Decentralized Robust Adaptive Control of Nonlinear Systems With Unmodeled Dynamics

Yusheng Liu and Xing-Yuan Li

***Abstract*—The authors present a decentralized robust adaptive output feedback control scheme for a class of large-scale nonlinear systems of the output feedback canonical form with unmodeled dynamics. A modified dynamic signal is introduced for each subsystem to dominate the unmodeled dynamics and an adaptive nonlinear damping is used to counter the effects of the interconnections. It is shown that under certain assumptions, the proposed decentralized adaptive control scheme guarantees that all the signals in the closed-loop system are bounded in the presence of unmodeled dynamics, high-order interconnections and bounded disturbances. Furthermore, by choosing the design constants appropriately, the tracking error can be made arbitrarily small regardless of the interconnections, disturbances, and unmodeled dynamics in the system. An illustration example demonstrates the effectiveness of the proposed scheme.**

***Index Terms*—Adaptive, decentralized control, large-scale systems, nonlinear, robust.**

I. Introduction

Much progress has been made in the field of decentralized adaptive control, see e.g., [1]–[8], [18], [19], and the references therein. Particularly, a decentralized adaptive output control scheme was presented in [8] for a class of large-scale nonlinear systems that are transformable via a global diffeomorphism into the output feedback canonical form. The scheme guarantees global uniform boundedness of the tracking error and all the states of the closed-loop system in the presence of parametric and dynamic uncertainties in the interconnections and bounded disturbances. However, the scheme cannot apply to the systems with unmodeled dynamics. The work in [18] presented a decentralized adaptive output feedback control scheme for large-scale systems with nonlinear interconnections. The scheme of [18] has several advantages: 1) it achieves asymptotic tracking; 2) the considered large-scale systems may possess an unknown, nonzero equilibrium. But, the scheme cannot apply to the systems with unmodeled dynamics and disturbances.

On the other hand, the robust adaptive control of nonlinear systems has emerged as an active research area recently, e.g., [10]–[13]. Especially, the problem of robust adaptive control of nonlinear systems with unmodeled dynamics was studied in [13]. Based on the concept of input-to-state practical stability, a dynamic signal was introduced in [13] to dominate the unmodeled dynamics. It has been shown that such a signal is a useful tool for handling unmodeled dynamics. Using a combined backstepping and small-gain approach, paper [20] presented an adaptive output feedback control scheme for nonlinear systems with unmodeled dynamics. As an extension of the centralized case in [20], a decentralized robust adaptive output feedback regulation scheme was presented for a class of large-scale nonlinear systems in [19].

It should be pointed out that the class of large-scale nonlinear systems considered in [19] is broader than the class of nonlinear systems with polynomial bounds as in [7], [8], [16] and [15], in the sense that the interconnections are subject to general nonlinear bounds. Although, [17] studies a different problem—decentralized H_∞ almost disturbance decoupling for a class of large-scale nonlinear systems, it is also assumed that the uncertain interconnections are bounded by general nonlinear functions. Theoretically, to consider a broader class of nonlinear systems is important, but some other issues such as controller design and practical problems must be considered. With the general nonlinear bounds, the design of the controllers depends on a qualitative approach (as in [19] and [20]). For large-scale nonlinear systems with many decentralized controllers, it is difficult to choose so many design parameters and functions by qualitative approach. Furthermore, with the general nonlinear bounds, the design of the ith decentralized controller is related to that of the rest decentralized controllers (see [19, (25) and (10)]). This implies that redesigns of controllers are needed when subsystems are appended to original system or taken offline (e.g., during faults in a power system), which is neither economical nor feasible for the control of large-scale systems. Besides, many practical nonlinear interconnected systems do have some uncertainties and interconnections subject to polynomial bounds, e.g., robot manipulators with the presence of Coriolis and centripetal terms have uncertainties bounded by second-order polynomials [16] and the electric power deviations between subsystems are bounded by first order polynomials [21]. Therefore, for the practical purpose and the above reasons, we still consider the case where uncertainties and interconnections are subject to polynomial bounds in this work. As in [8], with our approach, no controller redesign is needed if subsystems are added online or taken offline as long as the order of the interconnections of the appended system is less or equal to that of the original system.

This work presents a decentralized robust adaptive control scheme for large-scale nonlinear systems of the output feedback canonical form with unmodeled dynamics. First, in each subsystem, a modified dynamic signal is introduced to dominate the unmodeled dynamics and a nonlinear adaptive damping is used to counter the effects of the interconnections. Then, we employ the systematic design procedure to obtain the decentralized robust adaptive output feedback controllers. It is shown that under certain assumptions, the proposed decentralized adaptive control scheme guarantees that all the signals in the closed-loop system are bounded in the presence of unmodeled dynamics, high order interconnections and bounded disturbances. Furthermore, the tracking error can be made arbitrarily small by choosing the design constants appropriately.

II. Problem Statement

Consider a large-scale nonlinear system of the output feedback canonical form with unmodeled dynamics given by the following equations:

$$\begin{aligned}
\dot{\zeta}_i &= q_i(\zeta_i, y_i)\\
\dot{z}_{i1} &= z_{i2} + \phi_{i1}(y_1,\ldots,y_N)\\
&\quad + \xi_{i1}(y_1,\ldots,y_N)\,\omega(t) + \Delta_{i1}(\zeta_i, y_i)\\
&\vdots\\
\dot{z}_{i,\rho_i-1} &= z_{i,\rho_i} + \phi_{i,\rho_i-1}(y_1,\ldots,y_N)\\
&\quad + \xi_{i,\rho_i-1}(y_1,\ldots,y_N)\,\omega(t) + \Delta_{i,\rho_i-1}(\zeta_i, y_i)\\
\dot{z}_{i,\rho_i} &= z_{i,\rho_i+1} + \phi_{i,\rho_i}(y_1,\ldots,y_N)\\
&\quad + \xi_{i,\rho_i}(y_1,\ldots,y_N)\,\omega(t)\\
&\quad + b_{i,k_i-\rho_i}\delta_i(y_i)\,u_i + \Delta_{i,\rho_i}(\zeta_i, y_i)\\
&\vdots\\
\dot{z}_{i,k_i} &= \phi_{i,k_i}(y_1,\ldots,y_N) + \xi_{i,k_i}(y_1,\ldots,y_N)\,\omega(t)\\
&\quad + b_{i0}\delta_i(y_i)\,u_i + \Delta_{i,k_i}(\zeta_i, y_i)\\
y_i &= z_{i1} \quad 1 \le i \le N
\end{aligned} \tag{1}$$

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where $z_i = [z_{i1}, \ldots, z_{i,k_i}]^T$, $y_i \in R$ and $u_i \in R$ are the state, the output and the control input of the ith subsystem, respectively; $\omega(t)$ is a bounded unmeasurable disturbance; $\phi_{ij}(y_1, \ldots, y_N)$ and $\xi_{ij}(y_1, \ldots, y_N)$ are the dynamic and parameter uncertainties related to the interconnections between subsystems; ζ_i is the unmeasurable unmodeled dynamics and $\Delta_{ij}(\zeta_i, y_i)$ represents the unmeasurable uncertainties related to the unmodeled dynamics. We assume that ϕ_{ij}, ξ_{ij} and Δ_{ij} are unknown, but satisfy

$$\begin{aligned} \|\phi_{ij}(y_1, \ldots, y_N)\| &\le \sum_{k=1}^{p_{ij}} \sum_{l=1}^{N} \varsigma_{ijl}^k \|y_l\|^k \\ \|\xi_{ij}(y_1, \ldots, y_N)\| &\le \sum_{k=1}^{p_{ij}} \sum_{l=1}^{N} \vartheta_{ijl}^k \|y_l\|^k \\ \|\Delta_{ij}(\zeta_i, y_i)\| &\le \sum_{k=1}^{p_{ij}} \left(\eta_{ijk} \|\zeta_i\|^k + \varphi_{ijk} \|y_i\|^k\right) \end{aligned} \tag{2}$$

where ς_{ijl}^k, ϑ_{ijl}^k, η_{ijk} and φ_{ijk} are unknown constants, $p = \max\{p_{ij}, 1 \le i \le N, 1 \le j \le k_i\}$ is known. To highlight the main idea of this work, it is assumed that the parameters b_{ij}, $1 \le i \le N$, $0 \le j \le k_i - \rho_i$, are also known. As indicated in [8], the case of unknown b_{ij} can be treated identically.

