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Research Article

Decision-Making Approach with Complex Bipolar Fuzzy N-Soft Sets

Adeel Farooq,[1] **Mohammed M. Ali Al-Shamiri**,[2,3] **Mohammed M. Khalaf**,[4] **and Umaira Amjad**[5]

[1]*Department of Mathematics, COMSATS University Islamabad, Lahore Campus, Lahore 54590, Pakistan*
[2]*Department of Mathematics, Faculty of Science and Arts, King Khalid University, Muhayl Assir, Saudi Arabia*
[3]*Department of Mathematics and Computer, Faculty of Science, Ibb University, Ibb, Yemen*
[4]*Faculty of Engineering, Arab Academy for Science & Technology and Maritime Transport (AASTMT), Aswan Branch, Egypt*
[5]*Department of Mathematics, University of the Punjab, New Campus, Lahore, Pakistan*

Correspondence should be addressed to Adeel Farooq; adeelfarooq@cuilahore.edu.pk

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The primary aim of this article is to extend the bipolar fuzzy N-soft sets with the concern of pursuing the periodicity involved real-world problems and introduce a new multiskilled hybrid model, namely, complex bipolar fuzzy N-soft sets. The novel model possesses the parametric characteristics of the versatile N-soft set and enjoys the distinguished attributes of a complex bipolar fuzzy set to handle the double-sided periodic vague data. We illustrate that the innovative model assists as a proficient mechanism for grading-based parameterized two-dimensional bipolar fuzzy information. We present some elementary operations and results for a complex bipolar fuzzy N-soft environment. Further, we establish the three dexterous algorithms to find the optimal solution to multiattribute decision-making problems. Moreover, the algorithms are supported with the robust assessment of a real-world application. Lastly, a comparison with existent decision-making techniques, such as choice values, weighted choice values, and $\mathscr{D}$-choice of values of bipolar fuzzy N-soft sets, is also conducted to manifest the phenomenal accountability and authenticity of the presented decision-making approaches.

1. Introduction

The primary concept of fuzzy set ($\mathscr{F}\mathscr{S}$) theory was presented by Zadeh [1] in 1965, which is a generalization of classical set theory to handle vague and uncertain information. In ($\mathscr{F}\mathscr{S}$), the value of membership degree (MD) lies in $[0, 1]$. In 1983, Atanassov [2] suggested the intuitionistic fuzzy set ($\mathscr{I}\mathscr{F}\mathscr{S}$) by adding the nonmembership degree (NMD) having the property that the sum of MD and NMD should be less than or equal to 1. Among many generalizations of $\mathscr{F}\mathscr{S}$ theory, our main focus is on the bipolar fuzzy set put forward by Zhang [3].

The traditional models of $\mathscr{F}\mathscr{S}$ are incapable of dealing with the periodic information in any meaningful way. Hence, another ground-breaking model was established by Ramot et al. [4] that can tackle the two-dimensional uncertain information. Among many other extensions of CFS, our focus is on complex bipolar fuzzy sets ($\mathscr{C}\mathscr{B}\mathscr{F}\mathscr{S}$), which was initiated by Akram et al. [5]. The rationale of $\mathscr{C}\mathscr{B}\mathscr{F}\mathscr{S}$ is to represent the bipolar information having vagueness and periodicity in complex geometry, as shown in Figures 1 and 2. By implementing the concept of bipolar fuzzy set in complex geometry to portray diverse phenomena at various phases, the notion of complex bipolar fuzzy set expresses bipolar behavior of uncertainty and periodicity simultaneously.

It has been a dilemma for researchers, as well as decision makers, to deal with the imprecision and ambiguity as it shows up in every discipline of life including social sciences, information technology, economics, business management, and so forth. Many attempts have been made to confront this concern. Molodtsov [6, 7] gave the idea of a soft set ($\mathscr{S}_f\mathscr{S}$) in 1999. Maji et al. [8] illustrated the application of $\mathscr{S}_f\mathscr{S}$ for

