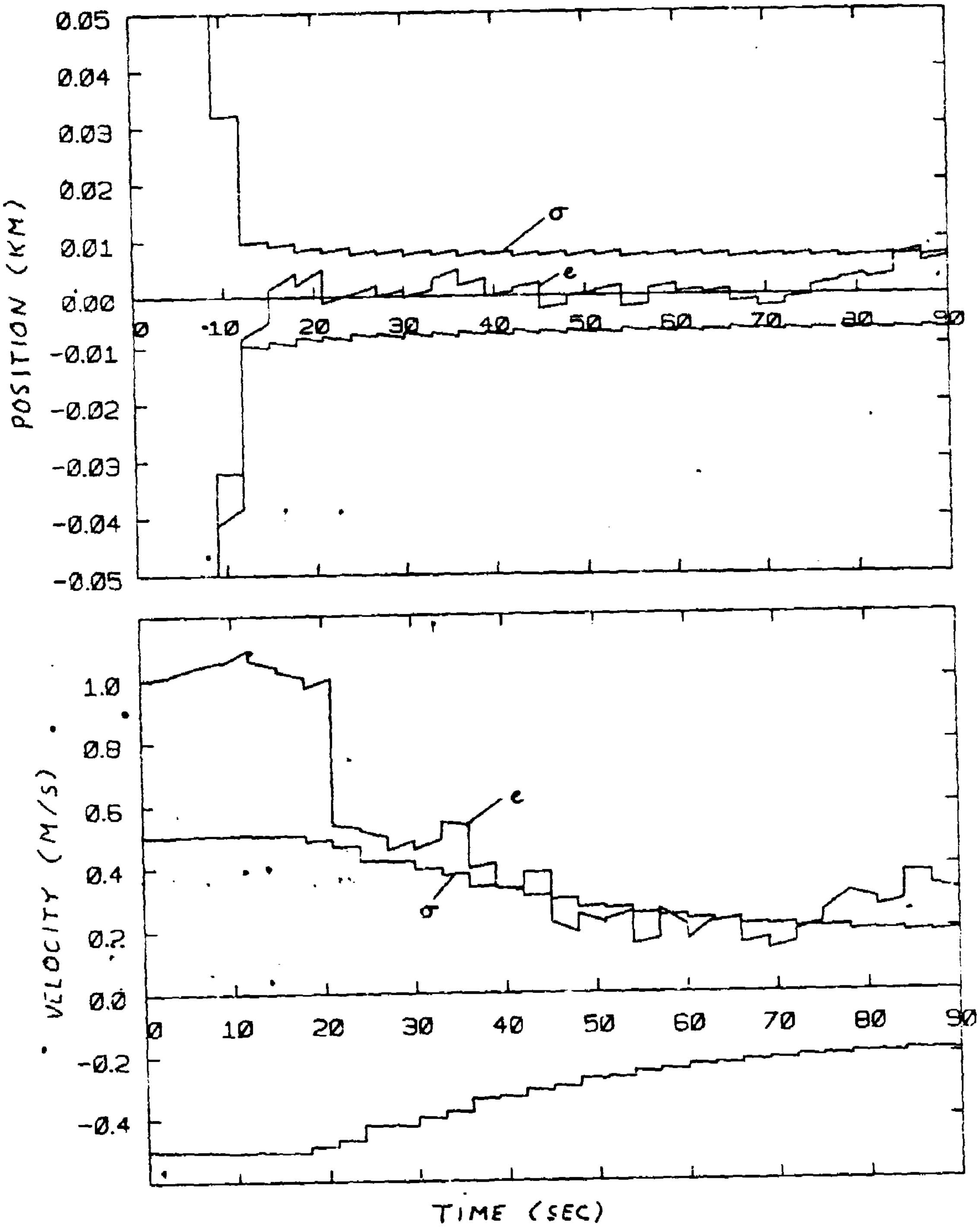


Fig. 4.3 Optimal Estimate of Member 4 East Errors

filter of member 3 has 11 states (x3,y3,vx3,vy3,t3,f3; x2,t2; x4,y4,t4). The suboptimal filter of member 4 has 11 states (x4,y4,vx4,vy4,t4,f4; x2,t2; x3,y3,t3). The absence of a rate state is compensated for by adding a driving noise in the related positional or clock error state. The variance parameter of the resulting random walk model is set to $(10\ \mathrm{m})^2/\mathrm{sec}$. The resulting standard deviation of the random walk at 400 sec is 200 meters, which matches the level of the increased position error due to a constant 0.5 m/sec velocity error.

The performances of the suboptimal filters are shown in Figs. 4.4, 4.5, and 4.6. The performances are again excellent, coming close to the optimal filter performance. The suboptimal filters take somewhat longer to converge to positional accuracy of 10 m, but they do achieve that level. Velocity convergence also is somewhat slower, but again the steady state accuracy is of the same order as that of the optimal filter. For both the position estimates and the velocity estimates the filter computed uncertainties are in good agreement with the single case error traces. Deleting the rate states of the others appears to have little effect on estimating ones own errors.

4.4 Measurement Sharing Conclusions

The proposed measurement sharing organization has excellent performance characteristics. Member estimates of their own errors approach the theoretical optimal. Positions and clock errors are successfully estimated in one or two cycles of measurements. The accuracy is at the level of the measurement

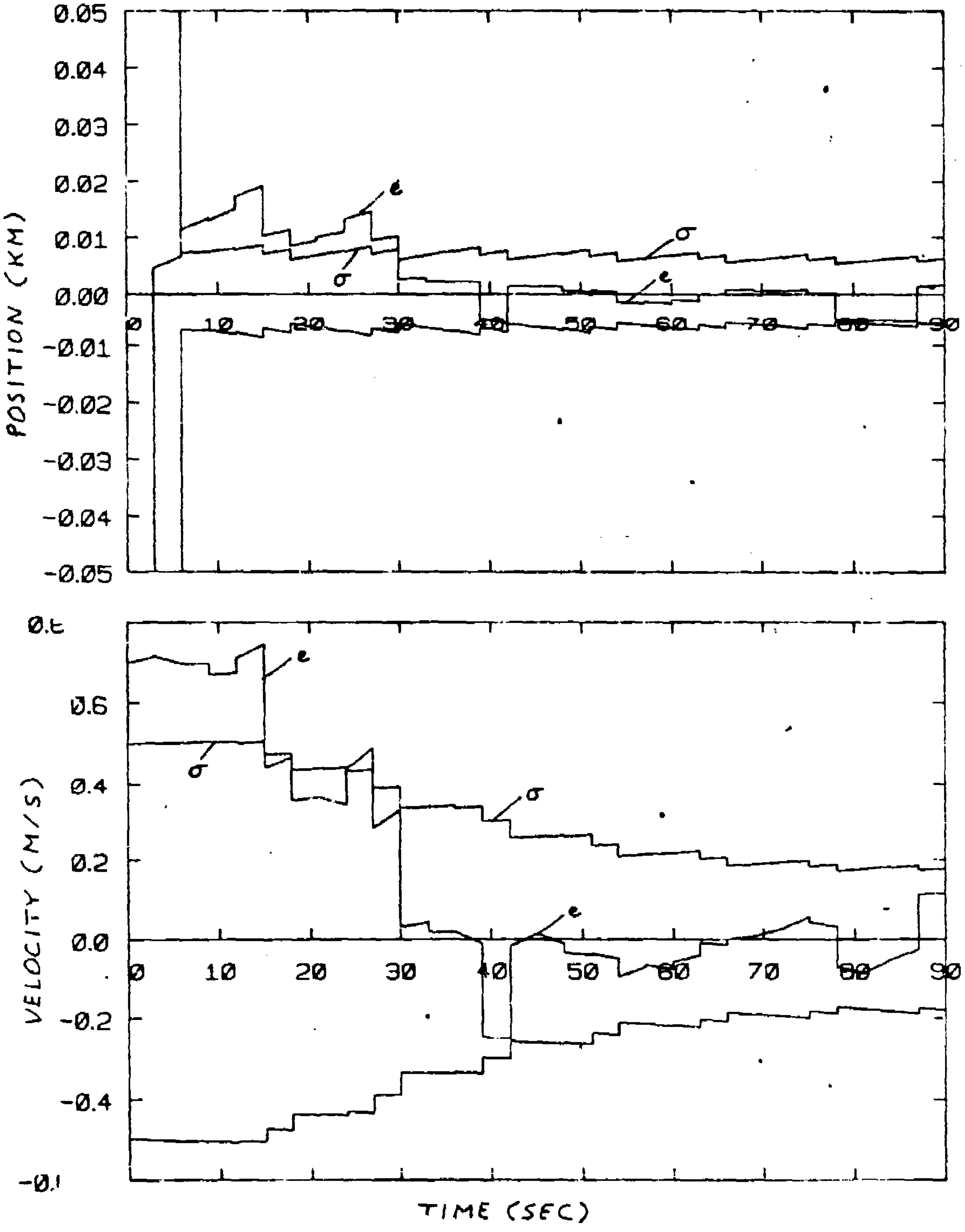


Fig. 4.4 Suboptimal Filter of Member 2, Estimates of Own East Errors

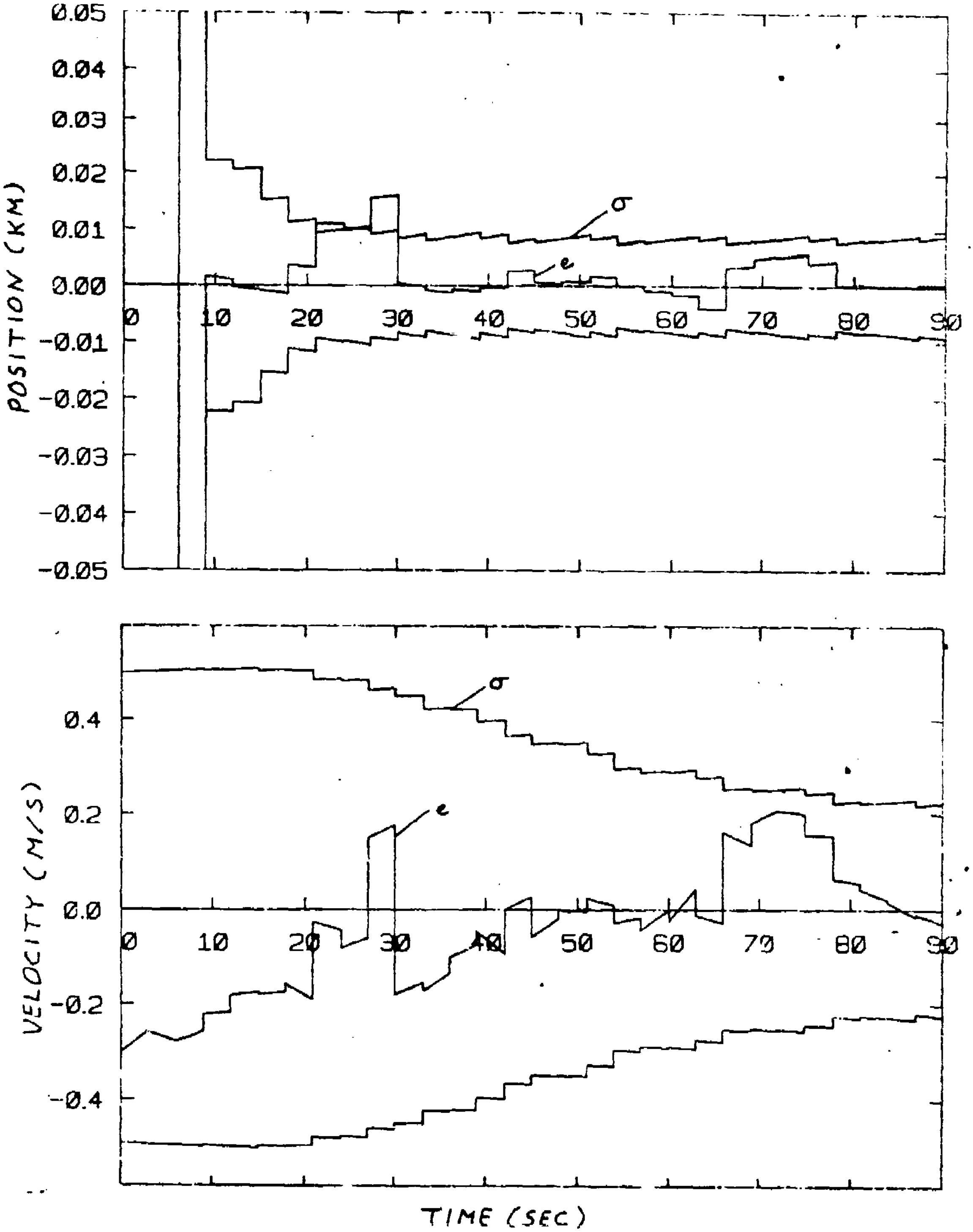


Fig. 4.5 Suboptimal Filter of Member 3 Estimates of Own East Errors

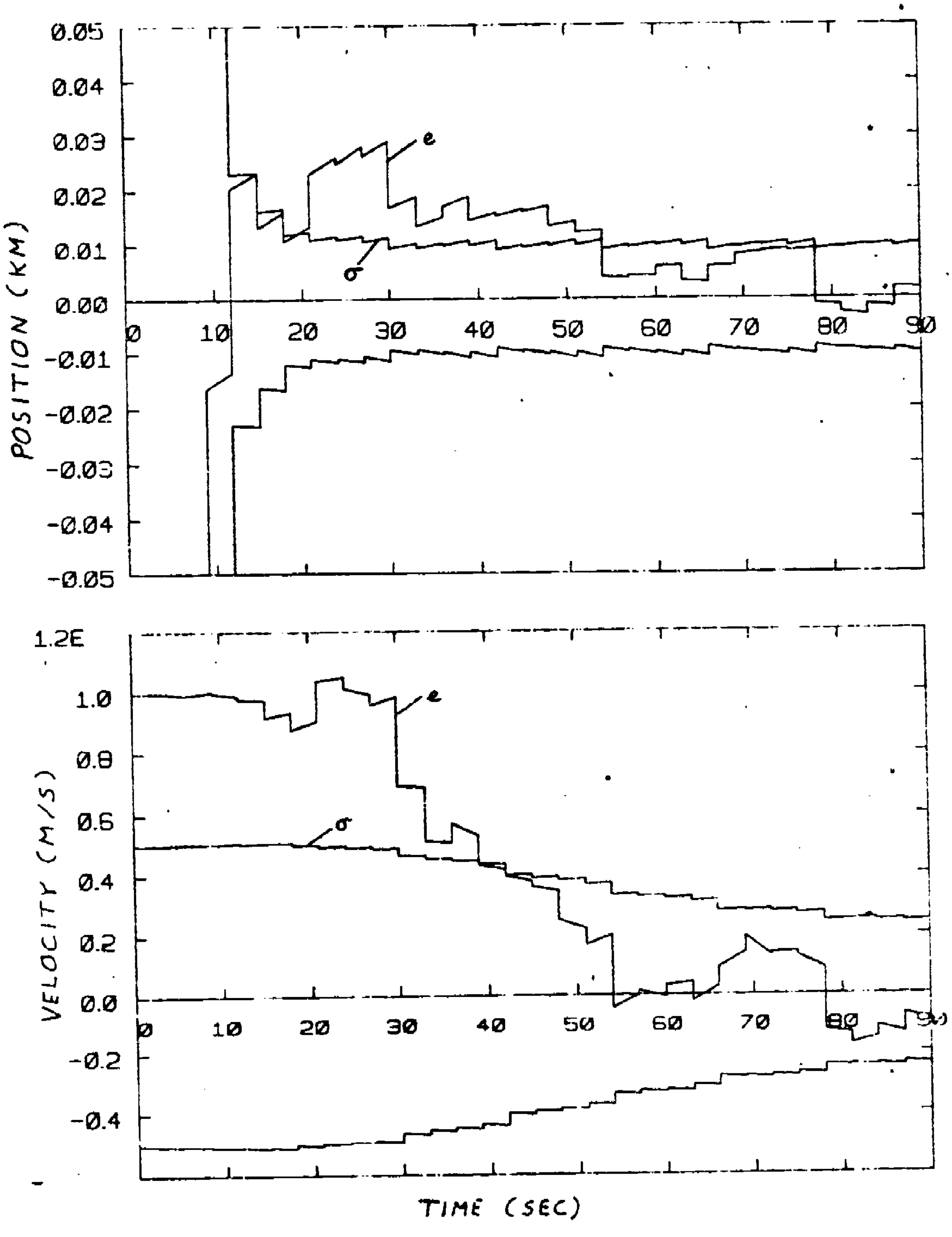


Fig. 4.6 Suboptimal Filter of Member 4 Estimates of Own East Errors

noise.

The measurement sharing organization decouples the estimation processes of the members. Should one member's filter be poorly implemented and be producing bad estimates, this will have no effect on the estimates of other members. Similarly there is no possibility of an instability similar to the interaction instability of ownstate democratic organizations.

To limit the navigation message traffic to acceptable levels it is necessary to introduce the concept of primary members and limit measurement sharing to these members. Further research is needed to explore the necessary number of primary members. Net management rules for dropping a primary member and introducing a new primary member need to be explored.

A more precise estimate of the number of bits to be transmitted needs to be worked out. The issue of whether or not reset information must be broadcast must be considered.

The measurement sharing approach simplifies the source selection logic. There is no need for the covariance based hierarchy source selection logic.

The number of measurement types is reduced. There is only one time of arrival measurement type. Compare this with the current JTIDS baseline software, with its geodetic update type and relative grid update type. Also there is no need for the JTIDS grid offset measurement type, which has two components.

A suboptimal filter can be implemented in each member, deleting the rate states of the other members. Performance is nearly optimal, according to our 2-D low state small number of members simulations. These performance tests should be repeated in a 3-D simulator with more state variables in the truth and filter models and with more members.

CHAPTER 5

JTIDS/GPS/INS INTEGRATION

In Section 2.3 we reviewed the JTIDS/GPS/INS integration literature. The CNPI study conducted by Draper Lab and the MFBARS studies conducted by ITT and TRW concentrated on hardware integration issues. There has also been some qualitative discussion in the literature of the functional and performance benefits that may be realized with integration of JTIDS, GPS, and INS data. In this chapter, Section 5.1 discusses further these functional and performance benefits. Section 5.2 discusses the additions and changes required to the network data transmissions and to the member software to obtain the desired benefits. Section 5.3 comments on the tradeoffs between improved performance and system complexity.

5.1 Benefits of JTIDS/GPS/INS Integration

GPS/INS Integration. First we review some of the benefits of GPS and INS data integration. A GPS receiver can operate very well without an inertial system provided it is operating in a low dynamic environment. But if it is to be used in a high dynamic environment such as in a tactical aircraft, then optimal integration with an inertial system becomes highly desireable if not essential. Some of the integrated functions and benefits are:

1. Dead reckoning. The tracking of the GPS signals from the satellites can be broken due to jamming or due to wing shadowing of the line of sight from the antenna to a satellite. During loss of GPS tracking, the inertial system provides a dead reckoning capability, which accurately follows the vehicle maneuvers. Navigation accuracy continues to be quite good for a moderate period of time.

2. Reacquisition aiding. The inertial indicated position and velocity is used to aid the GPS signal search algorithm. With the significantly smaller position/velocity uncertainty than would be the case without inertial data, the search algorithm reacquires the GPS signals faster.

3. Tracking aiding. During tracking, inertial aiding of the tracking loops permits narrower tracking loop bandwidths. This improves the jamming rejection ability of the system.

4. Antenna beam pointing. The inertial indicated attitude can be used to point the beams of a phased array antenna. By permitting the use of high gain antennas, additional jamming protection is achieved.

5. Sequential receiver aiding. The availability of inertial position and velocity data relaxes the requirement for simultaneous tracking of the code from four satellites to obtain a fix. A single receiver channel can be time shared among the satellites being tracked.

JTIDS/INS Integration. In the case of JTIDS and INS data integration, some of the functions and benefits are the following:

1. Dead reckoning. The inertial system again provides a dead reckoning capability. This can maintain the JTIDS relative navigation accuracy during a brief period of loss of signal or poor measurement geometry. The provision for loss of signal is not as essential a function as in the case of GPS/INS navigation because of the higher JTIDS signal strength. On the other hand JTIDS Relnav accuracy can suffer from significant geometric dilution of precision. If poor geometry occurs only for a short period of time, then the inertial dead reckoning capability provides accurate navigation during this period.

2. Sequential measurement processing. Even with good signal tracking and good measurement geometry, the inertial data makes possible more accurate relative navigation. The position messages received from the various members of the community occur at different instants. The inertial data is at the heart of the interpolation and extrapolation needed to reconcile these asynchronous measurements.

3. Inertial error estimating. The inertial navigation of a member planning to leave the net benefits from the estimation of the INS errors by its Kalman filter during net operations. More accurate inertial navigation is possible after leaving the net.

<u>JTIDS/GPS/INS Integration</u>. In addition to the above benefits of inertial integration with GPS or JTIDS receivers, there are significant benefits to integrating the data from all three systems. Some of the benefits are from GPS aiding the

JTIDS functions and some are from JTIDS aiding GPS functions.

In the GPS aiding JTIDS category, a basic benefit is:

1. Relative grid stabilization. A few GPS equipped members can act as geodetic position references for the JTIDS network. This can anchor the relative navigation grid, provided the dead reckoning errors of the navigation controller are properly modeled.

If JTIDS net time is synchronized to GPS time and if all GPS equipped members know this then some additional benefits are possible:

2. Net entry aiding. A GPS equipped unit can enter the JTIDS net faster because net time is known.

3. Passive relative navigation. A GPS equipped unit at the fringe of the JTIDS community can maintain radio silence and still do accurate relative ranging and navigation. Round trip timing is not needed to resolve the fundamental time/distance ambiguity.

4. Relative navigation accuracy improvement. Similarly the general relative navigation accuracy throughout the community might improve somewhat because the measurement innovations are concentrated on position fixing rather than spread between position and time fixing.

Some of the benefits of JTIDS aiding GPS functions are the following:

1. Geodetic navigation during jamming. Reception of the relatively weak GPS satellite signals is more susceptible to jamming than the reception of the shorter range JTIDS signals.

There are likely to be target areas where jamming denies GPS reception. In these areas accurate geodetic navigation can be maintained by the members, provided the grid is anchored by GPS equipped members outside the jammed area.

2. Reacquisition aiding. When a GPS equipped member emerges from the jammed area, reacquisition of the GPS signal can be rapid because the receiver need only search a relatively small pseudorange and pseudorange rate space. If the period of jamming is brief, the inertial aiding alone would be sufficient to provide this benefit. But if the period of jamming is long, the pure inertial errors and the GPS receiver clock errors would grow, so the JTIDS relaying of geodetic information becomes important. If the clock error is as significant as the inertial error, then the GPS/JTIDS time synchronization information is also important.

3. Non-coherent tracking support. Accurate GPS navigation is based in part on knowing the exact satellite positions and satellite clock errors. Normally this data is obtained from the data modulation on the GPS signal. However to decode this data modulation the receiver must be able to maintain carrier tracking in the phase locked Costas loop. In a jamming environment, carrier tracking (and therefore the data demodulation) is lost before code tracking is lost. Transferring the needed GPS satellite data over the JTIDS link may permit GPS operation in the code tracking only mode (non coherent mode) in regions where data demodulation is impossible.

4. Geodetic navigation with all members partially jammed.

In the absence of jamming, GPS navigation is based on tracking the four satellites that have the best geometry. With some jamming present, some of the best geometry satellites may be denied, in which case an alternate quadruple is selected and somewhat lower accuracy will result. With more jamming, the user may be reduced to tracking only three satellites. A geodetic navigation fix is still possible if known altitude (ships) or measured altitude (aircraft) is also used. In the case of using barometric rather than radar altimeter data, the altitude accuracy is low and this may cause a significant degradation in the horizontal accuracy as well. If the jamming is such that only two or one satellites can be tracked, then a position fix is no longer possible.

Now suppose that all GPS members are partially jammed but three members can each track a pair of satellites. Each of these three can compute a geodetic line of position using the two pseudorange plus altitude data. If these three members share their computed lines of position and if there is good horizontal relative navigation, then in theory there is sufficient information to fix the origin and orientation of the relative navigation grid. This is illustrated in Figure 5.1. It is necessary that all three lines of position not be parallel. This condition is satisfied if the lines of position are not based on the same pair of satellites.

Note to make a useful contribution, a member must be able to track a pair of satellites. A single satellite measurement is useless to other members because of the unknown and unobservable receiver clock error.

The paper by Rome, Reilly, and Ward (Ref. 23) includes the suggestion that GPS measurement sharing can make possible geodetic fixing of the relative grid even if only the same two GPS satellites are tracked by three or more members. No accuracy analysis is presented in support of this suggestion. We believe the accuracy would be so poor as to be useless. The method relies on the lines of position (at the different members) having different azimuths. Not discussed is the fact that members must be widely separated in order for these azimuths to be significantly different. Since the member separation is small compared with the distance to the satellites, the azimuths will not be very different and the grid fix accuracy will be very poor.

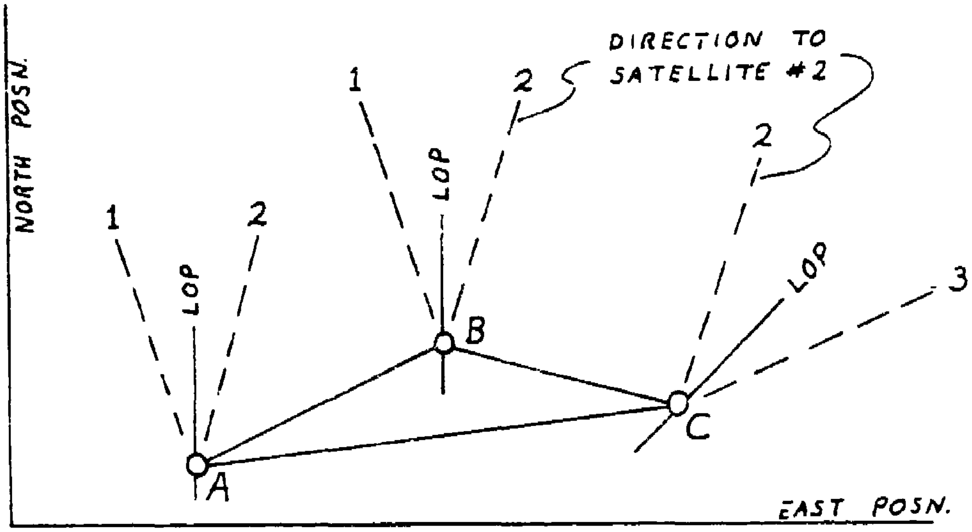


Fig. 5.1 Relative Grid Geodetic Fix by Shared GPS Lines of Position

5.2 Approaches to Data Integration

To accomplish full JTIDS/GPS/INS integration requires hardware and software modifications. Hardware integration has been studied by Draper Lab, ITT, and TRW. In this subsection we discuss some of the software requirements. Changes or additions are required both in the data transmitted on the JTIDS net and in the processing of the data and measurements by the individual members.

GPS/INS Integration. First we review some of the software features of an integrated GPS/INS navigation system. At the heart of an optimally integrated system is a Kalman filter that estimates the inertial navigation errors and other state variables using the selected GPS measurements. A typical set of state variables to be estimated by the Kalman filter is shown in Table 5.1.

Table 5.1 GPS/INS Filter States

INS geodetic position error	3
INS geodetic velocity error	3
INS geodetic misalignment	3
Altimeter error	1
GPS receiver clock phase error	1
GPS receiver clock frequency error	1
Total state variables	12

If long periods without GPS are expected or if the inertial system is of low quality, then additional INS states may be beneficial. The designer might add one vertical acceleration error state and two or three gyro drift rate states.

There are three measurement types processed by the filter:

1. GPS pseudorange. A GPS pseudorange measurement is incorporated by first forming the difference between the measurement and the a priori estimate of the measurement, which is based in part on the inertial indicated position and the filter estimate of the inertial position error. In terms of the filter state variables, the difference is a function of the errors in the estimates of inertial position error and GPS receiver clock phase error.

2. GPS pseudorange rate. Similarly a GPS pseudorange rate measurement is differenced with the a priori estimate of the measurement. In terms of filter state variables, this difference is a function of the errors in the estimates of inertial velocity error and GPS receiver clock frequency error.

3. Altimeter. An altimeter measurement is incorporated also as a difference measurement. The a priori estimate of the measurement is formed from the inertial indicated altitude and the filter estimates of the inertial altitude error and the altimeter error. In terms of filter state variables, the difference is a function of the errors in the estimates of inertial altitude error and altimeter error. In some systems the altimeter data may be directly blended with the inertial data within the inertial navigation equations. This is a non optimal utilization of the data. One settles for this configuration when there is an operational requirement that a stable baro-inertial navigation capability be maintained independent of the workings of the Kalman filter.

The GPS system, as currently planned, will have 18 satellites in orbit. A good fraction of these will be visible at any time above the local horizon. Since only four pseudorange measurements are required to obtain a position/time fix, measurement selection logic selects the four satellites that can most help reduce the navigation errors. The algorithms used are typically based on a static position fix error analysis which provides the geometric dilution of precision (GDOP).

JTIDS/INS Integration. Kalman filter software is also used for the integration of JTIDS and INS data. A typical JTIDS/INS filter, designed to operate in an ownstate community organization, might estimate the states shown in Table 5.2.

Table 5.2 JTIDS/INS Filter States

INS geodetic horiz. position error	2
INS geodetic horiz. velocity error	2
INS geodetic misalignment	3
JTIDS receiver clock phase error	1
JTIDS receiver clock frequency error	1
Relative grid origin position error	2
Relative grid origin velocity error	2
Relative grid azimuth error	1
Total state variables	14

Note that INS altitude and vertical velocity errors and altimeter error are not estimated. These are normally unobservable errors because of the normally nearly horizontal JTIDS measurement geometry. The vertical channel of the inertial navigation equations must be stabilized using the barometric altimeter data. A conventional constant gain third order mechanization can blend the baro and inertial data. Little or no performance benefit can be expected by adding the vertical channel error states to the filter and by incorporation the altimeter data through the filter.

To obtain a more accurate calibration of the inertial navigation sources of error in anticipation of leaving the JTIDS network, the designer may choose to add two or three gyro drift rate states to the filter state vector. Three gyro drift state are included in the Hughes JTIDS/INS filter design (Ref. 18).

Note, in a filter designed for an ownstate organization, there are no states associated with the positional and time errors of the other active members, except for the relative grid error states which are related to the navigation controller geodetic errors. These grid states are needed to permit dual grid navigation.

There are four measurement incorporation types utilized by the Hughes JTIDS/INS filter (Ref. 18):

1. JTIDS grid pseudorange. This is the basic relative navigation measurement incorporation in which a source with good quality grid position serves to improve the user knowledge of grid position. A grid pseudorange measurement is one where the JTIDS measured time of arrival is differenced with an a priori estimate of the TOA that is computed from the source grid position in the P message and the internal estimate of own grid position and grid time. In terms of filter state variables, the difference is a function of the errors in the estimates of inertial position error, grid origin position error, and JTIDS

receiver clock phase error. It is also a function of the grid position and time errors of the source, but since these are not states in the filter their effect is modeled as an additive random error with variance related to the relative position quality word and the time quality word in the P message.

2. JTIDS geodetic pseudorange. This is an alternate way of processing the same physical time of arrival measurement. A source having good quality geodetic information serves to improve the user knowledge of geodetic position. A geodetic pseudorange measurement is one where the measured time of arrival is differenced with an a priori estimate of the TOA that is computed from the source geodetic position and the internal estimate of own geodetic position and time. In terms of filter states, this difference is a function of the errors in the estimates of the inertial position error and the JTIDS receiver clock phase error. It is also a function of the error in the geodetic position and the error in the time of the source, but since these are not states in the filter they are modeled as an additive random error of variance related to the geodetic position quality word and the time quality word.

3. JTIDS grid/geodetic offset. This type provides a means of transferring from the source to the user information about the relative grid to geodetic transformation. A grid/geodetic offset measurement is not directly related to any measured time of arrival. It is computed from the source geodetic position transformed through the internal estimate of own grid/geodetic transformation and the source grid position. The offset is a

horizontal vector with two components.

4. JTIDS round trip timing. A round trip timing measurement serves to update the estimate of the receiver clock error.

It is interesting to note that there is fundamentally only one physical measurement type in the JTIDS community, namely pseudorange (time of arrival). Navigation based on measurement sharing would use only one measurement incorporation type. This would be the difference between a measured pseudorange and the a priori estimate of the pseudorange. The state vector of the filter would include all the horizontal position and clock error states of all primary members of the community. In terms of the filter state variables, the pseudorange difference measurement would be a function of the errors in the estimates of the inertial position errors and clock phase errors of both the source and the receiver of the measurement.

The measurement selection logic in ownstate JTIDS/INS navigation software is complicated. The hierarchy rules concerning who may use measurements off of whom are imbedded in the logic. The number of possible measurement incorporations may be quite large not only because there may be many active members but also because there are the three different ways of constructing measurements from a single time of arrival and position message event. The measurement selection logic must screen all these possibilities and and select subsets that will in do the most to improve the navigation estimates. Multiple criteria are used in the Hughes logic. These test the ability of a possible measurement incorporation to improve the accuracy of:

grid level position, geodetic level position, vertical position, time, and grid azimuth.

JTIDS/GPS/INS Integration. We now discuss the changes or additions required to achieve full integration of the JTIDS, GPS, and INS data.

Among the hardware changes, we assume a timing line is provided between the GPS receiver clock and the JTIDS receiver clock so that the clock difference can be observed precisely.

In discussing network and software changes, we assume that the ownstate organization concept is being retained.

First we discuss the impact on the network data transmissions. The JTIDS position message format already provides for the source geodetic position estimate and its quality. The present format is adequate to accomplish the propagation of GPS derived geodetic information throughout the network. This facilitates the relative grid stabilization, the geodetic navigation during jamming, and the GPS reacquisition aiding.

In order to achieve the benefits associated with synchronized GPS and JTIDS time, GPS equipped members must know that the synchronization exists. First consider the case where the net time controller is GPS equipped and actively maintains JTIDS time in sync with GPS time. If this is known to a GPS equipped user before net entry, then GPS time can be used to achieve faster net entry. If it is not known in advance, then JTIDS net entry is accomplished in the normal way. The net time controller should broadcast the syncronization fact so that users who did not know of this in advance learn of it after net entry.

A more general organization would allow for the possibility that the JTIDS time controller is not GPS equipped and can not maintain JTIDS time in sync with GPS time. In this case the GPS equipped members might estimate the time offset of the two systems and would transmit the offset estimate and its error variance (a quality word) for the benefit of other GPS equipped members. The benefits for GPS equipped members include possible passive relative navigation at the fringe of the community, a general relative navigation accuracy improvement, and improved GPS reacquisition aiding.

GPS satellite data messages may be added to the network to facilitate continued GPS navigation in the non coherent mode in moderately jammed regions.

GPS lines of position can be added to the network to facilitate geodetic position fixing in the partially jammed situation where no member has succeeded in obtaining by itself a GPS fix. To specify a line of position requires three parameters such as the longitude and latitude of one point on the line plus the azimuth of the line. A quality word for the line must also be sent. It would be related to the satellite pair geometry, pseudorange measurement accuracy, and altimeter accuracy. An alternate organization would transmit the raw pair of pseudorange measurements plus the altimeter measurement plus the altimeter quality. This eliminates some of the computation burden from the source member and shifts it to any user member that chooses to incorporate a line of position measurement type.

Now we discuss the impact of full integration on the software of the individual members. At the heart of the navigation software of each member will be a Kalman filter. The filter provides the optimal time varying combination of the dissimilar data from the JTIDS, GPS, and INS subsystems. Time varying rather than constant gains are needed because of the time varying JTIDS and GPS measurement geometries and because of varying measurement availability such as due to jamming or due to members joining or leaving the network.

Kriegsman and Stonestreet (Ref. 21) briefly discuss the possibility that separate GPS/INS and JTIDS/INS filters might be maintained, with their outputs combined in some way. An advantage of this approach might be that the existing software of these currently developed configurations might be useable without significant modification. Disadvantages are that redundant states must be carried in both filters, the INS must interact with two filters, and special filter updates must be defined to transfer geodetic and time information from the GPS filter to the JTIDS filter, all of which increase the software complexity and tax the computer resources. The authors recommend implementing a single integrated filter.

An additional disadvantage of the two filter approach, not discussed by Kriegsman and Stonestreet, is that additional special filter updates must be defined to transfer JTIDS relayed geodetic fix information to the GPS filter to obtain the benefits noted for the GPS jammed environment. Also not discussed is the fundamental fact that the two filter mechanization is not the optimal estimator. Performance will be degraded compared with the performance of the single optimal estimator. These additional considerations furthur support the choice of a single integrated JTIDS/GPS/INS filter rather than the patching together of a JTIDS/INS filter and a GPS/INS filter.

An integrated JTIDS/GPS/INS filter might model and estimate the states shown in Table 5.3. This set of state variables is essentially the same as the set suggested by Kriegsman and Stonestreet (Ref. 21). The filter states do not include two states related to the JTIDS net time controller clock phase and frequency errors relative to GPS time. It is assumed that the JTIDS/GPS time offset can be estimated in a direct manner without adding states to the filter.

Table 5.3 JTIDS/GPS/INS Filter States

INS geodetic position error	3
INS geodetic velocity error	3
INS geodetic misalignment	3
Altimeter error	1
GPS receiver clock phase error	1
GPS receiver clock frequency error	1
JTIDS receiver clock phase error	1
JTIDS receiver clock frequency error	1
Relative grid origin position error	2
Relative grid origin velocity error	2
Relative grid azimuth error	1
	--
Total state variables	19

The measurement types to be processed by the integrated Kalman filter include those found in a GPS/INS filter plus those found in a JTIDS/INS filter plus possibly a shared GPS line of position measurement. A typical set is shown in Table 5.4.

Table 5.4 Measurements Incorporated by filter

GPS pseudorange
GPS pseudorange rate
Altimeter
JTIDS grid pseudorange
JTIDS geodetic pseudorange
JTIDS grid/geodetic offset (2 components)
JTIDS round trip timing
(Shared GPS line of position)

A shared GPS line of position measurement would be incorporated by the filter in the following way. One first computes the displacement of the measured line of position from the a priori estimated line of position. This a priori estimate is constructed from the source indicated relative grid horizontal position and the user estimate of the grid geodetic transformation. In terms of filter state variables, the displacement (measurement residual) is a function of the errors in the estimates of grid origin and orientation. The line of position error (due to source GPS pseudorange error and geometry and altimeter error) is treated as an additive random error of variance related to the quality word.

Practical measurement selection logic will probably utilize the present GPS logic plus the present JTIDS logic without significant modification to either. An available shared GPS line of position would be utilized if the user has been unable to fix the geodetic location and orientation of the relative grid by any other method.

5.3 Complexity/Performance Tradeoffs

It is evident that to achieve all of the possible benefits of full JTIDS and GPS data integration there is a significant increase in the quantity of the data traffic between members and in the complexity of the member software. The designers of the network and the system software must conduct tradeoff studies to determine whether or not each added complication brings a sufficiently significant benefit. In some cases accuracy analysis or simulation is needed to quantify some of the performance benefits.

The ability to share geodetic information is already included in the JTIDS network data and member software. The performance benefits from using this capability seem significant enough, so this should be retained.

To obtain the benefits of JTIDS time being synchronized to GPS time, a small message must be added to the network stating the time offset. Members must have a hardwired timing line connecting their GPS and JTIDS clock functions. Software additions are needed to handle the timing information. The benefits are sufficiently worthwhile to justify these increases in complexity.

To share the GPS satellite data messages will add a significant amount of data traffic to the network. The benefit of continued GPS code tracking long after carrier tracking has been lost due to jamming may not be judged worth the traffic.