The objective here is to design a decentralized robust adaptive output controller for each subsystem such that the output $y_i(t)$ tracks a given reference signal $y_{i,r}(t)$ and all the signals of the closed-loop system are bounded in the presence of unmodeled dynamics, high order interconnections and bounded disturbances. We need the following assumptions.

Assumption 1: The zero dynamics of system (1) is exponentially stable, i.e., the polynomial $P_i(s) = b_{i,k_i-\rho_i} s^{k_i-\rho_i} + \cdots + b_{i1}s + b_{i0}$ is strict Hurwitz.

Assumption 2: $\delta_i(y_i) \neq 0, \forall t \ge 0$.

Assumption 3: The reference signal $y_{i,r}(t)$ is bounded with bounded derivatives up to the ρ_ith order and $y_{i,r}^{(\rho_i)}(t)$ is piecewise continuous.

Assumption 4: The unmodeled dynamics is exponentially input-to-state practically stable (exp-ISpS) [13]; i.e., the system $\dot{\zeta}_i = q_i(\zeta_i, y_i)$ has an exp-ISpS Lyapunov function $V_i(\zeta_i)$ which satisfies

$$\alpha_{i1}(\|\zeta_i\|) \le V_i(\zeta_i) \le \alpha_{i2}(\|\zeta_i\|) \tag{3}$$

$$\frac{\partial V_i(\zeta_i)}{\partial \zeta_i} q_i(\zeta_i, y_i) \le -c_{i0} V_i(\zeta_i) + \gamma_i(\|y_i\|) + d_{i0} \tag{4}$$

where α_{i1}, α_{i2} are known functions of class K_∞ and $c_{i0} > 0$, $d_{i0} \ge 0$ are known constants. Without loss of generality, we assume $\gamma_i(\cdot)$ has the following form $\gamma_i(s) = s^2 \gamma_{i0}(s^2)$ where γ_{i0} is a nonnegative smooth function. Otherwise, as indicated in [13], it suffices to replace γ_i in (4) by $\|y_i\|^2 \gamma_{i0}(\|y_i\|^2) + \bar{\varepsilon}_{i0}$ with $\bar{\varepsilon}_{i0} > 0$ being a sufficiently small-real number.

III. The Design of Decentralized Robust Adaptive Output Feedback Controllers

First, we rewrite the ith subsystem (1) as follows:

$$\begin{aligned} \dot{\zeta}_i &= q_i(\zeta_i, y_i) \\ \dot{z}_i &= A_i z_i + \bar{k}_i y_i + \phi_i(y_1, \ldots, y_N) \\ &\quad + \xi_i(y_1, \ldots, y_N)\omega(t) \\ &\quad + \Delta_i(\zeta_i, y_i) + b_i \delta_i(y_i) u_i \end{aligned} \tag{5}$$

where

$$A_i = \begin{bmatrix} -k_{i1} & \\ \vdots & I \\ -k_{i,k_i} & 0 \cdots 0 \end{bmatrix} \quad \bar{k}_i = \begin{bmatrix} k_{i1} \\ \vdots \\ k_{i,k_i} \end{bmatrix}$$

$$\phi_i = \begin{bmatrix} \phi_{i1} \\ \vdots \\ \phi_{i,k_i} \end{bmatrix} \quad \xi_i = \begin{bmatrix} \xi_{i1} \\ \vdots \\ \xi_{i,k_i} \end{bmatrix}$$

$$\Delta_i = \begin{bmatrix} \Delta_{i1} \\ \vdots \\ \Delta_{i,k_i} \end{bmatrix} \quad b_i = [0 \quad \cdots \quad 0 \quad b_{i,k_i-\rho_i} \quad \cdots \quad b_{i0}]^T$$

and $\bar{k}_i$ is chosen such that A_i is a strict Hurwitz matrix. Thus, given a $Q_i > 0$, there exists a $P_i > 0$ satisfying

$$A_i^T P_i + P_i A_i = -Q_i. \tag{6}$$

To estimate the states of the system, we use the following observer for the ith subsystem:

$$\dot{\hat{z}}_i = A_i \hat{z}_i + \bar{k}_i y_i + b_i \delta_i(y_i) u_i. \tag{7}$$

Let $e_i = z_i - \hat{z}_i$. Then

$$\begin{aligned} \dot{e}_i = A_i e_i &+ \phi_i(y_1, \ldots, y_N) \\ &+ \xi_i(y_1, \ldots, y_N)\omega(t) + \Delta_i(\zeta_i, y_i). \end{aligned} \tag{8}$$

Define a dynamic signal

$$\dot{\bar{r}}_i = -\bar{c}_{i0}\bar{r}_i + z_{i1}^2 \gamma_{i0}\left(z_{i1}^2\right) + d_{i0} \quad \bar{r}_i(t_0) = \bar{r}_i^0 > 0 \tag{9}$$

where $\bar{c}_{i0} \in (0, c_{i0})$ The properties of the dynamic signal are given by the following lemma.

Lemma 1 [13]: If the system $\dot{\zeta}_i = q_i(\zeta_i, y_i)$ is exp-ISpS, then for any constant $\bar{c}_{i0} \in (0, c_{i0})$, any initial instant $t_0 \ge 0$, any initial condition $\zeta_i^0 = \zeta_i(t_0)$ and $\bar{r}_i^0 > 0$, for any function γ_{i0} such that $\gamma_{i0}(y_i) \ge \gamma_i(\|y_i\|)$, there exist a finite $T_i^0 = T_i^0\left(\bar{c}_{i0}, \bar{r}_i^0, \zeta_i^0\right) \ge 0$ and a nonnegative function $D_i(t_0, t)$ defined for all $t \ge t_0$ such that $D_i(t_0, t) = 0, \forall t \ge t_0 + T_i^0$ and $V_i(\zeta_i(t)) \le \bar{r}_i(t) + D_i(t_0, t)$ for all $t \ge t_0$ where the solutions are defined.

In this work, we introduce the following modified dynamic signal in the design of decentralized robust adaptive feedback controllers:

$$\begin{aligned} \dot{r}_i &= \begin{cases} -\bar{c}_{i0} r_i + z_{i1}^2 \gamma_{i0}\left(z_{i1}^2\right) + d_{i0} & \text{if } m_{ri} > 0 \\ 0, & \text{if } m_{ri} \le 0 \end{cases} \\ r_i(t_0) &= r_i^0 > 0 \end{aligned} \tag{10}$$

where $m_{ri} = -\bar{c}_{i0} r_i + z_{i1}^2 \gamma_{i0}\left(z_{i1}^2\right) + d_{i0}$. It can be seen that $\dot{r}_i \ge 0$ and $r_i(t) \ge \bar{r}_i(t), \forall t \ge t_0$ if $r_i^0 = \bar{r}_i^0$. Using Lemma 1, we have

$$V_i(\zeta_i(t)) \le r_i(t) + D_i(t_0, t), \tag{11}$$

where $D_i = (t_0, t) = 0, \forall t \ge t_0 + T_i^0$.

Next, we employ the systematic design procedure of [9], [14], and [8] to obtain the decentralized robust adaptive feedback controllers. To reduce notational complexity, as in [8], we assume that all subsystems have a uniform relative degree, i.e., $\rho_i = \rho$, $1 \le i \le N$. However, it can be treated similarly without any difficulties when the subsystems have different relative degrees.

