Decimation for Bandpass Sigma-Delta Analog-to-Digital Conversion

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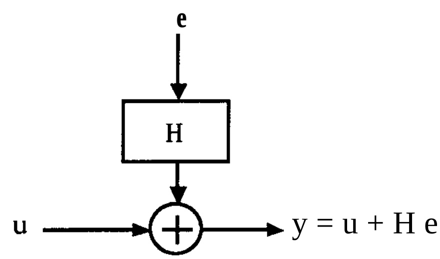
Abstract

An efficient means for constructing a narrow bandpass filter for an analog-to-digital converter based on a bandpass sigma-delta modulator is presented. It is shown that the filter can be made with very simple hardware, comparable to that required by an analog-to-digital converter using standard lowpass sigma-delta modulation.

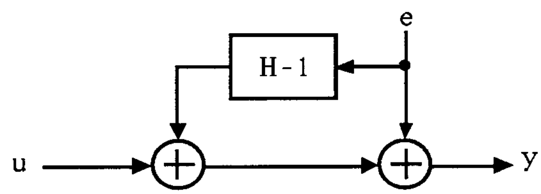
Introduction

Bandpass sigma-delta modulation [1] is a variant of the more usual kind of sigma-delta modulation, lowpass sigma-delta modulation [2,3,4,5,6]. Lowpass sigma-delta modulation achieves an accurate representation of the input from DC to some small fraction of the sampling rate, whereas bandpass sigma-delta modulation is accurate in a narrow frequency band, typically at a significant fraction of the sampling rate. It is particularly suited to the simultaneous demodulation and analog-to-digital conversion of AM signals.

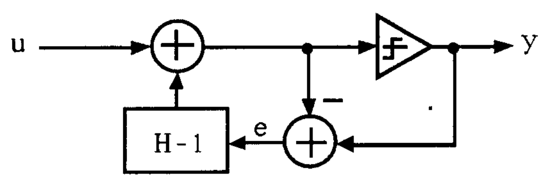
The principles underlying bandpass sigma-delta modulation are identical to those of lowpass sigma-delta modulation [2]. Given a structure, one analyzes it by assuming that the quantizer is a source of additive white noise and deriving the transfer function, H, from this noise source to the output using ordinary linear system analysis. If we reverse this procedure, we turn analysis into synthesis. Figure 1 illustrates the process by which we convert H, the desired error transfer function, into a $\Sigma\Delta$ n modulator The only prerequisite for this process is that H-l be strictly causal, or in terms of its z-transform, that $H(\infty) = 1$.



The standard linear model



H separated into two parts



The true structure of the modulator

Figure 1: Synthesis of a $\Sigma\Delta$ modulator from H, the desired error transfer function.

Figure 2 contrasts H for a lowpass modulator with that of a bandpass modulator. For lowpass $\Sigma\Delta$ modulation, H has zeros at or near DC and consequently the passband is centred around $\omega_0=0$. In the bandpass case, H has zeros at a large fraction of the sampling rate, $\frac{f_s}{8}$ for the modulator illustrated in Figure 2, and so the passband is centred at this frequency.

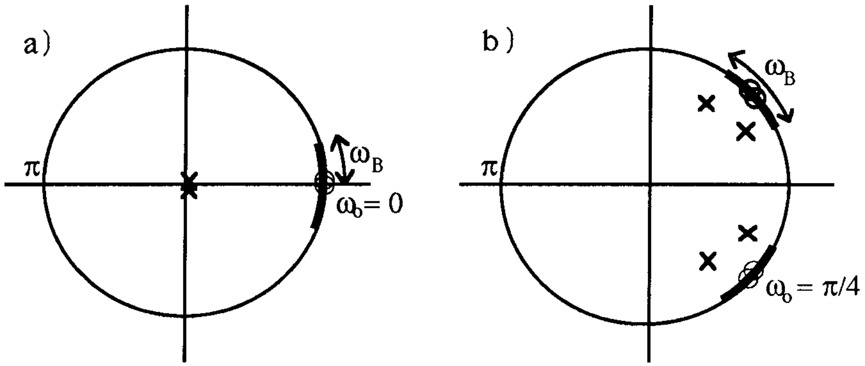


Figure 2: The error transfer functions and passbands for a) lowpass and b) bandpass ΣΔ modulation.

In an analog-to-digital converter, the modulator is only half the system. The other half is a digital filter which converts the high speed bit stream into multi-bit output at the Nyquist rate, as shown in Figure 3. In lowpass modulation, the first stage of filtering and decimation can be done with a $\mathbf{sinc^k}$ filter fashioned out of very simple logic: a counter and k- 1 adders [3]. However, for bandpass modulation, we need to do narrowband filtering on a high-speed bit stream. At first glance, this appears to be a major hurdle, but we shall shortly see how to accomplish this feat.