To share GPS lines of position may not add a great amount to the network data traffic. But it does increase the member software complexity in the Kalman filter and in the measurement selection logic. It seems to us that the benefit is not great because the only situation where this sharing is useful is in the unlikely event that no two GPS equipped members can get a GPS fix and yet three members can each track pairs of satellites. Because of the poor benefit to complexity ratio we recommend that sharing of GPS lines of position not be implemented.

CHAPTER 6

JTIDS/GPS/INS SIMULATOR

While much insight can be obtained with a two dimensional simulator with low order error models and few members, for a convincing demonstration of the community performance one needs a simulator with much higher fidelity. This chapter provides a summary of the design and capabilities of the M.I.T. JTIDS/GPS/INS simulator, developed to support this research.

6.1 Simulator Design

The simulator is capable of modeling as many as 12 members in the community. At present these are all aircraft. Realistic mission scenarios can be simulated, including typical aircraft trajectories. The simulator is three dimensional and includes a curved earth model. The navigation equipment in each aircraft includes JTIDS receiver, GPS receiver, inertial measurement unit, barometric altimeter, and navigation computer. The simulator is capable of demonstrating either JTIDS/INS relative navigation or integrated JTIDS/GPS/INS dual grid relative and geodetic navigation.

Before undertaking the development of the simulator, we explored the availability and suitability of other JTIDS navigation simulators. One approach considered was to obtain, modify, and use the Dynamics Research Corporation JTIDS Relnav simulator. The development of this simulator has been sponsored by the Naval Air Development Center. Unfortunately the Navy felt it was premature to release a preliminary version of this simulator to M.I.T.

We next explored the availability and suitability of the Hughes/Intermetrics JTIDS Relnav simulator. The Hughes effort has been sponsored by the Air Force. Hughes and the Air Force were willing to have M.I.T. utilize the simulator. We obtained the simulator documentation and Fortran source code from the JTIDS Joint Program Office. We determined that the trajectory generator would be directly applicable to our work with some modification. The balance of the simulator was less applicable to our purposes. The Hughes/Intermetrics simulator is designed in part for a different purpose than navigation research and analysis. It faithfully simulates the high data rate traffic between the inertial system and the JTIDS terminal. The simulator can be used in a laboratory environment to drive an actual JTIDS terminal to check out the operational computer program. As a result of the necessary simulation complexity and short time steps, the simulator is costly to operate. For navigation research and analysis we require a simulation that is less costly to operate. We decided therefore to utilize the trajectory generator as a starting point but not attempt to utilize the balance of the simulator source code.

Some of the modifications to the Hughes/Intermetrics trajectory generator are the following: The inertial navigation system simulations have been removed from the trajectory generator. We have moved this function to the navigation simulation so as to permit closed loop resetting of the navigation variables by the Kalman filters. The random time slot assignments have been eliminated. We are willing to assume that all assigned times occur on integer seconds. The trajectory data for each member is synchronized on the integer seconds. The high frequency at which trajectory data was provided has been eliminated. We have no requirement to provide high frequency inputs to an inertial navigation function. The rate at which we record trajectory data is one community data set per second. We have eliminated the ellipsoidal earth model and are using a spherical earth model. All of the design simplifications discussed above are thought to have no essential effect on the ability of the simulator to make realistic predictions of the actual JTIDS/GPS/INS navigation errors.

A second trajectory generator program has been developed to provide GPS satellite positions. The present capability provides four GPS satellites that provide good measurement geometry.

The navigation portion of the simulation operates on the precomputed member and satellite trajectories. It simulates the member inertial navigation errors. It generates simulated measurements and mimics the processing of the measurements by the data processors of each member. Output records are written on the disc for later editing, printing, or plotting.

One simplifying assumption is that all broadcast and measurement events occur at integer seconds. This avoids the need for interpolation between trajectory data points. Another simplifying assumption is that all member data processors are infinitely fast. Thus for example the member who is about to broadcast is able to incorporate any measurements that were taken at the last integer second, read its INS indicated position at the broadcast time, update the Kalman filter state to the broadcast time, and include current estimate and variance data in the broadcast position message. This assumption greatly simplifies the simulation logical flow. There is no need to simulate a lagging filter process.

To simulate the inertial navigation, INS error equations have been implemented. This permits longer time steps than the alternate approach of directly simulating the inertial navigation by whole value integration of the high dynamic specific force. The INS error equations implemented are from the Widnall and Grundy inertial navigation error models report (Ref. 31). A local level, rather than strapdown, mechanization is assumed.

The simulator implements the current JTIDS ownstate organization with estimate sharing, rather than an organization based on measurement sharing. We have relied on a JTIDS software specification by Hughes (Ref. 32) for details on the definition of the relative grid, navigation controller constraints, position message content, covariance based hierarchy source selection logic, and JTIDS measurement processing.

We have designed a Kalman filter for integrating the JTIDS, GPS, and local level INS data. The ownstate community organization is retained. To avoid nonlinear difficulties with JTIDS time of arrival measurements at short range between members, we have implemented the Gaussian second order filter equations (Ref. 33). These equations deweight short range measurements by increasing the assumed variance of the measurement as a function of the range and the positional uncertainty. The biasing effect of the nonlinearity is also compensated.

We have implemented the Bierman UDU' factored version of the Kalman filter measurement incorporation algorithm (Ref. 34). The Bierman algorithm splits the error covariance matrix P into factors U, D, and U transpose. D is a diagonal matrix. U is an upper triangular matrix with ones on the diagonal. Measurement incorporations updating U and D avoid the numerical difficulties experienced with Kalman's original algorithm. To evaluate filter performance, we require the filter computed one sigma values of the estimation errors. These are obtained as square roots of elements or combinations of elements of the covariance matrix P. Accordingly the P matrix is reformed from its U and D factors after the measurement incorporations. Covariance time updating is done in terms of the P matrix. The P matrix is refactored before incorporating the next set of measurements.

The simulator has been programmed in Fortran and runs on a Digital Equipment Corp. VAX 11/780 digital computer with VMS (virtual memory) operating system. The navigation simulation segment has been carefully implemented with a coherent subroutine structure to make it easier to understand, easier to test, and easier to modify. At least 50% of the lines in the source code are comment lines. An introductory set of comments in each subroutine states the subroutine function and lists the inputs and outputs.

There are no preprogrammed input statements to the simulator. We have found it to be straightforward to use the powerful file storage and text editing capability of the DEC VAX system to modify mission event parameters, number of members, etc. in a copy of the source code, then recompile the altered subroutines, link with the unaltered compiled programs, and run.

The overall structure of the navigation simulation is shown in Table 6.1. The indentation indicates which programs are called by which. For example NAVSIM calls INITAL and MAINLP; INITAL calls INITTR, INITDP, and OUTPUT; INITTR calls FRSREC; etc.

NAVSIM is the main program of the navigation simulation. It calls two subroutines, INITAL the initialization routine and MAINLP the main loop of the simulation.

INITAL first calls INITTR, which sets the parameters and initial values in the truth models. FRSREC reads in the first records to be used from the aircraft and satellite trajectory files. INITDP sets the parameters and initial values in the member data processor simulations. UDUFAC takes a covariance matrix and splits it into the Bierman UDU' factors.

OUTPUT supervises the outputting of the estimation errors and filter indicated one sigma values. It calls OUTDAT which supervises the outputting of data for a single member. OUTEVE writes an output record identifying the event, if any. OUTGEO writes an output record of the geodetic navigation errors and the

Table 6.1 Subroutine Structure of Navigation Simulator

- NAVSIM - Navigation simulator main program
  - INITAL - Initialization
    - INITTR - Initialize truth models
      - FRSREC - Read first trajectory record
    - INITDP - Initialize data processor simulations
      - UDUFAC - UDU' factorization of covariance matrix
    - OUTPUT - Output results for all members
      - OUTDAT - Output data for one member
        - OUTEVE - Output event record
        - OUTGEO - Output geo. errors and filter sigmas
        - OUTGRD - Output rel. grid errors and sigmas
          - RELCAL - Est. rel. posn. and sigmas calcs.
          - TRUREL - True relative position
  - MAINLP - Main loop
    - NEXTDT - Determine time of next event
      - NEXREC - Read next trajectory record
    - TRUTH - Truth models
      - TRUTUP - Truth models time update
        - EGPSUP - External GPS errors update
        - INSTUP - INS errors update
        - JTDTUP - JTIDS clock errors update
        - GPSTUP - GPS clock errors update
        - TRUGRD - True rel. grid origin and orientation
      - TRUMES - True measurements
        - JTDMES - JTIDS time of arrival measurement
        - RTTMES - Round trip timing measurement
        - GPSMES - GPS pseudorange measurement
        - BARMES - Barometric altimeter measurement
    - DPSIMS - Member data processor simulations
      - KFTIME - Kalman filter time update
        - DPPHI - Data processor state transition matrix
        - DPQMAT - Data processor driving noise Q matrix
        - KFTUP - Kalman filter state and P matrix update
          - UDUFAC - UDU' factorization routine
      - PMESSG - Position message
      - SORSEL - Source selection logic
      - OUTDAT - Output data for one member
      - KFMEAS - Kalman filter measurement incorporation
        - BARRES - Barometric altimeter meas. residual
        - RTTRES - Round trip timing meas. residual
        - GPSRES - GPS measurement residual
        - GEORES - TOA geodetic meas. residual
          - GEOSOF - Geo. TOA 2nd order filter eqs.
        - GRDRES - TOA relative grid meas. residual
          - GRDSOF - Grid TOA 2nd order filter eqs.
        - MESINC - UDU' measurement incorporation
        - KFMSNC - KFMEAS nav. controller constraints
      - RTTREQ - Round trip timing request
      - RESETS - INS and clock error resets
    - OUTPUT - Output results for all members
      - OUTDAT - Output data for one member

filter computed geodetic sigmas. OUTGRD writes an output record of the relative navigation errors and the filter computed relative sigmas. The computation of these relative data are complex since the relative navigation is implicitly imbedded in differences between geodetic position estimates of the member's position and the grid origin position. Two subroutines support these calculations: RELCAL provides the member's estimate of the relative position and the relative sigmas. TRUREL provides the true relative position.

MAINLP calls three subroutines. NEXTDT determines the time of the next event, such as a JTIDS broadcast. The simulation will step to the time of the next event. TRUTH updates the truth models (actual navigation errors) and provides the actual measurements. DPSIMS provides the member data processor simulations.

NEXREC, called by NEXTDT, finds the next trajectory records to be used. These records correspond to the time of the next event.

TRUTH calls two subroutines. TRUTUP supervises the time updating of the truth models. TRUMES supervises the preparation of the actual measurements.

TRUTUP calls five subroutines. EGPSUP is the external GPS errors time update. INSTUP is the inertial navigation system errors time update. JTDTUP is the JTIDS clock errors time update. GPSTUP is the GPS receiver clock errors time update. TRUGRD calculates the true geodetic position of the origin of the Relnav grid from the true geodetic position of the navigation controller plus the indicated relative position of the navigation controller and the true grid azimuth (beta angle).

TRUMES calls four subroutines. JTDMES provides the JTIDS time of arrival measurements obtained by each member. RTTMES provides round trip timing measurements. GPSMES provides GPS pseudo range measurements for any member to the four GPS satellites. BARMES provides barometric altimeter measurements. The current version of the simulator does not include GPS pseudorange rate measurements. The effect of this omission on navigation accuracy is small.

DPSIMS simulates the navigation software in the data processors of the members. It calls seven subroutines. KFTIME is the Kalman filter time update to the current event time. PMESSG prepares the position message of the broadcaster (if any). SORSEL is the measurement source selection logic which accepts or rejects available time of arrival measurements according to the current JTIDS covariance based hierarchy. OUTDAT outputs filter performance data, here just before measurement incorporations. KFMEAS is the Kalman filter measurement incorporation routine. RTTREQ is the round trip timing request logic. RESETS implements the resetting of the inertial and clock variables as a function of the Kalman filter estimates.

KFTIME calls three subroutines. DPPHI calculates the data processor Kalman filter state transition matrix (phi matrix). DPQMAT calculates the data processor Kalman filter driving noise covariance matrix (Q matrix). KFTUP completes the Kalman filter time update, advancing the filter estimates and the covariance matrices P using the Phi and Q matrices. UDUFAC is again called to factor the resulting P matrices.

KFMEAS calls seven subroutines. Five of these process the different measurement types, calculating the residual, setting the measurement gradient vector, and setting the measurement variance. The residual is the difference between the physical measurement and the filter predicted measurement. BARRES forms a barometric altimeter measurement residual and related data. RTTRES forms a round trip timing residual and related data. GPSRES forms a GPS pseudorange residual and related data.

GEORES forms a geodetic time of arrival residual and related data. Here the predicted TOA measurement is based on the source and user best estimates of geodetic position. GEOSOF implements the geodetic second order filter corrections to the residual and the measurement variance to compensate for the nonlinear elongation of the measured range. GRDRES forms a Relnav grid time of arrival residual and related data. Here the predicted TOA measurement is based on the source's indicated relative position and the user's indicated relative position, which is implicit in terms of own geodetic position and grid origin geodetic position. GRDSOF implements the grid second order filter corrections to the residual and variance.

The JTIDS grid/geodetic offset measurement type is not implemented in the current version of the simulator. Simulation results show that the omission of this measurement type is significant. This measurement type therefore should be added.

MESINC implements the Bierman measurement incorporation algorithm. It operates on the measurement residual, measurement gradient vector, and measurement variance to update the filter estimate and associated U and D factors of the error covariance matrix.

KFMSNC is a subroutine called by KFMEAS if the member is the navigation controller. It enforces the navigation controller constraint that requires that the indicated relative position of the navigation controller be continuous even when the nav. controller updates its estimates of INS geodetic position, velocity, and alignment errors. This requires adjusting the grid origin geodetic position, velocity, and azimuth estimates as a function of the changes to the navigation controller's own geodetic position, velocity, and alignment error estimates.

Following all measurement incorporations, KFMEAS reforms the P matrix by multiplying the U, D, and U transpose factors.

RTTREQ sets up a round trip timing request if the filter computed JTIDS clock phase error variance is a factor of 1.414 higher than the additive random error in a RTT measurement. The RTT event will take place at the next integer second.

RESETS resets the inertial variables and the JTIDS clock variables in response to nonzero filter estimates of the errors. Both the truth model errors and the filter estimates of the errors are changed at the same time point. At present, resets are applied only to the vertical channel variables of the inertial system and to the JTIDS clock variables.

OUTPUT is called at the end of the main loop to output the performance data after the measurement incorporations but before the next time update.

6.2 Truth Models of Sources of Error

The simulation navigation performance results are strongly related to models selected to represent the important sources of error. This section summarizes these simulator truth models.

The inertial navigation error model represents a local level three axis inertial system using the error dynamic equations from the Widnall/Grundy report (Ref. 31). The basic nine state variables and their initial one sigma values are

Error state	Initial 1 sigma value
Latitude error	1 km
Longitude error	1 km
Altitude error	200 m
East velocity error	1 m/s
North velocity error	1 m/s
Up velocity error	1 m/s
Tilt about east	0.5 arc min
Tilt about north	0.5 arc min
Azimuth error	1 millirad

The angular velocity error components, including gyro drift rates, are modeled as exponentially correlated first order Markov random processes. Similarly the acceleration measurement error components, due to accelerometer error and gravity model error, are modeled as first order Markov processes. The initial and steady state one sigma values and the correlation times of these processes are

Error state	One sigma	Correlation time
Gyro drift east	0.015 deg/hr	1 hr

Gyro drift north	0.015 deg/hr	1 hr
Gyro drift up	0.015 deg/hr	1 hr
Accel. error east	50 micro g	1 hr
Accel. error north	50 micro g	1 hr

The JTIDS clock error model has a first order Markov process for frequency error and the phase error is the integral of frequency error. The initial one sigma phase error and the initial and steady state one sigma frequency error and correlation time are

Error state	One sigma	Correlation time
JTIDS clock phase	0.1 milli sec	N.A.
JTIDS clock freq.	1 E-8 sec/sec	2 hr

The simulated JTIDS time of arrival measurements all have a scale factor error due to uncompensated atmospheric retardation of 50 parts per million. In addition there is an additive random error of 30 nano sec (10 m) one sigma. A round trip timing measurement has half the variance of a TOA measurement.

Atmos. delay	50 ppm of range
JTIDS TOA meas. noise	30 n sec (10 m) one sig
JTIDS RTT meas. noise	21 n sec (7 m) one sig

The GPS receiver clock error model is similar to the JTIDS clock error model

Error state	One sigma	Correlation time
GPS clock phase	0.1 milli sec	N.A.
GPS clock freq.	1 E-8 sec/sec	2 hr

The pseudo range measurement from each of the four GPS satellites is modeled as having a first order Markov error due to sources of error external to the receiver. External error sources can include satellite clock, satellite ephemeris, ionospheric retardation, and tropospheric retardation compensation errors. The model parameters are

Error state	One sigma	Correlation time
GPS external error	2 m	0.5 hr

The GPS receiver pseudorange tracking error is modeled as an additive random error in the measurements of 2 m one sigma.

GPS meas. noise	2 m one sigma

The barometric altimeter error model includes a scale factor error of 3% due to nonstandard day tempurature, a zero setting error of 50 m, a variation in height of a constant pressure surface of 0.2 meters/km in both the east and north direction, and an additive random noise of 3 meters one sigma

Baro scale factor	0.03
Zero setting	50 m
Weather slope east	0.2 m/km
Weather slope north	0.2 m/km
Baro noise	3 m one sigma

6.3 Kalman Filter Models of Sources of Error

The Kalman filter implements an error state formulation. The most significant errors of the subsystems are included in the filter state. The choice of state variables is always a compromise between model fidelity (many states) and filter practicality (fewer states). Our design for an integrated JTIDS/GPS/INS filter has the following 19 state variables:

1. INS longitude error
2. INS latitude error
3. INS altitude error
4. INS velocity error east
5. INS velocity error north
6. INS velocity error up
7. INS misalignment east
8. INS misalignment north
9. INS misalignment up
10. Barometric altimeter error
11. JTIDS clock phase error relative to net time ref.
12. JTIDS clock frequency error relative to NTR
13. JTIDS grid origin posn. error U
14. JTIDS grid origin posn. error V
15. JTIDS grid origin vel. error U
16. JTIDS grid origin vel. error V
17. JTIDS grid alignment (beta) error
18. GPS clock phase error
19. GPS clock frequency error

The initial estimate of these navigation errors is zero. The initial value of the estimation error covariance matrix P is non zero on the main diagonal. The square roots of these diagonal entries are the one sigma values of the initial estimation errors. These are

State	Initial one sigma est. err.
Latitude, longitude	1 km
Altitude	200 m
Velocity east, north, up	1 m/sec
Tip about east, north	0.5 arc min
Azimuth error	1 milli rad
Barometric altimeter	200 m
JTIDS rel. clock phase	0.1414 milli sec
JTIDS rel. clock freq.	1.414 E-8 sec/sec
Grid origin posn. u, v	1 km
Grid orientation	1 milli rad
GPS clock phase	0.1 milli sec
GPS clock frequency	1 E-8 sec/sec

If the member is the net time reference, a geodetic position reference, or a ground station, then special initialization is required. If the member is the net time reference, then the JTIDS relative clock phase and frequency variance is set to zero. If the member is a geodetic position reference, then the INS position, velocity, and alignment variances (9) are set to zero. If the member is a ground station, then the INS horizontal position (not altitude), velocity (3), and alignment variances (3) are set to zero.

The Kalman filter state transition matrix is block diagonal. The driving noise vector covariance matrix Q is diagonal. The dynamics of the inertial navigation errors are represented in the first nine by nine block of the state transition matrix. The elements are based on the same inertial navigation error model used in the simulation truth model. The Kalman filter model for inertial errors does not include gyro drift states or acceleration error states, so the effects of these errors are modeled as uncorrelated state driving noises. The spectral density of the white noises driving the three velocity error states has been set to

$$\text{Nve, Nvn, Nvz} \qquad (50 \text{ micro g})^2 * 800 \text{ sec}$$

These noises add a random walk component to the velocity errors. After 800 sec, the variance of the random walk matches the velocity variance that would have been caused by a 50 micro g bias acceleration error. The alignment error states are similarly driven by white noises. Their densities are

Nepse, Nepsn, Nepsz $(0.015 \text{ deg/hr})^2 * 800$ sec

This noise density causes a random walk component in the alignment errors (epsilons) who variance at 800 sec is the same as if there were a 0.015 deg/hr gyro drift rate.

The barometric altimeter error is modeled as an exponentially correlated first order Markov process. The state transition matrix uses the assumed correlation time of the error, which is

Tauba	1.25 hr

The spectral density of the white noise driving the baro error state is the level required to maintain the RMS amplitude of the stochastic process at the initial one sigma value of 200 m. This value is a function of the desired RMS level and the correlation time.

Nhb	$2(200 \text{ m})^2/\text{Tauba}$

The JTIDS relative clock phase error is modeled as the integral of the relative frequency error which is modeled as an exponentially correlated first order Markov process. The filter model parameters are

TauJTD	2 hr
NJTDf	$4(1\ \text{E-8})^2/\text{TauJTD}$

Note the factor of 4 rather than 2. The relative error is the difference between the member's clock absolute error and the net time reference clock absolute error. The variance of this difference is double the variance of either error.

The grid origin position errors are modeled as being the integrals of the grid origin velocity errors, which are modeled as a random walks. The spectral density of the white noises driving these random walks is

Nrgvu, Nrgvv	$(1\ \text{m/sec})^2 / (21\ \text{min})$

The grid origin errors are related to the inertial navigation errors of the navigation controller. This level of noise density matches the shift in grid velocity error due to one quarter cycle of a Schuler oscillation (21 min) of a 1 m/sec INS velocity error. The grid orientation error is modeled as a random walk driven by a white noise of density

Nrgb	$(0.015\ \text{deg})^2 / (1\ \text{hr})$

This matches the azimuth error change in the navigation controller due due a 0.015 deg/hr gyro drift rate acting for 1 hr.

The GPS receiver clock error model has the same parameters as in the truth model. The clock frequency error correlation time and driving noise density are

TauGPS	2 hr
NGPSf	$2(1\ E-8)^2$ / TauGPS

The Kalman filter uses several types of measurements to update its state vector estimate. The filter measurement incorporations require an assumed value for the variance of the additive random error in the measurement. The following values are assumed for the variances of the baro noise, the JTIDS TOA noise, the GPS pseudorange noise, and the JTIDS RTT noise:

Fbarvr	$(3\ m)^2$
Fatmsd	50 parts per million
FJTDvr	$(30\ nano\ sec)^2$
FGPSvr	$(2\ m)^2$
FRTTvr	FJTDvr / 2

The parameter Fatmsd is the assumed one sigma value of the atmospheric delay affecting the JTIDS TOA measurements. An additional increment of variance is assumed for the TOA measurements equal to the square of the one sigma atmospheric delay factor times the estimated range.

6.4 Simulator Running Speed

Many cases have been run using the simulator. The performance results are reported and analyzed in Chapters 7 and 8. We are pleased that the simulator has proved to be economical to operate.

For example, in a two member simulation with neither member using GPS measurements, the simulation required 9% of real time. That is, each 100 sec of flight time required 9 sec of computer time.

In a four member simulation with one member processing GPS measurements, the simulation required 38% of real time.

The time required seems to be proportional to the square of the number of members. The number of JTIDS time of arrival measurements to be processed each JTIDS net cycle also grows as the square of the number of members. It appears that the preparation of the simulated measurements and/or the execution of the Kalman measurement incorporation equations are dominating the consumption of computer time.

CHAPTER 7

JTIDS/INS SIMULATION RESULTS

7.1 Baseline Simulation Conditions

A series of simulations have been run using the M.I.T. simulator to explore the performance characteristics of JTIDS/INS relative navigation. No member has access to accurate geodetic information, such as could be obtained with a GPS receiver. The simulations explore the effect on performance of several variations in conditions. Unless otherwise stated, the baseline conditions are as summarized in this section.

The community organization has a single navigation controller who is also the time master. There is no secondary controller, such as an end of baseline member. The navigation controller is always member 1.

Additional members beyond the navigation controller are all designated primary members. This means among other things that they may use active round trip timing whenever their computed relative clock uncertainty exceeds a tight threshhold.

The organization is a covariance based hierarchy.

The Kalman filters include the Gaussian quadratic protection for the nonlinear elongation of the measured range.

The data processor algorithms simulation is carried out in single precision.

The time slot assignments are such that all members broadcast once per 12 sec cycle.

Some of the simulations have only two members, the navigation controller plus one other. Other simulations have four members, the navigation controller plus three others.

Different trajectories are flown. The dynamics are that of flying aircraft or hovering helicopters. In all of the trajectories, the members are flying at the same constant altitude.

7.2 Trajectory Effect on Performance

Several different trajectories have been flown to illustrate the effect of the trajectory on the relative navigation performance.

A two member boomerang shaped ground track trajectory is shown in Fig. 7.1. The navigation controller is hovering and member 2 initially is flying directly at the nav. controller (member 1) from the east. Member 2 turns north just before the navigation controller and flies straight north.

For the two member boomerang trajectory, the relative navigation results for member 2 are shown in Fig. 7.2. The relative position estimation error and the filter computed uncertainty (one sigma error) are plotted. Both the plus one sigma value and the minus one sigma value are plotted to form a symmetric band about zero error. The two horizontal components of relative position error are plotted. The U axis is nominally east. The V axis is nominally north. The actual orientation of these axes depends on the value of the azimuth alignment error of

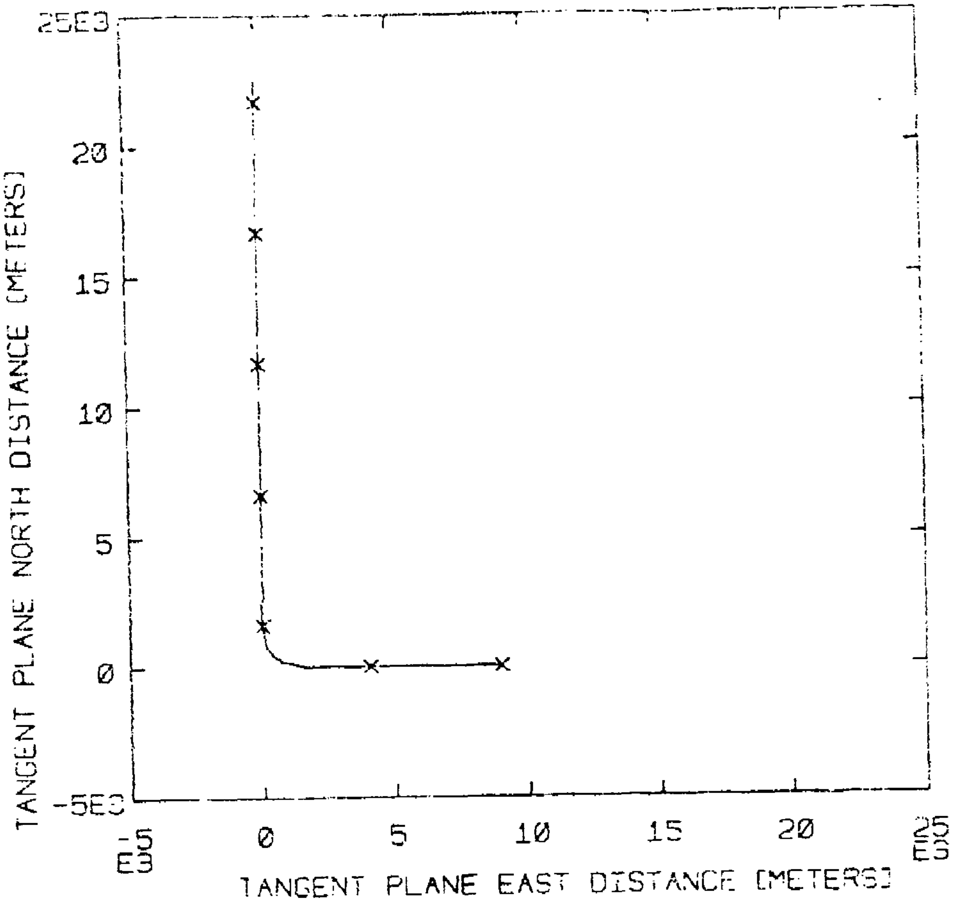


Fig. 7.1 Two Member Boomerang Trajectory

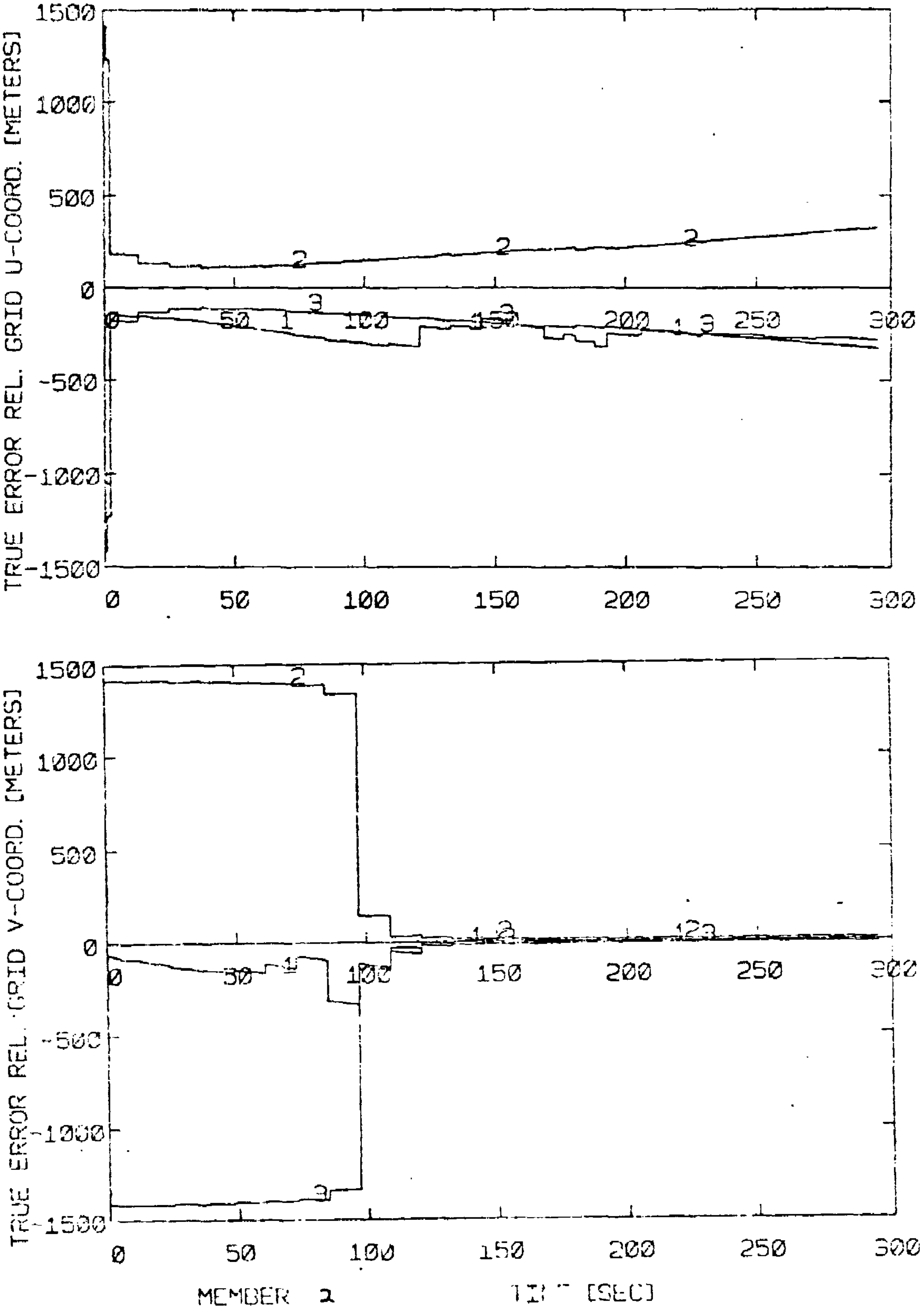


Fig. 7.2(a) Boomerang Trajectory, Relnav Results

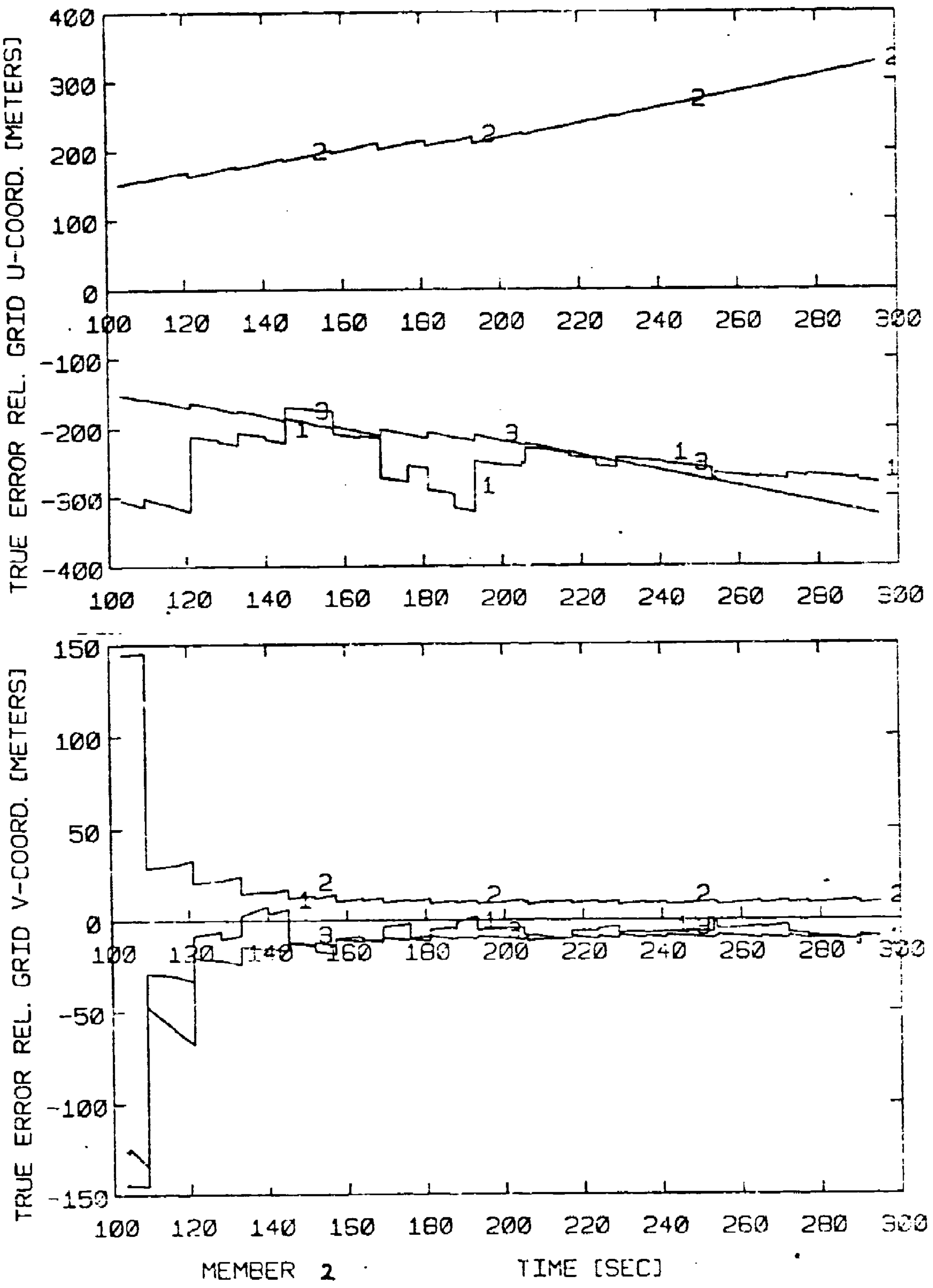


Fig. 7.2(b) Boomerang Trajectory, Relnav Results, Expanded Scale

the navigation controller inertial navigator.

The initial uncertainty of both components is computed to be 1400 meters. Recall the estimated relative position is the difference between the estimated geodetic position of the member and the estimated geodetic position of the relative grid origin. The initial covariance matrix assumes these errors are uncorrelated and each has a 1000 m uncertainty. The initial uncertainty of the relative position is the root sum square of these two uncertainties.

Member 2, being to the east, has good geometry for measuring easterly (U) relative position. After the first time of arrival measurement and round trip timing measurement, the U uncertainty has been reduced to about 200 m. Subsequent measurements reduce the computed uncertainty at 50 sec to about 100 m but the actual U error is about 200 m. The reason the filter is not even more optimistic about its accuracy is that the Gaussian quadratic equations protect the estimation process from divergence due to the nonlinear elongation of the measured time of arrival. Note there is no improvement in the knowledge of north (V) relative position during this initial period flying in from the east.

Member 2 is turning north at about 80 sec. The northerly (V) relative position is soon accurately estimated. Both the actual V error and the computed uncertainty are reduced to the order of 10 m. The easterly (U) error, however, gradually grows to about 300 m in 300 sec. Evidently the filter was unable to reduce significantly the initial 1.4 m/sec east relative velocity error during the initial inbound portion of the trajectory.

In the two member tear drop trajectory shown in Fig. 7.3, member 2 again approaches from the east and turns north. But then it turns southeast. The relative navigation results of member 2 are shown in Fig. 7.4. Up until the turn to the southeast, the results are similar to those of the boomerang trajectory. After the turn to the southeast at about 150 sec, the easterly relative accuracy improves, reaching the level of 10 m by 300 sec. The northerly accuracy deteriorates to about 80 m in the last 150 sec, a rate of about 0.5 m/sec. The filter was somewhat successful at reducing the initial northerly 1.4 m/sec relative velocity error.

In the two member fly around trajectory shown in Fig. 7.5, member 2 does a U turn around the hovering navigation controller. The relative navigation performance results are shown in Fig. 7.6. The results are similar to those of the tear drop trajectory. About 15 m accuracy is eventually achieved along the line of sight and the cross range accuracy deteriorates at about 0.5 m/sec.