Step 0: Define

$$\chi_{i1} = z_{i1} - y_{i,r},\ \bar{e}_i = \left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\}^{-1/2} e_i \quad (12)$$

where α_{i1}^{-1} is the inverse function of α_{i1}^{1} and is again a function of class K_∞. Then, the dynamics of the tracking error is

$$\dot{\chi}_{i1} = \hat{z}_{i2} - \dot{y}_{i,r} + \phi_{i1}(y_1,\ldots,y_N) + \xi_{i1}(y_1,\ldots,y_N)\,\omega(t) + \Delta_{i1}(\zeta_i, y_i) + \left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\}^{1/2} \bar{e}_{i2}. \quad (13)$$

Since $z_{i2} = \hat{z}_{i2} + e_{i2}$, we have

$$\begin{aligned}\dot{\bar{e}}_i =& A_i\bar{e}_i + \left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\}^{-1/2} \\ &\cdot\left[\phi_i(y_1,\ldots,y_N) + \xi_i(y_1,\ldots,y_N)\,\omega(t) + \Delta_i(\zeta_i,y_i)\right] \\ &- \left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\}^{-1} \\ &\cdot\left\{\sum_{k=1}^{p} 2^{2k}k\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k-1}\right\}\cdot\frac{\partial\alpha_{i1}^{-1}(2r_i)}{\partial r_i}2\dot{r}_i\bar{e}_i. \end{aligned} \quad (14)$$

We propose the following virtual decentralized control law for (13):

$$\begin{aligned}\hat{z}_{i2} =& -\pi_{i1}\chi_{i1} - (N+1)p\chi_{i1} \\ &- \frac{1}{2}\chi_{i1}\left\{\sum_{k=1}^{p} 2^{k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{k}\right\}^2 \\ &- \frac{1}{2}\chi_{i1}\left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\} \\ &- \hat{\beta}_i\sum_{k=1}^{p} 2^{2k}\chi_{i1}^{2k-1} + \dot{y}_{i,r} \\ \triangleq& M_{i1}\left(\chi_{i1},\hat{\beta}_i,r_i\right) + \dot{y}_{i,r} \end{aligned} \quad (15)$$

where $\pi_{i1} > 0$ is a design constant and $\hat{\beta}_i$ is an adaptive gain used to counter the effects of the interconnections between subsystems. We present the following adaptive law for $\hat{\beta}_i$:

$$\dot{\hat{\beta}}_i = \Gamma_i\sum_{k=1}^{p} 2^{2k}\chi_{i1}^{2k} - \sigma_i\Gamma_i\hat{\beta}_i \quad \hat{\beta}_i(t_0) > 0 \quad (16)$$

where $\Gamma_i > 0$, $\sigma_i > 0$ are design constants.

Since $\hat{z}_{i2}$ is not the actual control, we define

$$\chi_{i2} = \hat{z}_{i2} - M_{i1}\left(\chi_{i1},\hat{\beta}_i,r_i\right) - \dot{y}_{i,r}. \quad (17)$$

Thus

$$\begin{aligned}\dot{\chi}_{i1} =& \chi_{i2} + M_{i1}\left(\chi_{i1},\hat{\beta}_i,r_i\right) + \phi_{i1}(y_1,\ldots,y_N) + \Delta_{i1}(\zeta_i,y_i) \\ &+ \xi_{i1}(y_1,\ldots,y_N)\,\omega(t) \\ &+ \left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\}^{1/2}\bar{e}_{i2}. \end{aligned} \quad (18)$$

Choose the composite Lyapunov function candidate as follows:

$$V_0 = \sum_{i=1}^{N}\left\{\bar{e}_i^T P_i\bar{e}_i + \chi_{i1}^2 + \Gamma_i^{-1}\left(\hat{\beta}_i - \beta_i^*\right)^2\right\} \quad (19)$$

where β_i^* is the desired value of $\hat{\beta}_i$. Differentiating V_0 along the solutions of (14), (16). and (18) yields (20), as shown at the bottom of the page, where

$$\begin{aligned} d_{ik} =& \sum_{l=1}^{N}\|P_l\|^2\sum_{j=1}^{k_i}\left(\varsigma_{lji}^k\right)^2 \\ v_{ik} =& \sum_{l=1}^{N}\|P_l\|^2\sum_{j=1}^{k_i}\left(\vartheta_{lji}^k\omega_{\max}\right)^2 \\ \omega_{\max} =& \sup(|\omega(t)|),\ \bar{\eta}_{ik} = \sum_{j=1}^{k_i}\|P_i\|^2(\eta_{ijk})^2 \\ d_{1ik} =& \sum_{j=1}^{N}\left(\varsigma_{j1i}^k\right)^2 \\ \bar{\varphi}_{ik} =& \sum_{j=1}^{k_i}\|P_i\|^2(\varphi_{ijk})^2,\ \bar{\eta}_{i1} = \max(\eta_{i1k}, 1 \le k \le p_{i1}) \\ v_{1ik} =& \sum_{j=1}^{N}\left(\vartheta_{j1i}^k\omega_{\max}\right)^2,\ \bar{\eta}_{1ik} = (\eta_{i1k})^2,\ \bar{\varphi}_{1ik} = (\varphi_{i1k})^2. \end{aligned}$$

Differentiating (17) gives (21), shown at the bottom of the next page.

Step m ($1 \le m \le \rho - 2$): Assume that in step $m-1$, we designed a virtual control $M_{i,m}$ and defined

$$\chi_{i,m+1} = \hat{z}_{i,m+1} - M_{i,m}\left(y_i,\hat{z}_{i1},\ldots,\hat{z}_{i,m},\hat{\beta}_i,r_i,y_{i,r},\ldots,y_{i,r}^{(m)}\right). \quad (22)$$

$$\begin{aligned}\dot{V}_0 \le \sum_{i=1}^{N}\Bigg\{& -\left[\lambda_{\min}(Q_i) - 4\right]\|\bar{e}_i\|^2 + \sum_{k=1}^{p}2^{2k}\Big(d_{ik} + v_{ik} + \bar{\varphi}_{ik} \\ &+ d_{1ik} + v_{1ik} + \bar{\varphi}_{1ik}\Big)\left(\|\chi_{i1}\|^{2k} + \|y_{i,r}\|^{2k}\right) + \sum_{k=1}^{p}\bar{\eta}_{ik} + \bar{\eta}_{i1}^2 \\ &+ \sum_{k=1}^{p}(\bar{\eta}_{ik} + \bar{\eta}_{1ik})\,2^{2k}\left[\alpha_{i1}^{-1}(2D_i(t_0,t))\right]^{2k} \\ &+ 2\chi_{i1}\chi_{i2} - 2\pi_{i1}\chi_{i1}^2 \\ &- 2\beta_i^*\sum_{k=1}^{p}2^{2k}\chi_{i1}^{2k} - \sigma_i\left(\hat{\beta}_i - \beta_i^*\right)^2 + \sigma_i\beta_i^{*2}\Bigg\} \end{aligned} \quad (20)$$

Thus, (23), shown at the bottom of the next page, holds true, where the definitions of $\Omega_{i,m+1}$ and $H_{i,m+1}$ are similar to those of Ω_{i2} and H_{i2} given by (21), which are obtained by differentiating $M_{i,m}$ and $\hat{z}_{i,m+1}$ with respect to t. We choose the virtual control (24), shown at the bottom of the page.