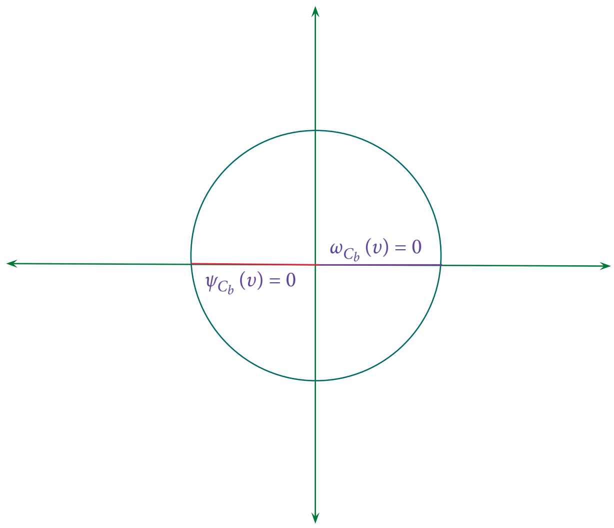

Figure 1: Phase term in BFS.

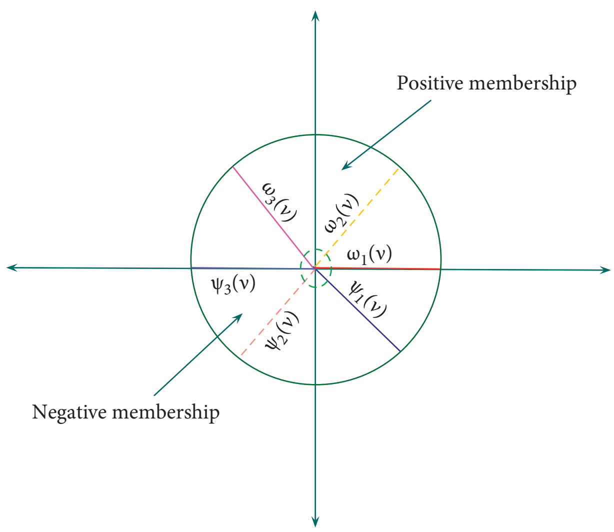

Figure 2: Graphical representation of $\mathscr{C}\mathscr{B}\mathscr{F}\mathscr{S}$.

choosing the best house. Maji et al. [9] introduced the fuzzy $\mathscr{S}_f\mathscr{S}$ and investigated its basic properties. Maji et al. [10] presented the intuitionistic fuzzy $\mathscr{S}_f\mathscr{S}$. Bipolar $\mathscr{S}_f\mathscr{S}$ was proposed by Shabir and Naz [11]. Later, Karaaslan and Karataş [12] redefined the bipolar $\mathscr{S}_f\mathscr{S}$ and worked for the decision-making method along with the application. Aslam et al. [13] put forth the idea of a hybrid model, namely, bipolar fuzzy $\mathscr{S}_f\mathscr{S}$ and defined its fundamental operations. Alghamdi et al. [14] used the various multicriteria decision techniques under a bipolar fuzzy environment. Later, Akram et al. [15] applied the TOPSIS and ELECTRE I approaches to diagnose medical diseases with the help of bipolar fuzzy data.

$\mathscr{S}_f\mathscr{S}$ theory is used to evaluate the binary evaluation based information. It is not helpful for nonbinary discreet evaluation based systems. Nowadays, mostly systems are being assessed on the basis of rating. In these systems, the rating of the alternatives is done by a number of stars, check marks, dots, numbers, et cetera. To overcome these obstacles, Fatimah et al. [16] developed the stimulated concept of N-soft set ($N\mathscr{S}_f\mathscr{S}$) along with set-theoretic operations and decision-making algorithms which are useful to capture the ordered graded information. Furthermore, Akram et al. [17] merged the novel models of $\mathscr{F}\mathscr{S}$ and $N\mathscr{S}_f\mathscr{S}$ to introduce the fuzzy $N\mathscr{S}_f\mathscr{S}$ ($\mathscr{F}N\mathscr{S}_f\mathscr{S}$). Akram et al. also proposed the intuitionistic fuzzy $N\mathscr{S}_f\mathscr{S}$ [18], complex Pythagorean fuzzy $N\mathscr{S}_f\mathscr{S}$ [19], complex spherical fuzzy $N\mathscr{S}_f\mathscr{S}$ [20], complex neutrosophic $N\mathscr{S}_f\mathscr{S}$ [21], and bipolar fuzzy $N\mathscr{S}_f\mathscr{S}$ ($\mathscr{B}\mathscr{F}N\mathscr{S}_f\mathscr{S}$) [22]. Pythagorean fuzzy $N\mathscr{S}_f\mathscr{S}$ was initiated by Zhang et al. [23]. Fatimah and Alcantud [24] put forward the multifuzzy $N\mathscr{S}_f\mathscr{S}$. Kamaci and Petchimuthu [25] introduced the hybrid model of bipolar $N\mathscr{S}_f\mathscr{S}$ along with practical applications.