The four member crossing trajectory shown in Fig. 7.7 is designed to give high angular velocities of the lines of sight. Members start at the corners of a 20 km square, flying in straight lines toward the center of the initial square (actually a little left of center). They all cross each other, crossing a few kilometers behind the member who had been coming from ones left. The relative navigation results for members 2, 3, and 4 are shown in Figs. 7.8, 7.9, and 7.10. The final accuracies are not as good as with the previous trajectories. Member 2 has a

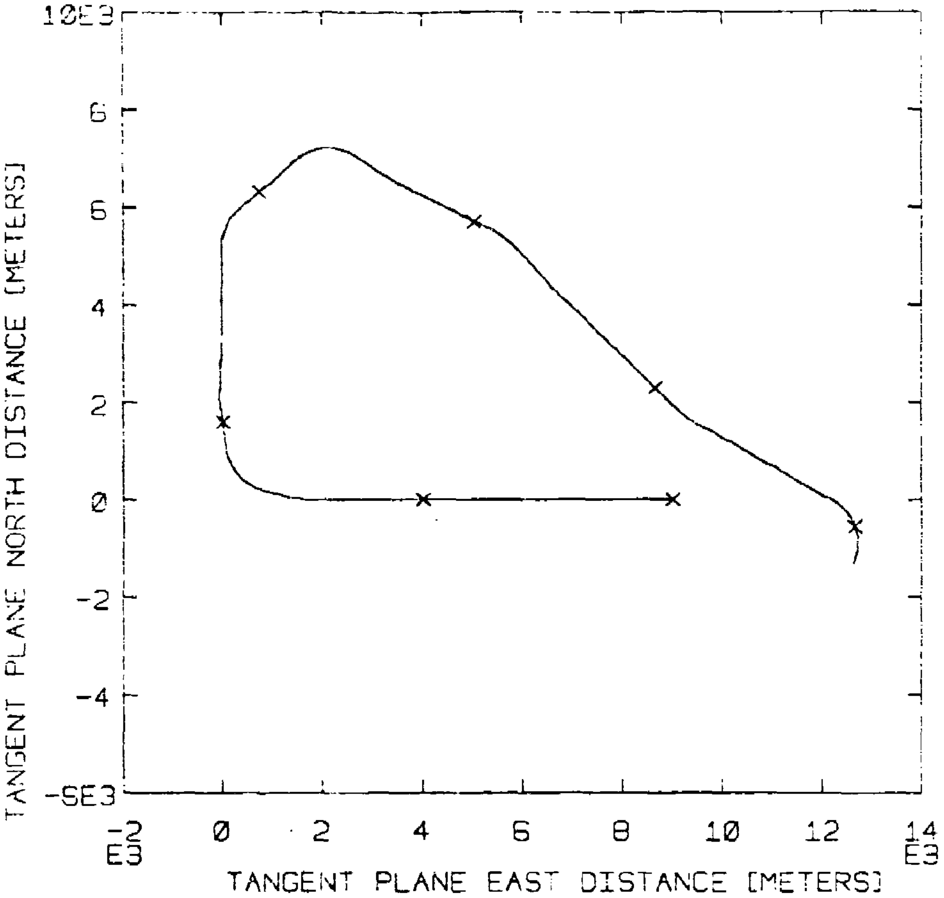


Fig. 7.3 Two Member Tear Drop Trajectory

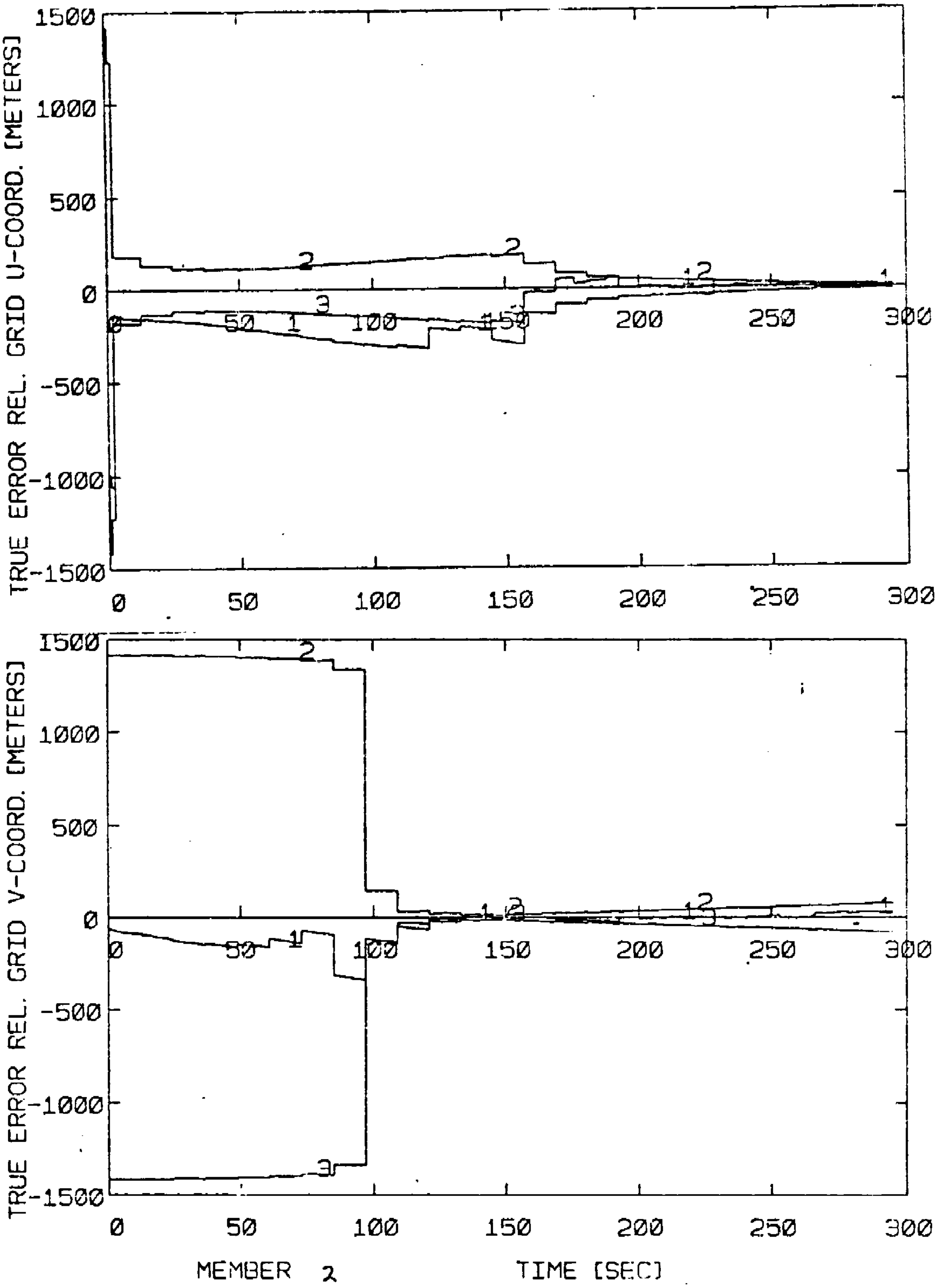


Fig. 7.4(a) Tear Drop Trajectory, Relnav Results

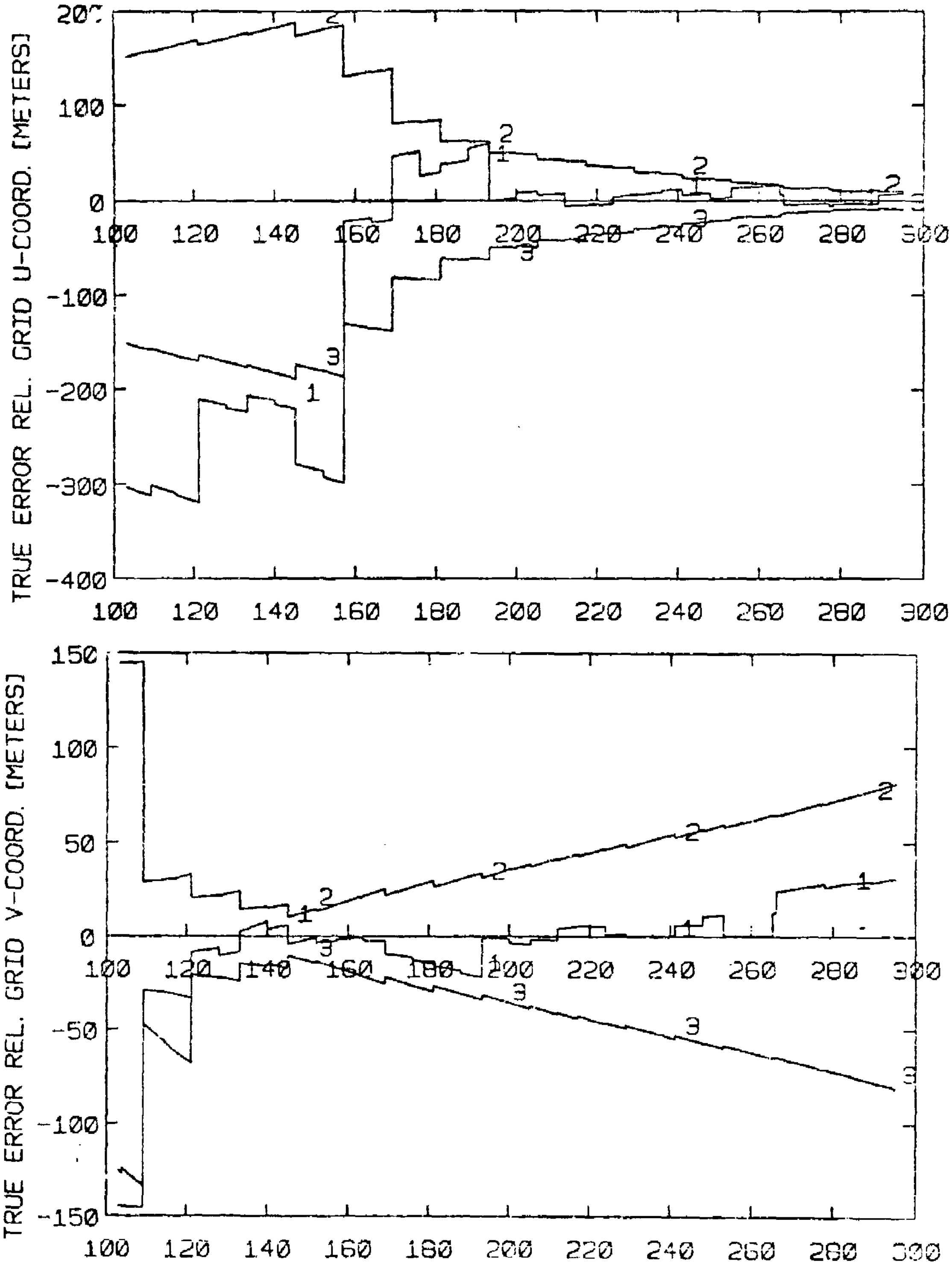


Fig. 7.4(b) Tear Drop Trajectory, Relnav Results, Expanded Scale

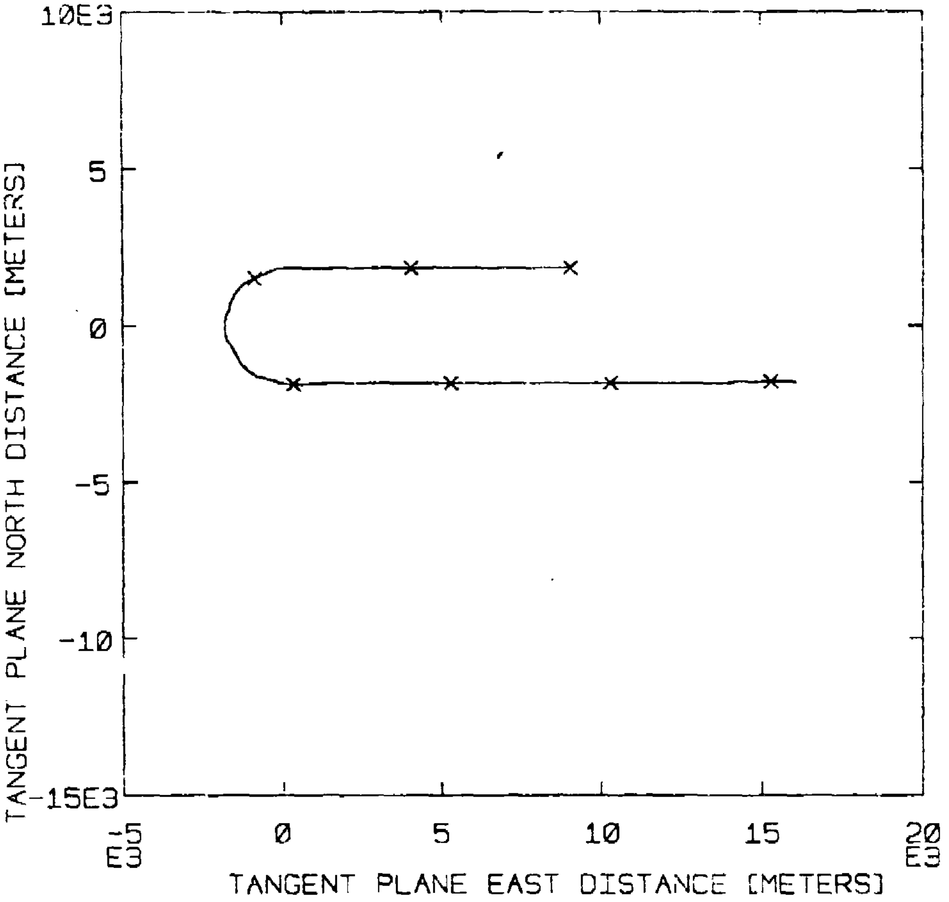


Fig. 7.5 Two Member Fly Around Trajectory

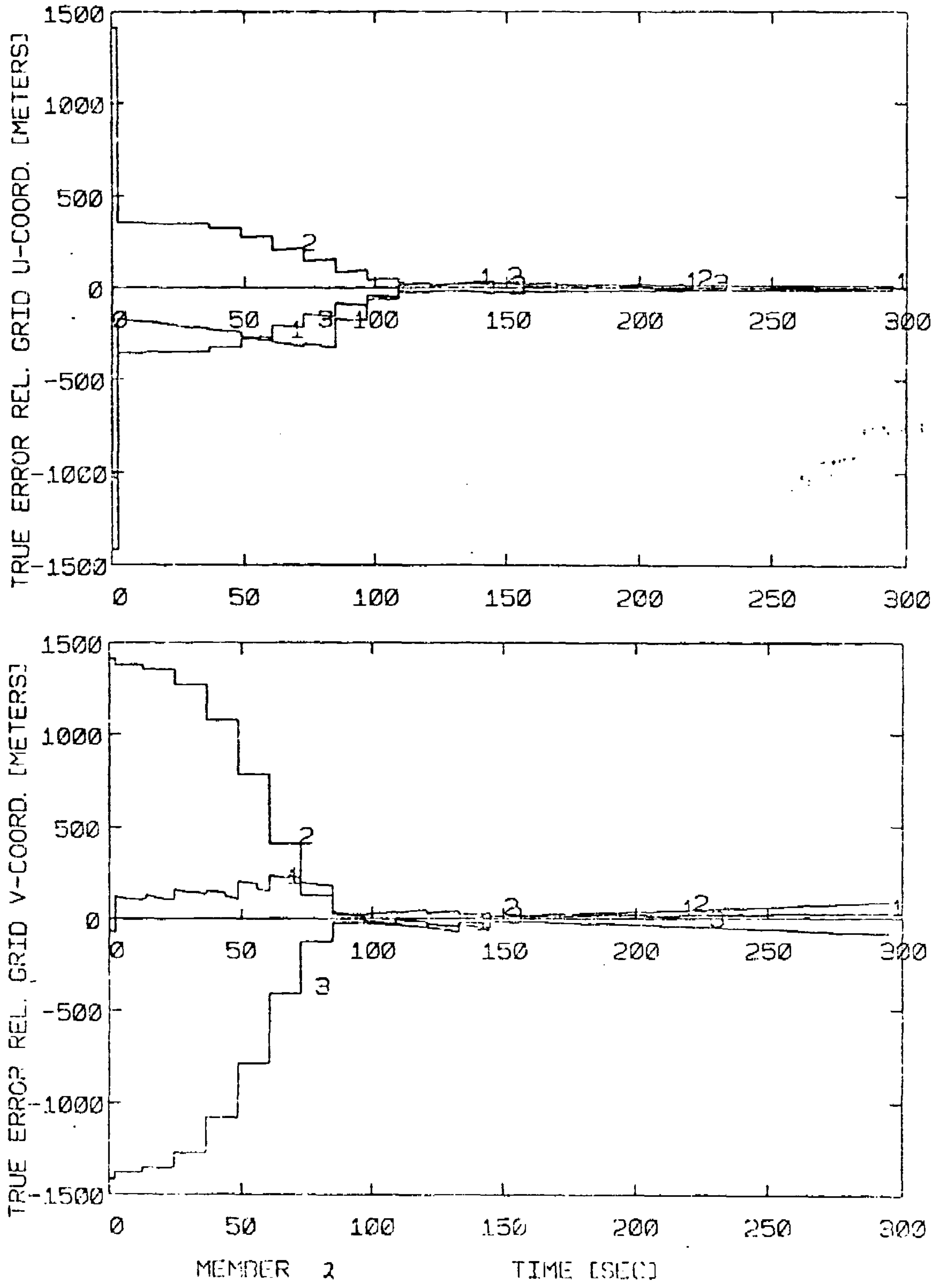


Fig. 7.6(a) Fly Around Trajectory, Relnav Results

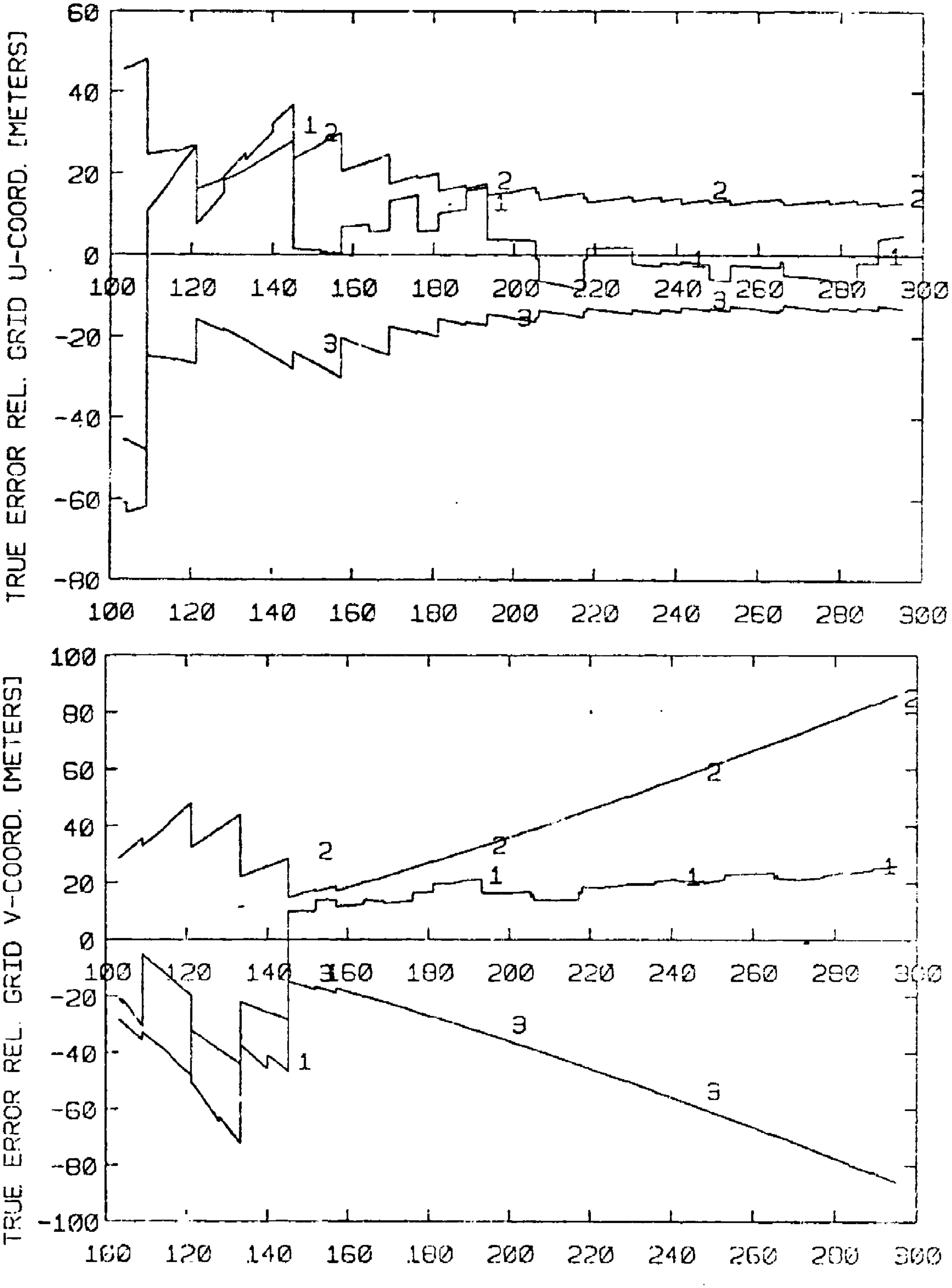


Fig. 7.6(b) Fly Around Trajectory, Relnav Results, Expanded Scale

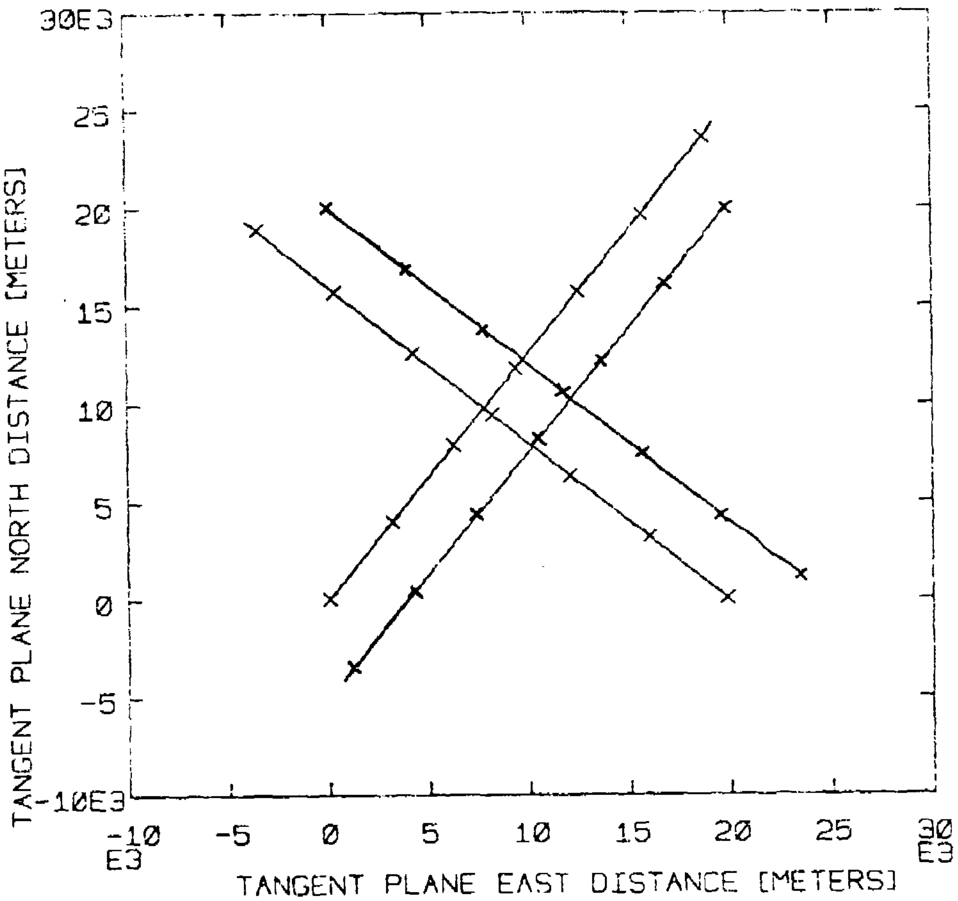


Fig. 7.7 Four Member Crossing Trajectory

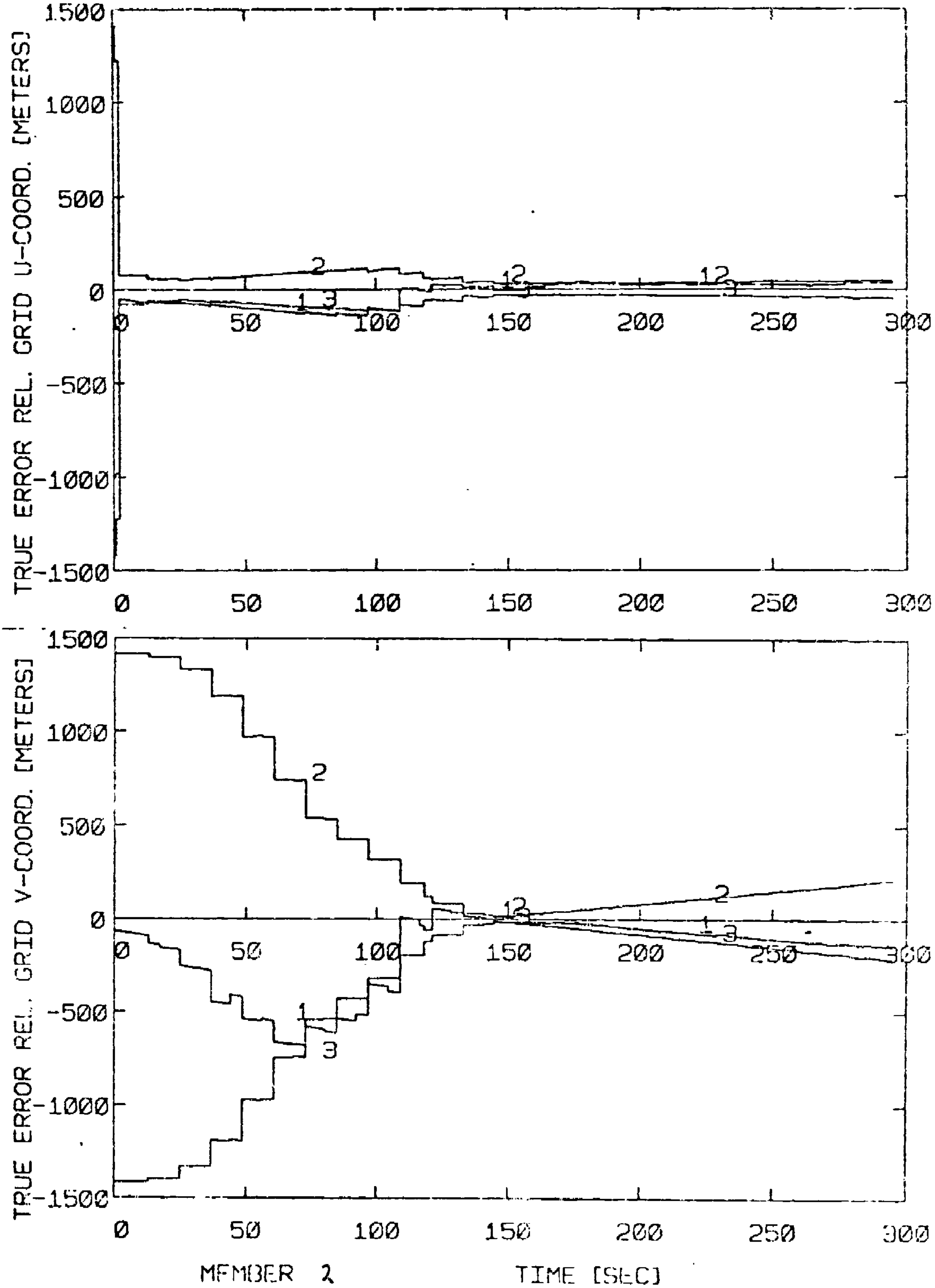


Fig. 7.8(a) Crossing Trajectory, Relnav Results, Member 2

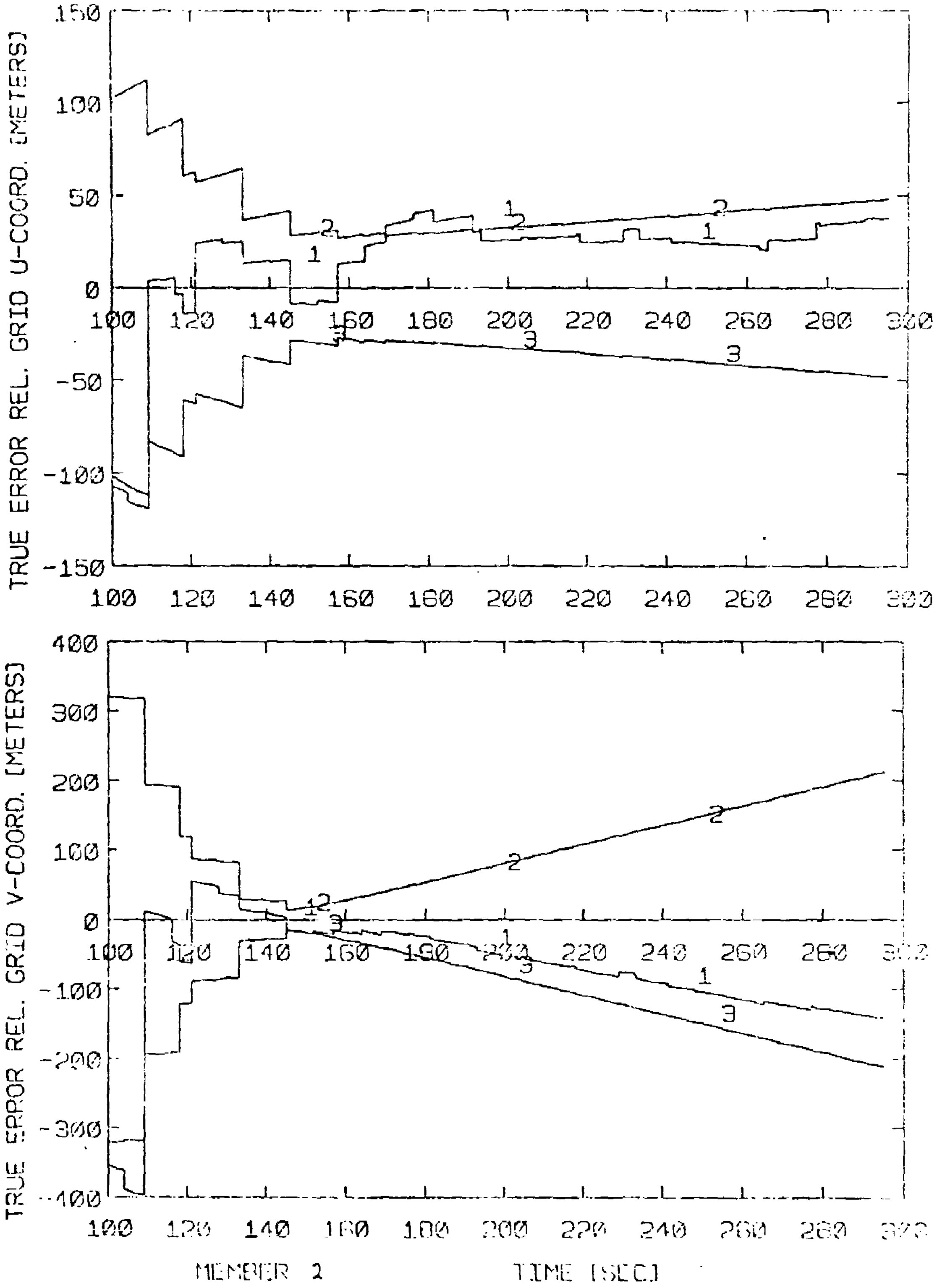


Fig. 7.8(b) Crossing Trajectory, Relnav Results, Member 2, Expanded Scale

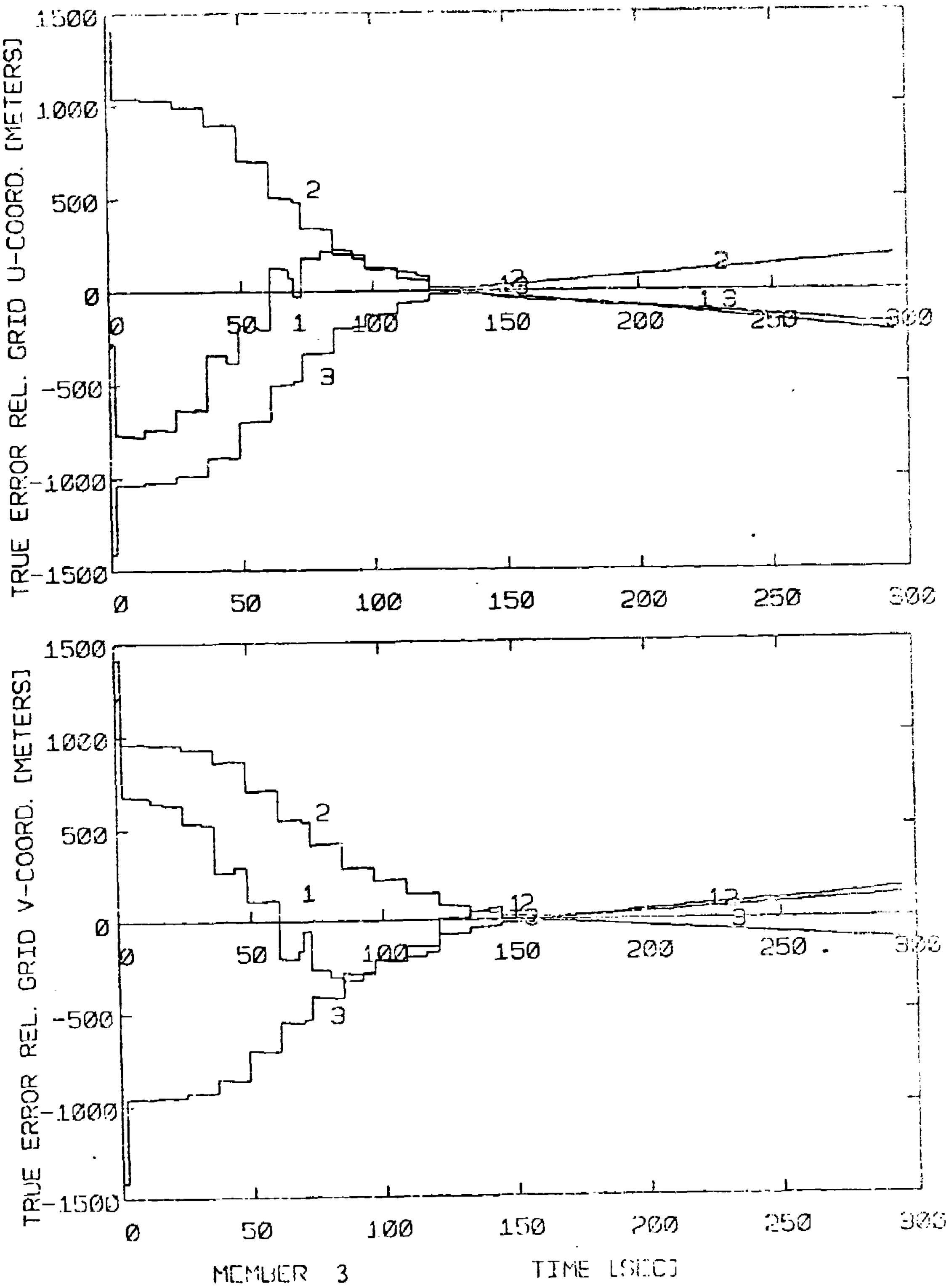


Fig. 7.9(a) Crossing Trajectory, Relnav Results, Member 3

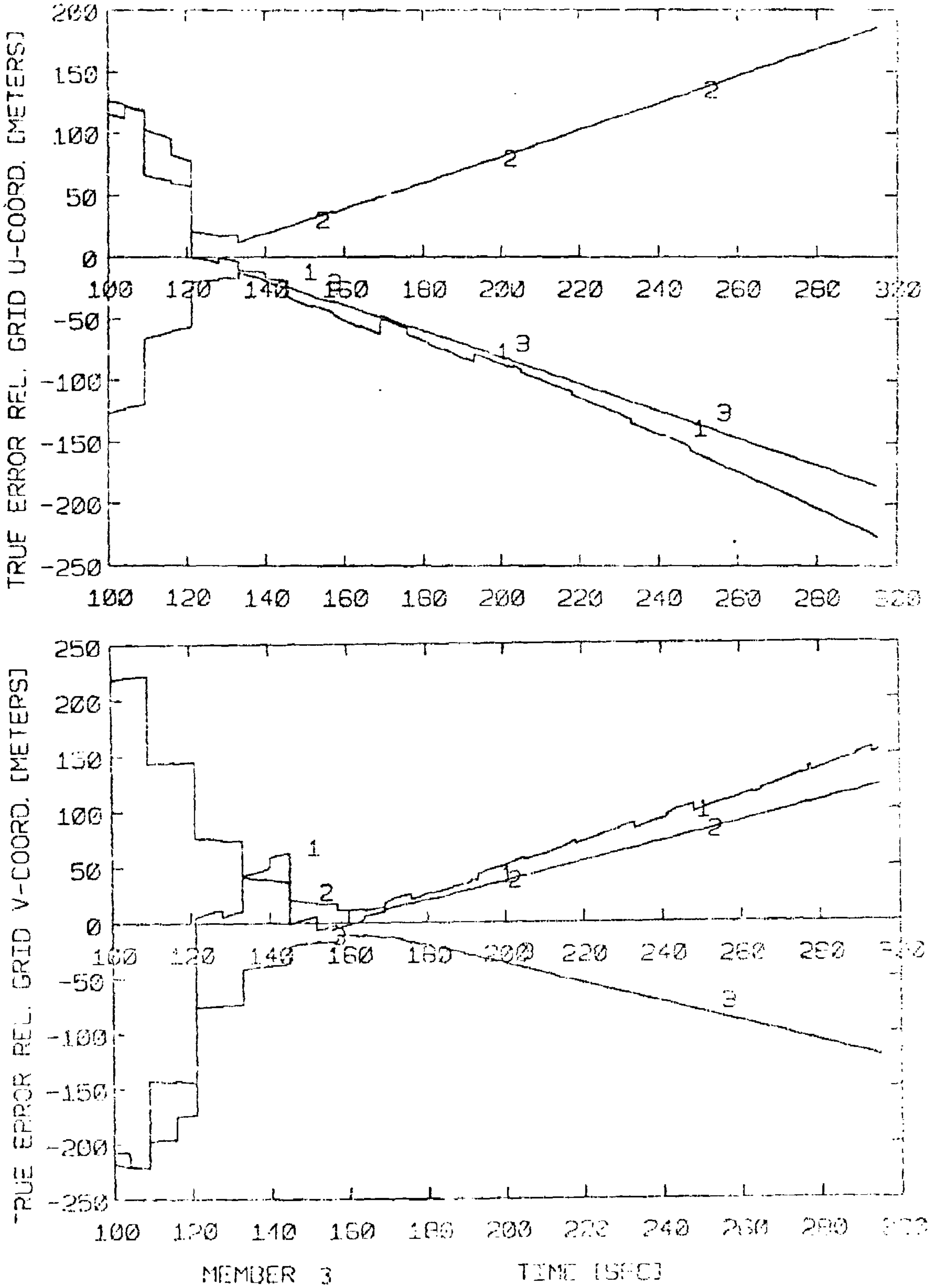


Fig. 7.9(b) Crossing Trajectory, Relnav Results, Member 3, Expanded Scale

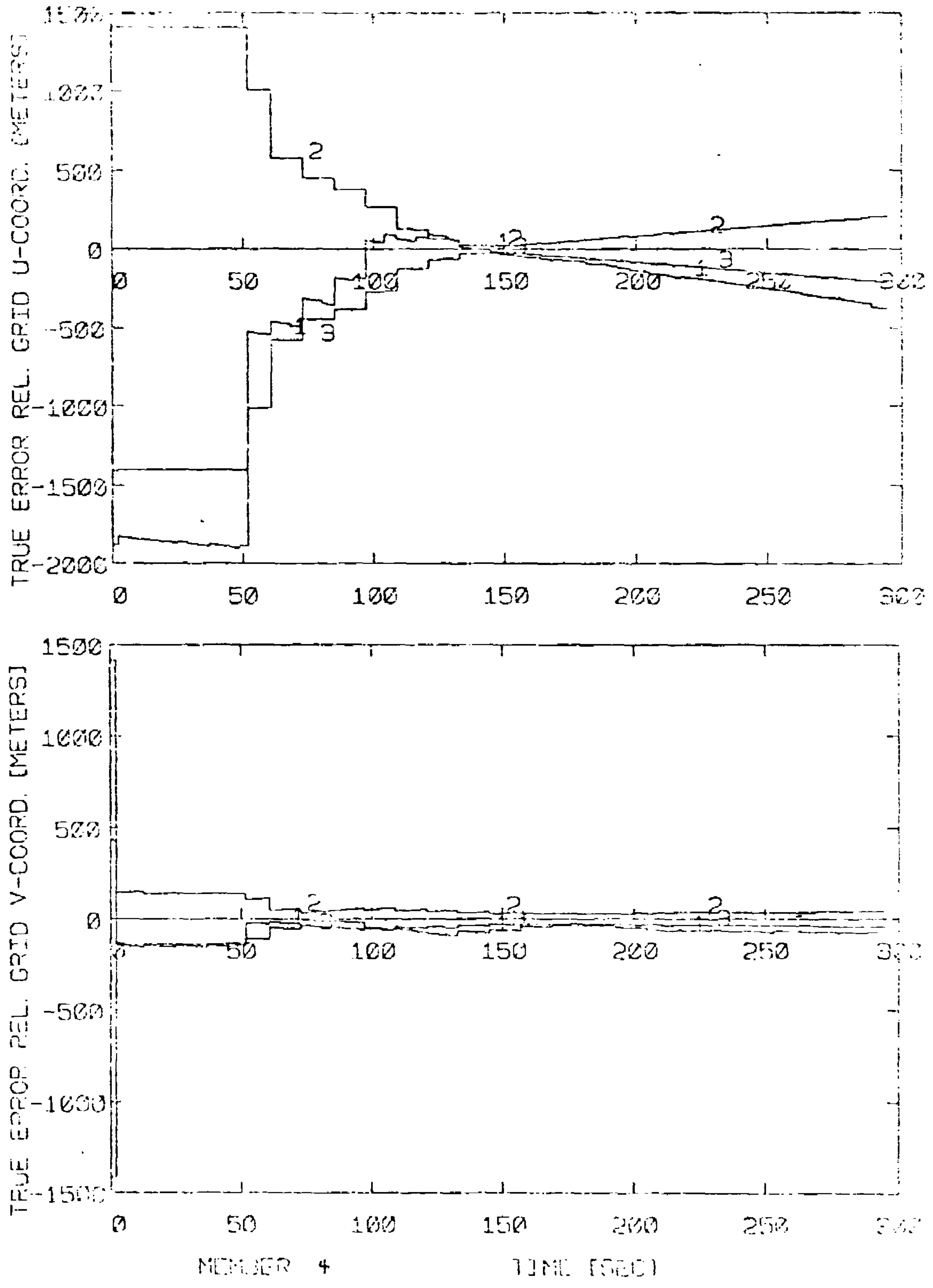


Fig. 7.10(a) Crossing Trajectory, Relnav Results, Member 4

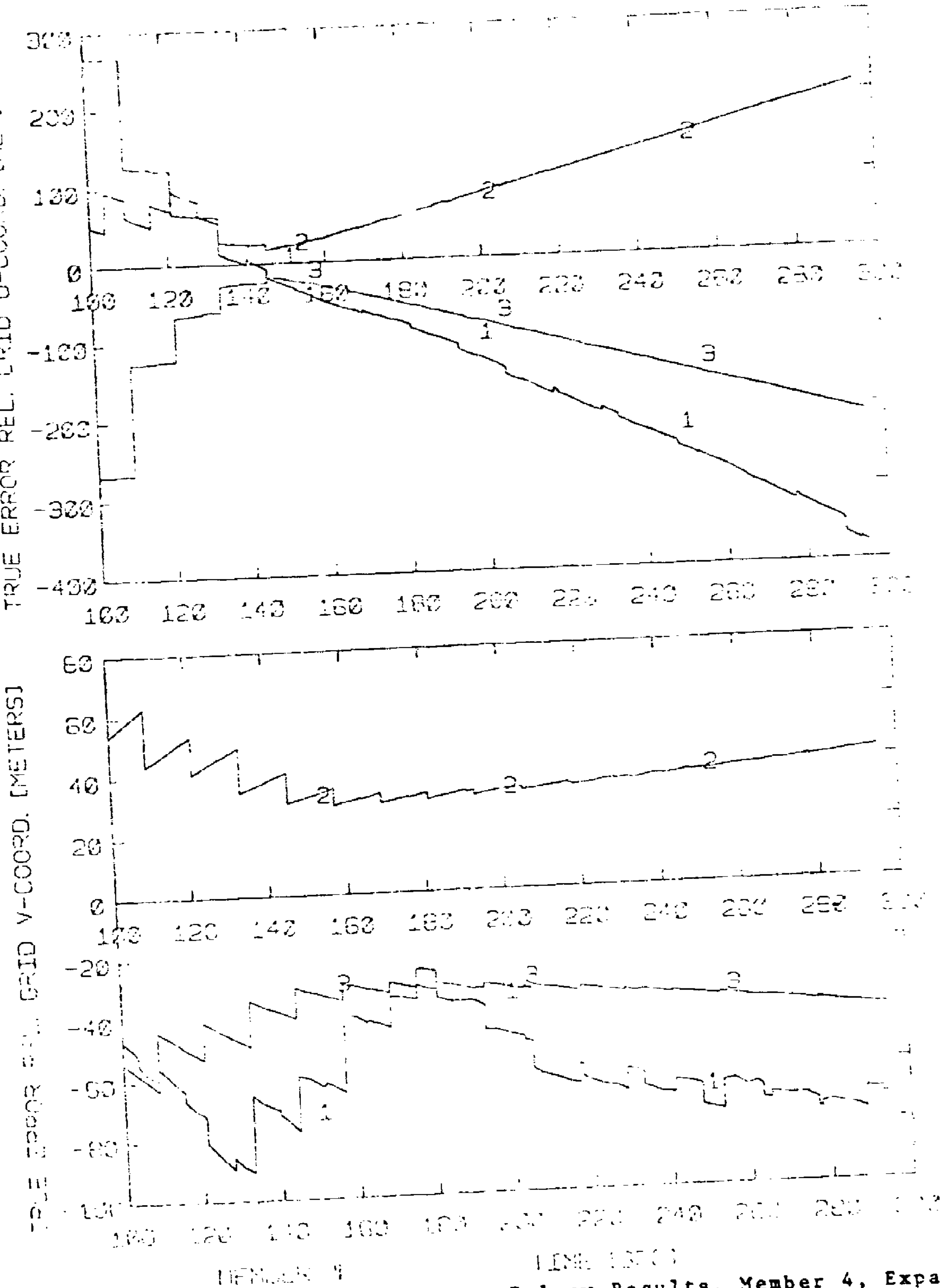


Fig. 7.10(b) Crossing Trajectory, Relnav Results, Member 4, Expanded Scale

200 m northerly uncertainty at the end, which is growing at 1.4 m/sec. Apparently the rapid pass behind the navigation controller did not last long enought to reduce the initial northerly relative velocity error. Member 3 has easterly and northerly errors both growing rapidly at about the same rate. Member 4 errors are similar to those of member 2, but with the roles of U and V reversed.

7.3 Round Trip Timing Effect on Performance

The two member fly around trajectory of Fig. 7.5 has been repeated but with round trip timing not allowed. The perfmance results, shown in Fig. 7.11, may be compared with the performance with round trip timing, which were shown in Fig. 7.6. Without RTT, on the inbound leg the member is not able to resolve its distance/time ambiguity. Not until the line of sight starts rotating noticeably does the easterly error come down. After flying around the navigation controller both components of error have been reduced to the order of 100 m. On the outbound leg these errors grow to about 200 m. This is significantly worse than with RTT. With RTT, at the end tne along range error was about 15 m and the cross range error was about 80 m