Since $\hat{z}_{i,m+2}$ is not the actual control, we define

$$\chi_{i,m+2} = \hat{z}_{i,m+2} - M_{i,m+1} \cdot \left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,m+1}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(m+1)}\right). \quad (25)$$

$$\begin{aligned}\dot{\chi}_{i2} =& \hat{z}_{i3} + k_{i2}(y_i - \hat{z}_{i1}) - \frac{\partial M_{i1}}{\partial \chi_{i1}}\left[\chi_{i2} + M_{i1}\left(\chi_{i1}, \hat{\beta}_i, r_i\right)\right] \\ & - \ddot{y}_{i,r} - \frac{\partial M_{i1}}{\partial \hat{\beta}_i}\dot{\hat{\beta}}_i - \frac{\partial M_{i1}}{\partial r_i}\dot{r}_i \\ & - \frac{\partial M_{i1}}{\partial \chi_{i1}}\Bigg\{\phi_{i1}(y_1, \ldots, y_N) + \xi_{i1}(y_1, \ldots, y_N)\,\omega(t) \\ & + \Delta_{i1}(\zeta_i, y_i) + \left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\}^{1/2} \bar{e}_{i2}\Bigg\} \\ & \triangleq \hat{z}_{i3} + \Omega_{i2}\left(y_i, \hat{z}_{i1}, \hat{z}_{i2}, \hat{\beta}_i, r_i, y_{i,r}, \dot{y}_{i,r}, \ddot{y}_{i,r}\right) \\ & + H_{i2}\left(\chi_{i1}, \hat{\beta}_i, r_i\right)\Bigg\{\phi_{i1}(y_1, \ldots, y_N) + \xi_{i1}(y_1, \ldots, y_N)\,\omega(t) \\ & + \Delta_{i1}(\zeta_i, y_i) + \left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\}^{1/2} \bar{e}_{i2}\Bigg\}.\end{aligned} \quad (21)$$

$$\begin{aligned}\dot{\chi}_{i,m+1} =& \hat{z}_{i,m+2} + \Omega_{i,m+1}\left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,m+1}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(m+1)}\right) \\ & + H_{i,m+1}\left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,m+1}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(m+1)}\right) \\ & \Bigg\{\phi_{i1}(y_1, y_2, \ldots, y_N) + \xi_{i1}(y_1, y_2, \ldots, y_N)\,\omega(t) \\ & + \Delta_{i1}(\zeta_i, y_i) + \left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\}^{\frac{1}{2}} \bar{e}_{i2}\Bigg\}\end{aligned} \quad (23)$$

$$\begin{aligned}\hat{z}_{i,m+2} =& -\Bigg\{\Omega_{i,m+1}\left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,m+1}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(m+1)}\right) + \chi_{i,m} \\ & + \pi_{i,m+1}\chi_{i,m+1} + \frac{1}{2}\chi_{i,m+1}\left\{\sum_{k=1}^{p} 2^{k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{k}\right\}^2 \\ & \cdot H_{i,m+1}^2\left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,m+1}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(m+1)}\right) \\ & + (p+1)\chi_{i,m+1}H_{i,m+1}^2\left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,m+1}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(m+1)}\right) \\ & + \frac{1}{2}\chi_{i,m+1}\left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\} \\ & \cdot H_{i,m+1}^2\left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,m+1}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(m+1)}\right)\Bigg\} \\ \triangleq & M_{i,m+1}\left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,m+1}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(m+1)}\right).\end{aligned} \quad (24)$$

Choose the composite Lyapunov function candidate as follows:

$$V_m = V_{m-1} + \sum_{i=1}^{N} \chi_{i,m+1}^2. \quad (26)$$

Then, (27), shown at the bottom of the page, holds.

Step $\rho - 1$: Assume that in step $\rho - 2$, we designed a virtual control $M_{i,\rho-1}$ for subsystem $(\bar{e}_i, \chi_{i1}, \ldots, \chi_{i,\rho-1}, \hat{\beta}_i)$ and defined

$$\chi_{i,\rho} = \hat{z}_{i,\rho} - M_{i,\rho-1}\left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,\rho-1}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(\rho-1)}\right). \quad (28)$$

Thus

$$\begin{aligned}\dot{\chi}_{i,\rho} =& b_{i,k_i-\rho}\delta_i(y_i)u_i + \hat{z}_{i,\rho+1} \\ &+ \Omega_{i,\rho}\left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,\rho}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(\rho)}\right) \\ &+ H_{i,\rho}\left(y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,\rho}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(\rho)}\right) \\ &\cdot \Bigg\{\phi_{i1}(y_1, \ldots, y_N) + \Delta_{i1}(\zeta_i, y_i) \\ &\quad + \xi_{i1}(y_1, \ldots, y_N)\omega(t) \\ &\quad + \left\{1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right\}^{\frac{1}{2}} \bar{e}_{i2}\Bigg\}. \end{aligned} \quad (29)$$

We choose the composite Lyapunov function candidate as follows

$$V_{\rho-1} = V_{\rho-2} + \sum_{i=1}^{N} \chi_{i,\rho}^2. \quad (30)$$

We have (31), shown at the bottom of the page, where $(\cdot) = (y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,\rho}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(\rho)})$.

We propose the following decentralized robust adaptive control for the ith subsystem:

$$\begin{aligned} u_i = & -\frac{1}{b_{i,k_i-\rho}\delta_i(y_i)} \\ & \cdot \Bigg\{\chi_{i,\rho-1} + \hat{z}_{i,\rho+1} + \Omega_{i,\rho}(\cdot) + \pi_{i,\rho}\chi_{i,\rho} \\ &\quad + (p+1)\chi_{i,\rho}H_{i,\rho}^2(\cdot) \\ &\quad + \frac{1}{2}\chi_{i,\rho}H_{i,\rho}^2(\cdot)\left(1 + \sum_{k=1}^{p} 2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right) \\ &\quad + \frac{1}{2}\chi_{i,\rho}H_{i,\rho}^2(\cdot)\left(\sum_{k=1}^{p} 2^{k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{k}\right)^2\Bigg\}. \end{aligned} \quad (32)$$

Theorem 3.1: Under Assumptions 1)–4), the decentralized adaptive control (32) guarantees that all the signals in the closed-loop system

$$\begin{aligned}\dot{V}_m \le \sum_{i=1}^{N}\Bigg\{& -\left[\lambda_{\min}(Q_i) - (m+4)\right]\|\bar{e}_i\|^2 - 2\sum_{j=1}^{m+1}\pi_{ij}\chi_{ij}^2 \\ & - 2\beta_i^*\sum_{k=1}^{p}2^{2k}\chi_{i1}^{2k} - \sigma_i\left(\hat{\beta}_i - \beta_i^*\right)^2 + 2\chi_{i,m+1}\chi_{i,m+2} \\ & + \sum_{k=1}^{p}2^{2k}\left[d_{ik} + v_{ik} + \bar{\varphi}_{ik} + (m+1)(d_{1ik} + v_{1ik} + \bar{\varphi}_{1ik})\right] \\ & \cdot\left(\|\chi_{i1}\|^{2k} + \|y_{i,r}\|^{2k}\right) + \sum_{k=1}^{p}\bar{\eta}_{ik} + (m+1)\bar{\eta}_{i1}^2 + \sigma_i\beta_i^{*2} \\ & + \sum_{k=1}^{p}2^{2k}\left[\bar{\eta}_{ik} + (m+1)\bar{\eta}_{1ik}\right]\cdot\left[\alpha_{i1}^{-1}(2D_i(t_0,t))\right]^{2k}\Bigg\}. \end{aligned} \quad (27)$$

$$\begin{aligned}\dot{V}_{\rho-1} \le \sum_{i=1}^{N}\Bigg\{& -\left[\lambda_{\min}(Q_i) - (\rho+3)\right]\|\bar{e}_i\|^2 - 2\sum_{j=1}^{\rho-1}\pi_{ij}\chi_{ij}^2 \\ & - 2\beta_i^*\sum_{k=1}^{p}2^{2k}\chi_{i1}^{2k} - \sigma_i\left(\hat{\beta}_i - \beta_i^*\right)^2 \\ & + \sum_{k=1}^{p}2^{2k}\left[d_{ik} + v_{ik} + \bar{\varphi}_{ik} + \rho(d_{1ik} + v_{1ik} + \bar{\varphi}_{1ik})\right]\left(\|\chi_{i1}\|^{2k} + \|y_{i,r}\|^{2k}\right) \\ & + \sum_{k=1}^{p}\bar{\eta}_{ik} + \sum_{k=1}^{p}2^{2k}(\bar{\eta}_{ik} + \rho\bar{\eta}_{1ik})\left[\alpha_{i1}^{-1}(2D_i(t_0,t))\right]^{2k} + (\rho-1)\bar{\eta}_{i1}^2 \\ & + \sigma_i\beta_i^{*2} + 2\chi_{i,\rho}\left\{\chi_{i,\rho-1} + b_{i,k_i-\rho}\delta_i(y_i)u_i + \hat{z}_{i,\rho+1} + \Omega_{i,\rho}(\cdot)\right\} \\ & + 2(p+1)\chi_{i,\rho}^2H_{i,\rho}^2(\cdot) + \chi_{i,\rho}^2H_{i,\rho}^2(\cdot)\left(1 + \sum_{k=1}^{p}2^{2k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{2k}\right) \\ & + 2\bar{\eta}_{i1}\chi_{i,\rho}H_{i,\rho}(\cdot)\sum_{k=1}^{p}2^{k}\left[\alpha_{i1}^{-1}(2r_i)\right]^{k}\Bigg\} \end{aligned} \quad (31)$$