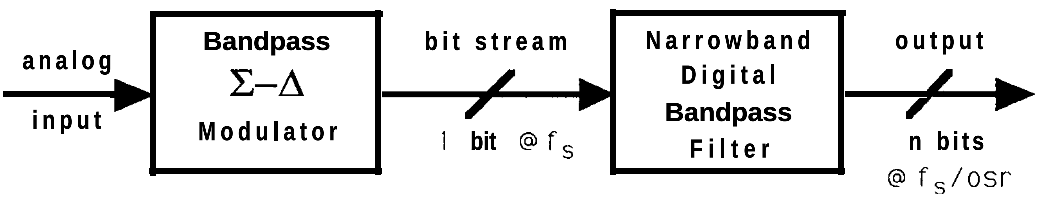
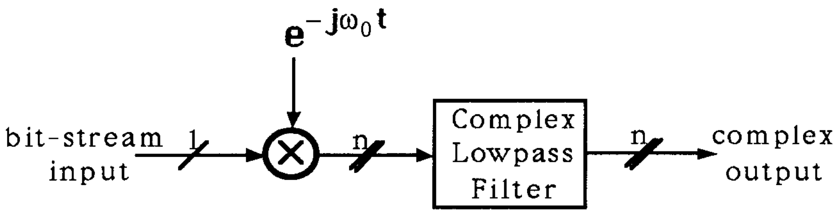


Figure 3: An analog-to-digital converter based on a bandpass ΣΔ modulator.

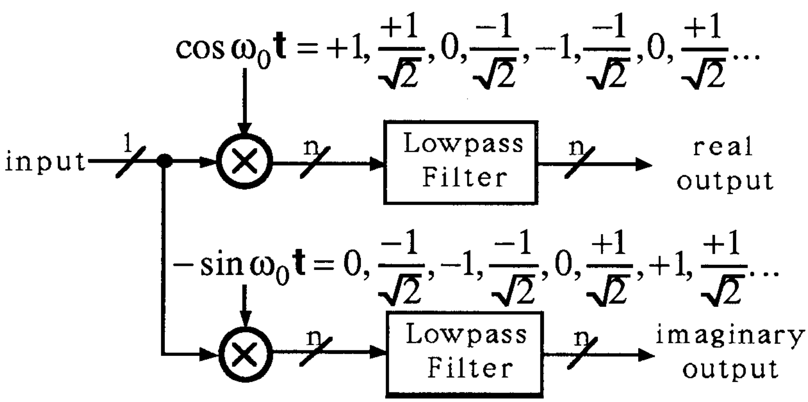
The proposed scheme

Theory

We can modulate the band of interest down to **DC** using a complex modulator, as shown in Figure 4. However, if we implement this scheme directly, we need two n-bit multipliers for the modulator, and since the lowpass filter no longer operates on a bit stream, it has to be a full-fledged filter.



A Complex Modulator and a Complex LPF



Modulator and Filter split in two,

$\omega_0 = \pi/4.$

Figure 4: A complex modulator followed by a complex lowpass filter.

Simplifications

We see from the above figure that if we choose $\omega_0=\pi/4$, then the sine and cosine sequences have a very simple structure. The terms of each sequence are 0, ± 1 and $\frac{\pm 1}{\sqrt{2}}$. Thus we see that we can separate the streams into integers and integers times $\frac{1}{\sqrt{2}}$:

$$
\begin{aligned}
\sin(\tfrac{\pi}{4}\ t) &= \left\{ 0,\ \tfrac{1}{\sqrt{2}},\ 1,\ \tfrac{1}{\sqrt{2}},\ 0,\ \tfrac{-1}{\sqrt{2}},\ -1,\ \tfrac{-1}{\sqrt{2}}\ \ldots \right\} \\
&= \left\{ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ -1,\ 0\ .. \right\} \\
&\quad + \frac{1}{\sqrt{2}} \times \left\{ 0, 1, 0, 1, 0, -1, 0, -1\ \ldots \right\}
\end{aligned}
$$

We can then use the linearity of the multiplication and filtering operations to separate the streams into rational and irrational parts, as illustrated in Figure 5. Note that the first stage of decimation is now done by lowpass filters operating on a bit stream, just as in the case of lowpass modulation. Also note that the multiplicands are now just 0 and ± 1, which can be effected by simple boolean operations. The last stage of decimation requires a more sophisticated filter, but it is now feasible because it operates at a low speed.

From this diagram, it appears that we now need four preliminary lowpass filter/decimators, but more simplifications are possible. Firstly, if the input is a double-sideband or vestigial sideband signal, then only the real or the imaginary half of' the system is necessary.

Secondly, we observe that the various sequences are full of zeros. If it were true that at any time instant only one of the sequences had a non-zero term, then it would be possible to multiplex a single set of adders to accumulate the results in all four channels. There are simultaneously non-zero terms in the irrational streams of sine and cosine, but if we instead use the sum and difference of these sequences, then there is no overlap and so a single set of adders is sufficient.