7.4 Democratic Organization Performance

In Chapter 3, using a simple two dimensional simulation with very few state variables in either the truth model or the filter models, we demonstrated that the democratic organization without round trip timing can be unstable. Recall the democratic

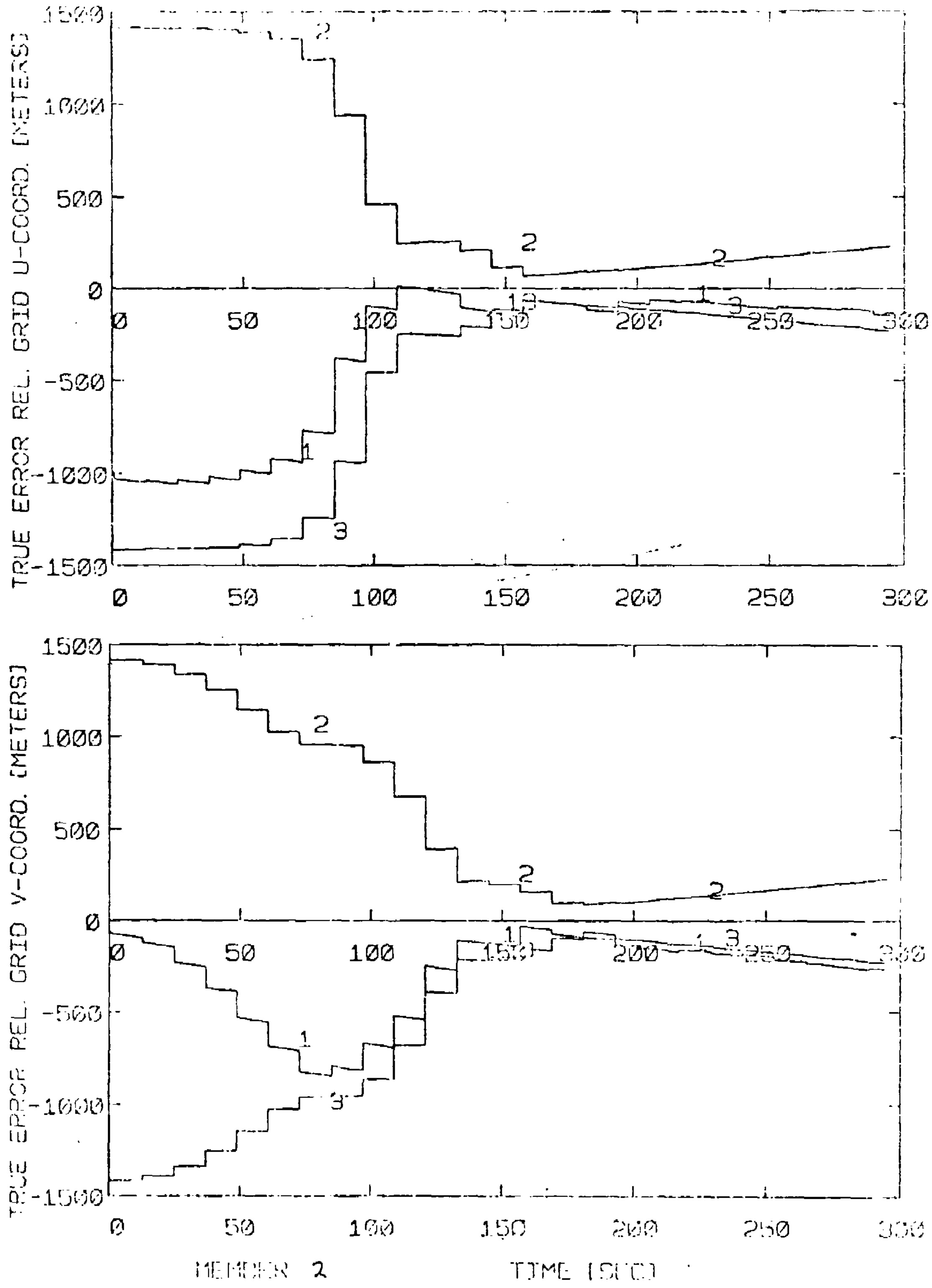


Fig. 7.11 Two Member Fly Around Trajectory, Relnav Results Without Round Trip Timing

organization is one where all time of arrival measurements are incorporated by the filter regardless of the reported accuracy of the source. It is of interest to demonstrate the instability of the democratic organization using our high fidelity simulator.

The trajectories of a four member community are shown in Fig. 7.12. The four members are flying north in formation maintaining constant separation at the corners of a 20 km square. The JTIDS source selection logic is disabled so that all time of arrival measurements are accepted as relative position updates. The use of the same measurements as geodetic updates has been inhibited. Also round trip timing has been inhibited. The relative navigation performance results for members 2, 3, and 4 are shown in Figs. 7.13, 7.14, and 7.15. The actual single case estimation errors are wildly unstable with errors as large as 6 or 8 km within 600 sec. The ownstate filter computed uncertainties are totally unaware of this instability. As more measurements are processed, the filters believe the accuracy is getting better and better.

The simulation has been repeated, but with round trip timing enabled. The results are shown in Figs. 7.16, 7.17, and 7.18. RTT seems to stop the oscillatory instability. However with the democratic organization the computed uncertainties again become very optimistic. With no relative motion, there is no way for the community to discover the orientation of the square. Relative position normal to the line of sight to the navigation controller is unobservable (even for a centralized all measurements optimal estimator). This lack of observability is

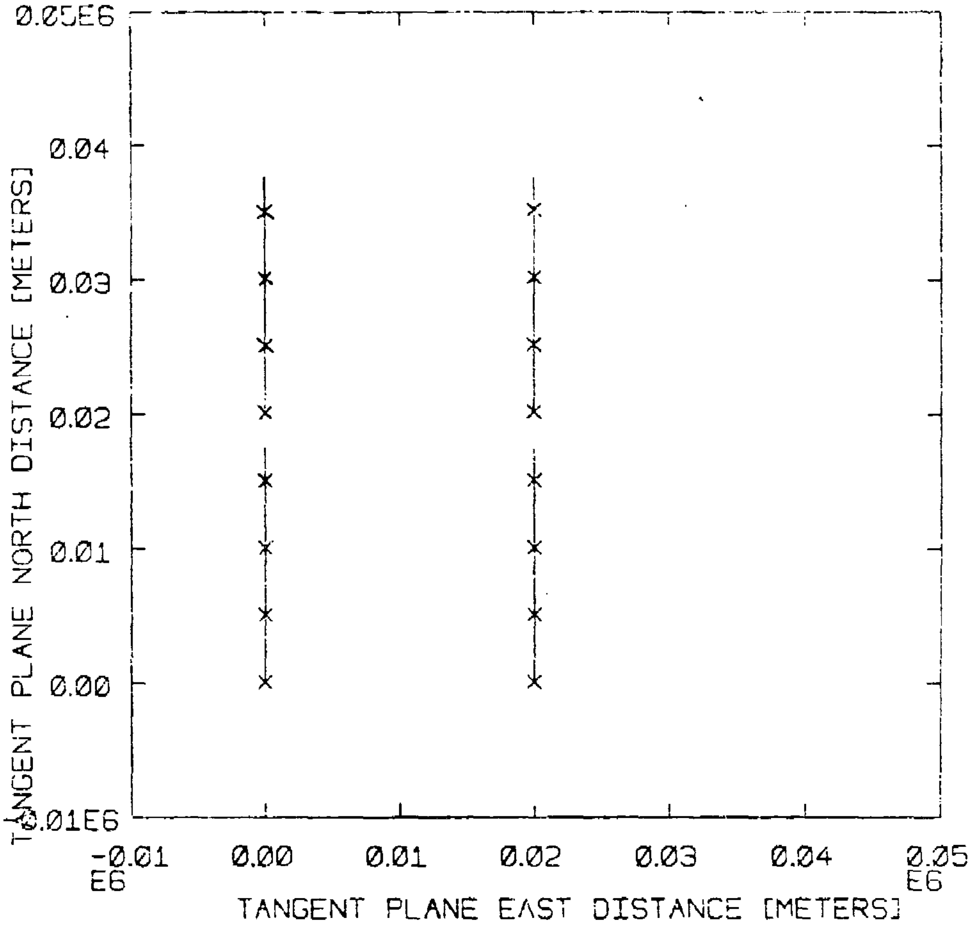


Fig. 7.12 Four Member Square Formation with no Relative Motion

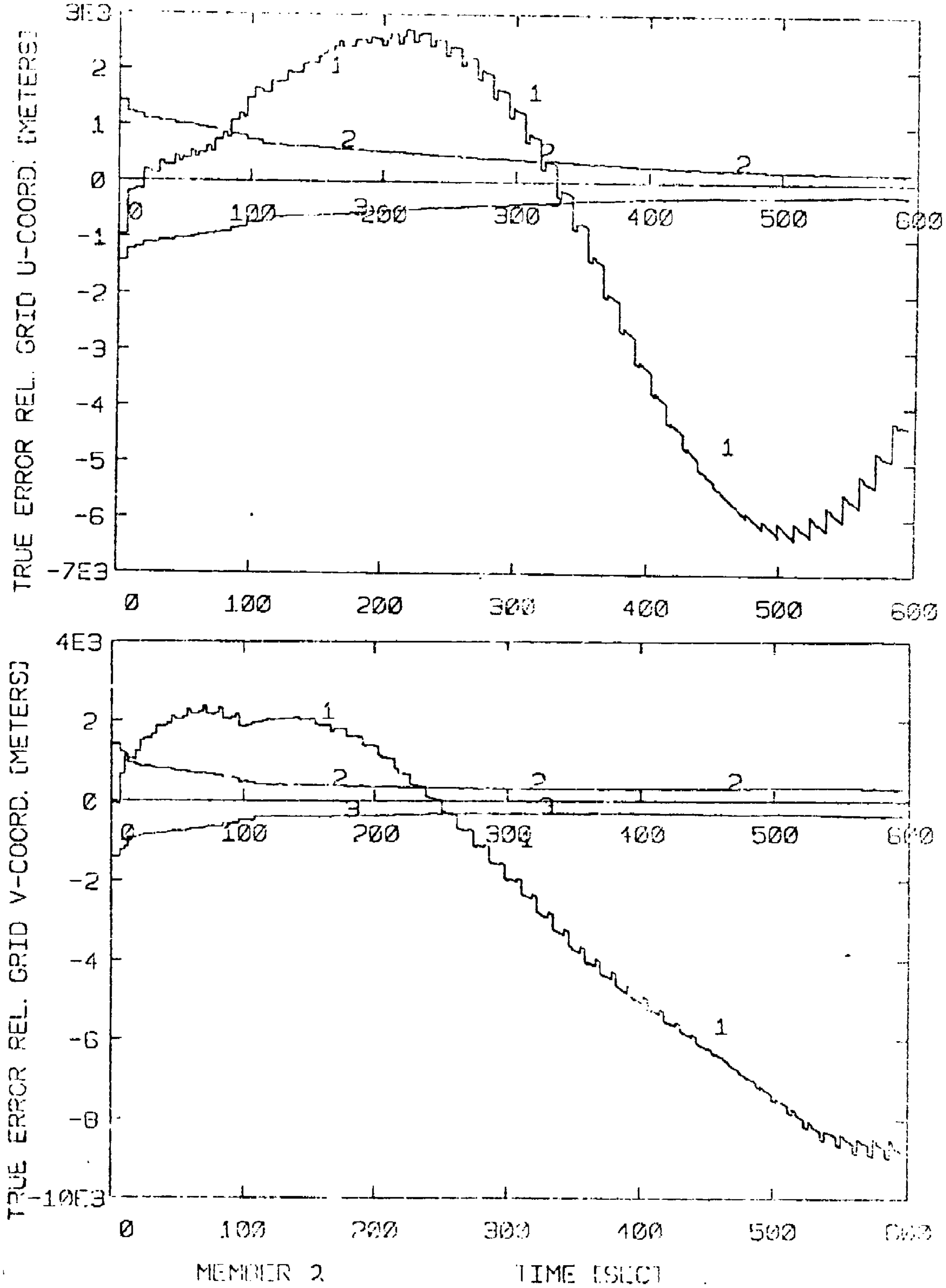


Fig. 7.13 Democratic Organization without RTT, Relnav Results, Member 2

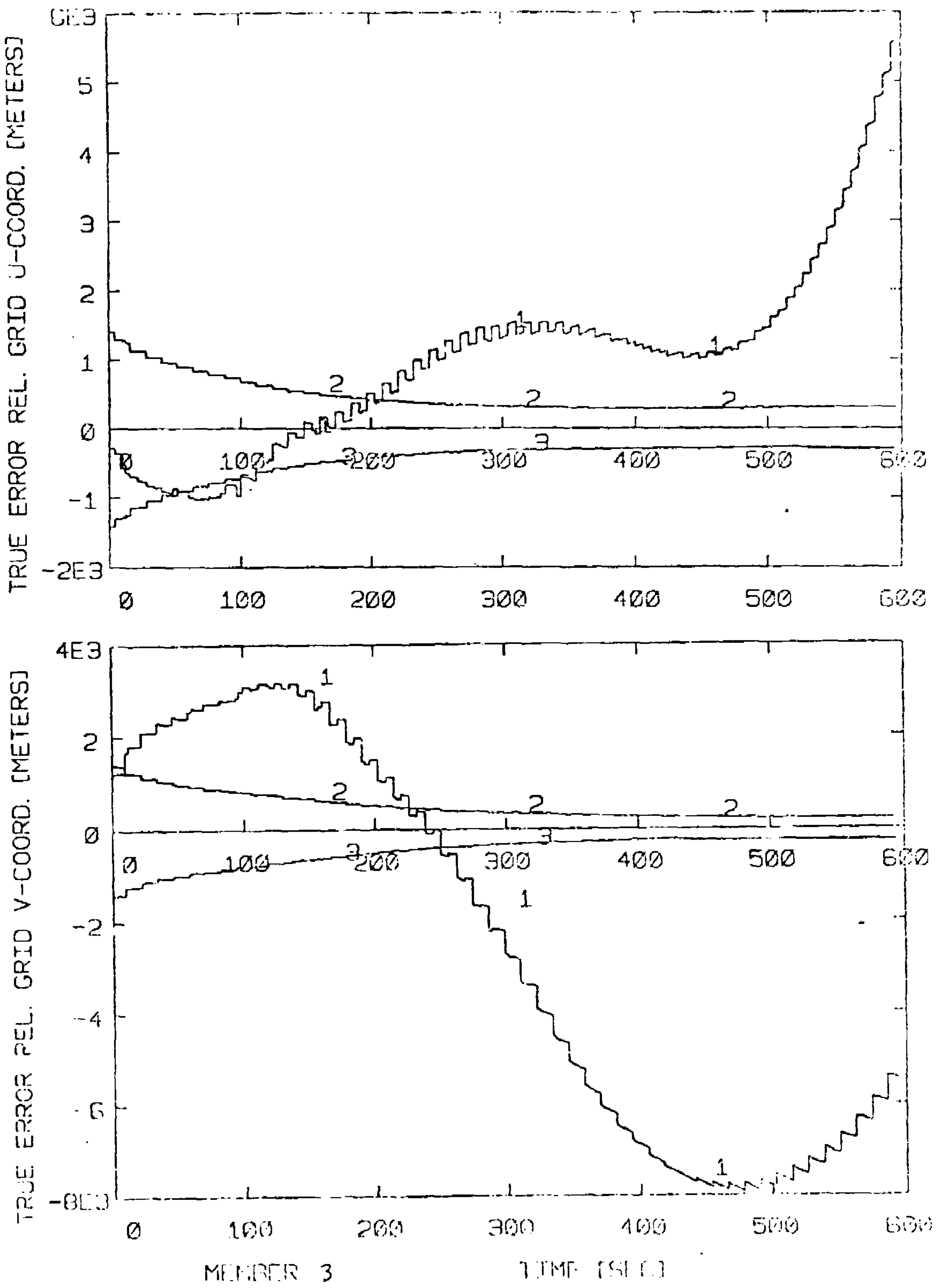


Fig. 7.14 Democratic Organization without RTT, Relnav Results, Member 3

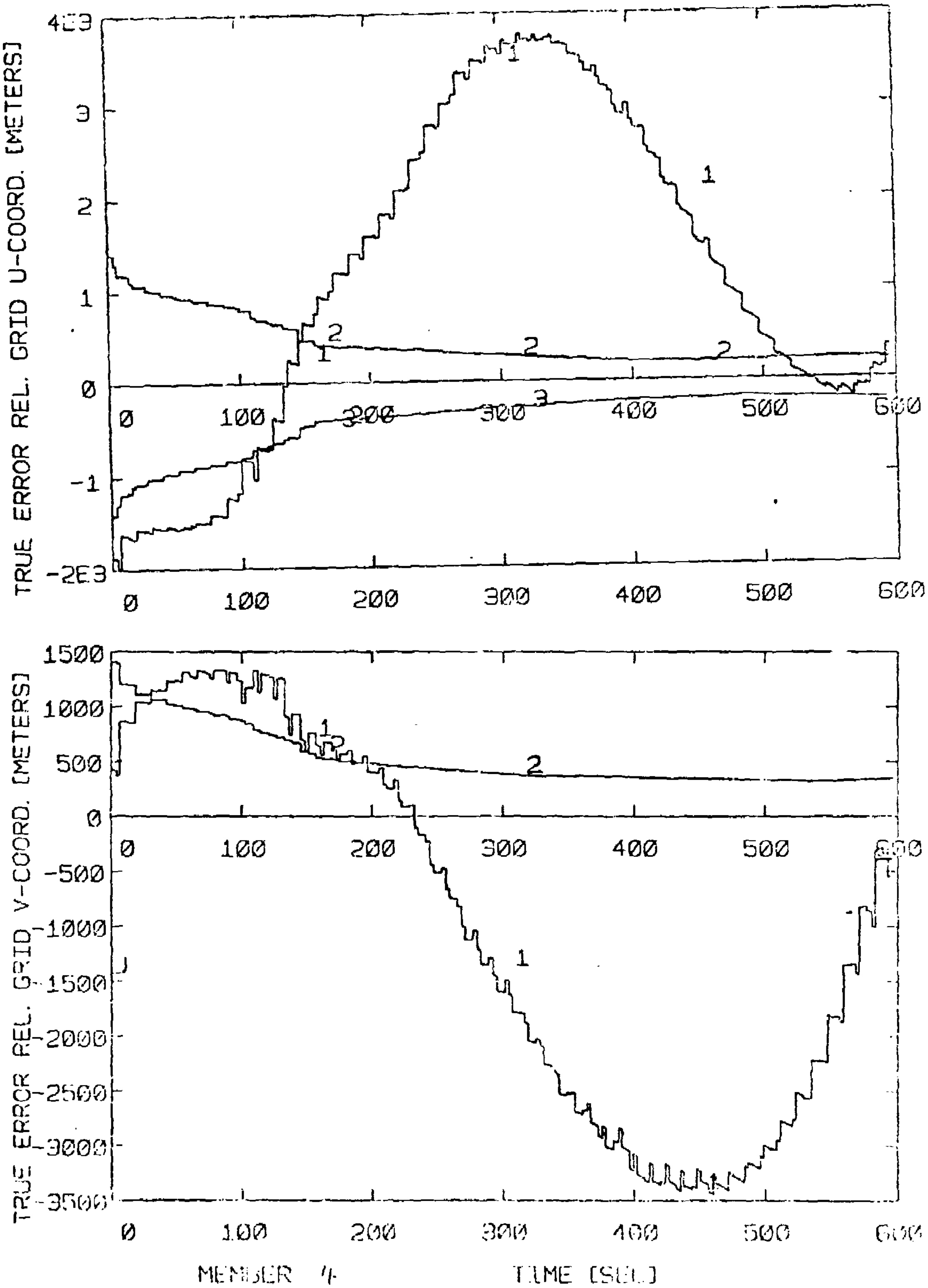


Fig. 7.15 Democratic Organization without RTT, Relnav Results, Member 4

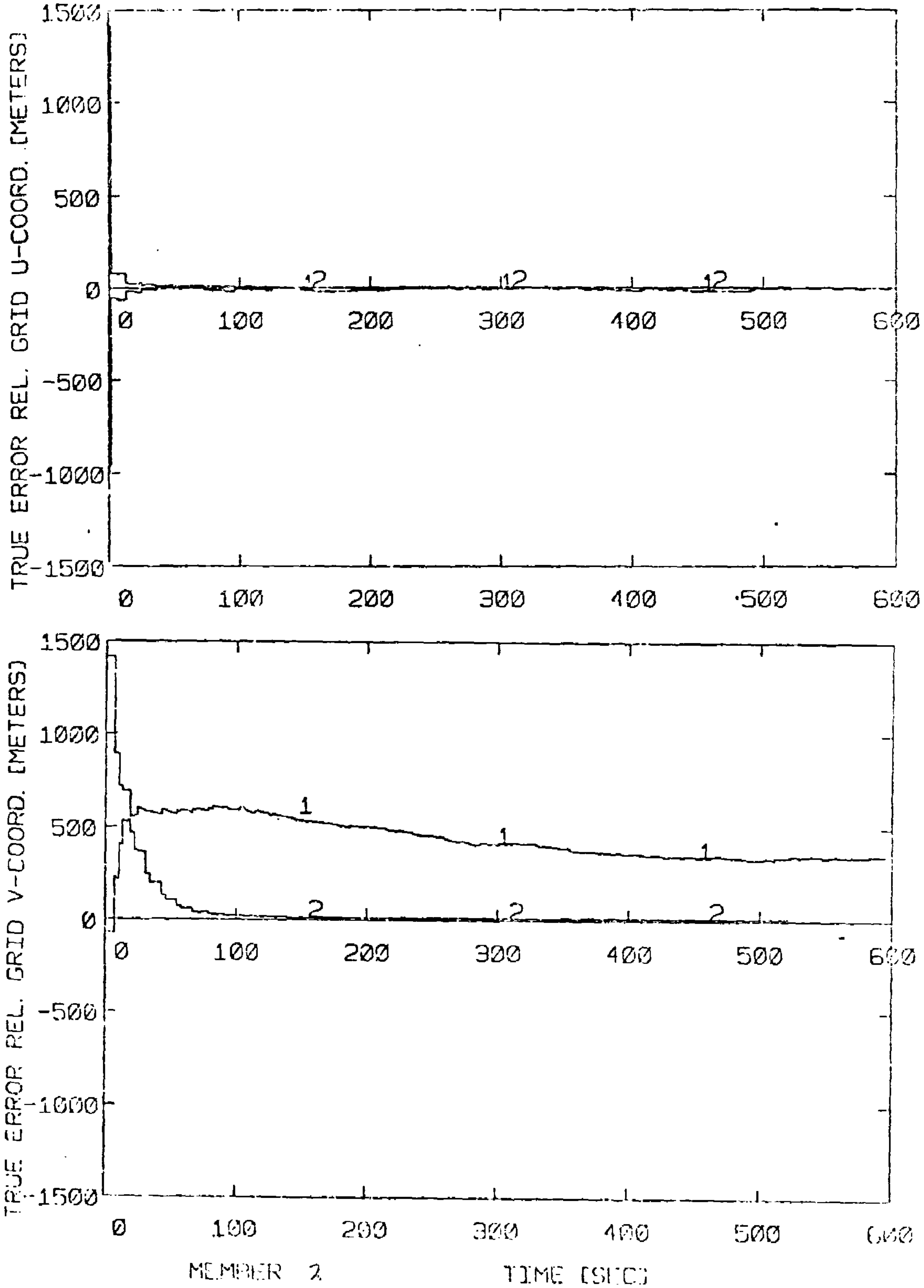


Fig. 7.16(a) Democratic Organization with RTT, Relnav Results, Member 2

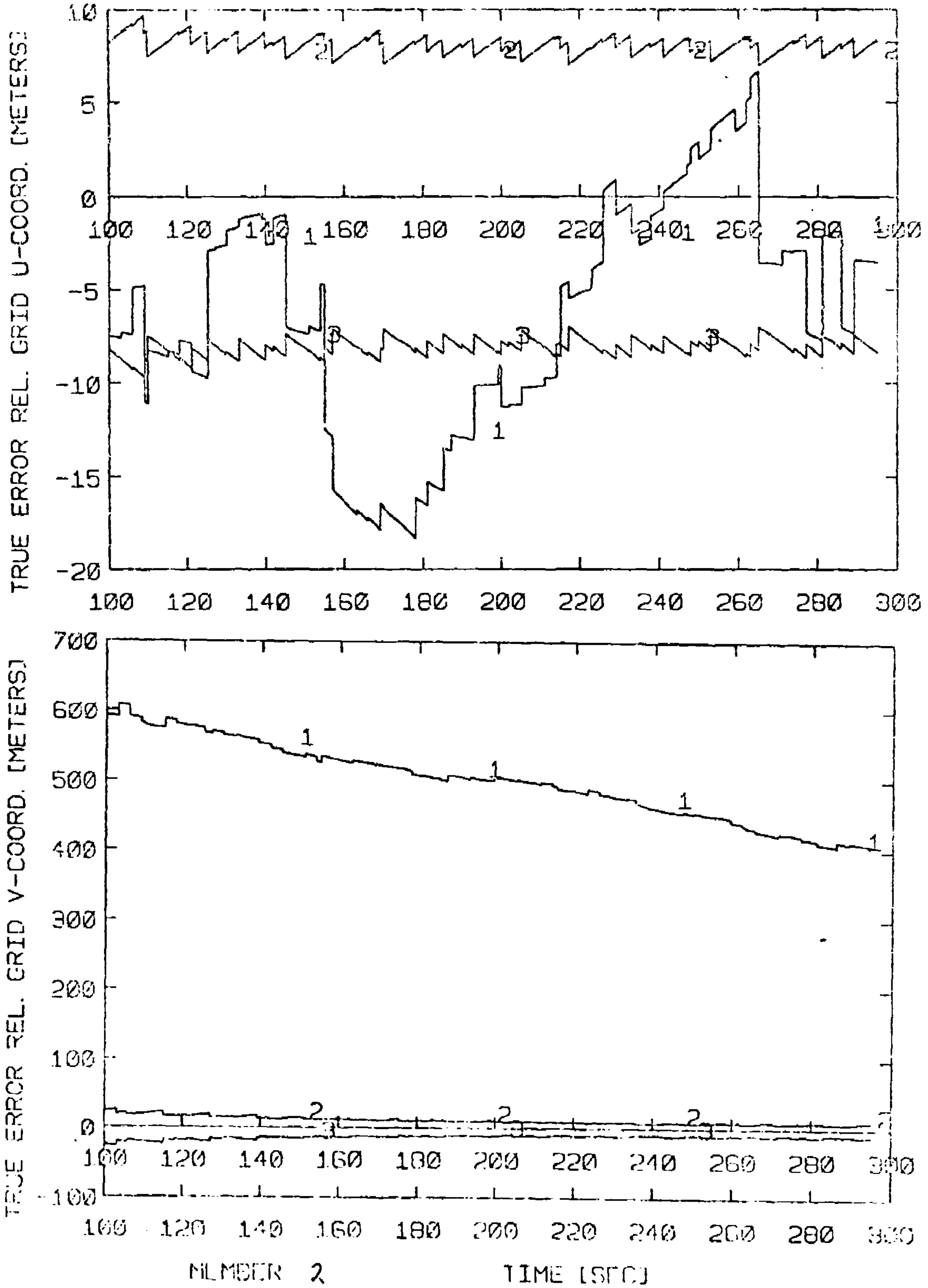


Fig.7.16.(b) Democratic Organization with RTT, Relnav Results, Member 2, Expanded Scale

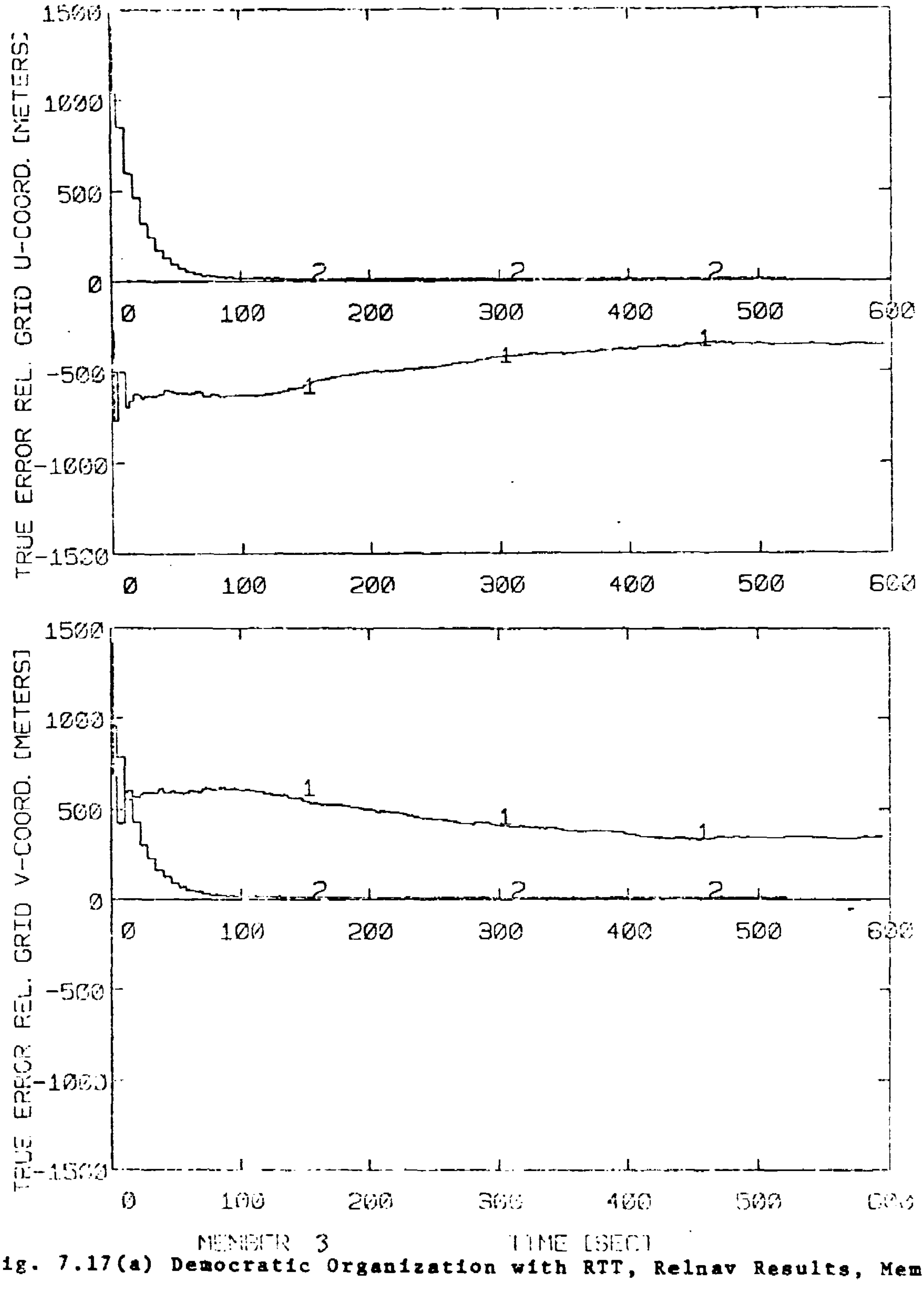


Fig. 7.17(a) Democratic Organization with RTT, Relnav Results, Member 3

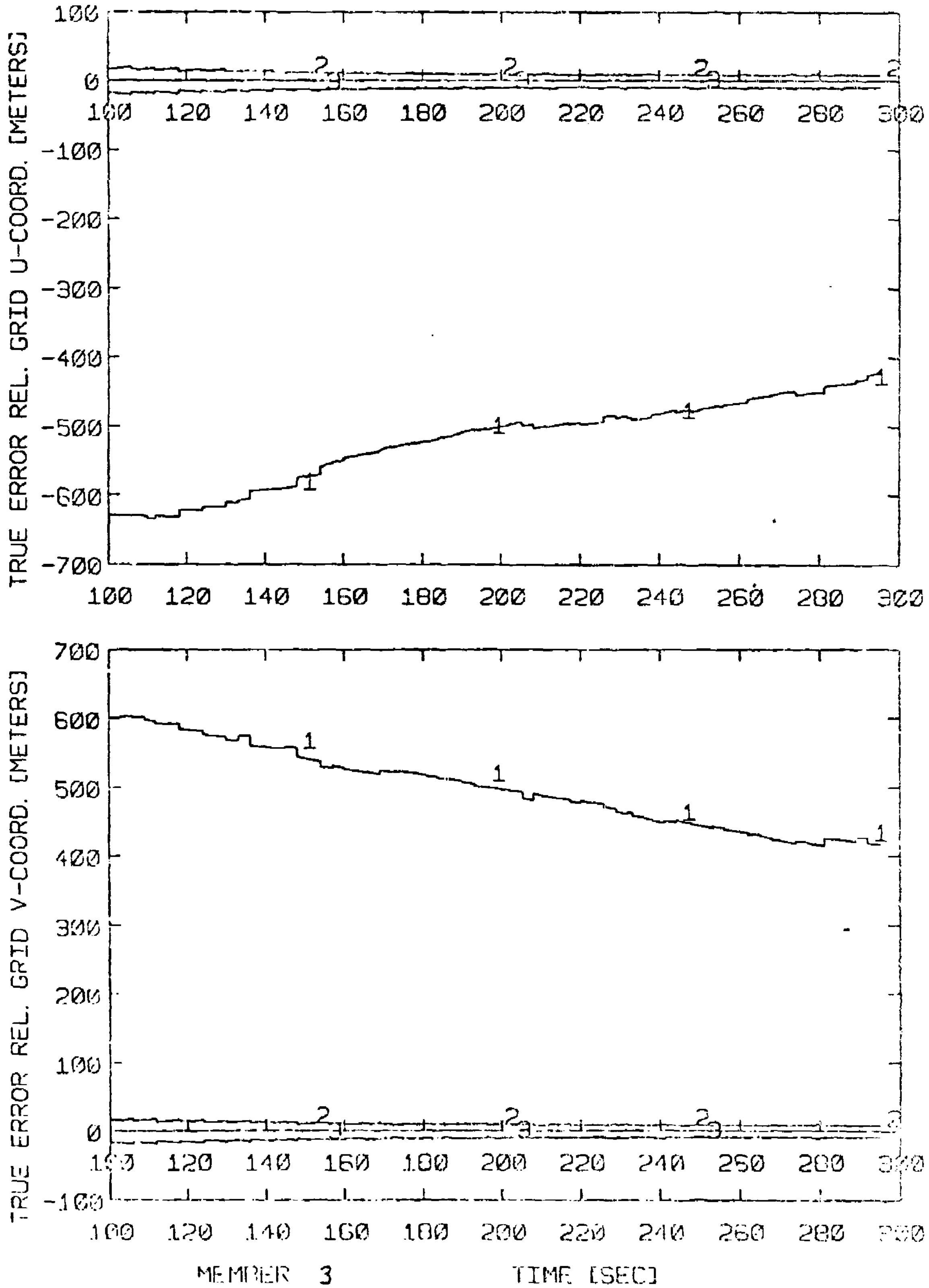


Fig. 7.17(b) Democratic Organization with RTT, Relnav Results, Member 3, Expanded Scale