consisting of (1), (7), (10), and (16) are bounded in the presence of unmodeled dynamics, high-order interconnections and bounded disturbances. Furthermore, by choosing the design constants σ_i, $\pi_{ij}(j = 0, 1, \ldots, \rho)$ and Γ_i appropriately, the tracking error can be made arbitrarily small regardless of the interconnections, disturbances and unmodeled dynamics in the system.

Proof: Choose the desired value β_i^* and Q_i satisfying

$$\beta_i^* \geq \frac{1}{2}\left[d_{ik} + v_{ik} + \bar{\varphi}_{ik} + \rho\left(d_{1ik} + v_{1ik} + \bar{\varphi}_{1ik}\right)\right] \tag{33}$$

$$\lambda_{\min}\left(Q_i\right) \geq 2\pi_{i0} + (\rho + 3) \tag{34}$$

where $\pi_{i0} > 0$ is a design constant. From (31)–(34), we get (35), shown at the bottom of the page, where $\bar{e} = [\bar{e}_1, \ldots, \bar{e}_N]^T \in R^n$, $n = \sum_{i=1}^N k_i, \hat{\beta} = [\hat{\beta}_1, \ldots, \hat{\beta}_N]^T$, $\chi = [\chi_{11}, \ldots, \chi_{1,\rho}, \ldots, \chi_{N1} \ldots, \chi_{N,\rho}]^T$.

Since $D_i(t_0, t) = 0, \forall t \geq t_0 + T_i^0$ (see Lemma 1) and α_{i1}^{-1} is a function of class K_∞, we have

$$\sum_{k=1}^{p} 2^{2k}\left(\bar{\eta}_{ik} + \rho\bar{\eta}_{1ik}\right)\left[\alpha_{i1}^{-1}\left(2D_i\left(t_0, t\right)\right)\right]^{2k} = 0, \quad \forall t \geq t_0 + T_i^0. \tag{36}$$

Denote (37) and (38), shown at the bottom of the page. Notice that since $y_{i,r}$ and $\alpha_{i1}^{-1}(2D_i(t_0, t))$ are bounded, S is bounded. Thus

$$\dot{V}_{\rho-1}\left(\bar{e}, \chi, \hat{\beta}\right) \leq -\mu V_{\rho-1}\left(\bar{e}, \chi, \hat{\beta}\right) + S. \tag{39}$$

Therefore, $V_{\rho-1}(\bar{e}, \chi, \hat{\beta})$ decreases monotonically until $(\bar{e}, \chi, \hat{\beta})$ reaches the compact set

$$R_s = \left\{\left(\bar{e}, \chi, \hat{\beta}\right) \in R^n \times R^{N\rho} \times R^N : V_{\rho-1}\left(\bar{e}, \chi, \hat{\beta}\right) \leq \mu^{-1} S\right\}. \tag{40}$$

This means that $\bar{e}, \chi, \hat{\beta}$ are bounded, i.e., $\bar{e}_i, \chi_{i1}, \ldots, \chi_{i,\rho}, \hat{\beta}_i, i = 1, \ldots, N$, are bounded. Thus, z_{i1} is bounded. Although $\dot{r}_i \geq 0, \forall t \geq t_0$, we can see from (10) that whenever $\dot{r}_i > 0$, we have $r_i < 1/\bar{c}_{i0}[z_{i1}^2 \gamma_{i0}(z_{i1}^2) + d_{i0}]$. Since $z_{i1}^2 \gamma_{i0}(z_{i1}^2)$ is bounded, it follows that r_i is bounded. The boundedness of e_i follows from that of $\bar{e}_i$ and r_i. From (15), we can see that $M_{i1}(\chi_{i1}, \hat{\beta}_i, r_i)$ is bounded since $\chi_{i1}, \hat{\beta}_i, r_i$ are bounded. Since χ_{i2} is bounded, the boundedness of $\hat{z}_{i2}$ is established from (17). Thus, by using (22) iteratively, the boundedness of $\hat{z}_{i,m+1}, 1 \leq m \leq \rho - 1$, is followed from the boundedness of $y_i, \hat{z}_{i1}, \ldots, \hat{z}_{i,m}, \hat{\beta}_i, r_i, y_{i,r}, \ldots, y_{i,r}^{(m)}$. Consequently, $z_{i1}, \ldots, z_{i,\rho}$ are bounded since e_i is bounded. In addition, since y_i is bounded and, according to Assumption 4), the unmodeled dynamics $\dot{\zeta}_i = q_i(\zeta_i, y_i)$ is exp-ISpS, ζ_i is bounded.

Next, we prove that $z_{i,\rho+1}, \ldots, z_{i,k_i}$ are bounded. Let

$$w_{ij} = z_{ij} - \frac{b_{i,k_i-j}}{b_{i,k_i-\rho}} z_{i,\rho} \quad \rho + 1 \leq j \leq k_i. \tag{41}$$

Then, we have

$$\begin{aligned}\dot{w}_{ij} =& w_{i,j+1} - \frac{b_{i,k_i-j}}{b_{i,k_i-\rho}} w_{i,\rho+1} \\ &+ z_{i,\rho}\left(\frac{b_{i,k_i-j-1}}{b_{i,k_i-\rho}} - \frac{b_{i,k_i-j} b_{i,k_i-\rho-1}}{b_{i,k_i-\rho}^2}\right) \\ &+ \left[\phi_{ij}\left(y_1, \ldots, y_N\right) - \frac{b_{i,k_i-j}}{b_{i,k_i-\rho}} \phi_{i,\rho}\left(y_1, \ldots, y_N\right)\right] \\ &+ \left[\xi_{ij}\left(y_1, \ldots, y_N\right) - \frac{b_{i,k_i-j}}{b_{i,k_i-\rho}} \xi_{i,\rho}\left(y_1, \ldots, y_N\right)\right]\omega(t) \\ &+ \left[\Delta_{ij}\left(\zeta_i, y_i\right) - \frac{b_{i,k_i-j}}{b_{i,k_i-\rho}} \Delta_{i,\rho}\left(\zeta_i, y_i\right)\right] \\ \rho + 1 \leq & j \leq k_i - 1,\end{aligned} \tag{42}$$

$$\begin{aligned}\dot{w}_{i,k_i} =& -\frac{b_{i0}}{b_{i,k_i-\rho}} w_{i,\rho+1} - z_{i,\rho} \frac{b_{i0} b_{i,k_i-\rho-1}}{b_{i,k_i-\rho}^2} \\ &+ \left[\phi_{i,k_i}\left(y_1, \ldots, y_N\right) - \frac{b_{i0}}{b_{i,k_i-\rho}} \phi_{i,\rho}\left(y_1, \ldots, y_N\right)\right] \\ &+ \left[\xi_{i,k_i}\left(y_1, \ldots, y_N\right) - \frac{b_{i0}}{b_{i,k_i-\rho}} \xi_{i,\rho}\left(y_1, \ldots, y_N\right)\right]\omega(t) \\ &+ \left[\Delta_{i,k_i}\left(\zeta_i, y_i\right) - \frac{b_{i0}}{b_{i,k_i-\rho}} \Delta_{i,\rho}\left(\zeta_i, y_i\right)\right].\end{aligned} \tag{43}$$