To sum up, the motivation of this article is given as follows:

> Although the traditional $N\mathscr{S}_f\mathscr{S}$ can capture the graded evaluation of parameters and is superior than $\mathscr{S}_f\mathscr{S}$; nevertheless it can not handle the fuzziness involved in the information.
>
> The $\mathscr{C}\mathscr{B}\mathscr{F}\mathscr{S}$ is beneficial to tackle the two-dimensional bipolar information, but it can not cope with the rating/ranking based parameterized information.
>
> $\mathscr{B}\mathscr{F}N\mathscr{S}_f\mathscr{S}$ can deal with graded parameterized double-sided ambiguous information, but still it has the inadequacy of phase term.

Motivated by the aforementioned concerns, this research article introduced a new hybrid model with multiple characteristics, namely, complex bipolar fuzzy N-soft sets. This methodology is designed to capture and interpret the two-dimensional bipolar fuzzy graded parameterized information. This article also describes the fundamental operations of the proposed model. Moreover, complex bipolar fuzzy N-soft number along with some algebraic operations and properties is also defined. Three algorithms for decision-making have been defined and ingeniously implemented on real-life problems. The rationality and applicability have been illustrated through comparative analysis with existing methodologies.

This research paper is organized as follows: Section 2 defines some preliminary concepts for the development of the new hybrid model. Section 3 describes the mathematical framework of the proposed complex bipolar fuzzy N-soft sets models and develops its operations. Section 4 provides the three algorithms for decision-making purposes. Section 5 illustrates the applicability of the presented novel algorithms. Section 6 demonstrates the comparative analysis of proposed techniques with existing decision-making methods. Section 7 sums up this article with some concluding remarks.

2. Preliminaries

Now, we will present some fundamental definitions that are essential for further developments.

Definition 2.1 (see [1]). Let V be a nonempty set. A *fuzzy set* μ over V is an object of the form:

$$\mu = \left\{\left(v, \alpha_{\mu}^{p}(v)\right) \mid v \in V\right\}, \tag{1}$$

where $\alpha_{\mu}^{p}\colon V \longrightarrow [0,1]$ denotes the degree of membership.

Definition 2.2 (see [3]) Let V be a nonempty set. A *bipolar fuzzy set* B over V is of the form:

$$B = \left\{\left(v, \alpha_{B}^{p}(v), \beta_{B}^{n}(v)\right) \mid v \in V\right\}, \tag{2}$$

where $\alpha_{B}^{p}\colon V \longrightarrow [0,1]$ and $\beta_{B}^{n}\colon V \longrightarrow [-1,0]$ represent the satisfaction function and dissatisfaction function, respectively. The satisfaction value $\alpha_{B}^{p}(v)$ indicates the strength of belongingness of element v to a certain property and the dissatisfaction value $\beta_{B}^{n}(v)$ indicates the belongingness of element v to some counter property of bipolar fuzzy set B.

Definition 2.3 (see [26]). Let $\ddot{b} = (\alpha^{p}, \beta^{n})$ be a bipolar fuzzy number. Then, the bipolar fuzzy score function f and accuracy function g are formulated as follows:

$$\begin{aligned} f(\ddot{b}) &= \frac{1 + \alpha^{p} + \beta^{n}}{2}, \\ g(\ddot{b}) &= \frac{\alpha^{p} - \beta^{n}}{2}, \end{aligned} \tag{3}$$

where $f(\ddot{b}), g(\ddot{b}) \in [0,1]$.