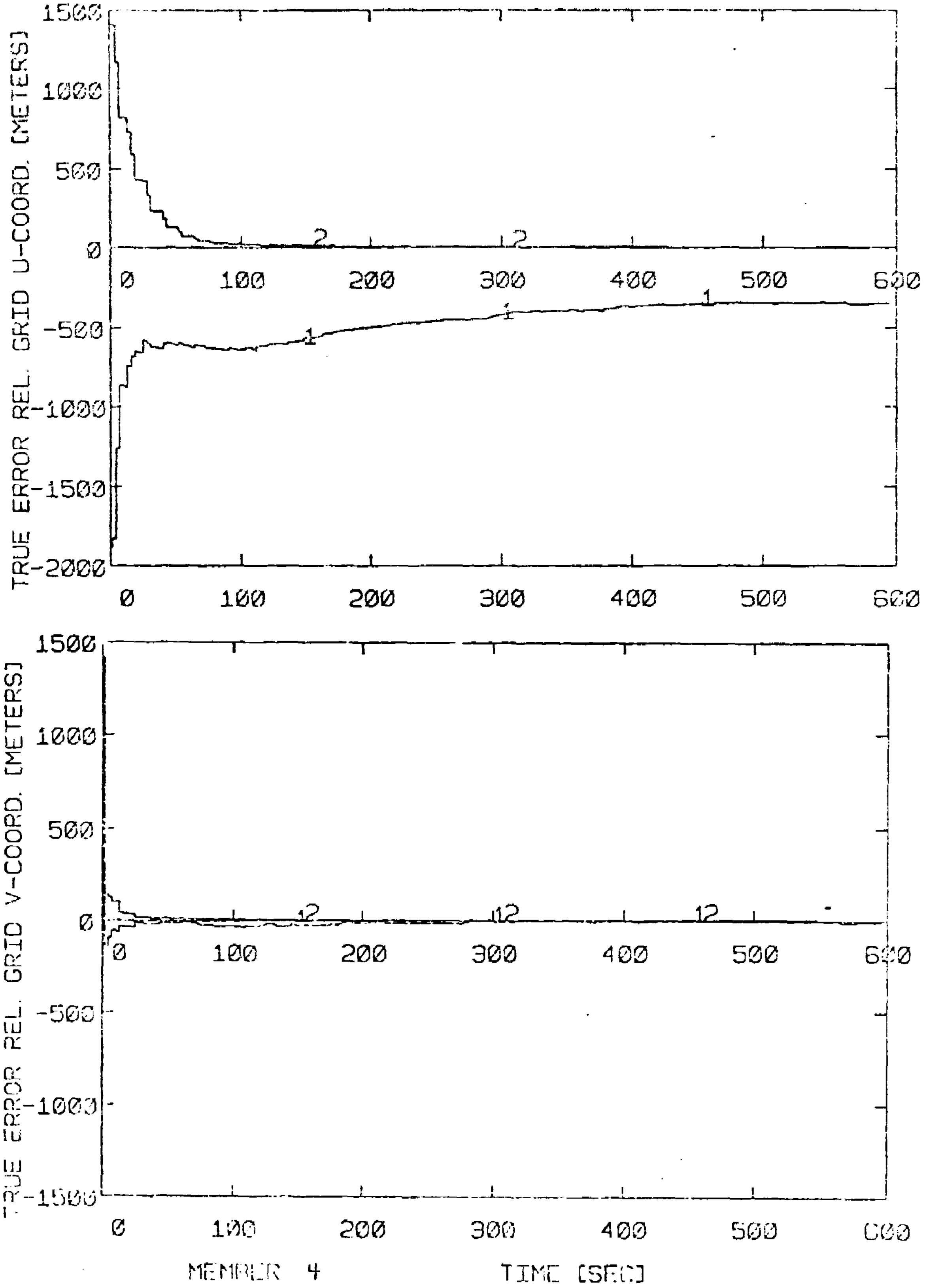


Fig. 7.18(a) Democratic Organization with RTT, Relnav Results, Member 4

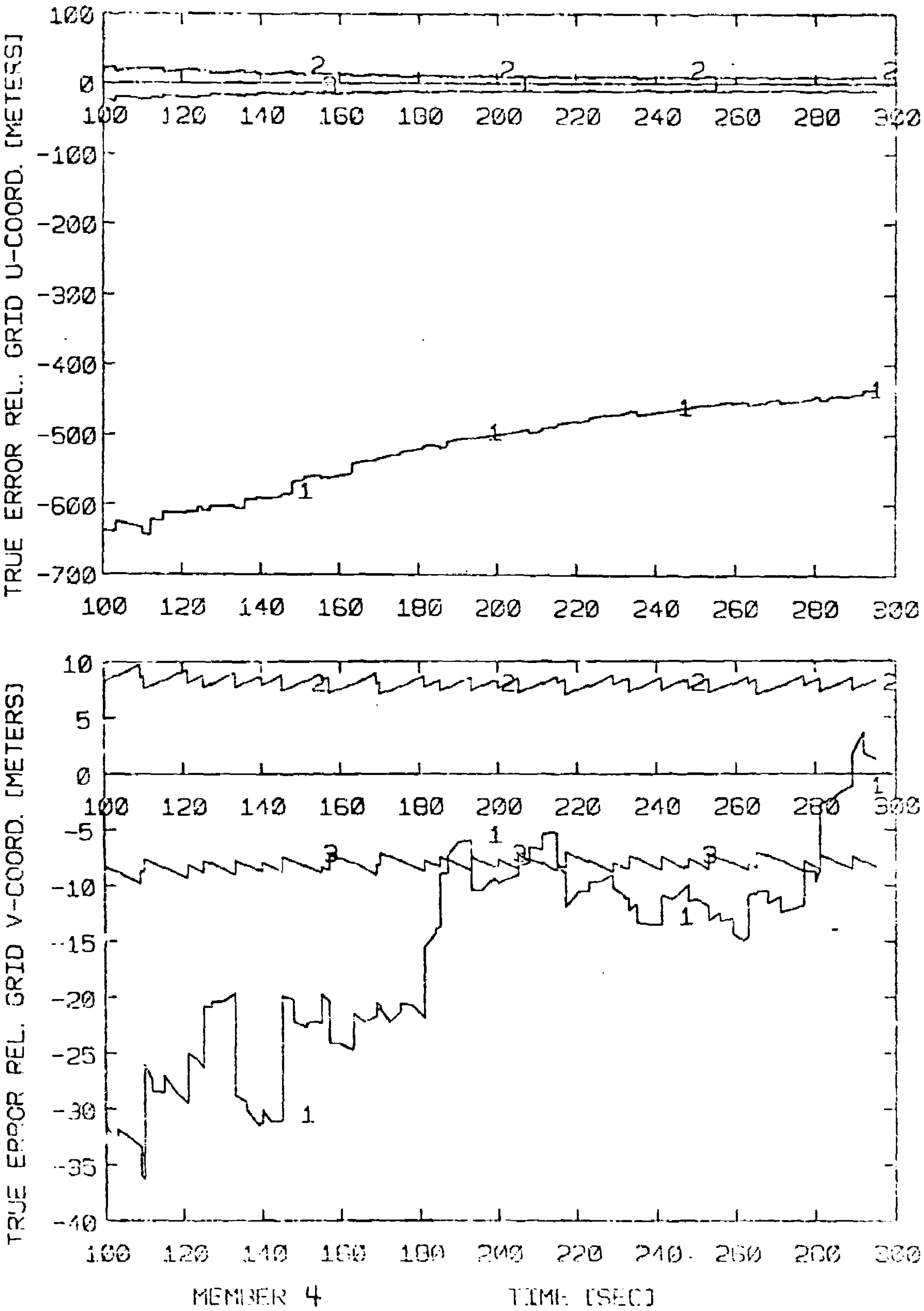


Fig. 7.18(b) Democratic Organization with RTT, Relnav Results, Member 4, Expanded Scale

evident in the single case estimation errors. And yet the ownstate filter computed uncertainties are gradually going to zero.

7.5 Nonlinear Protection Effect on Performance

Our Kalman filter design includes the Gaussian quadratic equations for protecting the filter from the effects of the nonlinear elongation of the measured time of arrival measurements. To show the importance of this design feature, we have run simulations with and without this protection.

A four member scenario was run with trajectories as shown in Fig. 7.19. This is similar to the previous trajectory except that there is some relative motion due to member two not flying in formation with the others. In theory this motion of member 2 provides observability. The relative navigation results with the nonlinear protection are shown in Figs. 7.20, 7.21, and 7.22. The results without the nonlinear protection are shown in Figs. 7.23, 7.24, and 7.25. With the nonlinear protection, the actual estimation errors are somewhat smaller and the filter computed uncertainties are in good agreement with the estimation errors. Without the nonlinear protection, the filters have optimistic computations of the uncertainties and the actual errors are larger. Filter divergence is most noticeable in the case of the easterly error of member 4.

Another pair of runs has been generated using the two member tear drop trajectory, previously shown in Fig. 7.3. The results with the nonlinear protection are shown in Fig. 7.26. The

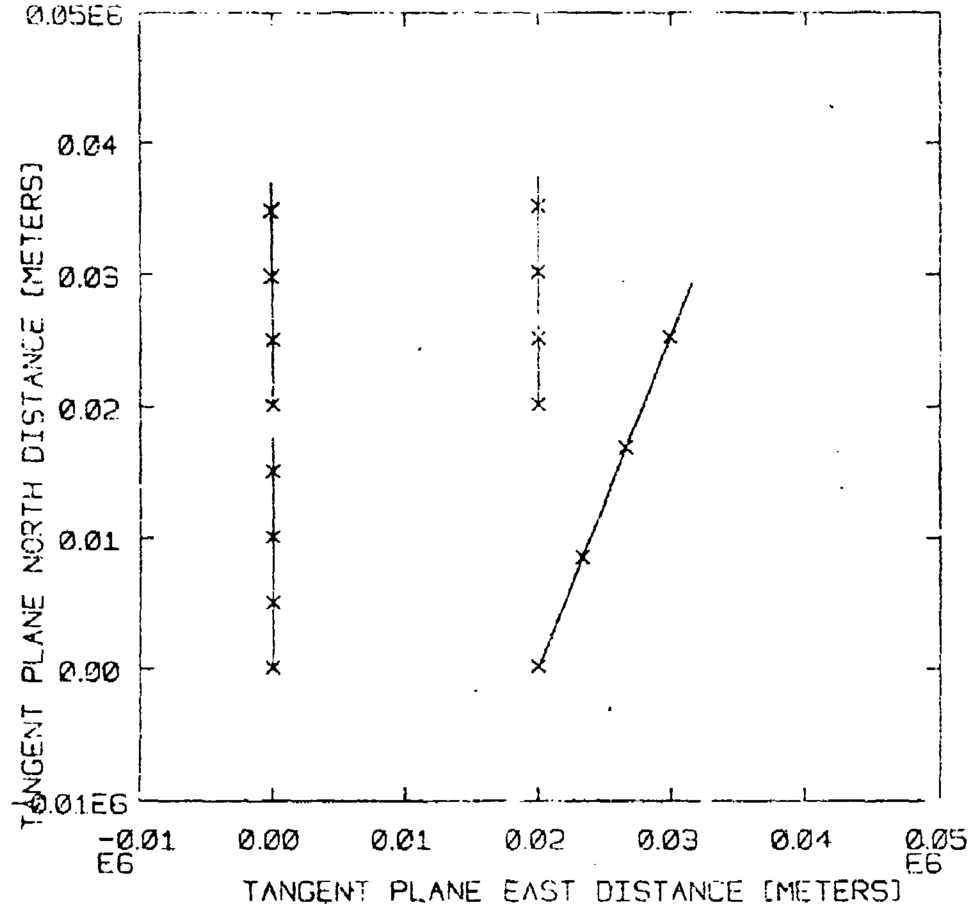


Fig. 7.19 Four Member Formation with Some Relative Motion

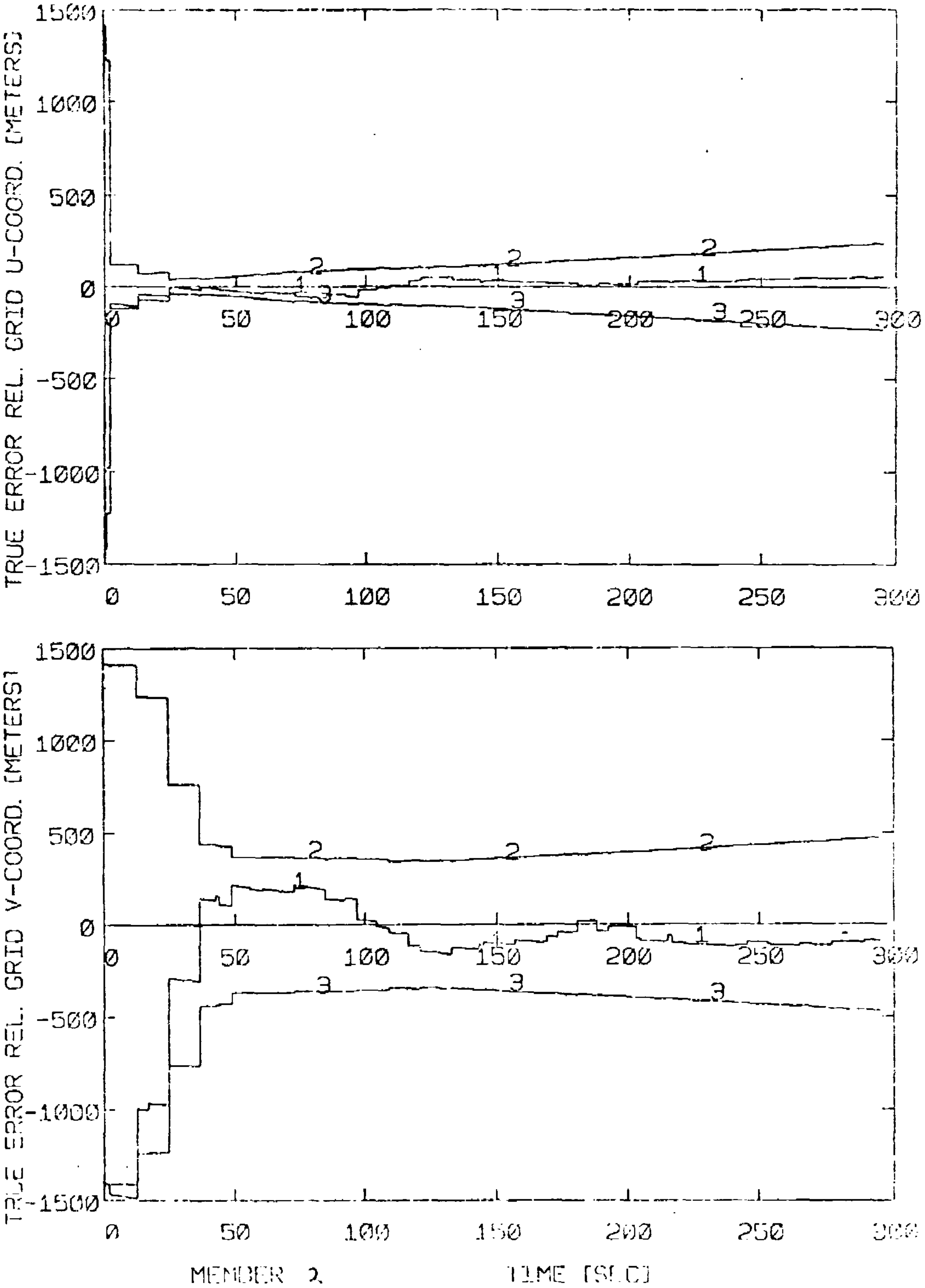


Fig. 7.20 Four Member Relnav Results with Nonlinear Protection, Member 2

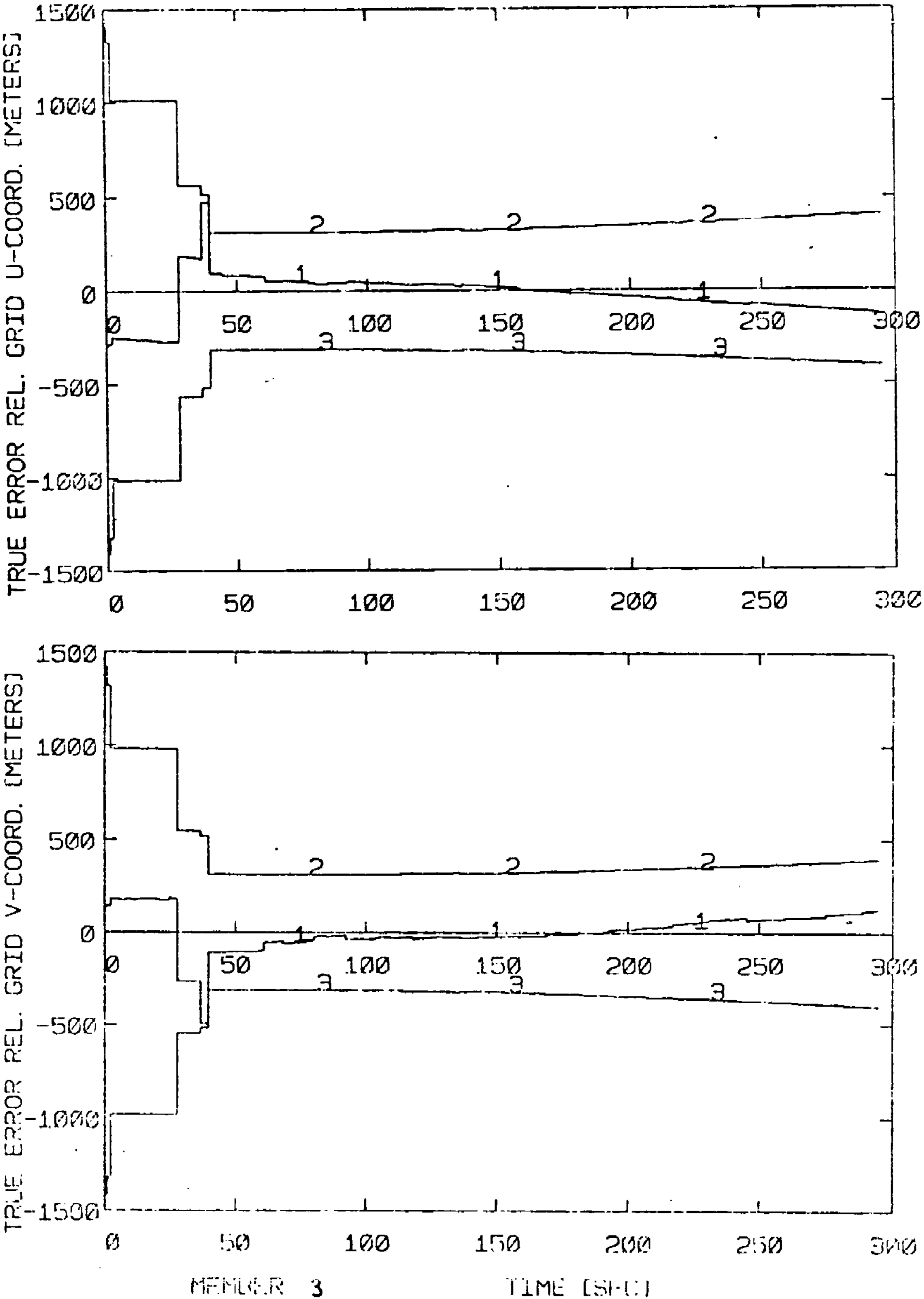


Fig. 7.21 Four Member Relnav Results with Nonlinear Protection, Member 3

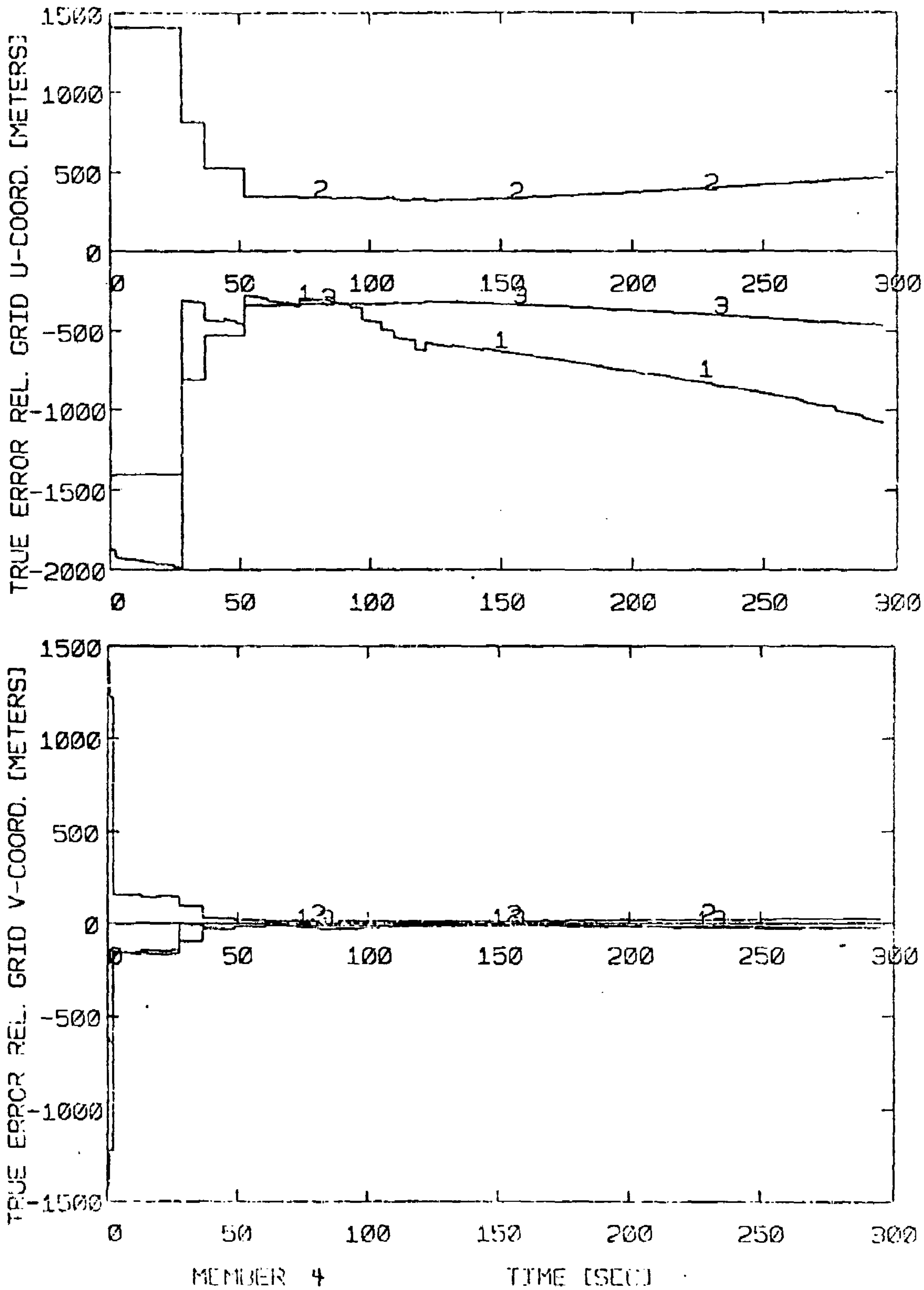


Fig. 7.22 Four Member Relnav Results with Nonlinear Protection, Member 4

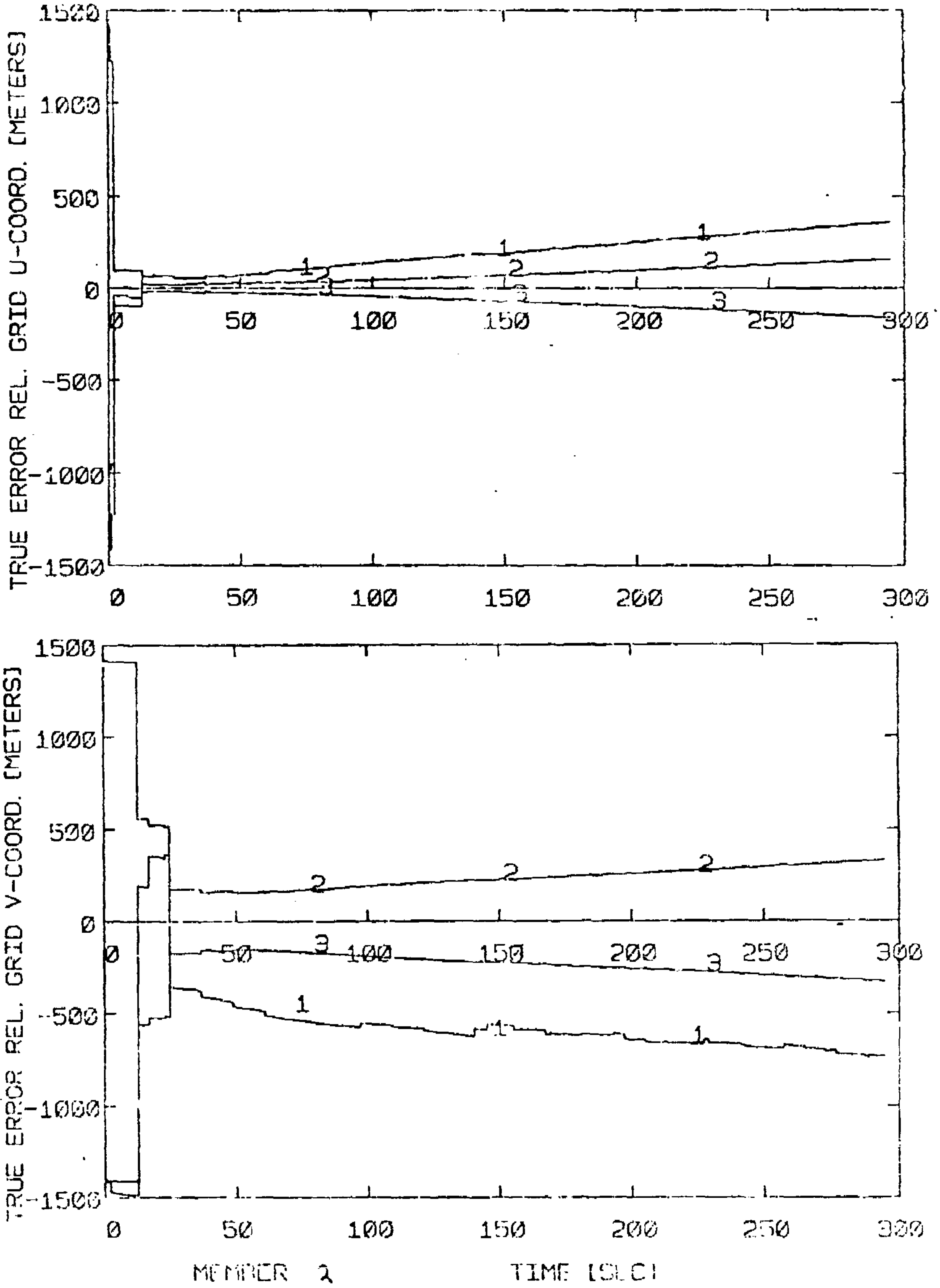


Fig. 7.23 Four Member Relnav Results without Nonlinear Protection, Member 2

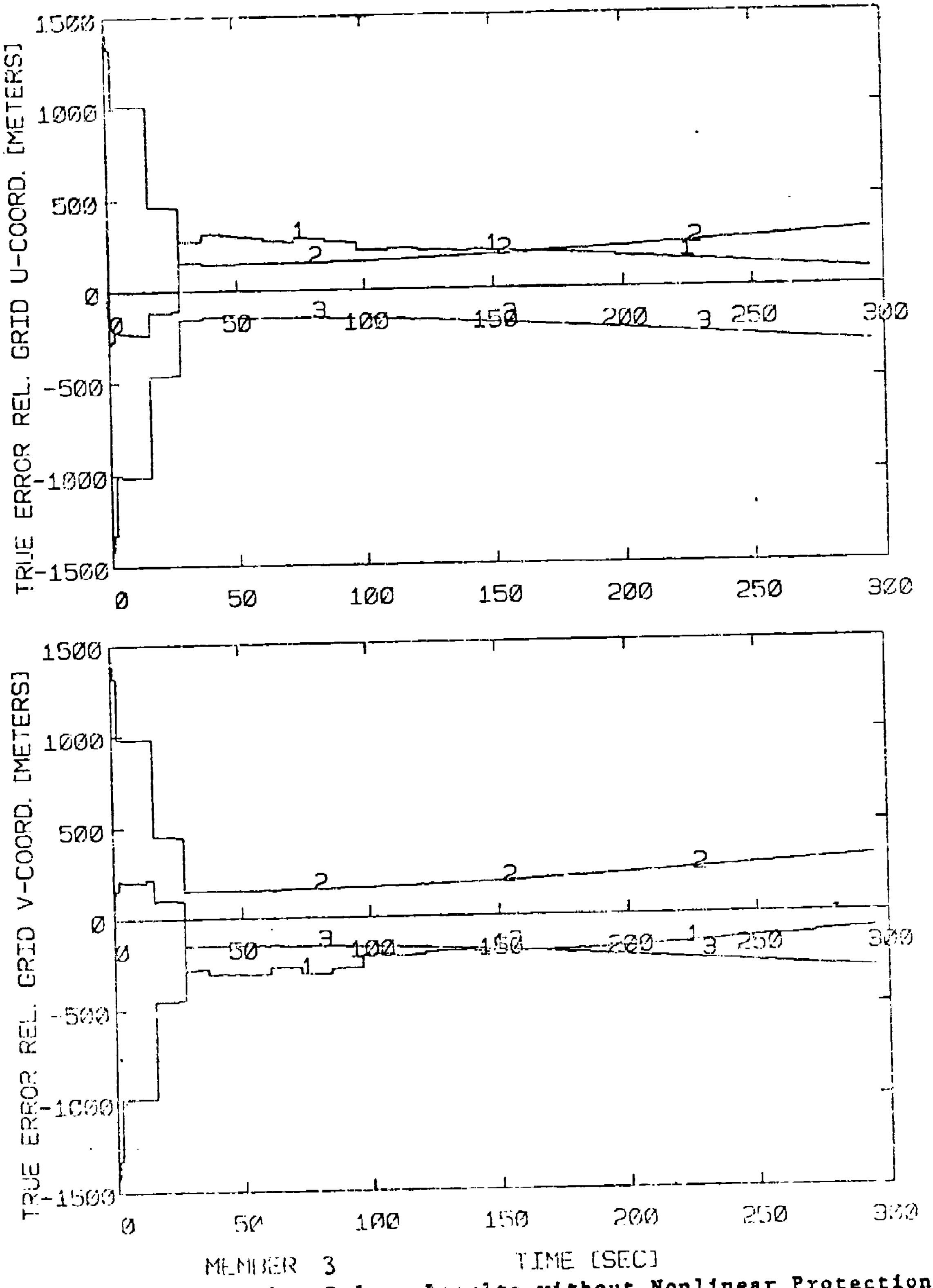


Fig. 7.24 Four Member Relnav Results without Nonlinear Protection, Member 3

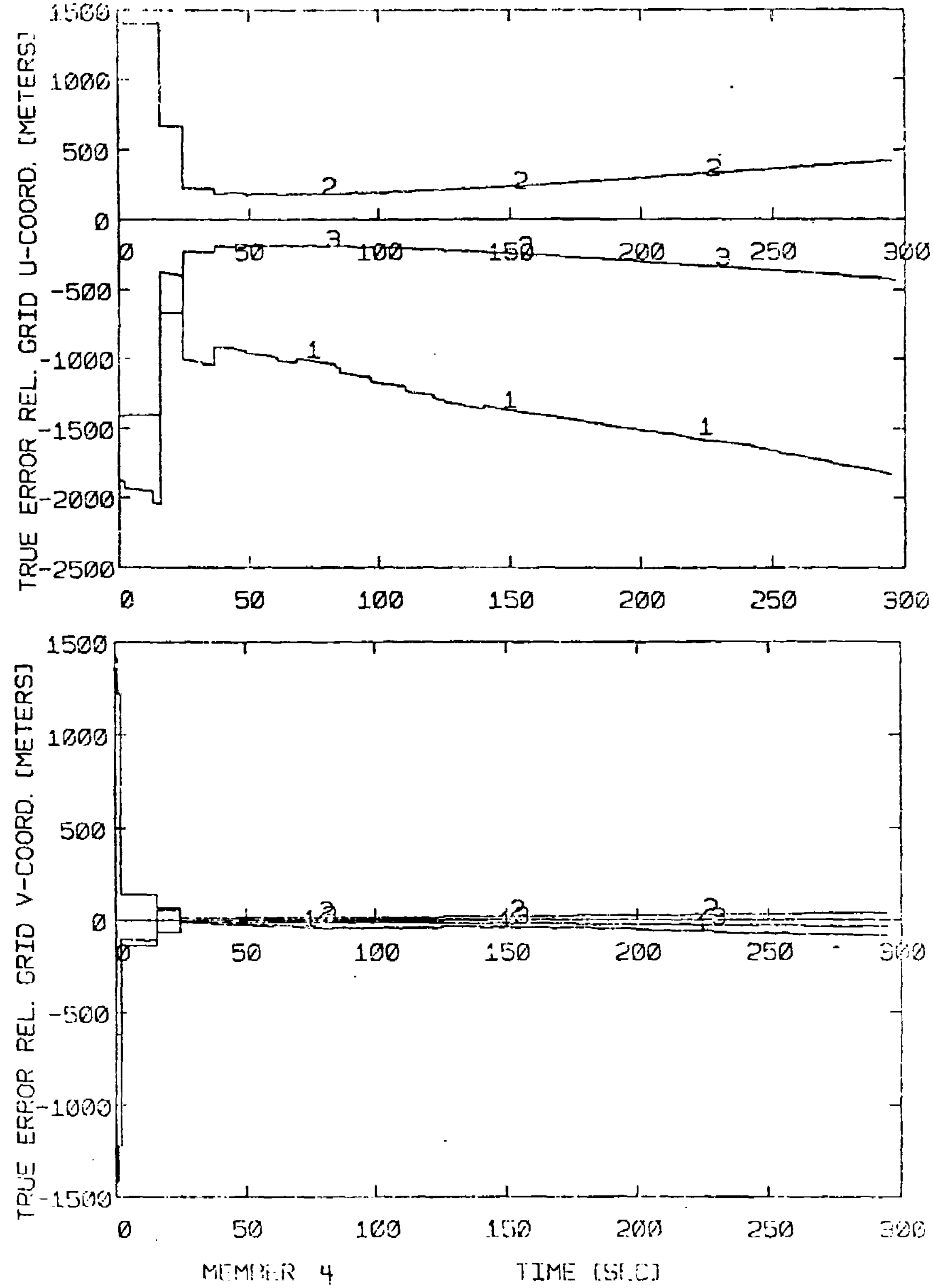


Fig. 7.25 Four Member Relnav Results without Nonlinear Protection, Member 4

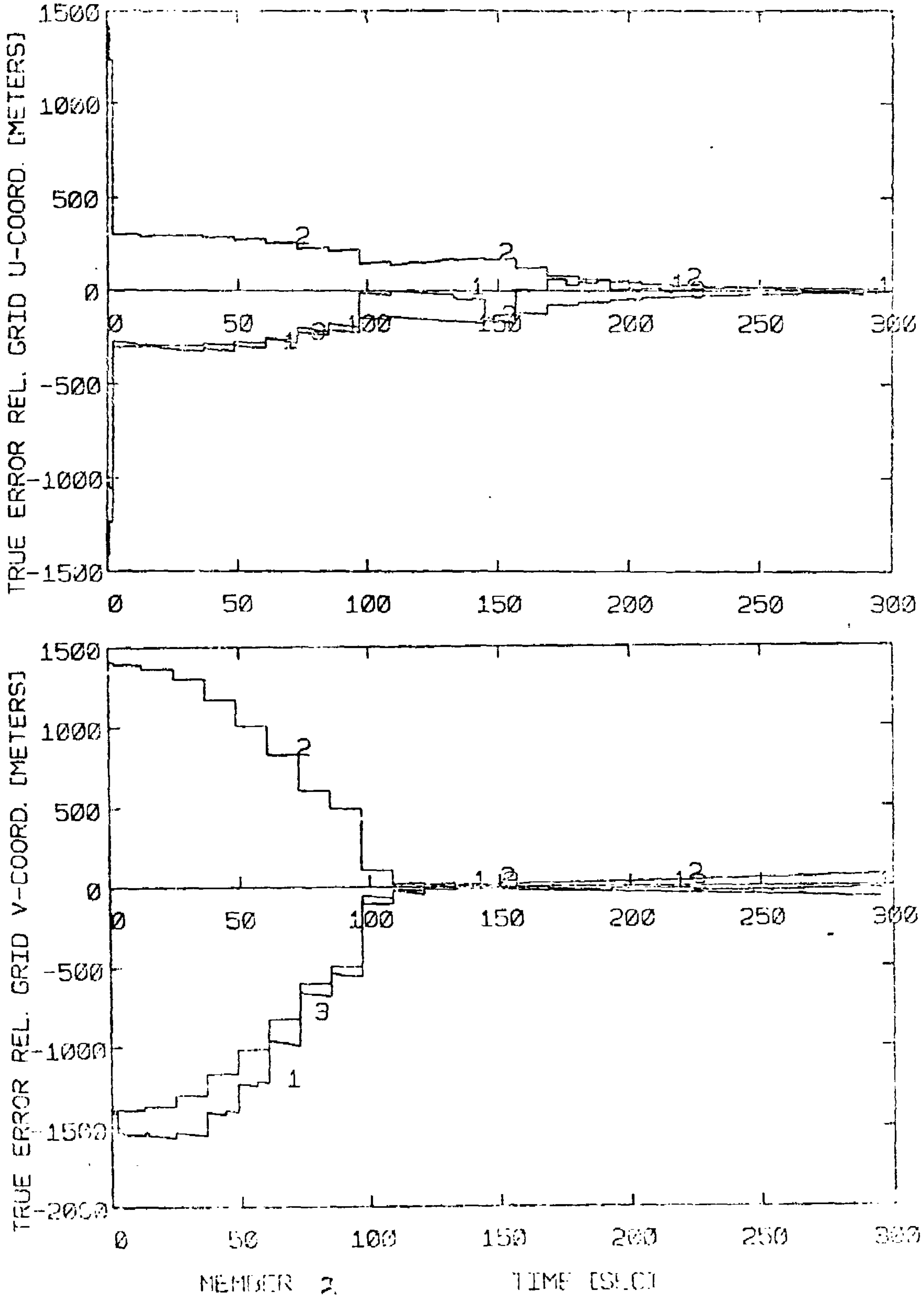


Fig. 7.26(a) Two Member Teardrop Trajectory with Nonlinear Protection, Member 2

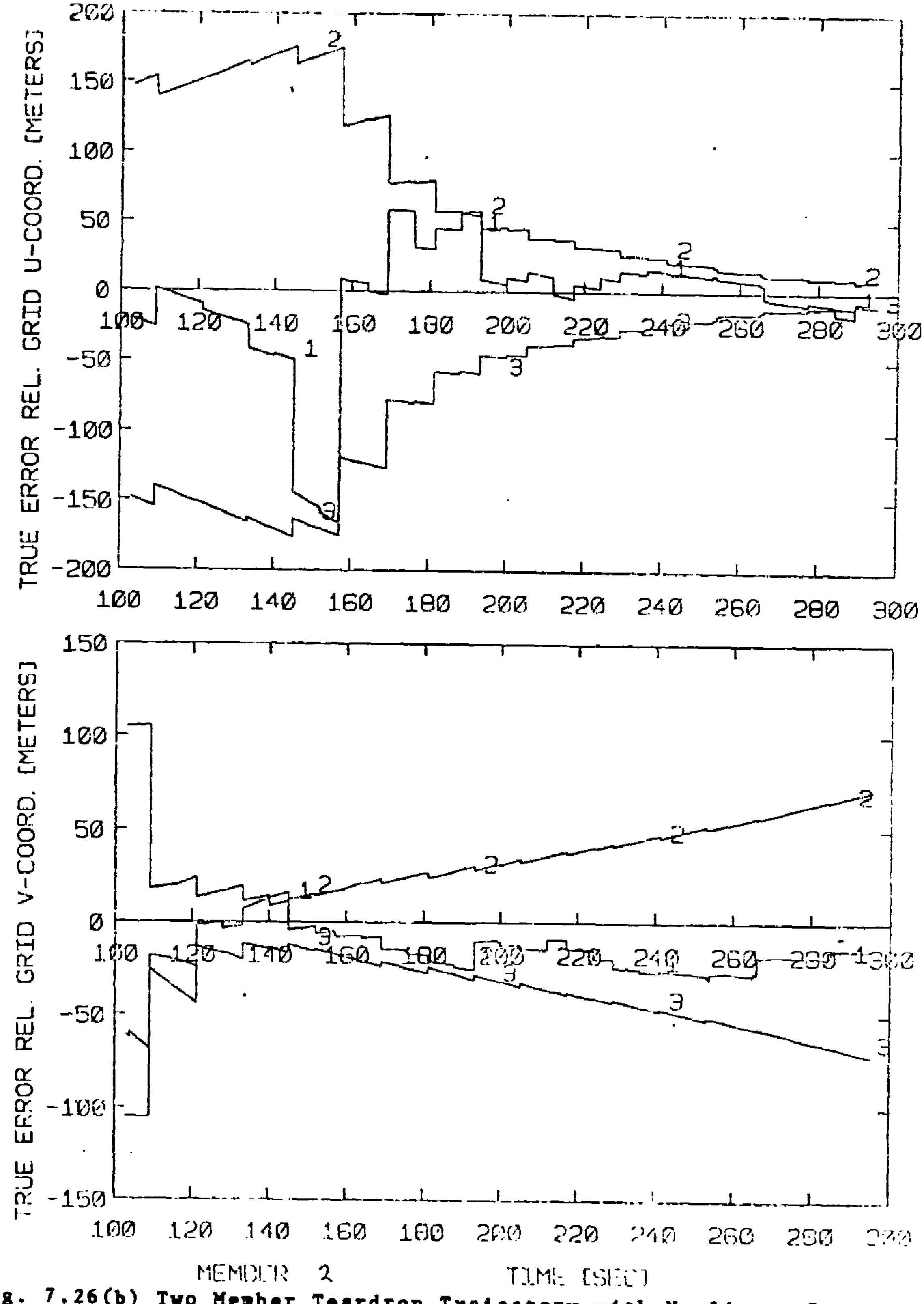


Fig. 7.26(b) Two Member Teardrop Trajectory with Nonlinear Protection, Member 2, Expanded Scale