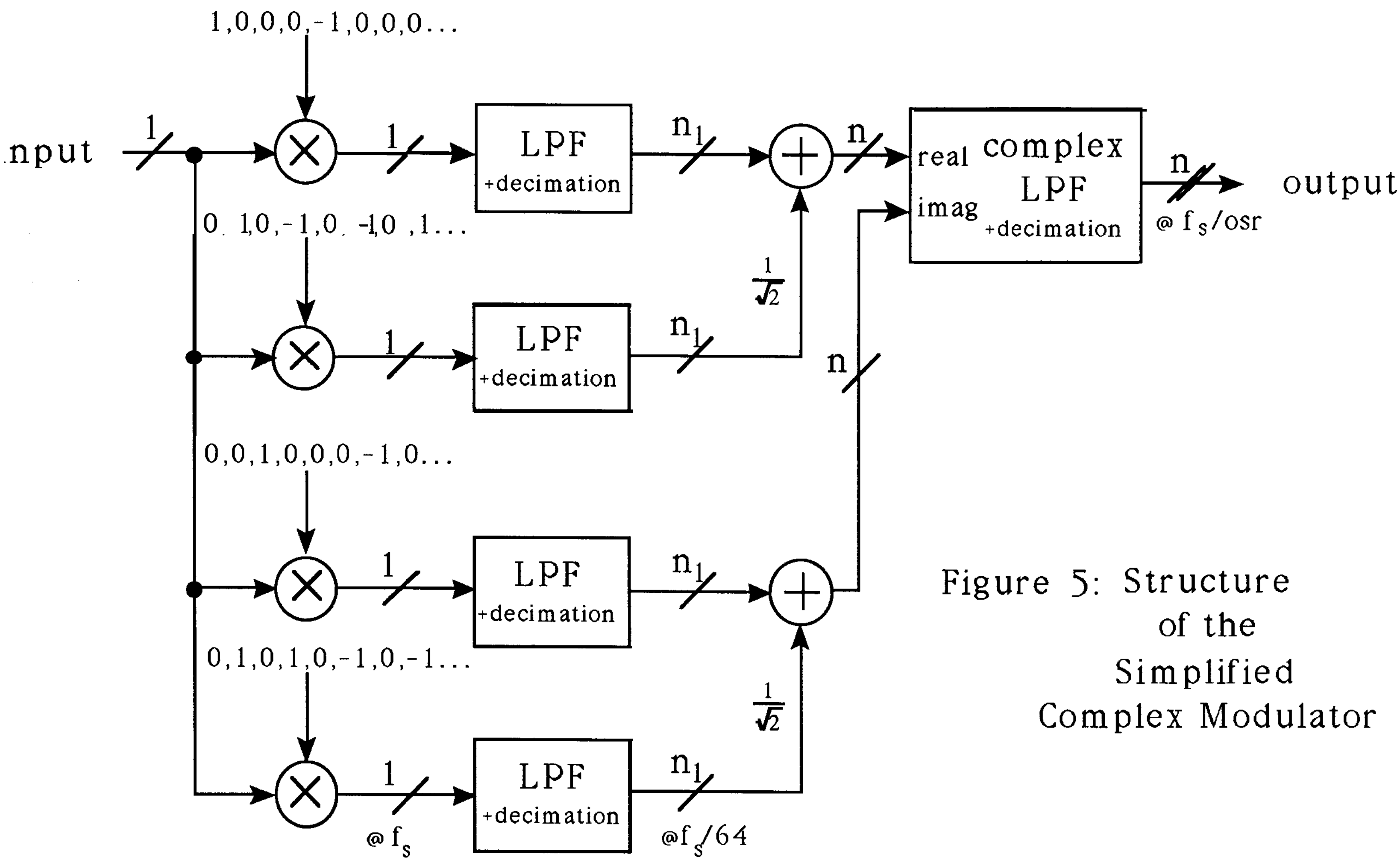


Figure 5: Structure of the Simplified Complex Modulator

Simulations

We have simulated second and fourth order modulators that yield 60dB and 99dB of signal-to-noise, respectively, for a centre frequency $\omega_0=\frac{\pi}{4}$, a bandwidth $\omega_B=\frac{\pi}{512}$ and an input level of -23dB. For this situation, the oversampling ratio, $\frac{\pi}{\omega_B}$, is 512. Triangular decimation by a factor of 64 reduces the SNR of the fourth order modulator to 88dB but the second order modulator's performance is not significantly degraded.

Summary

This paper gives a practical method for doing the narrowband bandpass filtering of a bit stream required by bandpass $\Sigma\Delta$ analog-to-digital converters. It is shown that the digital filtering can be done with almost the same level of complexity as that of ordinary lowpass $\Sigma\Delta$ converters.

The result is an analog-to-digital converter and an AM (DSB, VSB or SSB) demodulator all rolled in to one. The conversion is good for 16 bits and the demodulator achieves phenomenal selectivity with low-tolerance analog components.

Acknowledgment

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