Definition 2.4 (see [5]). Let V be a universal set. A $\mathscr{CBFS}$ C_b on a nonempty set V is an object of the form:

$$C_b = \left\{\left(v, \alpha_{C_b}^{p}(v)e^{i\omega_{C_b}(v)}, \beta_{C_b}^{n}(v)e^{i\psi_{C_b}(v)}\right) | v \in V\right\}, \tag{4}$$

where $i = \sqrt{-1}, \alpha_{C_b}^{p}\colon V \longrightarrow [0,1]$ and $\beta_{C_b}^{n}\colon V \longrightarrow [-1,0]$ are mappings, $\omega_{C_b}(v) \in [0,\pi]$ and $\psi_{C_b}(v) \in [-\pi,0]$. For any element $v \in V$, $\alpha_{C_b}^{p}(v)$ and $\beta_{C_b}^{n}(v)$ are known to be amplitude terms; $\omega_{C_b}(v)$ and $\psi_{C_b}(v)$ are phase terms.

Definition 2.5 (see [6]). Let V be a universe of discourse under consideration and A be the set of all attributes, $\mathbb{L} \subseteq A$. A pair $(\vartheta, \mathbb{L})$ is called $\mathscr{S}_f\mathscr{S}$ over V if $\vartheta\colon \mathbb{L} \longrightarrow P(V)$ where ϑ is a set-valued function.

Definition 2.6 (see [16]). Let V be a universe of discourse and A be the set of all attributes, $\mathbb{L} \subseteq A$. Consider $\mathscr{R} = \{0, 1, \ldots, N-1\}$ be a set of ordered grades where $N \in \{2, 3, \ldots\}$. A triple $(U, \mathbb{L}, N)$ is an $N\mathscr{S}_f\mathscr{S}$ on V if $U\colon \mathbb{L} \longrightarrow 2^{V\times\mathscr{R}}$, with the property that for each $k_j \in \mathbb{L}$, there exists a unique $(v_t, r_j^t) \in V \times \mathscr{R}$ such that $(v_t, r_j^t) \in U(k_j)$, $v_t \in V, r_j^t \in \mathscr{R}$.

Definition 2.7 (see [13]). Let V be a universe of discourse under consideration and A be the set of all attributes, $\mathbb{L} \subseteq A$. A pair $(\wp, \mathbb{L})$ is called $\mathscr{BFS}_f\mathscr{S}$ over V if $\wp\colon \mathbb{L} \longrightarrow \mathrm{BF}^V$, where BF^V is the collection of all bipolar fuzzy subsets of V. It is defined as follows:

$$(\wp, \mathbb{L}) = \left\{\left(v, \alpha_l^p(v), \beta_l^n(v)\right) | \forall v \in V \text{ and } l \in \mathbb{L}\right\}. \tag{5}$$

Definition 2.8 (see [22]). Let V be a universe of discourse and A be the set of all attributes under consideration, $\mathbb{L} \subseteq A$. Let $\mathscr{R} = \{0, 1, 2, \ldots, N-1\}$ be a set of ordered grades where $N \in \{2, 3, \ldots\}$. A triple $(\mathfrak{J}, \mathscr{P}, N)$ is called a $\mathscr{BFNS}_f\mathscr{S}$, when $\mathscr{P} = (U, \mathbb{L}, N)$ is an $N\mathscr{S}_f\mathscr{S}$ on V where $U\colon \mathbb{L} \longrightarrow 2^{V\times\mathscr{R}}$ and $\mathfrak{J}$ is a mapping such that $\mathfrak{J}\colon \mathbb{L} \longrightarrow 2^{V\times\mathscr{R}} \times \mathscr{BFV}$, which is as follows:

$$\begin{aligned} (\mathfrak{J}, \mathscr{P}, N) &= \left\{\langle l, (\mathscr{W}(l), \mathscr{Z}(l))\rangle | l \in \mathbb{L}, (\mathscr{W}(l), \mathscr{Z}(l)) \in 2^{V\times\mathscr{R}} \times \mathscr{BFV}\right\} \\ &= \left\{\left\langle l, (v, r_l^v), \alpha_l^p, \beta_l^n\right\rangle | l \in \mathbb{L}, v \in V, r_l^v \in \mathscr{R}\right\}, \end{aligned} \tag{6}$$

where

$$\mathscr{Z}, \mathscr{W}\colon \mathscr{K} \longrightarrow \mathscr{BFV}, \tag{7}$$

and $\mathscr{BFV}$ represents the collection of all bipolar fuzzy values on V.