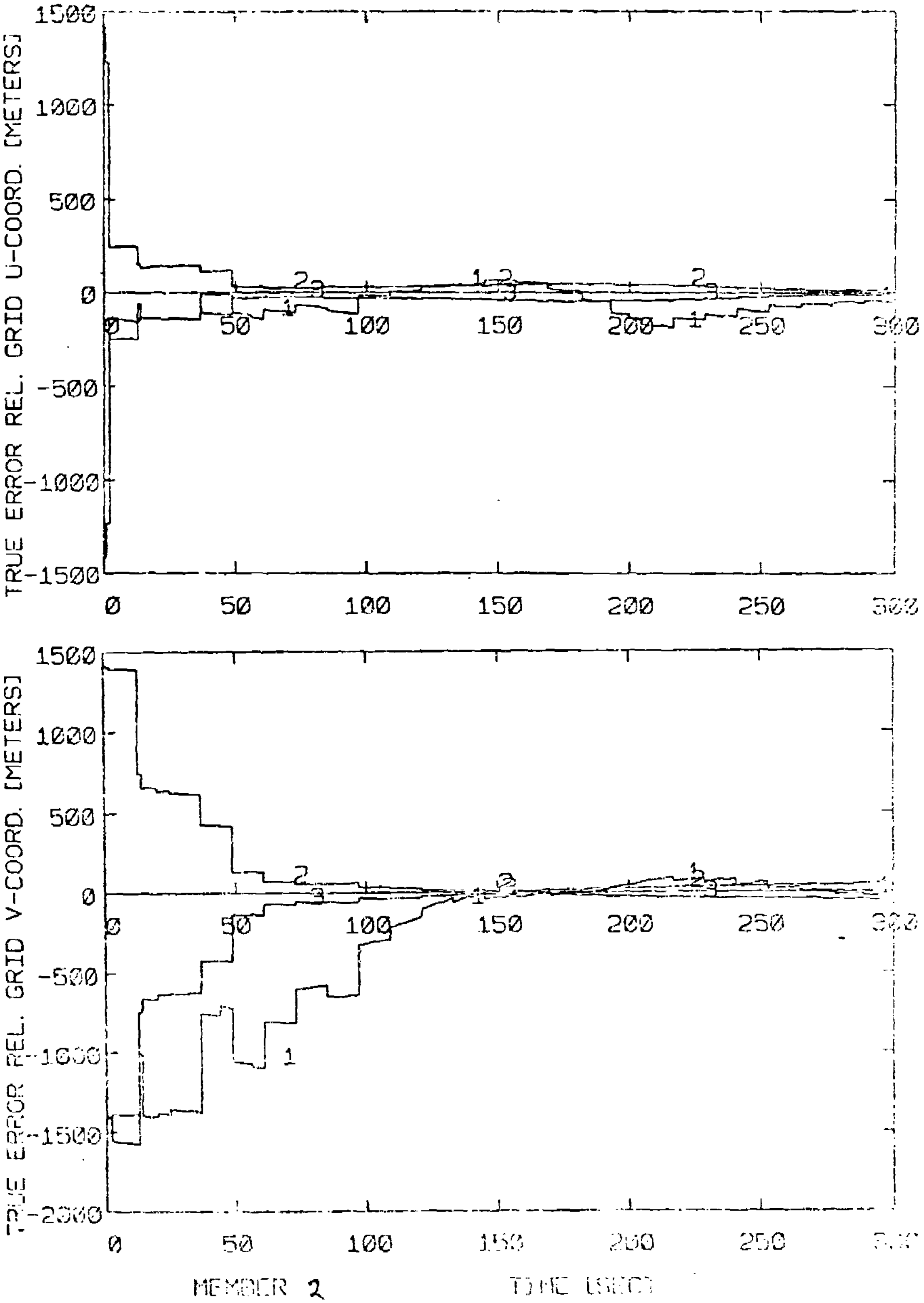


Fig. 7.27(a) Two Member Teardrop Trajectory without Nonlinear Protection, Member 2

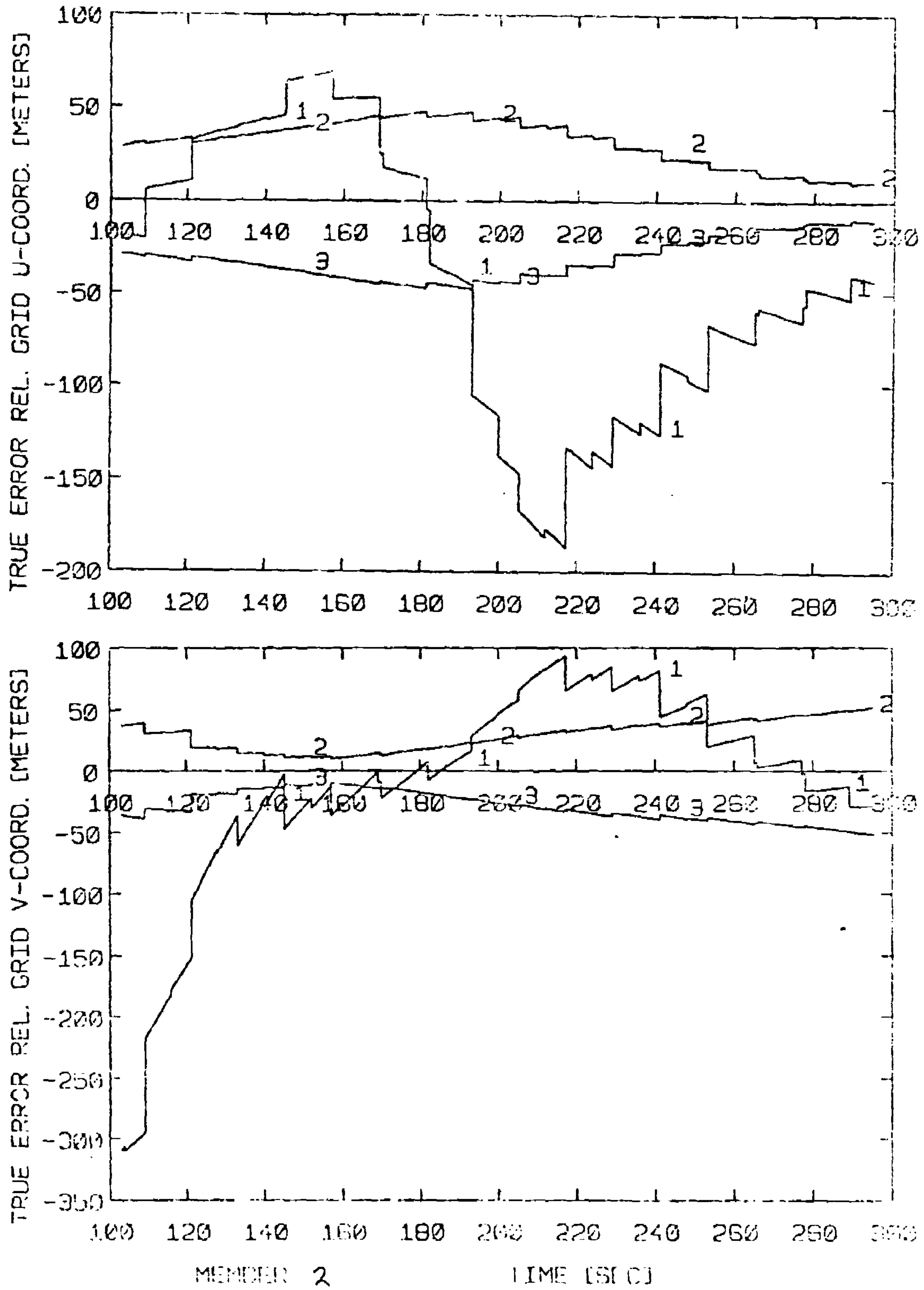


Fig. 7.27(b) Two Member Teardrop Trajectory without Nonlinear Protection, Member 2, Expanded Scale

results without the nonlinear protection are shown in Fig. 7.27. Again the nonlinear protection serves to prevent filter divergence.

Note the run with the nonlinear protection is not identical to the run shown earlier in Fig. 7.4. The difference is that here we have overriden the random number generator that sets the initial estimation errors and we have set the initial northerly relative error at 1400 m. This assures that the single case will exhibit nonlinear effects.

7.6 Measurement Noise Effect on Performance

It is of interest to know what effect reducing the measurement noise would have on performance. The baseline values for the measurement noises that we have been using in the simulator are 10 m and 7 m one sigma for the time of arrival noise and the round trip timing noise. Both the truth model and the filter model have assumed these levels. We have run some simulations with these noise levels reduced by a factor of 10, both in the truth model and in the filter model. The result, which at first was surprising to us, was that reducing the noise level had no noticeable effect on the performance.

After puzzling over these data for a while, we realized that the dominant source of the poor relative navigation accuracy was not the measurement noise. More significant sources of poor accuracy seem to be the single navigation controller method of grid setting, the low relative motion in some trajectories, and the measurement nonlinearity.

If one did not need to protect the estimation process from the effect of measurement nonlinearity, then the level of the measurement noises would be an important consideration. In the last section we showed that without the nonlinear protection the filter computed uncertainties do come down somewhat. We have rerun the four member low observability trajectory of Fig. 7.19, again without the nonlinear protection, but in addition with the measurement noises reduced by a factor of ten in both the truth model and the filter model. The results are shown in Figs. 7.28, 7.29, and 7.30. The filter computed uncertainties are further noticeably reduced.

The simulation results also show more severe filter divergence due to the neglected nonlinearity of the measurements. This is generally true, that nonlinear effects are more important when the measurement noise is small.

7.7 Filter Numerical Precision Effect

In the dual grid implementation of the JTIDS navigation software, the position in the geodetic grid is explicitly estimated, but the position in the relative grid is implicitly estimated. Relative position states do not appear in the Kalman filter. When an indication of relative position is wanted, it is obtained by differencing the estimated geodetic position and the estimated grid origin geodetic position. The uncertainties in geodetic position may be of the order of several kilometers. The uncertainties in relative position may be of the order of tens of meters, which is two or three orders of magnitude better. These

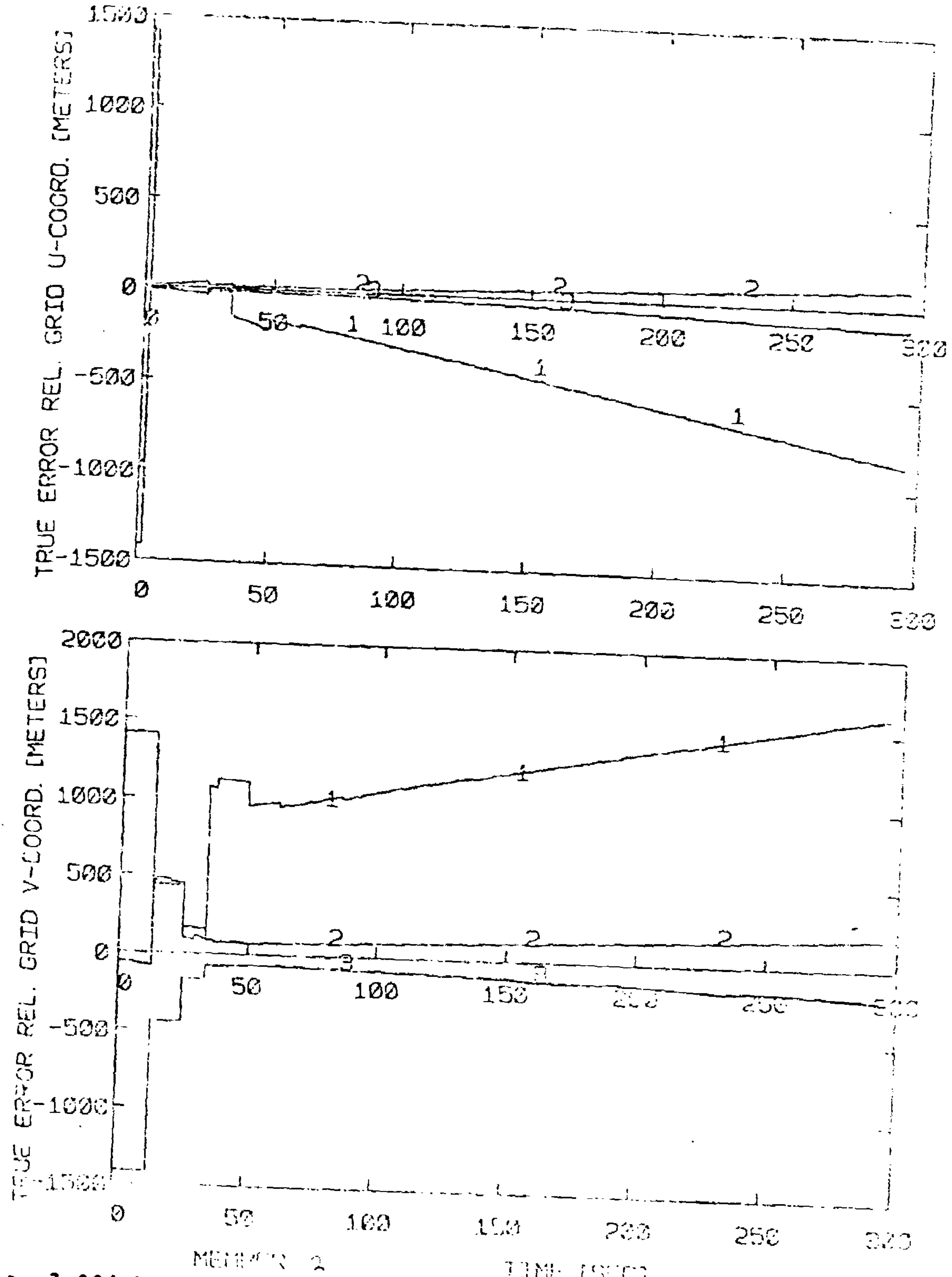


Fig. 7.28(a) Four Member Relnav Results, without Nonlinear Protection, and with More Accurate TOA Measurements (1m), Member 2

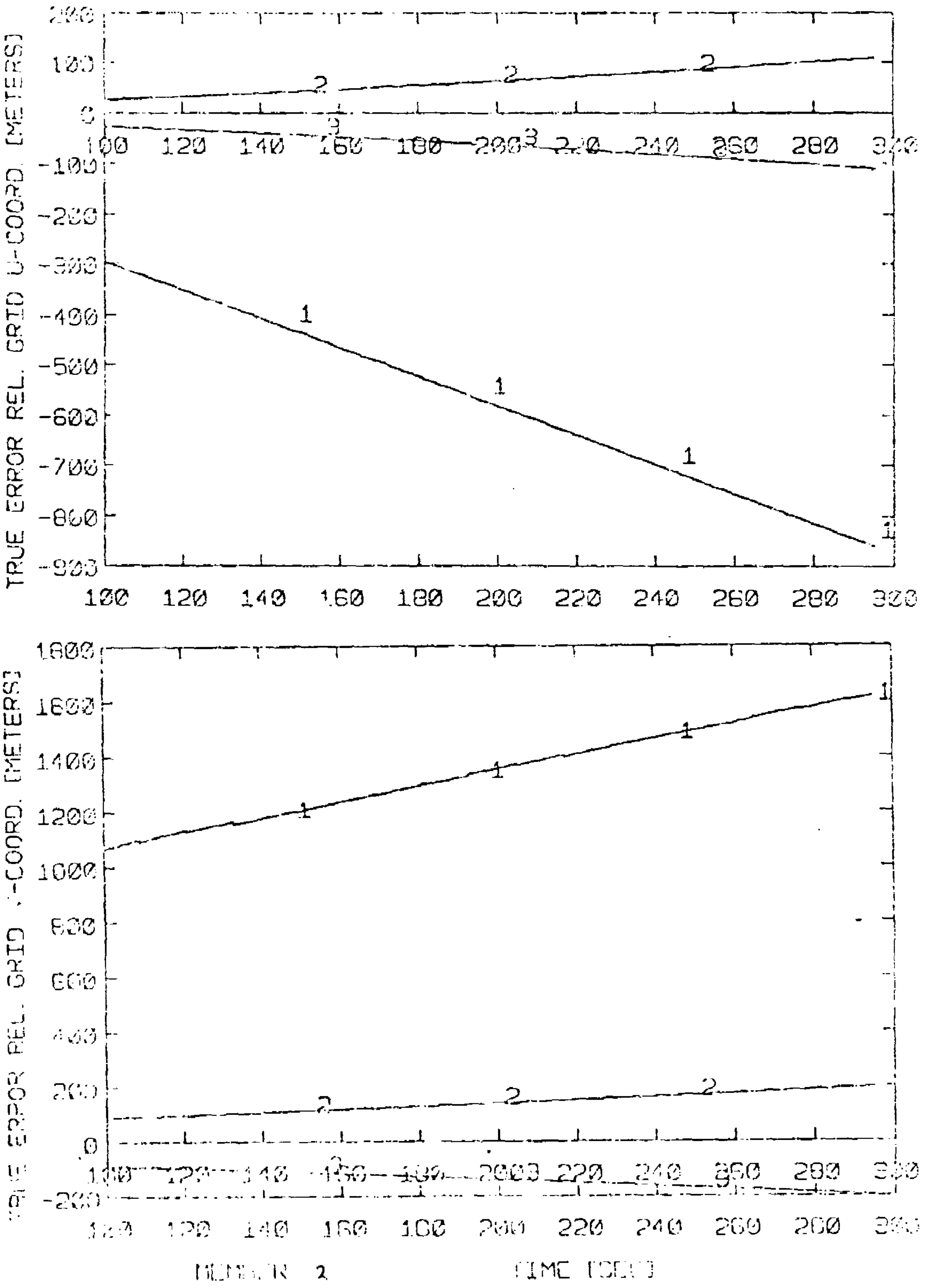


Fig. 7.28(b) Four Member Relnav Results, without Nonlinear Protection, and with More Accurate TOA Measurements (1m), Member 2, Expanded Scale

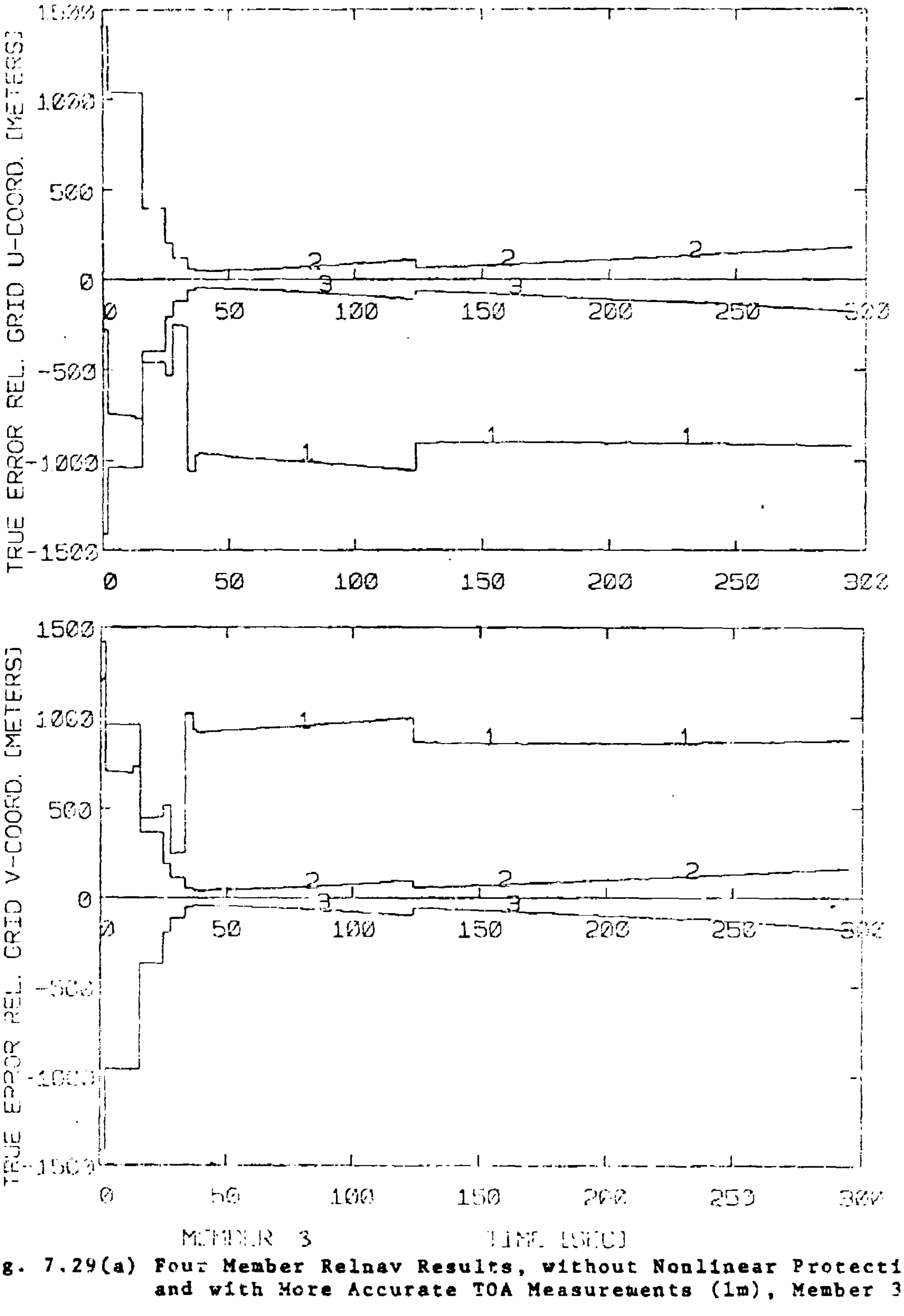


Fig. 7.29(a) Four Member Relnav Results, without Nonlinear Protection, and with More Accurate TOA Measurements (1m), Member 3

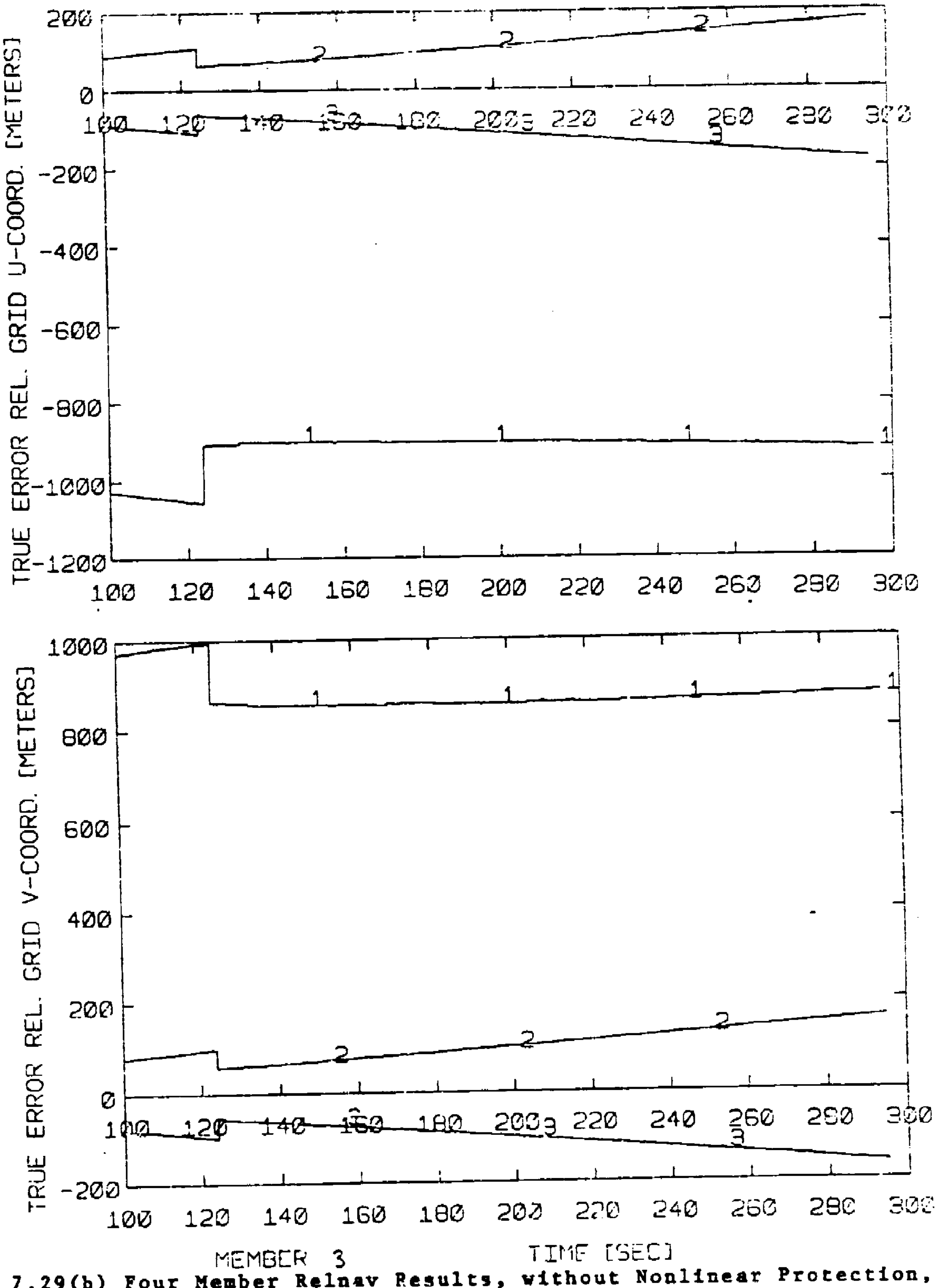


Fig. 7.29(b) Four Member Relnav Results, without Nonlinear Protection, and with More Accurate TOA Measurements (1m), Member 3, Expanded Scale

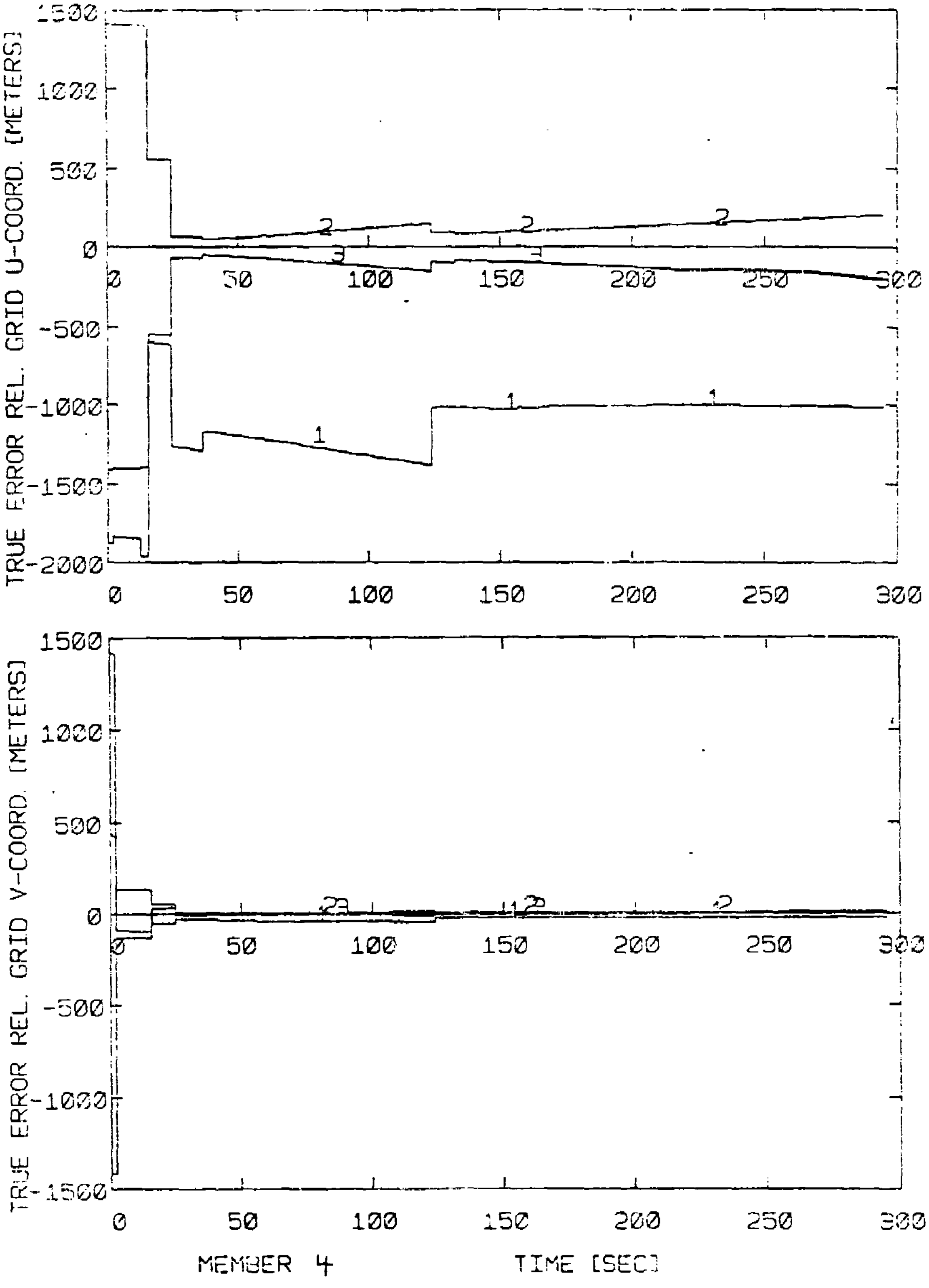


Fig. 7.30(a) Four Member Relnav Results, without Nonlinear Protection, and with More Accurate TOA Measurements (1m), Member 4

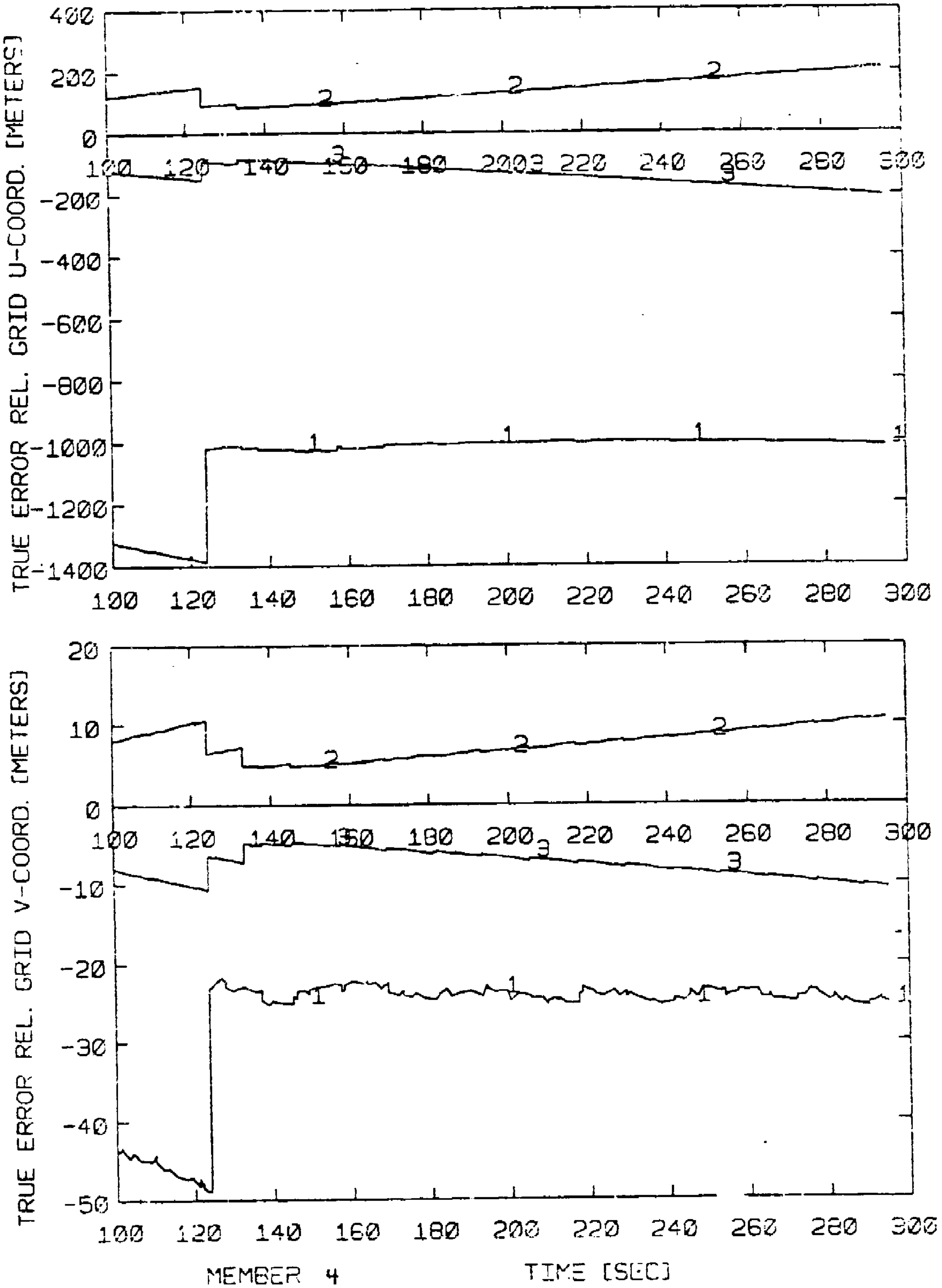


Fig. 7.30(b) Four Member Relnav Results, without Nonlinear Protection, and with More Accurate TOA Measurements (1m), Member 4, Expanded Scale

relationships are carried in the error covariance matrix. A consequence of this differencing of the two geodetic estimates with their highly correlated large errors is that the error covariance matrix is nearly singular. Therefore one must check to be sure that filter numerical precision is adequate.

We have generated two runs, one with single precision filter computations and one with double precision filter computations. On the computer being used to run the Fortran simulator, single precision has about 7 decimal digits of accuracy and double precision has about 16 digits accuracy. In each run, the four member trajectory of Fig. 7.19 was used. Table 7.1 shows some of the numerical results. The results are essentially the same. However the estimation errors and the filter computed uncertainties of the two cases differ in the second or third digit.

Table 7.1 Filter Precision Effect, Member 2
U Relative Position Error

	7 Digit Precision		16 Digit Precision	
Time (s)	True Error (m)	Filter Comp. Sigma (m)	True Error (m)	Filter Comp. Sigma (m)
0	-995.74	1414.23	-995.74	1414.23
2	-0.76	13.85	-0.76	13.83
13	-16.23	21.48	-16.23	21.46
52	-46.94	61.03	-47.00	60.97
58	-159.28	49.01	-161.26	48.91

We concluded that single precision was adequate for our simulation work. However the designers of flight software should consider carefully the effect of any shorter word length in the available flight computer.

7.8 JTIDS/INS Performance Conclusions

The effect of various trajectories on relative navigation accuracy was explored. All cases had a single navigation controller. It is important that there be relative motion between members providing angular velocity of the lines of sight. With round trip timing a member can quickly determine its range from the navigation controller. The initial accuracy is limited by nonlinear effects if the range is close and the cross range uncertainty is large. With angular velocity of the line of sight to the navigation controller, a member eventually estimates its other component of relative position. If the line of sight returns to the original direction, nonlinear limitations on accuracy are overcome and the positional accuracy along the line of sight is of the order of the 10 m noise in the time of arrival measurements. Generally members were not able to significantly reduce their initial relative velocity errors, so cross range accuracy deteriorates noticeably when the direction to the navigation controller stops changing.