$$\begin{aligned}\dot{V}_{\rho-1}\left(\bar{e}, \chi, \hat{\beta}\right) \leq \sum_{i=1}^{N}\Bigg\{& -2\pi_{i0}\left\|\bar{e}_i\right\|^2 - 2\sum_{j=1}^{\rho} \pi_{ij}\chi_{ij}^2 - \sigma_i\left(\hat{\beta}_i - \beta_i^*\right)^2 \\ &+ \sum_{k=1}^{p} 2^{2k}\left[d_{ik} + v_{ik} + \bar{\varphi}_{ik} + \rho\left(d_{1ik} + v_{1ik} + \bar{\varphi}_{1ik}\right)\right]\left\|y_{i,r}\right\|^{2k} \\ &+ \sum_{k=1}^{p} \bar{\eta}_{ik} + \sum_{k=1}^{p} 2^{2k}\left(\bar{\eta}_{ik} + \rho\bar{\eta}_{1ik}\right)\left[\alpha_{i1}^{-1}\left(2D_i\left(t_0, t\right)\right)\right]^{2k} \\ &+ \rho\bar{\eta}_{i1}^2 + \sigma_i\beta_i^{*2}\Bigg\}\end{aligned} \tag{35}$$

$$\mu = \min_{1 \leq i \leq N}\left\{\min\left(2\pi_{i0}\lambda_{\min}^{-1}\left(P_i\right), 2\pi_{i1}, \ldots, 2\pi_{i\rho}, \Gamma_i\sigma_i\right)\right\}, \tag{37}$$

$$\begin{aligned}S = \sum_{i=1}^{N}\Bigg\{& \sum_{k=1}^{p} 2^{2k}\left[d_{ik} + v_{ik} + \bar{\varphi}_{ik} + \rho\left(d_{1ik} + v_{1ik} + \bar{\varphi}_{1ik}\right)\right]\left\|y_{i,r}\right\|^{2k} \\ &+ \sum_{k=1}^{p} \bar{\eta}_{ik} + \sum_{k=1}^{p} 2^{2k}\left(\bar{\eta}_{ik} + \rho\bar{\eta}_{1ik}\right)\left[\alpha_{i1}^{-1}\left(2D_i\left(t_0, t\right)\right)\right]^{2k} + \rho\bar{\eta}_{i1}^2 + \sigma_i\beta_i^{*2}\Bigg\}.\end{aligned} \tag{38}$$

Since $\phi_{ij}(y_1,\ldots,y_N)$, $\xi_{ij}(y_1,\ldots,y_N)$, $\Delta_{ij}(\zeta_i,y_i)$, $z_{i,\rho}$ and $\omega(t)$ are bounded and, according to Assumption 1), the zero dynamics of system (1) is exponentially stable, it follows from (42) and (43) that $w_{i,\rho+1},\ldots,w_{i,k_i}$ are bounded. Thus, from (41), $z_{i,\rho+1},\ldots,z_{i,k_i}$ are bounded. Hence, $\hat{z}_{i,\rho+1},\ldots,\hat{z}_{i,k_i}$ are bounded since e_i is bounded. The boundedness of u_i is established by the fact that $\hat{z}_{i,\rho+1}$ is bounded and $b_{i,k_i-\rho}\delta_i(y_i)$ is bounded away from zero. Therefore, all the signals in the closed-loop system consisting of (1), (7), (10), and (16) are bounded.

Furthermore, it can be seen from (37)–(40) that reducing σ_i and increasing $\pi_{ij}(j=0,1,\ldots,\rho)$ and Γ_i will reduce the residual error bound $\mu^{-1}S$. This implies that by choosing the design constants σ_i, $\pi_{ij}(j=0,1,\ldots,\rho)$ and Γ_i appropriately, the tracking error can be made arbitrarily small. This completes the proof of Theorem 3.1.

From the above design procedure and Theorem 3.1, we can see easily that the proposed decentralized adaptive control scheme requires adaptation of only one scalar parameter for each subsystem and no controller redesign is needed if subsystems are added online or taken offline as long as the order of the interconnections of the appended system is less or equal to that of the original system.

IV. An Illustration Example

Consider an interconnected nonlinear system consisting of two subsystems

$$\begin{aligned}\dot{\zeta}_1 &= -\zeta_1 + y_1^2 + 0.5\\ \dot{z}_{11} &= z_{12} + \phi_{11}(y_1,y_2) + \xi_{11}(y_1,y_2)\omega(t) + \Delta_{11}(\zeta_1,y_1)\\ \dot{z}_{12} &= \phi_{12}(y_1,y_2) + \xi_{12}(y_1,y_2)\omega(t) + \Delta_{12}(\zeta_1,y_1) + u_1\\ y_1 &= z_{11}\\ \dot{\zeta}_2 &= -\zeta_2 + y_2^2 + 0.5\\ \dot{z}_{21} &= z_{22} + \phi_{21}(y_1,y_2) + \xi_{21}(y_1,y_2)\omega(t) + \Delta_{21}(\zeta_2,y_2)\\ \dot{z}_{22} &= \phi_{22}(y_1,y_2) + \xi_{22}(y_1,y_2)\omega(t) + \Delta_{22}(\zeta_2,y_2) + u_2\\ y_2 &= z_{21}\end{aligned} \tag{44}$$

where $\zeta_i, i=1,2$ is the unmeasurable and unmodeled dynamics, $\omega(t)=\sin^2(t)$ is the unmeasurable disturbance and

$$\begin{aligned}\phi_{11}(y_1,y_2) &= y_1^2 + y_1y_2, \xi_{11}(y_1,y_2) = y_1 + y_1y_2\\ \Delta_{11}(\zeta_1,y_1) &= 2\zeta_1^2, \phi_{12}(y_1,y_2) = y_2^2 + y_1\\ \xi_{12}(y_1,y_2) &= y_2 + y_1y_2, \Delta_{12}(\zeta_1,y_1) = 3\zeta_1^2\\ \phi_{21}(y_1,y_2) &= y_2^2\cos(t) + y_1, \xi_{21}(y_1,y_2) = y_2 + y_1y_2\\ \Delta_{21}(\zeta_2,y_2) &= 2\zeta_2^2\cos(t), \phi_{22}(y_1,y_2) = y_1^2 + y_2\\ \xi_{22}(y_1,y_2) &= y_1^2 + y_1y_2, \Delta_{22}(\zeta_2,y_2) = 3\zeta_2^2\sin(t).\end{aligned}$$

The objective is to design a decentralized robust adaptive output controller for each subsystem such that y_1 tracks $y_{1,r}=\sin(2t)$ and y_2 tracks $y_{2,r}=\cos(2t)$ and all the signals in the closed-loop system are bounded.

In the design of the controllers, we assume that $\phi_{ij}(y_1,y_2)$, $\xi_{ij}(y_1,y_2)$, $\Delta_{ij}(\zeta_i,y_i)(i,j=1,2)$ are unknown, but the upper bound of their orders is known, i.e., $p=2$. The relative degrees of the tow subsystems are $\rho_1=\rho_2=\rho=2$.