Here, $\alpha_l^p \in [0,1]$ and $\beta_l^n \in [-1,0]$ for all $v \in V$.

3. Theoretical Structure of Complex Bipolar Fuzzy N-Soft Set

Definition 3.1. Let V be the universal set and A be the set of parameters under examination, $\mathbb{L} \subseteq A$. Let $\mathscr{R} = \{0, 1, 2, \ldots, N-1\}$ be the set of ordered grades with $N \in \{2, 3, \ldots\}$. A triple $\chi = (\Upsilon_\Phi, \mathbb{T}, N)$ is called a *complex bipolar fuzzy N-soft set* ($\mathscr{CBFNS}_f\mathscr{S}$) on V, if $\mathbb{T} = (\Phi, \mathbb{L}, N)$ possessing $\Phi\colon \mathbb{L} \longrightarrow 2^{V\times\mathscr{R}}$ is an N-soft set ($N\mathscr{S}_f\mathscr{S}$) on V and $\Upsilon_\Phi\colon \mathbb{L} \longrightarrow 2^{V\times\mathscr{R}} \times \mathscr{CBFV}$, which is described as follows:

$$\begin{aligned} \chi &= \{\langle(\Phi(l_i), \Upsilon(l_i))\rangle | l_i \in \mathbb{L}\} \\ &= \{\langle l, ((v, r_l^v), \mu_l, \nu_l)\rangle | l \in \mathbb{L}, v \in V, r_l^v \in \mathscr{R}\} \\ &= \left\{\left\langle l, (v, r_l^v), \alpha_l^p e^{i\omega_l}, \beta_l^n e^{i\psi_l}\right\rangle | l \in \mathbb{L}, v \in V, r_l^v \in \mathscr{R}\right\}, \end{aligned} \tag{8}$$

where

$$\Upsilon\colon \mathbb{L} \longrightarrow \mathscr{CBFV}, \tag{9}$$

$\mathscr{CBFV}$ indicates the collection of all complex bipolar fuzzy values on V.

Here, $i = \sqrt{-1}$, $\alpha_l^p \in [0,1]$, $\beta_l^n \in [-1,0]$, $\omega_l \in [0,\pi]$ and $\psi \in [-\pi,0]$ for all $v \in V$.

Remark. Let $\chi(l_i) = \langle (v_y, r_i^y), \alpha_{yi}^p e^{i\omega_{yi}}, \beta_{yi}^n e^{\psi_{yi}} \rangle$ be a $\mathscr{CBFNS}_f\mathscr{S}$. Then, the complex bipolar fuzzy N-soft number ($\mathscr{CBFNS}_f\mathscr{N}$) is defined as follows:

$$\chi_{yi} = \langle r_i^y, (\alpha_{yi}^p e^{i\omega_{yi}}, \beta_{yi}^n e^{\psi_{yi}}) \rangle. \tag{10}$$

The following example of decision-making demonstrates the importance and significance of the proposed hybrid model. This example elaborates the shortcomings of the hybrid model $\mathscr{BFNS}_f\mathscr{S}$, put forward by Akram et al. [22], and illustrates the proficiency of the presented hybrid model in decision-making problems.

Example 3.2. Multiattribute decision-making procedures are considered as a primary tool for examining the patient's medical history in the medical symptomatic system and suggest the required tests. But most of the time, syndromes are not apparent and show bipolar behavior. In such circumstances, bipolar fuzzy sets are more capable of applying because it deals with positive along with negative behavior of an object towards a certain property. Suppose that a person with brain disorder visits a neurologist. A neurologist examines the symptoms first. The symptoms are as follows:

l_1: headache

l_2: seizures

l_3: fatigue

l_4: mood swings

The doctor examines the patient's history, but still, there is confusion about whether it is a traumatic brain injury or brain tumor. Hence, the doctor wants to suggest one of the following brain scans for clear examination.

v_1: computed tomography (CT) scan

v_2: magnetic resonance imaging (MRI)

v_3: positron emission tomography (PET)

v_4: single photon emission computed tomography (SPECT)

The neurologist will assign the $4\mathscr{S}_f$ grades to the alternatives in order to find out the best option for brain scanning according to the symptoms. The rating and associated $4\mathscr{S}_f\mathscr{S}$ of alternatives are given in Table 1, where

Table 1: Rating of brain scanning tests.