In some of the trajectories the navigation controller was not moving. This did not seem to prevent the other members from determining their relative position. If members did not have inertial systems, there would be an azimuth ambiguity with a fixed navigation controller. But with well aligned inertial systems, the members are able to determine their relative direction from the navigation controller.

In none of the trajectories examined could members simultaneously fix both components of horizontal position to the level of 10 m accuracy. A possible conclusion is that two navigation controllers (end of baseline concept) are needed to achieve accurate relative navigation.

Round trip timing has a significant beneficial effect on relative navigation accuracy. This was demonstrated with a two member trajectory comparing the performance with and without RTT. With RTT the initial along range position can be determined. Without RTT no component of position is determined until there is significant variation in the line of sight to the navigation controller. With RTT the member has some success at estimating both components of relative velocity. As a result good navigation accuracy is held longer after the line of sight stops changing. Without RTT the member had no success at estimating relative velocity. Both components of position error at the end are growing at the rate of the initial relative velocity error.

A simulation with a democratic organization of four members and without round trip timing was shown to be wildly unstable. When RTT was added, the instability was eliminated. The ownstate filters still had hopelessly optimistic computed uncertainties.

The Gaussian quadratic equations, for protecting the filter from the effect of the nonlinear elongation of the time of arrival measurements, have been shown to be important for preventing filter divergence.

The 10 m and 7 m one sigma noises in the time of arrival and round trip timing measurements are not significant contributors to the relative navigation error. The more important sources of error seem to be the single navigation controller method of establishing the relative navigation grid, the low angular velocities of the lines of sight on some trajectories, and the measurement nonlinearity. If nonlinearity were not a significant problem, then the measurement noise would be a significant source of error. Other methods of grid setting, such as the end of baseline method can eliminate nonlinear difficulties. Under these organizations, the measurement noise level would be important.

The effect of filter precision was explored. There was a difference in numerical results observed in the second or third digit comparing a single precision run and a double precision run. Single precision accuracy here had 7 decimal digits. The sensitivity to precision is due to the way that relative navigation is carried implicitly as the difference between geodetic position estimates. Some flight computers, with single precision accuracy less than our 7 digits, may have to implement the Kalman filter in double precision.

CHAPTER 8

JTIDS/GPS/INS/SIMULATION RESULTS

8.1 One Member GPS Equipped

Two runs have been generated showing navigation performance with some members having a GPS receiver in addition to the JTIDS receiver and inertial system. In both of these runs the trajectory is the four member low observability trajectory in which member 2 angles out from the initial square (Fig. 7.19). In the first run only member 1 is GPS equipped. In the second run both member 1 and member 2 are GPS equipped.

There are four GPS satellites providing excellent geometry. The azimuths and elevations of the satellites at the beginning and end of the 300 sec simulations, as seen from one reference point in the community, are

Time (sec)	Satellite	Azimuth (deg)	Elevation (deg)
0	1	32.8	14.4
	2	18.1	87.0
	3	-146.5	19.1
	4	129.3	7.1
300	1	33.1	12.4
	2	14.2	84.3
	3	-145.2	21.2
	4	130.1	5.5

With only the navigation controller (member 1) being GPS equipped, the community geodetic navigation results are shown in Figs. 8.1 through 8.4.

Fig. 8.1 is for the navigation controller. The excellent navigation accuracy provided by GPS is shown. Easterly and northerly geodetic position errors are only 3 to 5 meters. The filter computed uncertainty of 2 m is somewhat optimistic. This discrepancy is due to the external GPS errors (satellite clock, satellite ephemeris, ionospheric retardation, and tropospheric retardation) which are not modeled in the Kalman filter state.

The simulation incorporates a set of four measurements once each 12 sec cycle. No pseudorange rate measurements are simulated. An actual GPS navigator processing measurements more often and also using pseudorange rate measurements can be expected to have slightly better accuracy.

To be able to show the expanded scale, the processing on the first set of four measurements is not plotted. This initial fix reduced the initial 1000 m error down to the levels shown. After the second fix, the velocity also is accurately known, so the sawtooth amplitude is reduced.

As soon as member 1 has fixed its own geodetic position it can assist others in determining their geodetic positions. The results for members 2, 3, and 4 at first seem disappointing, until one realizes that they can do no better than their ability to determine their relative position. Thus we see geodetic navigation errors in some components of 200 m or more.

At the same time that the members are trying to determine their geodetic position, they are also implicitly trying to determine their relative grid position (dual grid concept). Time of arrival measurements are being processed both as geodetic

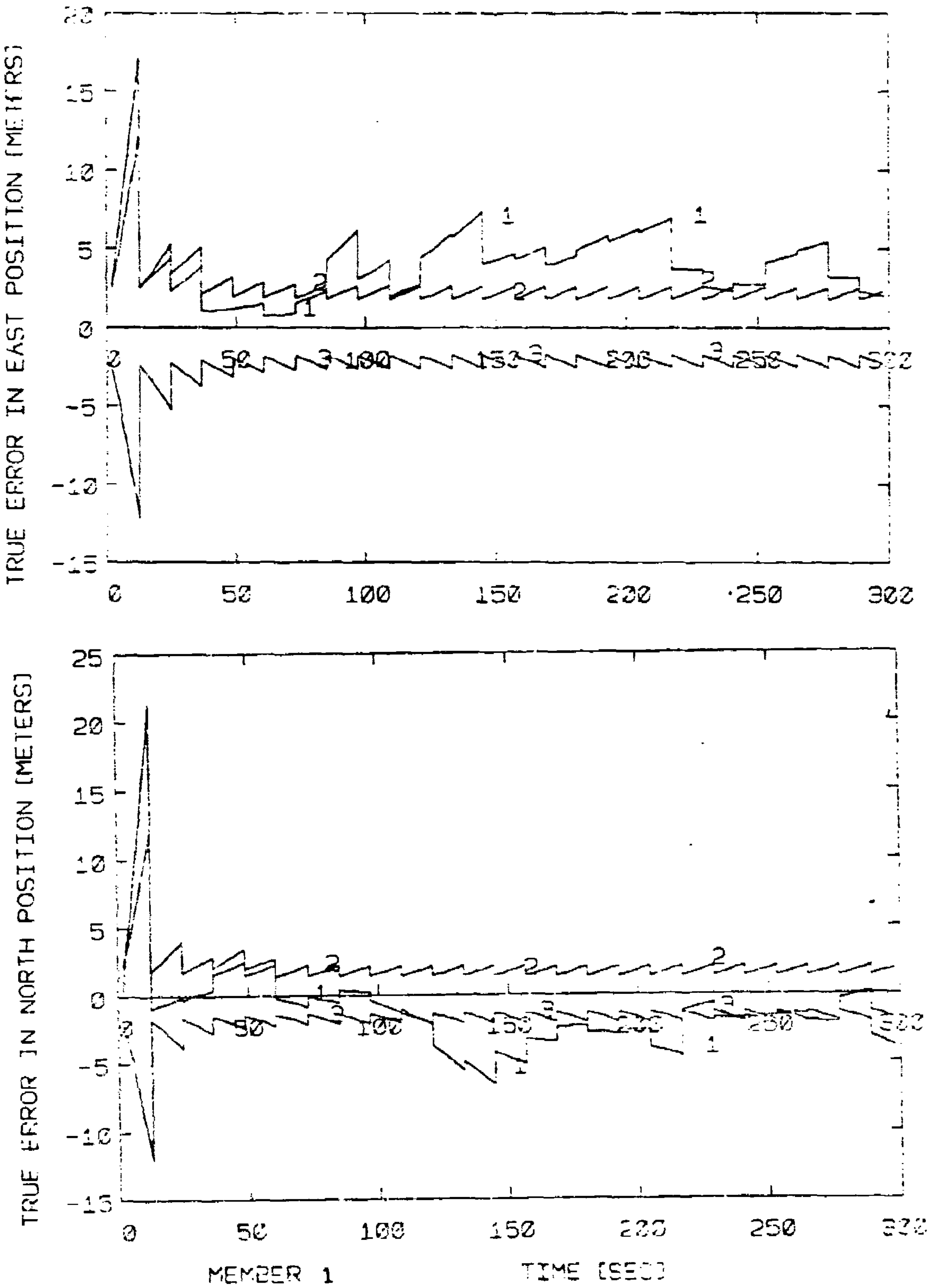


Fig. 8.1 Only Member 1 GPS Equipped, Member 1 Geodetic Nav. Results

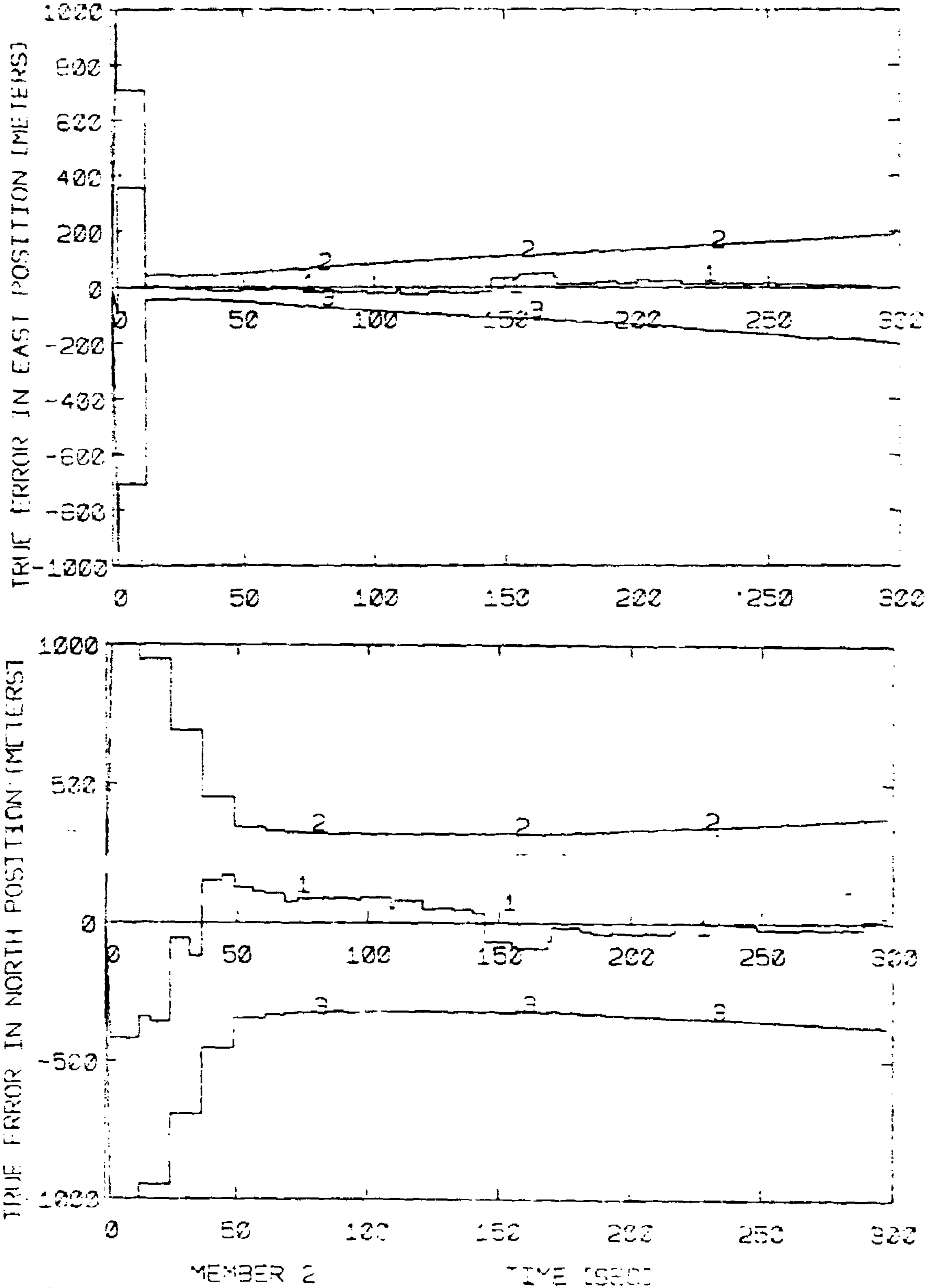


Fig. 8.2 Only Member 1 GPS Equipped, Member 2 Geodetic Nav. Results

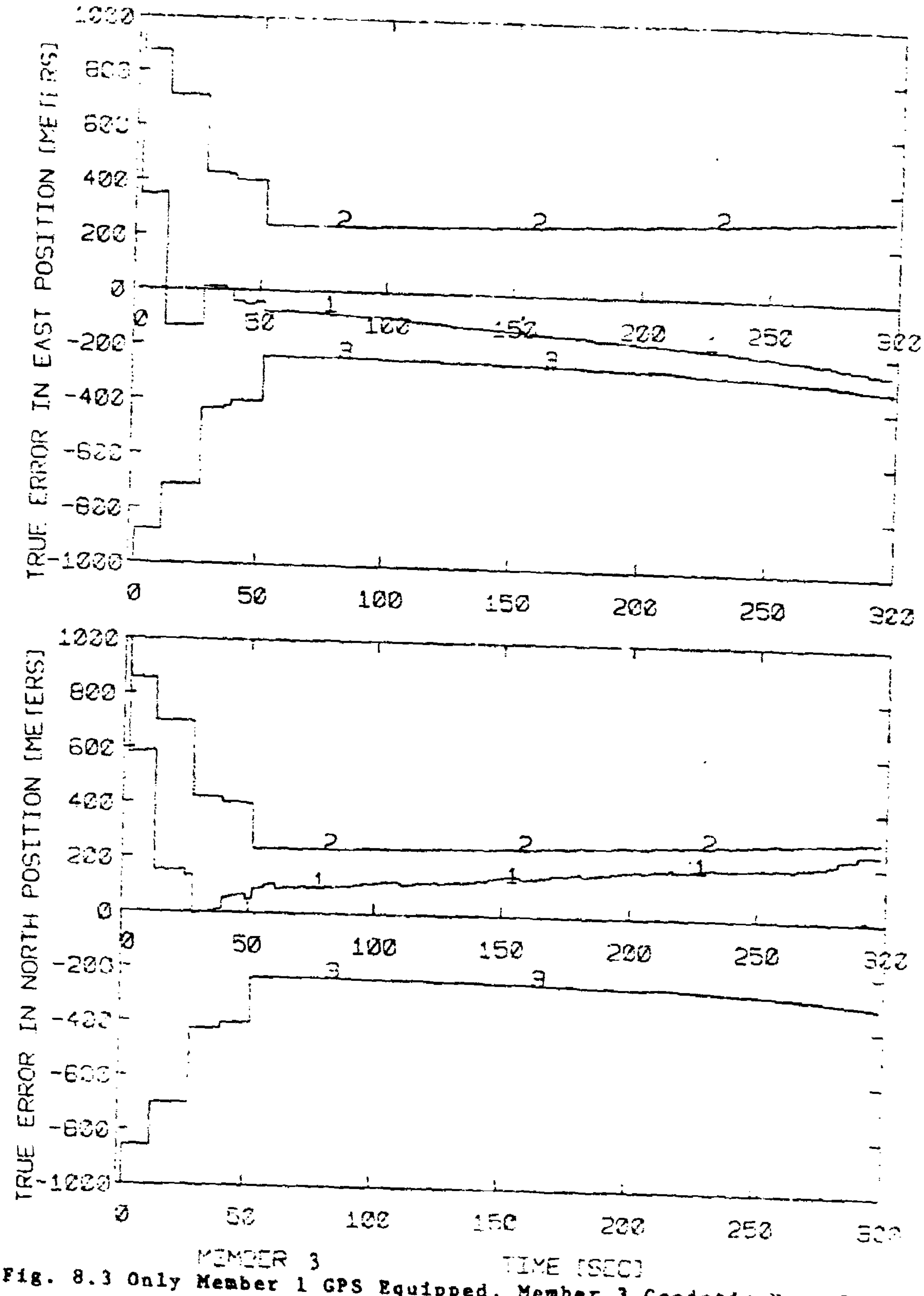


Fig. 8.3 Only Member 1 GPS Equipped, Member 3 Geodetic Nav. Results

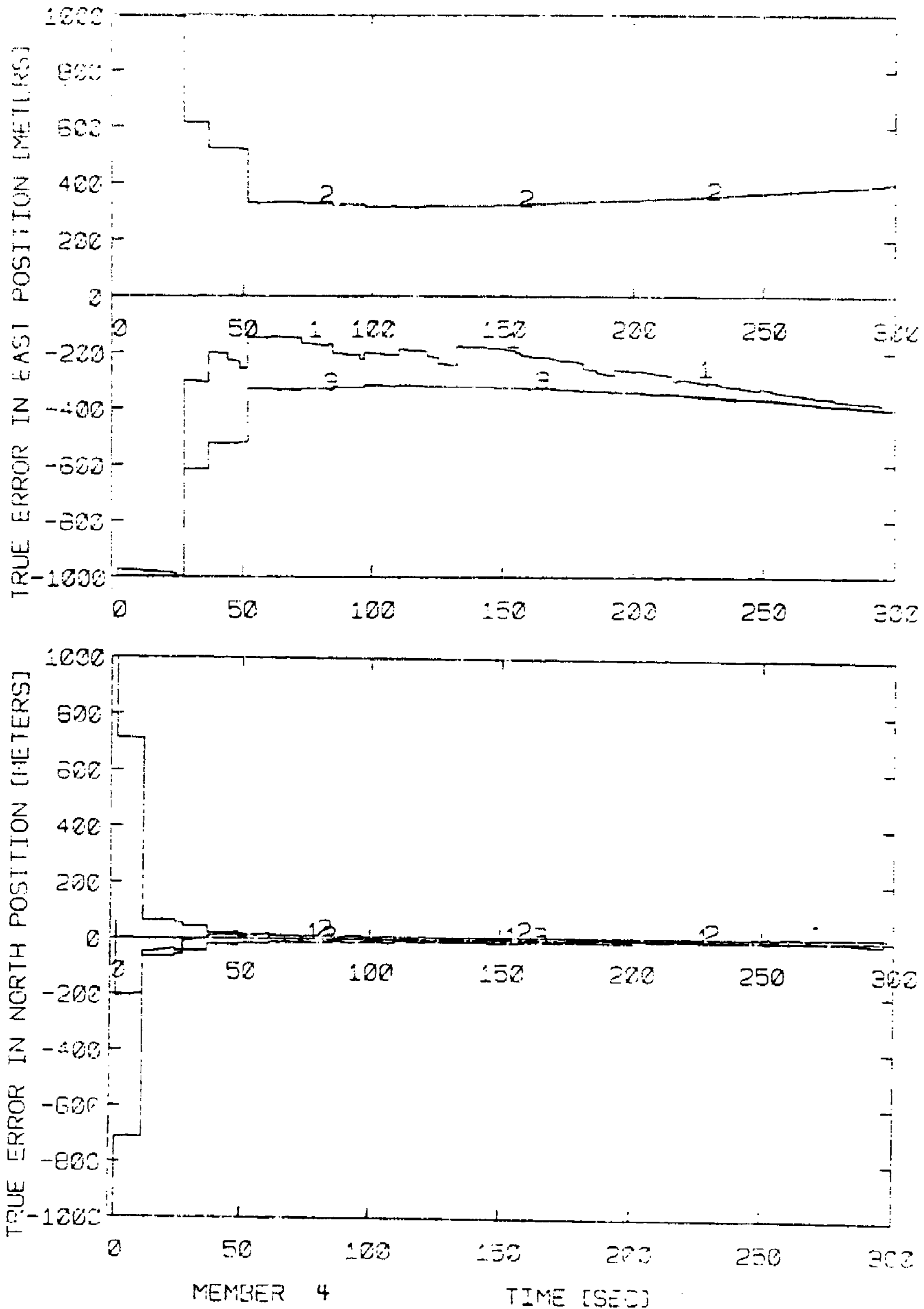


Fig. 8.4 Only Member I GPS Equipped, Member 4 Geodetic Nav. Results

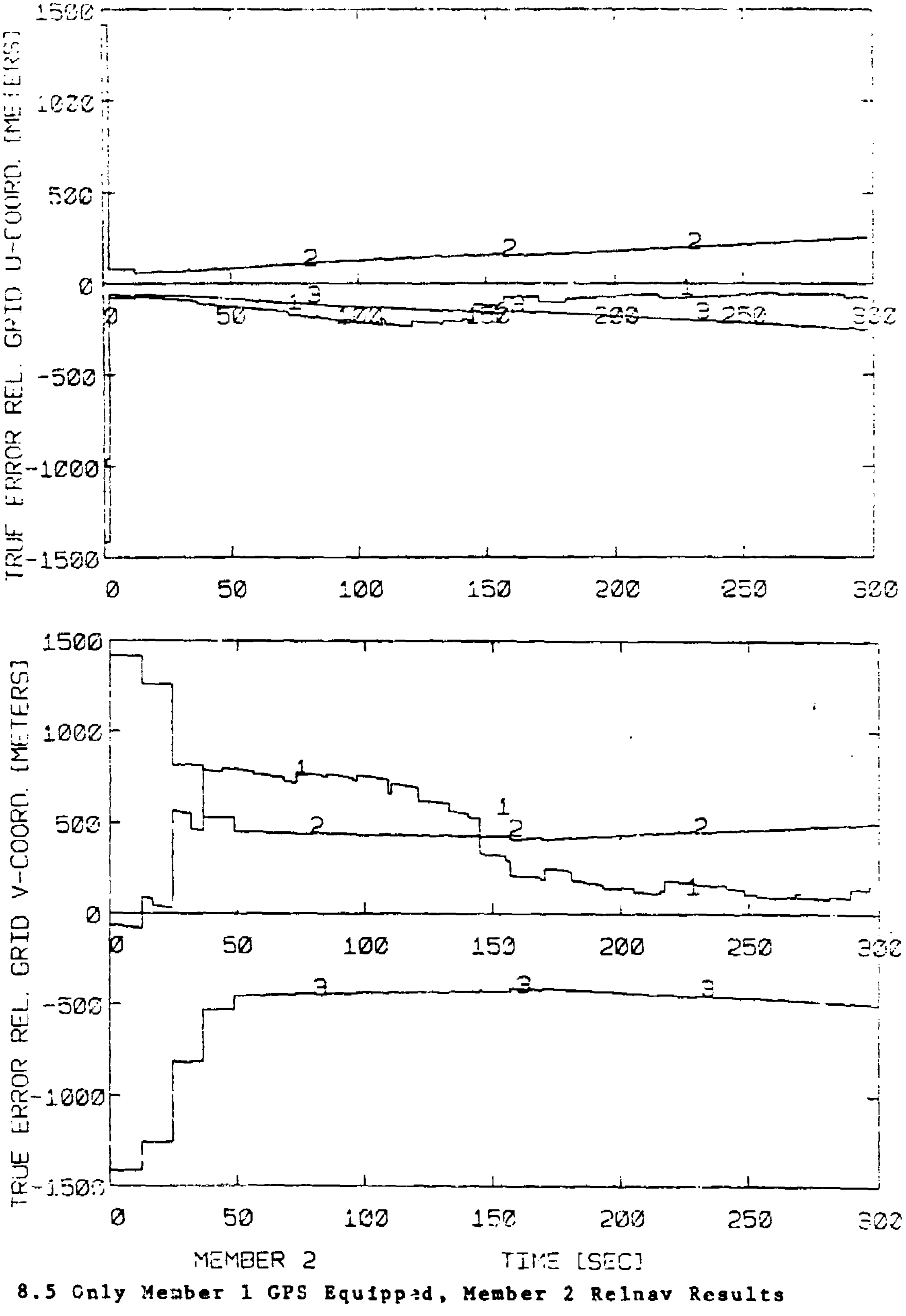


Fig. 8.5 Only Member 1 GPS Equipped, Member 2 Relnav Results

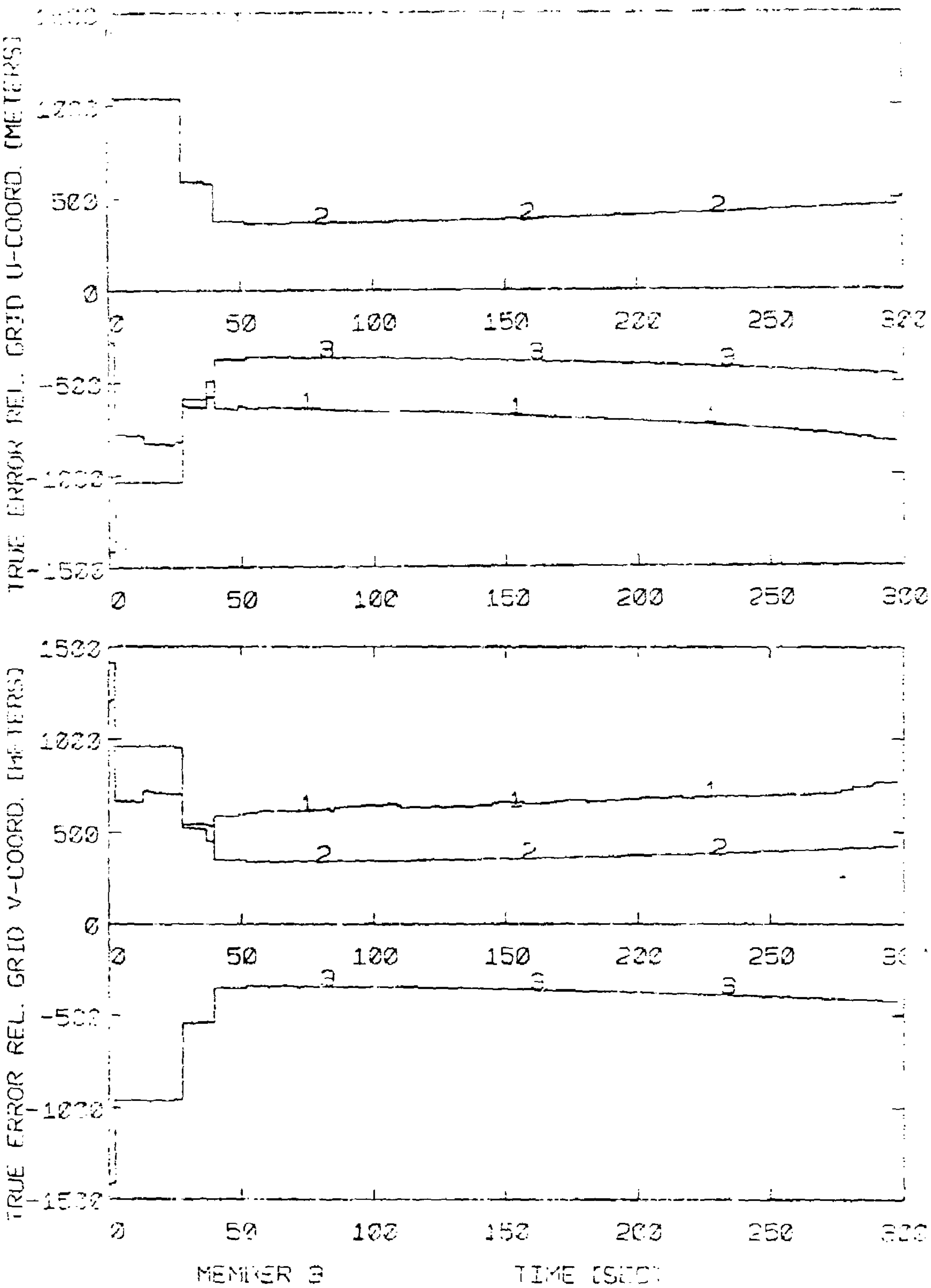


Fig. 8.6 Only Member 1 GPS Equipped, Member 3 Relnav Results

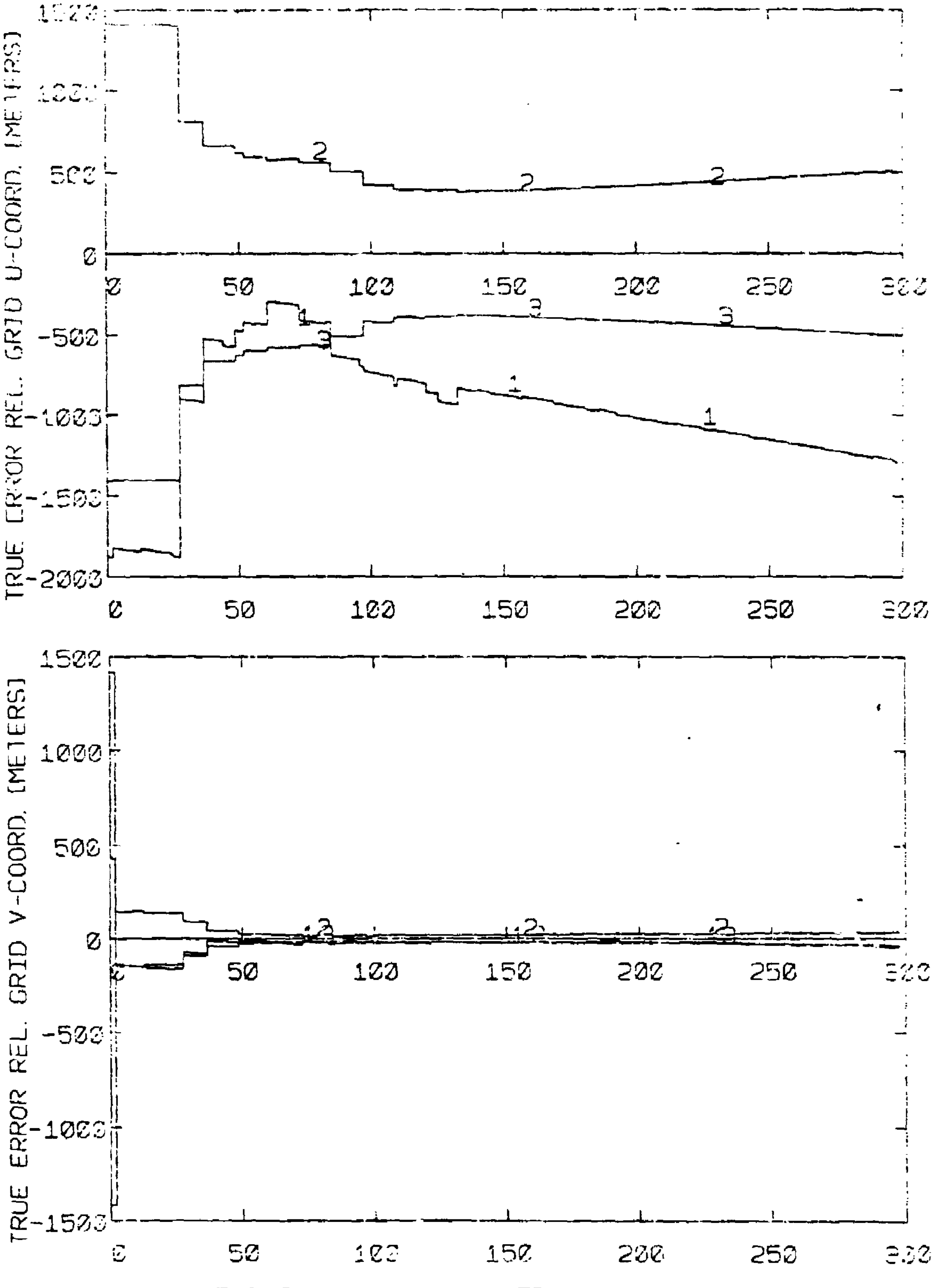


Fig. 8.7 Only Member 1 GPS Equipped, Member 4 Relnav Results

updates and relative grid updates, following the covariance based hierarchy logic of JTIDS. The relative navigation results for members 2, 3, and 4 are shown in Figs. 8.5, 8.6, and 8.7. The results are similar to the geodetic results.

8.2 Two Members GPS Equipped

The geodetic navigation capability of the community is enormously improved if two members are GPS equipped and are tracking. In this second simulation, members 1 and 2 are GPS equipped. The geodetic navigation results for all members are shown in Figs. 8.8 through 8.11. Both members 1 and 2 have the excellent accuracy associated with GPS. Members 3 and 4 are able to fix their own positions from the JTIDS measurements off of members 1 and 2. The accuracy of their fixes is limited by the 10 m one sigma time of arrival noise plus any geometric dilution of precision.

The relative navigation results are shown in Figs. 8.12, 8.13, and 8.14. The surprising result shown is that the navigation accuracy in the relative grid is much worse than the navigation accuracy in the geodetic grid. How can this be?

The explanation is that in the current version of the simulator we have not yet implemented the grid offset measurement type. The grid offset measurement is not a physical measurement but is prepared from data in the JTIDS position message. Using both the geodetic position and the relative position reported by a source, one can update ones own estimate of the location of the origin of the relative grid. Without this pseudo measurement,