First, we show that the unmodeled dynamics fulfils the Assumption 4). Let $V_i(\zeta_i)=\zeta_i^2$. Then $\dot{V}_i(\zeta_i)=-2\zeta_i^2+2\zeta_iy_i^2+\zeta_i$. Using [13, Lemma 3.2], we have $2\zeta_iy_i^2+\zeta_i \le (1/4\varepsilon_1)(2\zeta_i)^2+\varepsilon_1y_i^4+(1/4\varepsilon_2)+\varepsilon_2\zeta_i^2$. Taking $\varepsilon_1=2.5, \varepsilon_2=0.4$, we get $2\zeta_iy_i^2+\zeta_i \le 0.8\zeta_i^2+2.5\|y_i\|^4+0.625$. Thus, $V_i(\zeta_i)\le -1.2\zeta_i^2+2.5\|y_i\|^4+0.625$, i.e., the unmodeled dynamics is exp-ISpS with $c_{i0}=1.2, d_{i0}=0.625, \gamma_i(\|z_{i1}\|)=2.5z_{i1}^4$ and $\gamma_{i0}(\|z_{i1}\|)=2.5z_{i1}^2$. Taking $\bar{c}_{i0}=0.6\in(0,c_{i0})$, we define the modified dynamic signal as follows:

$$\begin{aligned}\dot{r}_i &= \begin{cases}-0.6r_i+2.5z_{i1}^4+0.625 & \text{if} \quad m_{ri}\ge 0\\ 0 & \text{if} \quad m_{ri}<0\end{cases}\\ r_i(0) &= r_i^0 > 0\end{aligned}$$

where $m_{ri}=-0.6r_i+2.5z_{i1}^4+0.625$. Take $\alpha_{i1}(\|\zeta_i\|)=0.8\zeta_i^2\le V_i(\zeta_i)$, then $\alpha_{i1}^{-1}(2r_i)=\sqrt{2r_i/0.8}$. Define the observer for the ith subsystem as follows:

$$\dot{\hat{z}}_i = \begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}\hat{z}_i + \begin{bmatrix}3\\2\end{bmatrix}y_i + \begin{bmatrix}0\\1\end{bmatrix}u_i.$$

Applying the decentralized robust adaptive output control scheme presented herein, we have

$$\begin{aligned}\chi_{i1} &= z_{i1} - y_{i,r}\\ M_{i1} &= -\pi_{i1}\chi_{i1} - \hat{\beta}_i\left(4\chi_{i1}+16\chi_{i1}^2\right)\\ &\quad -\frac{1}{2}\chi_{i1}\left(\sqrt{10r_i}+10r_i\right)^2 - 6\chi_{i1}\\ &\quad -\frac{1}{2}\chi_{i1}\left(1+10r_i+100r_i^2\right)\\ \dot{\hat{\beta}}_i &= \Gamma_i\left(4\chi_{i1}^2+16\chi_{i1}^4-\sigma_i\hat{\beta}_i\right), \qquad \hat{\beta}_i(0)>0\\ \chi_{i2} &= \hat{z}_{i2} - M_{i1} - \dot{y}_{i,r}\\ \Omega_{i2} &= 2(y_i-\hat{z}_{i1}) - \frac{\partial M_{i1}}{\partial\chi_{i1}}(\chi_{i2}+M_{i1})\\ &\quad - \ddot{y}_{i,r} - \frac{\partial M_{i1}}{\partial\hat{\beta}_i}\dot{\hat{\beta}}_i - \frac{\partial M_{i1}}{\partial r_i}\dot{r}_i\\ H_{i2} &= -\frac{\partial M_{i1}}{\partial\chi_{i1}}\end{aligned}$$

where

$$\begin{aligned}\frac{\partial M_{i1}}{\partial\chi_{i1}} &= -\pi_{i1} - 6 - \hat{\beta}_i(4+48\chi_{i1}) - \frac{1}{2}\left(\sqrt{10r_i}+10r_i\right)^2\\ &\quad -\frac{1}{2}\left(1+10r_i+100r_i^2\right)\\ \frac{\partial M_{i1}}{\partial\hat{\beta}_i} &= -\left(4\chi_{i1}+16\chi_{i1}^3\right)\\ \frac{\partial M_{i1}}{\partial r_i} &= -\frac{1}{2}\chi_{i1}\left(10+200r_i+30\sqrt{10r_i}\right)\\ &\quad -\frac{1}{2}\chi_{i1}(10+200r_i).\end{aligned}$$

The decentralized adaptive control for the ith subsystem is

$$\begin{aligned}u_i = -\Bigg\{&\chi_{i1}+\pi_{i2}\chi_{i2}+\Omega_{i2}+3\chi_{i2}H_{i2}^2\\ &+\frac{1}{2}\chi_{i2}H_{i2}^2(1+10r_i+100r_i^2)\\ &+\frac{1}{2}\chi_{i2}H_{i2}^2(\sqrt{10r_i}+10r_i)^2\Bigg\}, \quad i=1,2.\end{aligned}$$

With the following choice of the initial conditions and design constants:

$$\begin{aligned}\zeta_1(0) &= 0, z_{11}(0)=1, z_{12}(0)=0, \hat{z}_{11}(0)=0\\ \hat{z}_{12}(0) &= 0, r_1(0)=0.1, \hat{\beta}_1(0)=1\\ \zeta_2(0) &= 0, z_{21}(0)=2, z_{22}(0)=0\\ \hat{z}_{21}(0) &= 0, \hat{z}_{22}(0)=0, r_2(0)=0.1\\ \hat{\beta}_2(0) &= 1, \Gamma_1=1, \sigma_1=0.1\\ \pi_{11} &= 1, \pi_{12}=1, \Gamma_2=1, \sigma_2=0.1\\ \pi_{21} &= 1, \pi_{22}=1\end{aligned} \tag{45}$$

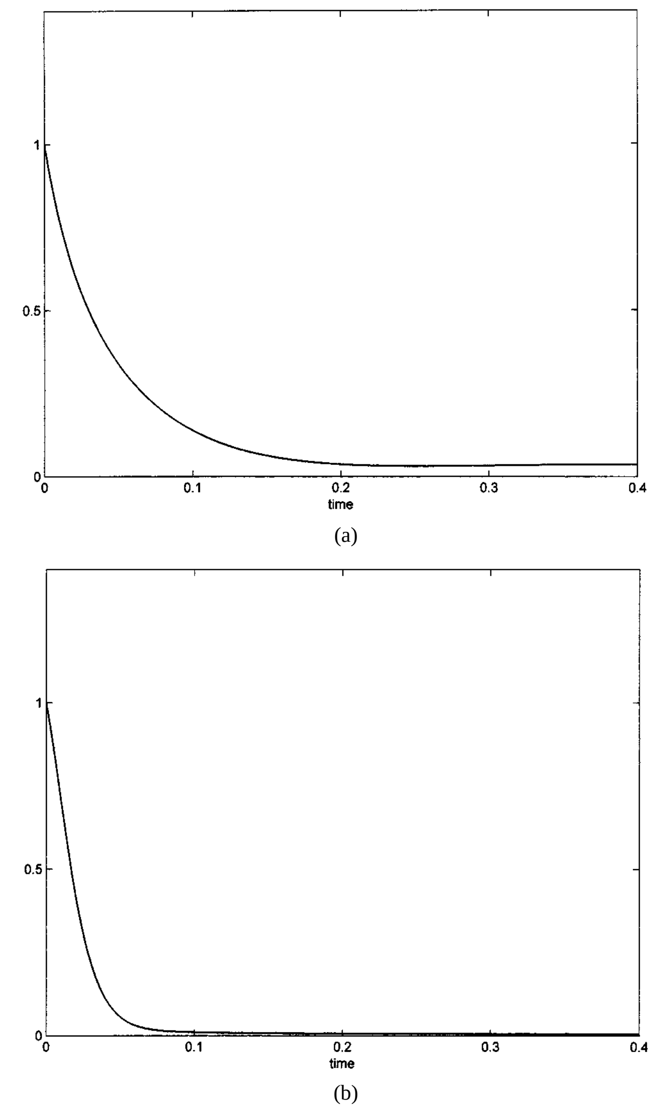

Fig. 1. Simulation results with the control scheme of this work. (a) Tracking error χ_{11}. (b) Tracking error χ_{21}.