$V/\mathbb{L}$	l_1	l_2	l_3	l_4
v_1	2 = ●●	2 = ●●	1 = ●	2 = ●●
v_2	3 = ●●●	3 = ●●●	3 = ●●●	3 = ●●●
v_3	1 = ●	2 = ●●	1 = ●	1 = ●
v_4	2 = ●●	3 = ●●●	0=°	1 = ●

Three ● represent outstanding

Two ● represent superb

One ● represents good

° represents average

The corresponding $8\mathscr{S}_f\mathscr{S}$ can be associated as follows:

<°' indicates 0

<●' indicates 1

<●●' indicates 2

<●●●' indicates 3

Corresponding to the grades, $\mathscr{CBF}8\mathscr{S}_f\mathscr{N}s$ is assigned to the criteria of vaccines by utilizing the following grading criteria:

$$\begin{aligned} 0.0 \le f(\ddot{b}_{yi}) &< 0.25 \quad \text{when grade } 0, \\ 0.25 \le f(\ddot{b}_{yi}) &< 0.50 \quad \text{when grade } 1, \\ 0.50 \le f(\ddot{b}_{yi}) &< 0.75 \quad \text{when grade } 2, \\ 0.75 \le f(\ddot{b}_{yi}) &\le 1.00 \quad \text{when grade } 3. \end{aligned} \tag{11}$$

where $\ddot{b}_{yi} = (\alpha_{yi}^p, \beta_{yi}^n)$, $f(\ddot{b}_{yi}) = \alpha_{yi}^p + \beta_{yi}^n + 1/2$ and $y = 1, 2, 3, 4;\ i = 1, 2, 3, 4$.

According to the aforementioned criteria, the corresponding grading criteria are represented in Table 2.

Thereby, the $\mathscr{BF}4\mathscr{S}_f\mathscr{S}$ can be defined as follows:

$$\begin{aligned} \Upsilon_\Phi(l_1) &= \{\langle (v_1, 2), 0.55, -0.33\rangle, \langle (v_2, 3), 0.81, -0.15\rangle, \langle (v_3, 1), 0.46, -0.52\rangle, \langle (v_4, 2), 0.65, -0.29\rangle, \} \\ \Upsilon_\Phi(l_2) &= \{\langle (v_1, 2), 0.71, -0.44\rangle, \langle (v_2, 3), 0.89, -0.13\rangle, \langle (v_3, 2), 0.68, -0.31\rangle, \langle (v_4, 3), 0.91, -0.17\rangle\}, \\ \Upsilon_\Phi(l_3) &= \{\langle (v_1, 1), 0.42, -0.57\rangle, \langle (v_2, 3), 0.86, -0.08\rangle, \langle (v_3, 1), 0.39, -0.64\rangle, \langle (v_4, 0), 0.15, -0.89\rangle\}, \\ \Upsilon_\Phi(l_4) &= \{\langle (v_1, 2), 0.69, -0.45\rangle, \langle (v_2, 3), 0.79, -0.05\rangle, \langle (v_3, 1), 0.31, -0.59\rangle, \langle (v_4, 1), 0.27, -0.68\rangle\}. \end{aligned} \tag{12}$$

The tabular representation of $\mathscr{BF}4\mathscr{S}_f$ decision matrix is given in Table 3.

The doctor assigns the $\mathscr{BFN}s$ corresponding to each grade value of the alternatives. Now, suppose that the doctor observes "Initially, almost 3 months the headache was bearable, but from the last 8 months it is severe." Then the bipolar fuzzy framework is ambiguous and fails to handle the complete situation. Hence, it is necessary to use the complex bipolar fuzzy environment in order to deal with the periodicity of the data. Therefore, we set up the $\mathscr{CBFNS}_f\mathscr{S}$ instead of $\mathscr{BFNS}_f\mathscr{S}$ for the assessment of such problems. The phase term in data will indicate the information related to the reference frame of time under consideration. Now, the grading criteria are redefined according to the framework of $\mathscr{CBFS}$.