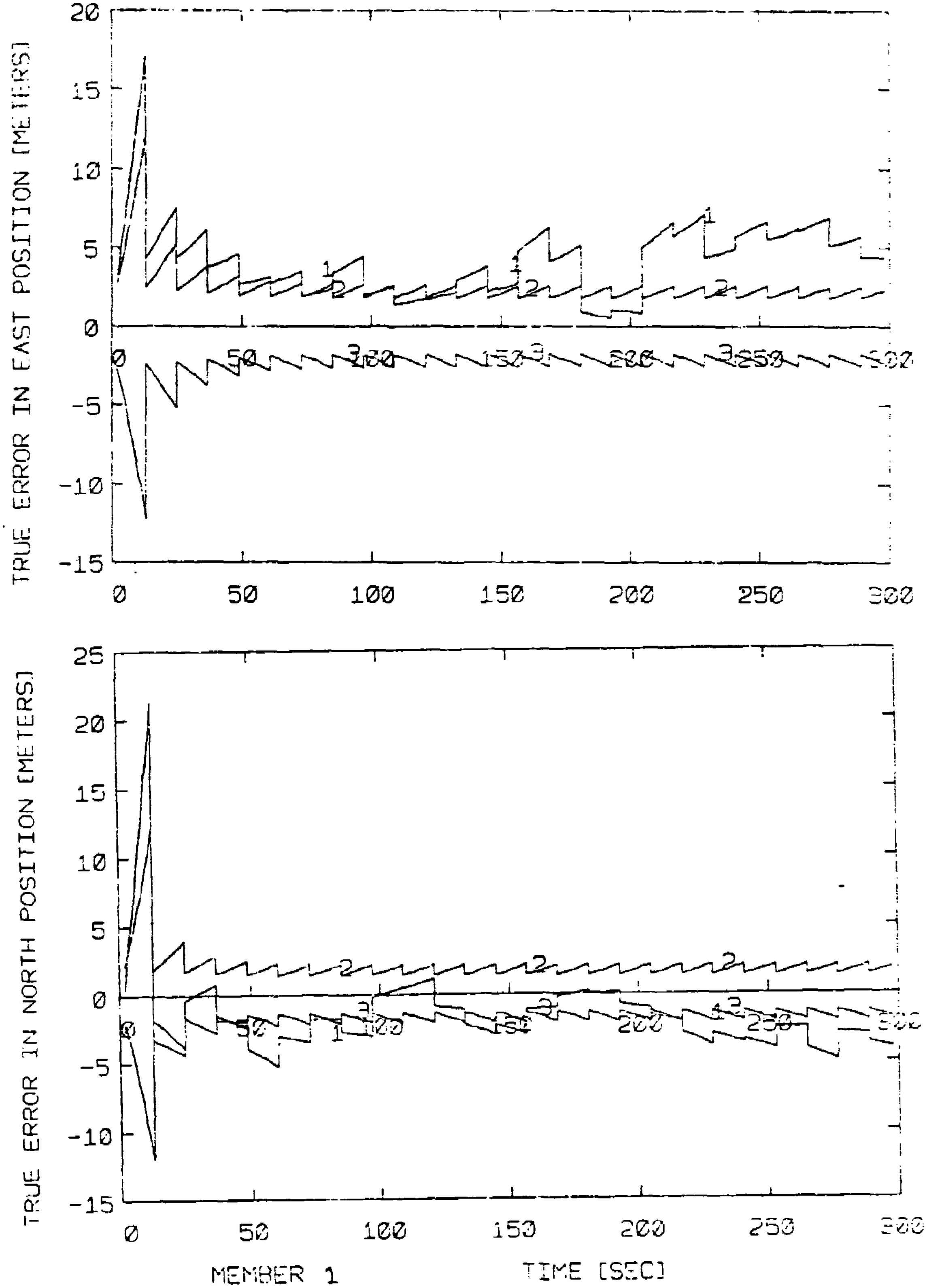


Fig. 8.8 Members 1 and 2 GPS Equipped, Member 1 Geodetic Nav. Results

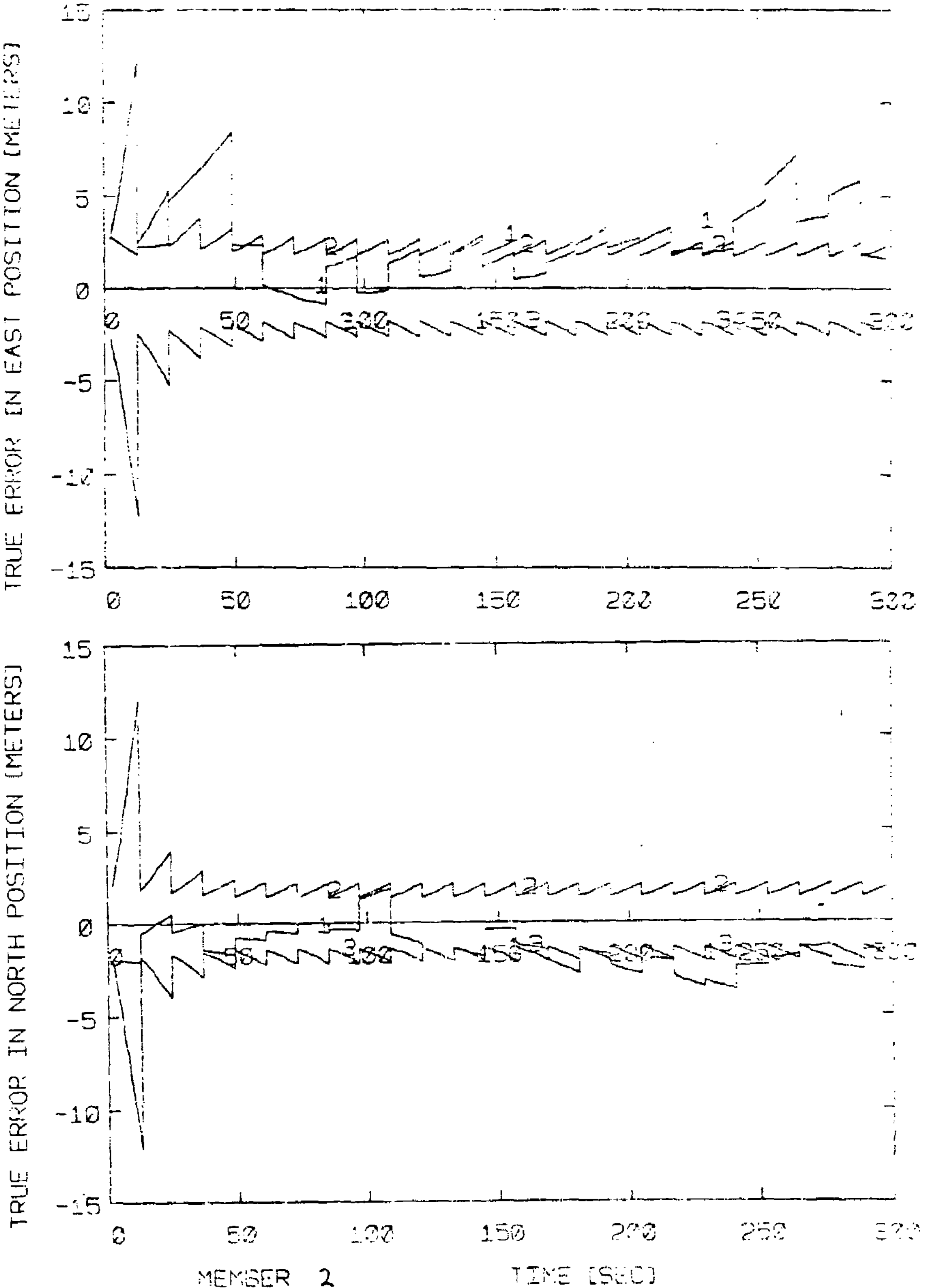


Fig. 8.9 Members 1 and 2 GPS Equipped, Member 2 Geodetic Nav. Results

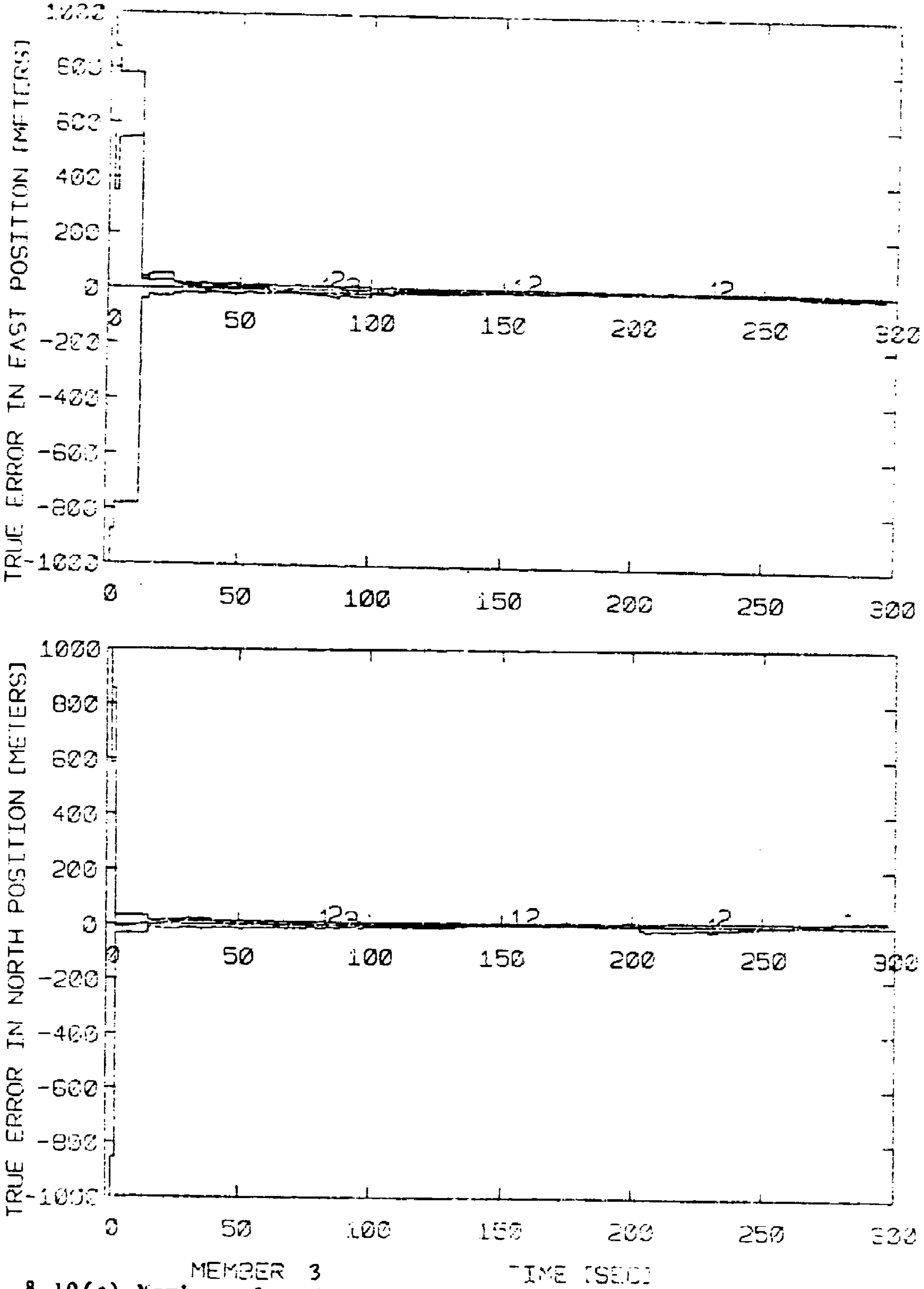


Fig. 8.10(a) Members 1 and 2 GPS Equipped, Member 3 Geodetic Nav. Results

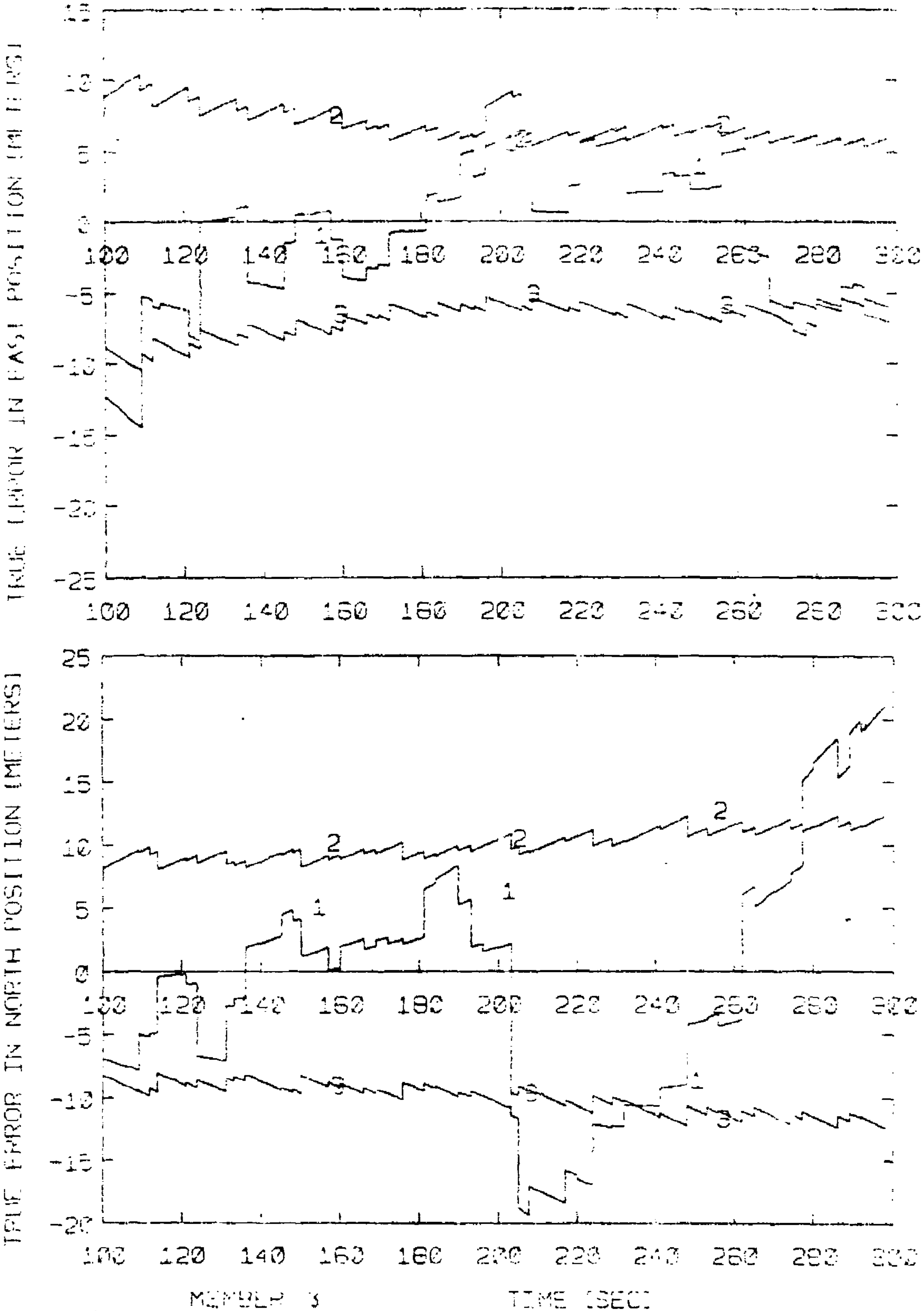


Fig. 8.10(b) Members 1 and 2 GPS Equipped, Member 3 Geodetic Nav. Results, Expanded Scale

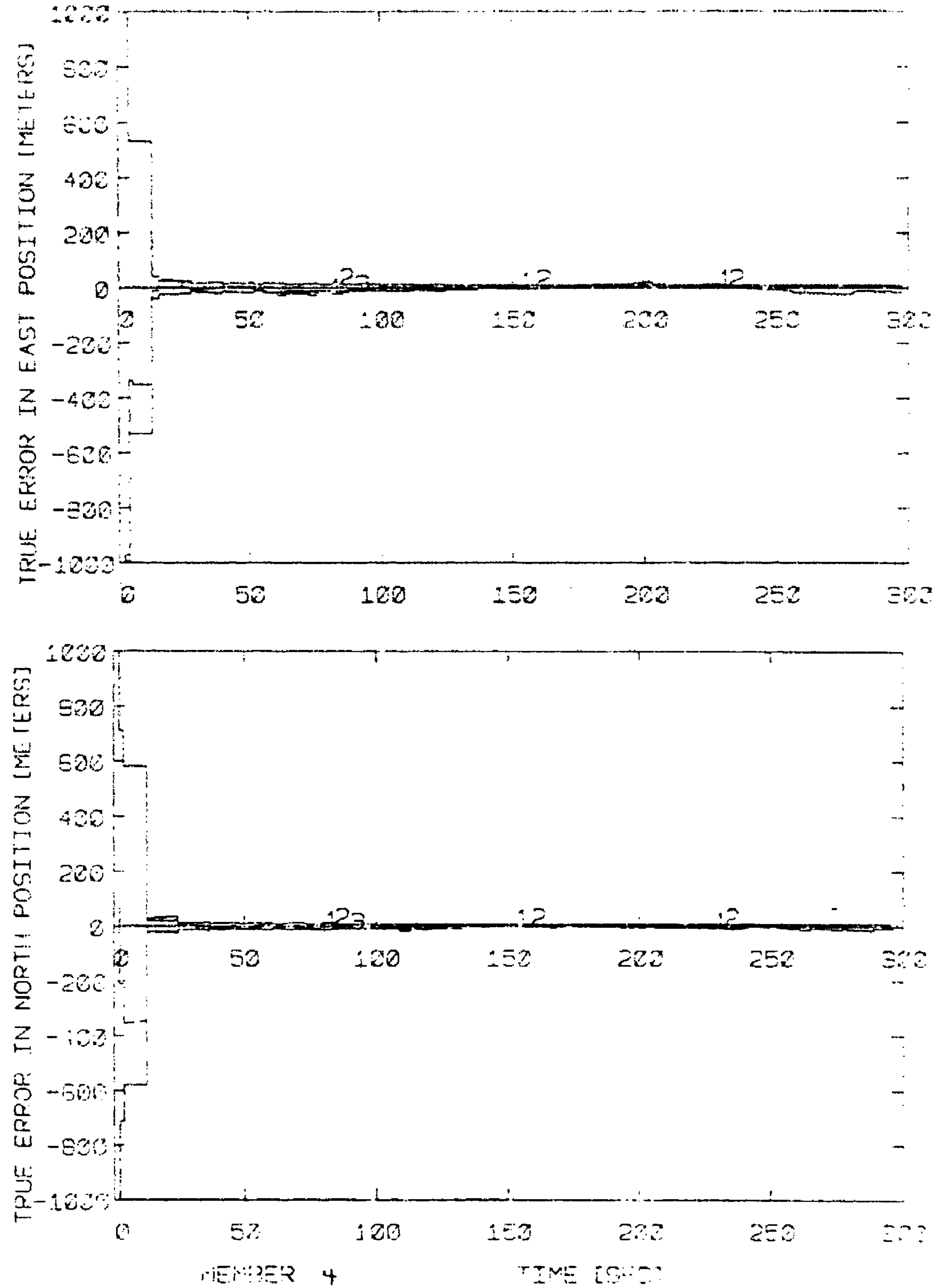


Fig. 8.11(a) Members 1 and 2 GPS Equipped, Member 4 Geodetic Nav. Results

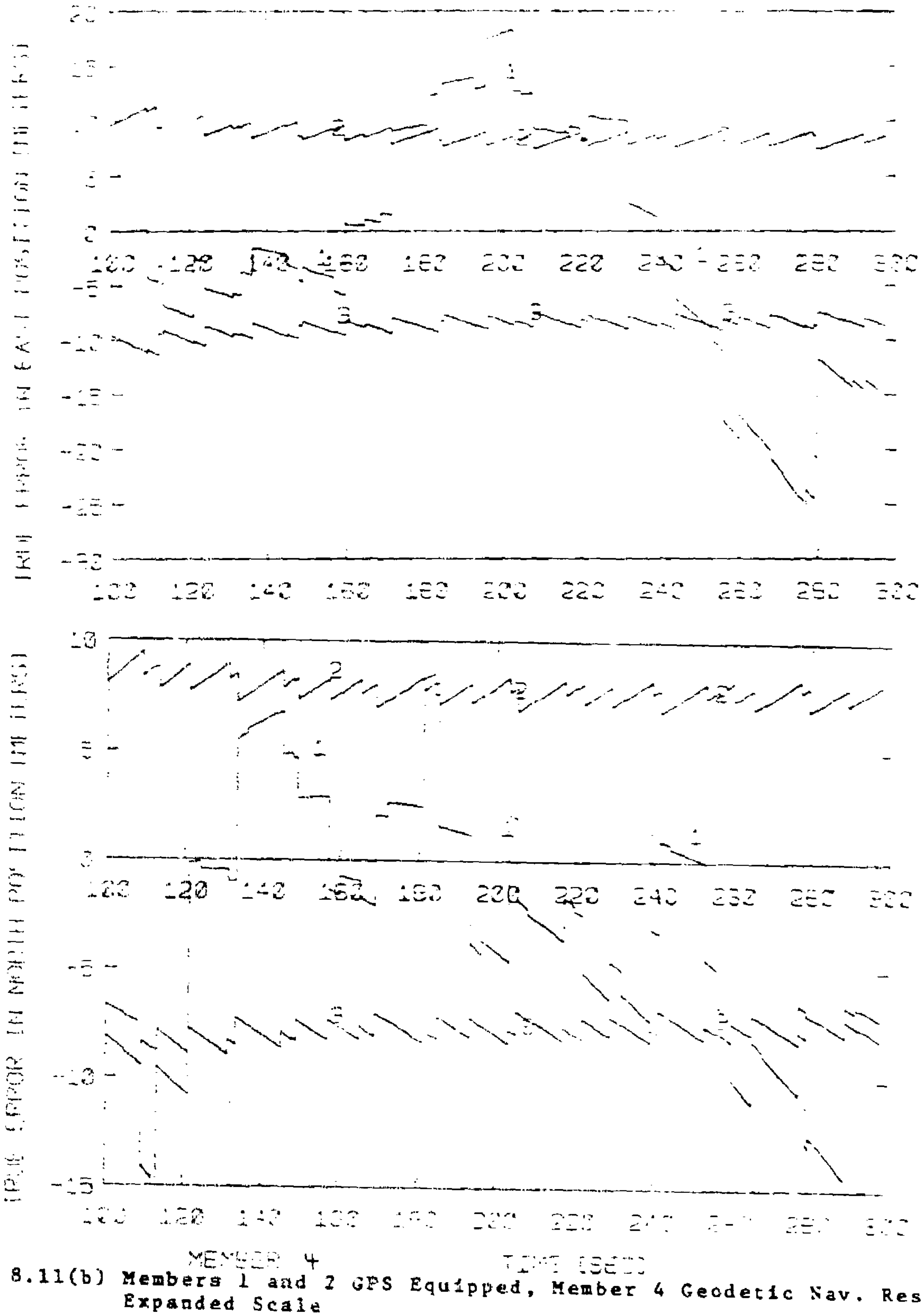


Fig. 8.11(b) Members 1 and 2 GPS Equipped, Member 4 Geodetic Nav. Results, Expanded Scale

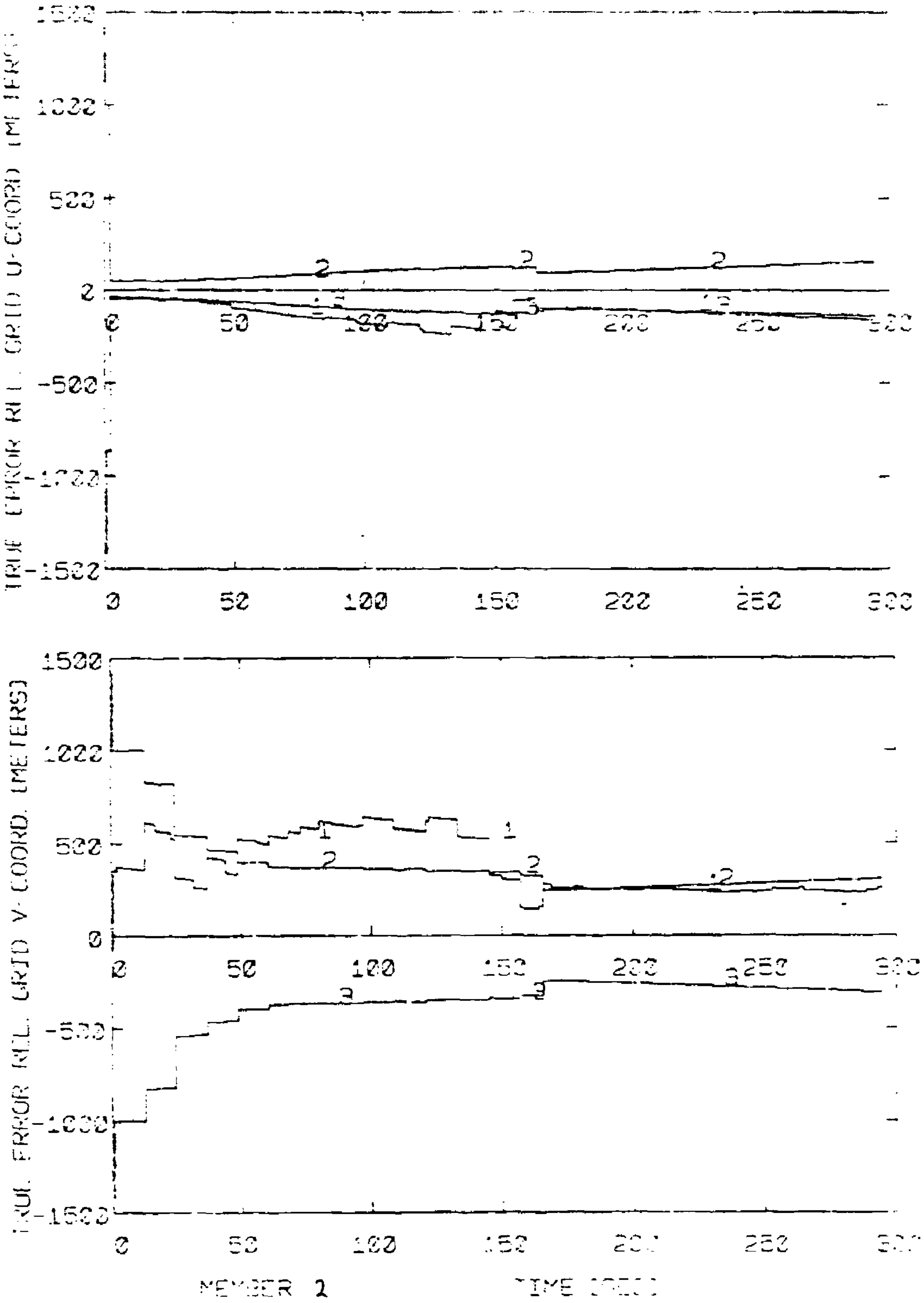


Fig. 8.12 Members 1 and 2 GPS Equipped, Member 2 Relnav Results

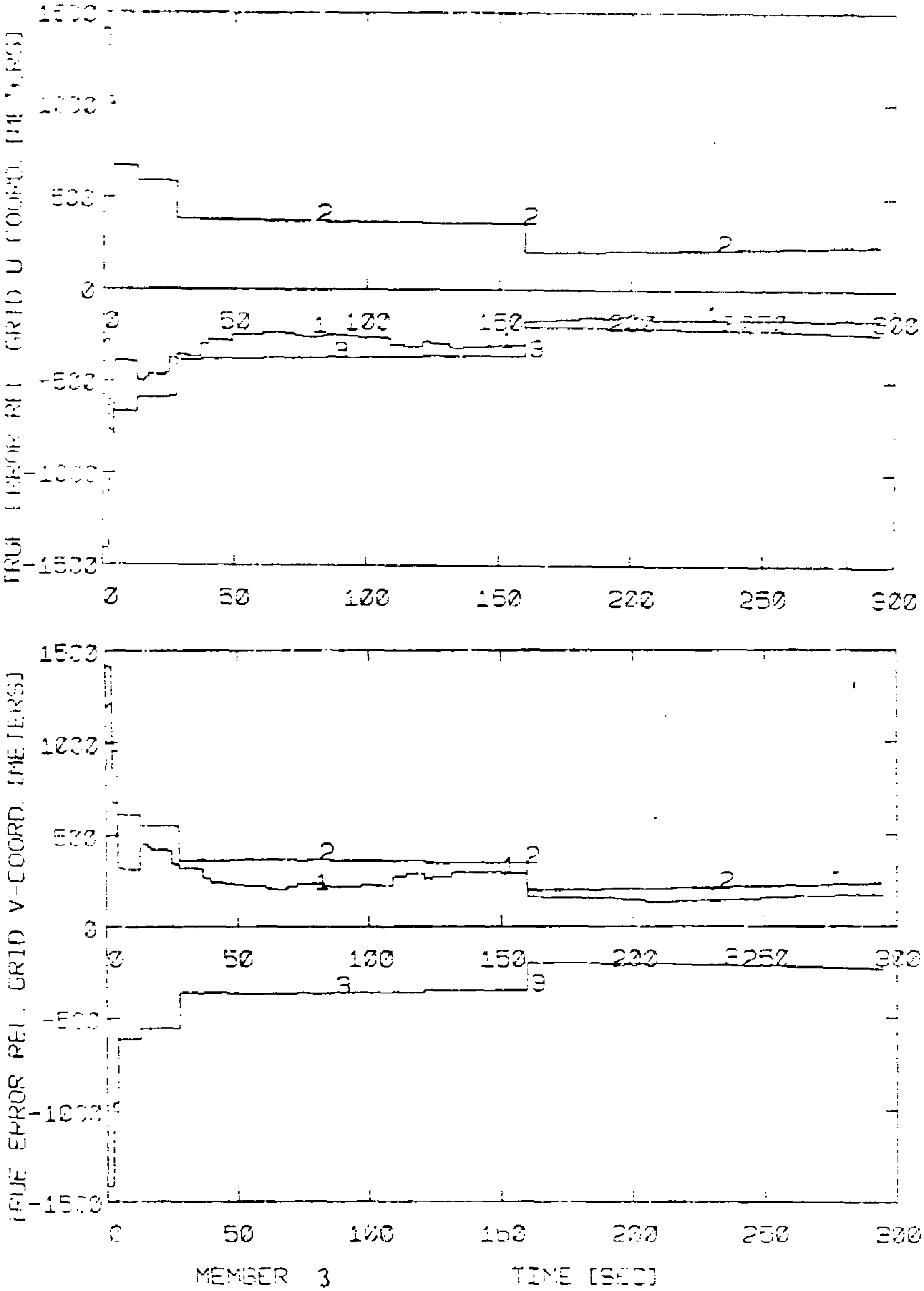


Fig. 8.13 Members 1 and 2 GPS Equipped, Member 3 Relnav Results

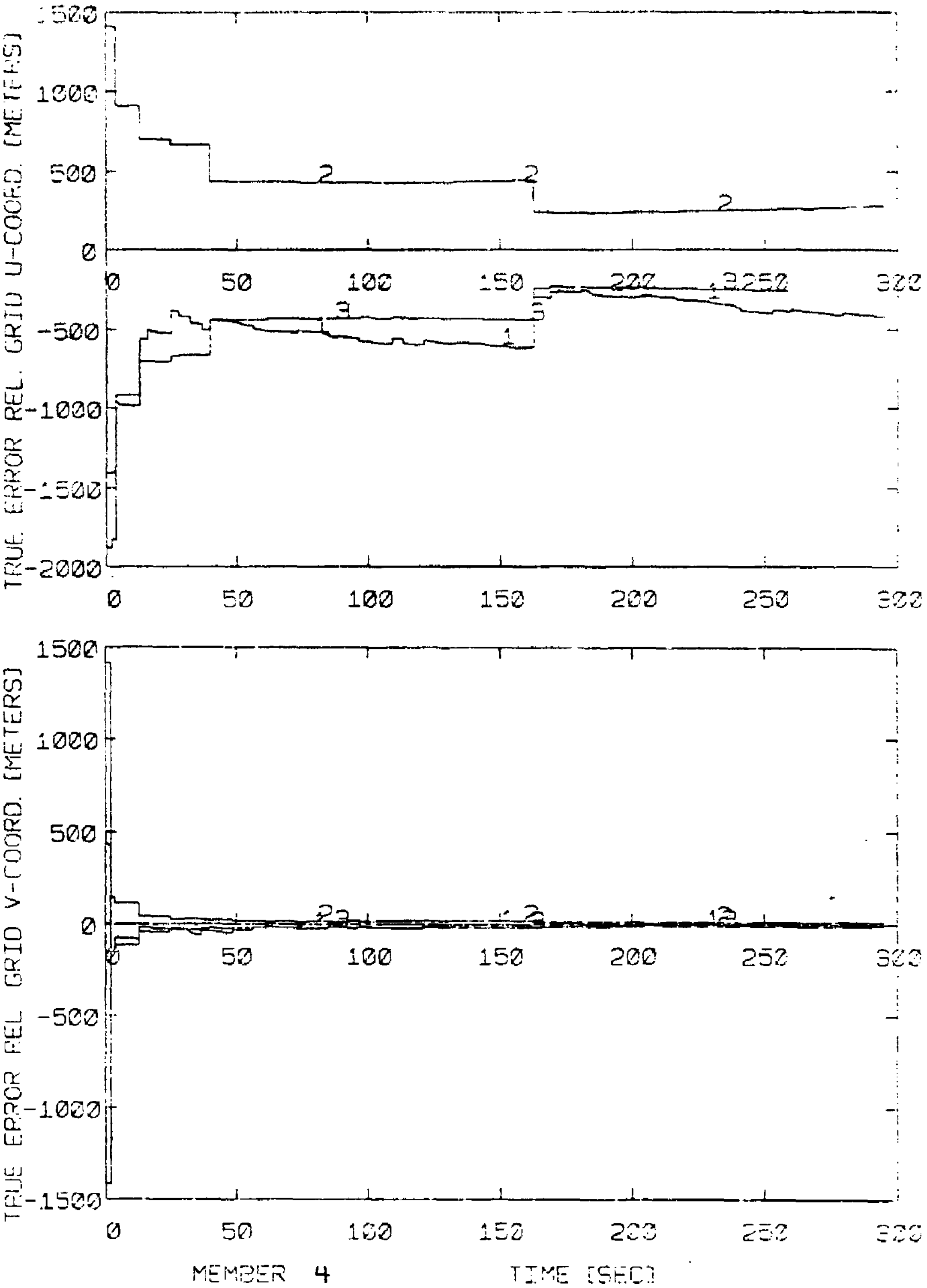


Fig. 8.14 Members 1 and 2 GPS Equipped, Member 4 Relnav Results

members 2, 3, and 4 in our simulation never successfully estimated the location of the origin of the relative grid, so they were unable to convert their accurate knowledge of geodetic position into accurate knowledge of relative position.

8.3 JTIDS/GPS/INS Performance Conclusions

The simulations show that a community with two GPS equipped members will have excellent geodetic navigation accuracy. The accuracy will be limited by the JTIDS measurement noise and the geometric dilution of precision.

Accurate position in the relative grid should follow. If every member knows their own geodetic position to 10 or 20 m accuracy, then they should also know their positions relative to each other to the same level of accuracy. With the ownstate formulation, to achieve this one must have a grid offset pseudo measurement type.

If only one member is GPS equipped, then the community geodetic accuracy is no better than the usual relative navigation accuracy. Relative motion is required to provide observability and measurement nonlinearily limits the accuracy that can be obtained.

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

9.1 Stability of Decentralized Navigation

We have made significant progress in understanding the stability of the ownstate organization of the decentralized navigation problem. The prior work of others reported in the literature was largely based on simulations. It was generally agreed that democratic organizations had a tendency to be unstable. The covariance based hierarchy was proposed to prevent instability. In evaluating simulations of covarianced based hierarchies, different authors have reached different conclusions. Some authors claim the covariance based hierarchy is stable and other authors say it is unstable. Lacking has been an analytic (as opposed to simulation) approach to understanding and proving stability.

We have succeeded in proving analytically the stability of one particular form of ownstate organization, namely the fixed rank hierarchy. The community errors are stable if the individual member ownstate filters are stable with respect to their suboptimal model. Sufficient conditions for individual filter stability are controllability of the suboptimal state by the assumed driving noises and observability of the suboptimal state. Controllability is built in by the filter designer.

Observability depends on the available measurements and their time varying geometry. For some members observability may depend on there being motion or on being allowed to use round trip timing.

The disadvantage of the fixed rank hierarchy is that there is no way of assigning fixed ranks that will be good for all missions. We gave a four member example in which for a certain assignment of the rankings each member had the necessary observability. The community navigation solution was guaranteed stable. But for an alternate assignment of the rankings, two of the members failed to have the necessary observability. Stability could not be guaranteed.

The covariance based hierarchy overcomes the disadvantage of the fixed rank hierarchy, assuring information from members with accurate navigation will propagate throughout the community. However it is not clear that this organization can be proven stable. Rank reversals can occur because after processing many measurements from supposedly more accurate sources, a member will believe it now has better accuracy than one or more of its previous sources. The role of source and user then reverses. Thus closed loop information paths do exist. It seems reasonable to predict that if the rank reversals occur infrequently relative to the settling times of the individual filters, then the community will be stable. But if the rank reversals are frequent and the settling times of the filters are large, then the community might be unstable.

Furthur research is needed to establish the conditions under which the covariance based hierarchy can be guaranteed stable. It may be necessary to enforce some rules concerning the allowable rate of rank reversals.

9.2 Navigation Based on Measurement Sharing

We have proposed an alternative to the ownstate organization of the community navigation problem. We call this navigation based on measurement sharing.

The proposed measurement sharing organization has excellent performance characteristics. Member estimates of their own errors approach the theoretical optimal. Positions and clock errors are successfully estimated in one or two cycles of measurements. The accuracy is at the level of the measurement noise.

The measurement sharing organization decouples the estimation processes of the members. Should one member's filter be poorly implemented and be producing bad estimates, this will have no effect on the estimates of other members. Similarly there is no possibility of an instability similar to the interaction instability of ownstate democratic organizations.

To limit the navigation message traffic to acceptable levels it is necessary to introduce the concept of primary members and limit measurement sharing to these members. Further research is needed to explore the necessary number of primary members. Net management rules for dropping a primary member and introducing a new primary member need to be explored.

A more precise estimate of the number of bits to be transmitted needs to be worked out. The issue of whether or not reset information must be broadcast must be considered.

The measurement sharing approach simplifies the source selection logic. There is no need for the covariance based hierarchy source selection logic.

The number of measurement types is reduced. There is only one time of arrival measurement type. Compare this with the current JTIDS baseline software, with its geodetic update type and relative grid update type. Also there is no need for the JTIDS grid offset measurement type, which has two components.

A suboptimal filter can be implemented in each member, deleting the rate states of the other members. Performance is nearly optimal, according to our 2-D low state small number of members simulations. These performance tests should be repeated in a 3-D simulator with more state variables in the truth and filter models and with more members.

9.3 JTIDS/GPS/INS Integration

Another objective of our research has been to explore the integration of JTIDS, GPS, and INS data. We reviewed the benefits of JTIDS/GPS/INS integration. We discussed the changes needed to the communication traffic and to the member software.

It is evident that to achieve all of the possible benefits of full JTIDS and GPS data integration there is a significant increase in the quantity of the data traffic between members and in the complexity of the member software. The designers of the network and the system software must conduct tradeoff studies to determine whether or not each added complication brings a sufficiently significant benefit. In some cases accuracy analysis or simulation is needed to quantify some of the performance benefits. From our own assessment of the costs and benefits, we have made some recommendations.

The ability to share geodetic information is already included in the JTIDS network data and member software. The performance benefits from using this capability seem significant enough, so this should be retained.

To obtain the benefits of JTIDS time being synchronized to GPS time, a small message must be added to the network stating the time offset. Members must have a hardwired timing line connecting their GPS and JTIDS clock functions. Software additions are needed to handle the timing information. The benefits are sufficiently worthwhile to justify these increases in complexity.

To share the GPS satellite data messages will add a significant amount of data traffic to the network. The benefit of continued GPS code tracking long after carrier tracking has been lost due to jamming may not be judged worth the traffic.

To share GPS lines of position may not add a great amount to the network data traffic. But it does increase the member software complexity in the Kalman filter and in the measurement selection logic. It seems to us that the benefit is not great because the only situation where this sharing is useful is in the unlikely event that no two GPS equipped members can get a GPS fix and yet three members can each track pairs of satellites.

Because of the poor benefit to complexity ratio we recommend that sharing of GPS lines of position not be implemented.

A key element in the member software is the Kalman filter. We designed a 19 state Kalman filter to integrate the JTIDS, GPS, INS, and barometric altimeter data. The Bierman UDU' factored algorithm was used to incorporate the measurements. We have added software protection for the nonlinear elongation of the measured times of arrival. This protection is based on Gaussian quadratic nonlinear filter theory. This protection is important for JTIDS because of the possible close ranges and moderately large estimation errors. Further development work is needed to simplify the protection equations for use in the flight computers.

In practical integrated navigation system design, an important design requirement is to prevent failures of one data source from corrupting what would otherwise be a good navigation solution based on the other data sources. For example in the design of integrated JTIDS/GPS/INS navigation systems one should ensure that a JTIDS data failure does not prevent GPS/INS navigation, and that a GPS data failure does not prevent JTIDS/INS navigation. What method of data integration can meet this partitioning requirement? Given sufficient computer capacity, a straightforward mechanization is to run three Kalman filters. One is the JTIDS/INS filter. The second is the GPS/INS filter. The third is the fully integrated JTIDS/GPS/INS filter. However if the available computer resources do not permit this solution, perhaps an alternate approach can be developed. Would it be possible to run two filters (the JTIDS/INS filter and the GPS/INS filter) and combine their estimates with an algorithm requiring less computer time than that of a third Kalman filter? Ideally the estimates from such an algorithm would be theoretically equivalent to that of the optimal filter. This is an important topic for future research.

9.4 Performance Simulations

We have developed a faithful simulation of integrated JTIDS/GPS/INS navigation. The simulator has been carefully designed to provide accurate predictions of navigation performance while not requiring excessive computer time. The simulator is implemented in Fortran. Careful attention was devoted to partitioning the simulation into separate functional subroutines, using the principles of top-down structured programming. Approximately half of the lines of source code are comment lines.

The JTIDS ownstate organization with covariance based hierarchy is simulated. The Kalman filter in each member is our 19 state Kalman filter design for integrating the JTIDS, GPS, INS, and barometric altimeter data.

Many simulations were run without the GPS data, to explore JTIDS/INS performance. The effect of various trajectories on relative navigation accuracy was explored. All cases had a single navigation controller. Several conclusions are supported by the simulation results. It is important that there be relative motion between members providing angular velocity of the lines of sight. With round trip timing a member can quickly determine its range from the navigation controller. The initial accuracy is limited by nonlinear effects if the range is close and the cross range uncertainty is large. With angular velocity of the line of sight to the navigation controller, a member eventually estimates its other component of relative position. If the line of sight returns to the original direction, nonlinear limitations on accuracy are overcome and the positional accuracy along the line of sight is of the order of the 10 m noise in the time of arrival measurements. Generally members were not able to significantly reduce their initial relative velocity errors, so cross range accuracy deteriorates noticeably when the direction to the navigation controller stops changing.

In some of the trajectories the navigation controller was not moving. This did not seem to prevent the other members from determining their relative position. If members did not have inertial systems, there would be an azimuth ambiguity with a fixed navigation controller. But with well aligned inertial systems, the members are able to determine their relative direction from the navigation controller.

In none of the trajectories examined could members simultaneously fix both components of horizontal position to the level of 10 m accuracy. A possible conclusion is that two navigation controllers (end of baseline concept) are needed to achieve accurate relative navigation. An important topic for further investigation would be the design and evaluation of alternate methods of establishing the relative grid. How does one implement a two navigation controller concept? How are the filters initialized and what are the navigation controller constraints? Performance evaluations with the detailed simulator most likely can show superior navigation accuracy and without the need for relative motion.

Round trip timing has a significant beneficial effect on relative navigation accuracy. This was demonstrated with a two member trajectory comparing the performance with and without RTT. With RTT the initial along range position can be determined. Without RTT no component of position is determined until there is significant variation in the line of sight to the navigation controller. With RTT the member has some success at estimating both components of relative velocity. As a result good navigation accuracy is held longer after the line of sight stops changing. Without RTT the member had no success at estimating relative velocity. Both components of position error at the end are growing at the rate of the initial relative velocity error.

A simulation with a democratic organization of four members and without round trip timing was shown to be wildly unstable. When RTT was added, the instability was eliminated. The ownstate filters still had hopelessly optimistic computed uncertainties.

The Gaussian quadratic equations, for protecting the filter from the effect of the nonlinear elongation of the time of arrival measurements, have been shown to be important for preventing filter divergence.

The 10 m and 7 m one sigma noises in the time of arrival and round trip timing measurements are not significant contributors to the relative navigation error. The more important sources of error seem to be the single navigation controller method of establishing the relative navigation grid, the low angular velocities of the lines of sight on some trajectories, and the measurement nonlinearity. If nonlinearity were not a significant problem, then the measurement noise would be a significant source of error. Other methods of grid setting, such as the end of baseline method can eliminate nonlinear difficulties. Under these organizations, the measurement noise level would be important.

The effect of filter precision was explored. There was a difference in numerical results observed in the second or third digit comparing a single precision run and a double precision run. Single precision accuracy here had 7 decimal digits. The sensitivity to precision is due to the way that relative navigation is carried implicitly as the difference between geodetic position estimates. Some flight computers, with single precision accuracy less than our 7 digits, may have to implement the Kalman filter in double precision.

Additional simulations were run using the GPS data. The simulations show that a community with two GPS equipped members will have excellent geodetic navigation accuracy. The accuracy will be limited by the JTIDS measurement noise and the geometric dilution of precision.

Accurate position in the relative grid should follow. If every member knows their own geodetic position to 10 or 20 m accuracy, then they should also know their positions relative to each other to the same level of accuracy. With the ownstate formulation, to achieve this one must have a grid offset pseudo measurement type.

If only one member is GPS equipped, then the community geodetic accuracy is no better than the usual relative navigation accuracy. Relative motion is required to provide observability and measurement nonlinearily limits the accuracy that can be obtained.

Our Kalman filter design for integrating the JTIDS, GPS, INS, and altimeter data performed entirely satisfactorily in the simulations.

The detailed simulator that we have developed has provided significant insight into the performance characteristics of both JTIDS/INS navigation and JTIDS/GPS/INS navigation. We expect that this tool will be a significant aid in future research concerning decentralized community navigation systems.

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