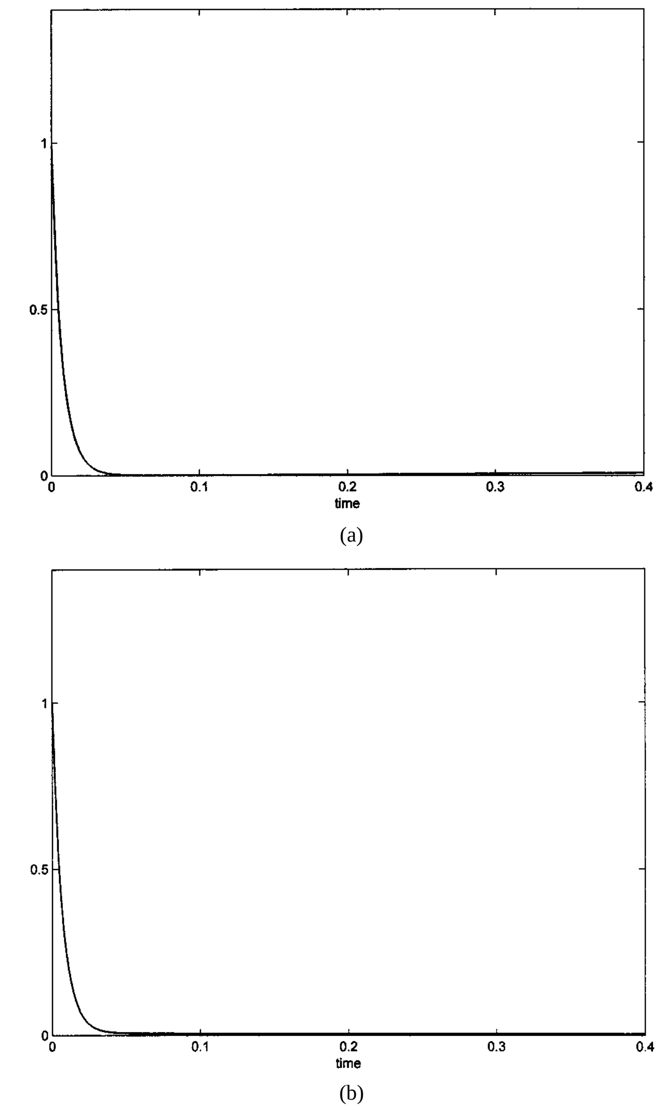

Fig. 2. Simulation results with the control scheme of this work by choosing the design constants appropriately. (a) Tracking error χ_{11}. (b) Tracking error χ_{21}.

we performed the simulation using MATLAB and obtained the results shown in Fig. 1. It can be seen from Fig. 1 that the decentralized controllers are robust to the unmodeled dynamics, bounded disturbances and high order interconnections with good tracking performance.

With the same initial conditions as in (45), we chose the design constants as follows:

$$\begin{aligned} \Gamma_1 =& 100, \sigma_1 = 0.001, \pi_{11} = 100, \pi_{12} = 100, \Gamma_2 = 100 \\ \sigma_2 =& 0.001, \pi_{21} = 100, \pi_{22} = 100 \end{aligned} \tag{46}$$

and obtained the simulation results shown in Fig. 2. The tracking errors χ_{11}, χ_{21} in Fig. 2(a) and (b) are much smaller than those in Fig. 1(a) and (b). This demonstrates that by choosing the design constants σ_i, π_{ij} and Γ_i appropriately, the tracking errors can be made arbitrarily small.

Finally, to compare the decentralized robust adaptive output control scheme presented in this work with that in [8], we applied the decentralized adaptive output control scheme presented in [8] to system (44) with the following initial conditions and design constants (using the same notations as in [8]):

$$\begin{aligned} \zeta_1(0) =& 0, z_{11}(0) = 1, z_{12}(0) = 0, \hat{z}_{11}(0) = 0, \hat{z}_{12}(0) = 0 \\ \hat{\beta}_1(0) =& 1, \zeta_2(0) = 0, z_{21}(0) = 2, z_{22}(0) = 0, \hat{z}_{21}(0) = 0 \\ \hat{z}_{22}(0) =& 0, \hat{\beta}_2(0) = 1, \ell_1 = 1, \Gamma_1 = 1, \sigma_1 = 0.1, \pi_{11} = 1 \\ \pi_{12} =& 1, \ell_2 = 1, \Gamma_2 = 1, \sigma_2 = 0.1, \pi_{21} = 1, \pi_{22} = 1. \end{aligned} \tag{47}$$

The simulation results are shown in Fig. 3, from which we can see that the tracking errors are unbounded and the decentralized adaptive output control scheme presented in [8] is not robust to unmodeled dynamics.

V. Conclusion

A new decentralized robust adaptive control scheme is presented for a class of large-scale nonlinear systems of the output feedback canonical form. The scheme can be used in the systems with unmodeled dynamics, high order interconnections and bounded disturbances. Under certain assumptions, it is shown that the scheme guarantees that all the signals in the closed-loop system are bounded. By choosing the design constants appropriately, the tracking error can be made arbitrarily small regardless of the interconnections, disturbances and unmodeled

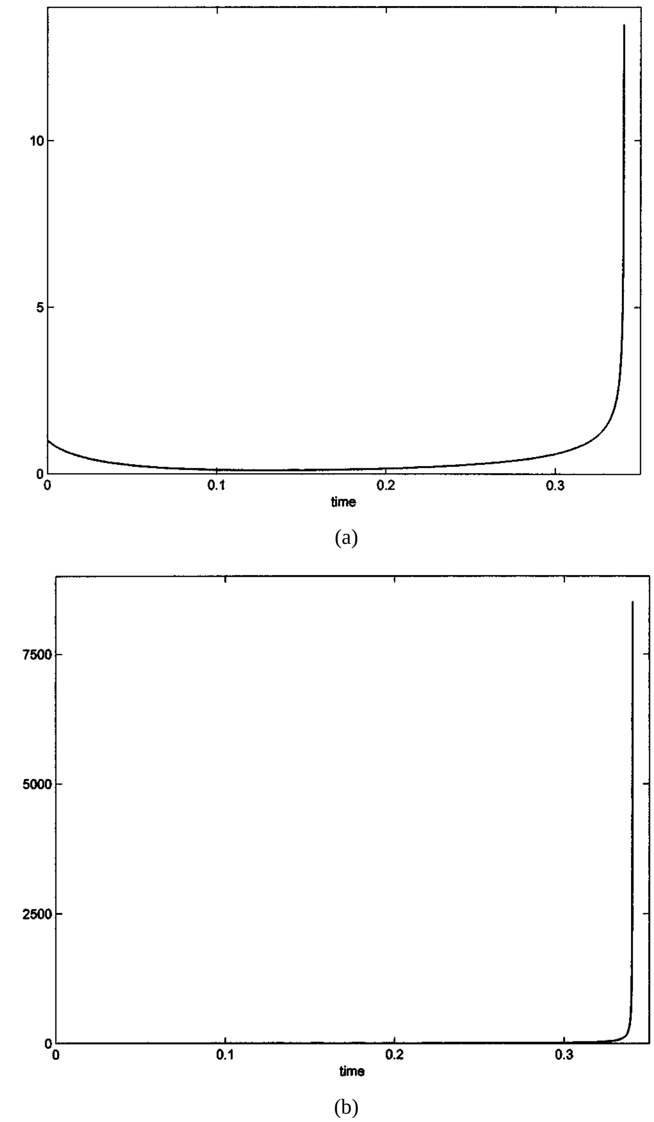

Fig. 3. Simulation results with the control scheme in [8]. (a) Tracking error χ_{11}. (b) Tracking error χ_{21}.

dynamics in the system. As an extension of the work in [8] to the case of unmodeled dynamics, the proposed decentralized adaptive control scheme of this work retains all the advantages of the scheme in [8]. The effectiveness of the proposed scheme is demonstrated by simulation results.

It should be pointed out that the unmodeled dynamics described in this work do not depend on the outputs of other subsystems. For the more general case where the unmodeled dynamics depends on the outputs of other subsystems, how to generate decentralized dynamic signals to dominate the unmodeled dynamics is a subject for further research.

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