Table 2: Grading criteria.

Grades r_i^y	Positive membership α_{yi}^p	Negative membership β_{yi}^n
$r_i^y = 0$	[0, 0.25)	[−1, −0.75)
$r_i^y = 1$	[0.25, 0.50)	[−0.75, −0.50)
$r_i^y = 2$	[0.50, 0.75)	[−0.50, −0.25)
$r_i^y = 3$	[0.75, 1.00]	[−0.25, −0.00]

Table 3: Tabular form of the $\mathscr{BF}4\mathscr{S}_f\mathscr{S}$.

$(\Upsilon_\Phi, \mathbb{T}, 4)$	l_1	l_2	l_3	l_4
v_1	⟨2, (0.55, −0.33)⟩	⟨2, (0.71, −0.44)⟩	⟨1, (0.42, −0.57)⟩	⟨2, (0.69, −0.45)⟩
v_2	⟨3, (0.81, −0.15)⟩	⟨3, (0.89, −0.13)⟩	⟨3, (0.86, −0.08)⟩	⟨3, (0.79, −0.05)⟩
v_3	⟨1, (0.46, −0.52)⟩	⟨2, (0.68, −0.31)⟩	⟨1, (0.39, −0.64)⟩	⟨1, (0.31, −0.59)⟩
v_4	⟨2, (0.65, −0.29)⟩	⟨3, (0.91, −0.17)⟩	⟨0, (0.15, −0.89)⟩	⟨1, (0.27, −0.68)⟩

$$\begin{aligned} 0.0 \le S(\mathscr{B}_{yi}) < 0.50 &\quad \text{when grade } 0,\\ 0.50 \le S(\mathscr{B}_{yi}) < 1.00 &\quad \text{when grade } 1,\\ 1.00 \le S(\mathscr{B}_{yi}) < 1.50 &\quad \text{when grade } 2,\\ 1.50 \le S(\mathscr{B}_{yi}) \le 2.00 &\quad \text{when grade } 3. \end{aligned} \tag{13}$$

where $\mathscr{B}_{yi} = (\alpha_{yi}^p e^{i\omega_{yi}\pi}, \beta_{yi}^n e^{\psi_{yi}})$, $S(\mathscr{B}_{yi}) = \alpha_{yi}^p + \beta_{yi}^n + 1/2 + 1/2(\omega_{yi}/\pi + \psi_{yi}/\pi + 1)$ and $y = 1, 2, 3, 4$; $i = 1, 2, 3, 4$.

According to the aforementioned criteria, the corresponding grading criteria are represented in Table 4.

Hence, $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$ can be obtained by applying Definition 3.1 as follows:

