
Decision Forests

**A Unified Framework for
Classification, Regression, Density
Estimation, Manifold Learning
and Semi-Supervised Learning**

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A Unified Framework for Classification, Regression, Density Estimation, Manifold Learning and Semi-Supervised Learning

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Foundations and Trends[®] in Computer Graphics and Vision

Published, sold and distributed by:

now Publishers Inc.
PO Box 1024
Hanover, MA 02339
USA
Tel. +1-781-985-4510
www.nowpublishers.com
sales@nowpublishers.com

Outside North America:

now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is A. Criminisi, J. Shotton and E. Konukoglu, Decision Forests: A Unified Framework for Classification, Regression, Density Estimation, Manifold Learning and Semi-Supervised Learning, Foundations and Trends[®] in Computer Graphics and Vision, vol 7, nos 2–3, pp 81–227, 2011

ISBN: 978-1-60198-540-8

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Foundations and Trends[®] in Computer Graphics and Vision, 2011, Volume 7, 4 issues. ISSN paper version 1572-2740. ISSN online version 1572-2759. Also available as a combined paper and online subscription.

Foundations and Trends[®] in
Computer Graphics and Vision
Vol. 7, Nos. 2–3 (2011) 81–227
© 2012 A. Criminisi, J. Shotton and E. Konukoglu
DOI: 10.1561/06000000035



Decision Forests: A Unified Framework for Classification, Regression, Density Estimation, Manifold Learning and Semi-Supervised Learning

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Abstract

This review presents a unified, efficient model of random decision forests which can be applied to a number of machine learning, computer vision, and medical image analysis tasks.

Our model extends existing forest-based techniques as it unifies classification, regression, density estimation, manifold learning, semi-supervised learning, and active learning under the same decision forest framework. This gives us the opportunity to write and optimize the core implementation only once, with application to many diverse tasks.

The proposed model may be used both in a discriminative or generative way and may be applied to discrete or continuous, labeled or unlabeled data.

The main contributions of this review are: (1) Proposing a unified, probabilistic and efficient model for a variety of learning tasks; (2) Demonstrating margin-maximizing properties of classification forests; (3) Discussing probabilistic regression forests in comparison with other nonlinear regression algorithms; (4) Introducing density forests for estimating probability density functions; (5) Proposing an efficient algorithm for sampling from a density forest; (6) Introducing manifold forests for nonlinear dimensionality reduction; (7) Proposing new algorithms for transductive learning and active learning. Finally, we discuss how alternatives such as random ferns and extremely randomized trees stem from our more general forest model.

This document is directed at both students who wish to learn the basics of decision forests, as well as researchers interested in the new contributions. It presents both fundamental and novel concepts in a structured way, with many illustrative examples and real-world applications. Thorough comparisons with state-of-the-art algorithms such as support vector machines, boosting and Gaussian processes are presented and relative advantages and disadvantages discussed. The many synthetic examples and existing commercial applications demonstrate the validity of the proposed model and its flexibility.

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1

Overview and Scope

This review presents a unified, efficient model of random decision forests which can be used in a number of applications such as scene recognition from photographs, object recognition in images, automatic diagnosis from radiological scans and semantic text parsing. Such applications have traditionally been addressed by different, supervised or unsupervised machine learning techniques.

In this review, we formulate diverse learning tasks such as regression, classification and semi-supervised learning as instances of the same general decision forest model. The unified framework further extends to novel uses of forests in tasks such as density estimation and manifold learning. The underlying unified framework gives us the opportunity to implement and optimize the general algorithm for all these tasks only once, and then adapt it to individual applications with relatively small changes.

This review is directed at engineers and PhD students who wish to learn the basics of decision forests as well as more senior researchers interested in the new research contributions.

We begin by presenting a roughly chronological, non-exhaustive survey of decision trees and forests, and their use in the past two decades. Further references will be available in the relevant sections.

1.1 A Chronological Literature Review

One of the earlier works on decision trees is the seminal “Classification and Regression Trees (CART)” book by Breiman et al. [12], where the authors describe the basics of decision trees and their use for both classification and regression problems. Following that publication researchers then focused on algorithms for constructing (learning) optimal decision trees for different tasks using available training data. For this purpose, one of the most popular algorithms is “C4.5” of Quinlan [81]. Although, decision trees were proven to be useful, their application remained limited to relatively low dimensional data.

In the nineties, researchers discovered how using ensembles of learners (e.g., generic “weak” classifiers) yields greater accuracy and generalization. This seems particularly true for high dimensional data, as often encountered in real life applications. One of the earliest references to ensemble methods is the boosting algorithm of Schapire [87], where the author discusses how iterative re-weighting of training data can be used to build “strong” classifiers as linear combination of many weak ones.

Combining the ideas of decision trees and ensemble methods gave rise to decision forests, that is, ensembles of randomly trained decision trees. The idea of constructing and using ensembles of trees with randomly generated node tests was introduced for the first time in the work of Amit and Geman [1, 2] for handwritten digit recognition. In that work the authors also propose using the mean of the tree probabilities as output of the tree ensemble.

In the subsequent work of Ho [47] tree training via randomized partitioning of the feature space is discussed further, and in [48] forests are shown to yield superior generalization to both boosting and pruned C4.5-trained trees, on some tasks. The author also shows comparisons between different split functions in the tree nodes.

Breiman’s later work in [10, 11] further consolidated the role of random forests and popularized their use. There, the author introduces a different way of injecting randomness in the forest by randomly sampling the labeled training data (namely “bagging”). The author

also describes techniques for predicting the forest test error based on measures of tree strength and correlation.

In computer vision, ensemble methods became popular with the seminal face and pedestrian detection papers of Viola and Jones [107, 108]. Random decision forests were used in [63] for image classification and in [60] for keypoint tracking in videos. Recent years have seen an explosion of forest-based techniques in the machine learning, vision and medical imaging literature [9, 15, 25, 31, 35, 37, 58, 59, 65, 67, 68, 69, 74, 79, 89, 92, 97, 110]. Decision forests compare favorably with respect to other techniques [15] and have led to one of the biggest success stories of computer vision in recent years: the Microsoft Kinect for XBox 360 [39, 91, 66].

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