$$\begin{aligned} \Upsilon_\Phi(l_1) &= \Big\{\big\langle (v_1, 2), 0.55e^{i0.62\pi}, -0.33e^{-i0.29\pi}\big\rangle, \big\langle (v_2, 3), 0.81e^{i0.89\pi}, -0.15e^{-i0.12\pi}\big\rangle, \big\langle (v_3, 1), 0.46e^{i0.43\pi}, -0.52e^{-i0.62\pi}\big\rangle,\\ &\quad \big\langle (v_4, 2), 0.65e^{i0.58\pi}, -0.29e^{-i0.34\pi}\big\rangle\Big\},\\ \Upsilon_\Phi(l_2) &= \Big\{\big\langle (v_1, 2), 0.71e^{i0.63\pi}, -0.44e^{-i0.38\pi}\big\rangle, \big\langle (v_2, 3), 0.89e^{i0.81\pi}, -0.13e^{-i0.07\pi}\big\rangle, \big\langle (v_3, 2), 0.68e^{i0.66\pi}, -0.31e^{-i0.37\pi}\big\rangle,\\ &\quad \big\langle (v_4, 3), 0.91e^{i0.78\pi}, -0.17e^{-i0.11\pi}\big\rangle\Big\},\\ \Upsilon_\Phi(l_3) &= \Big\{\big\langle (v_1, 1), 0.42e^{i0.38\pi}, -0.57e^{-i0.65\pi}\big\rangle, \big\langle (v_2, 3), 0.86e^{i0.93\pi}, -0.08e^{-i0.17\pi}\big\rangle, \big\langle (v_3, 1), 0.39e^{i0.34\pi}, -0.64e^{-i0.57\pi}\big\rangle,\\ &\quad \big\langle (v_4, 0), 0.15e^{i0.21\pi}, -0.89e^{-i0.92\pi}\big\rangle\Big\},\\ \Upsilon_\Phi(l_4) &= \Big\{\big\langle (v_1, 2), 0.69e^{i0.65\pi}, -0.45e^{-i0.42\pi}\big\rangle, \big\langle (v_2, 3), 0.79e^{i0.82\pi}, -0.05e^{-i0.21\pi}\big\rangle, \big\langle (v_3, 1), 0.31e^{i0.29\pi}, -0.59e^{-i0.72\pi}\big\rangle,\\ &\quad \big\langle (v_4, 1), 0.27e^{i0.31\pi}, -0.68e^{-i0.61\pi}\big\rangle\Big\}. \end{aligned} \tag{14}$$

The tabular representation of $\mathscr{CBF}4\mathscr{S}_f$ decision matrix is depicted in Table 5.

Definition 3.3. Let $\chi_{yi} = \langle r_i^y, (\alpha_{yi}^p e^{i\omega_{yi}}, \beta_{yi}^n e^{\psi_{yi}})\rangle$ be a $\mathscr{CBFNS}_f\mathscr{N}$ over V. The *score function* is defined as follows:

$$S(\chi_{yi}) = \frac{r_i^y}{N-1} + \frac{\alpha_{yi}^p + \beta_{yi}^n + 1}{2} + \frac{1}{2}\left(\frac{\omega_{yi}}{\pi} + \frac{\psi_{yi}}{\pi} + 1\right), \tag{15}$$

where $S(\chi_{yi}) \in [0, 3]$.

Definition 3.4. Let $\chi_{yi} = \langle r_i^y, (\alpha_{yi}^p e^{i\omega_{yi}}, \beta_{yi}^n e^{\psi_{yi}})\rangle$ be a $\mathscr{CBFNS}_f\mathscr{N}$ over V. The *accuracy function* is defined as follows:

$$A(\chi_{yi}) = \frac{r_i^y}{N-1} + \alpha_{yi}^p - \beta_{yi}^n + \frac{\omega_{yi}}{\pi} - \frac{\psi_{yi}}{\pi}, \tag{16}$$

where $A(\chi_{yi}) \in [0, 4]$.

Definition 3.5. For any two distinct $\mathscr{CBFNS}_f\mathscr{N}$s $\chi_{yi} = \langle r_i^y, (\alpha_{yi}^p e^{i\omega_{yi}}, \beta_{yi}^n e^{\psi_{yi}})\rangle$ and $\chi_{xi} = \langle r_i^x, (\alpha_{xi}^p e^{i\omega_{xi}}, \beta_{xi}^n e^{\psi_{xi}})\rangle$, we have the following:

(1) If $S(\chi_{yi}) < S(\chi_{xi})$, then $\chi_{yi} \prec \chi_{xi}$ (χ_{xi} is superior to χ_{yi})
(2) If $S(\chi_{yi}) > S(\chi_{xi})$, then $\chi_{yi} \succ \chi_{xi}$ (χ_{xi} is inferior to χ_{yi})
(3) If $S(\chi_{yi}) = S(\chi_{xi})$, then
  (a) If $A(\chi_{yi}) > A(\chi_{xi})$, then $\chi_{yi} \prec \chi_{xi}$ (χ_{xi} is superior to χ_{yi})
  (b) If $A(\chi_{yi}) < A(\chi_{xi})$, then $\chi_{yi} \succ \chi_{xi}$ (χ_{xi} is inferior to χ_{yi})

Table 4: Grading criteria.

Grades r_i^y	Amplitude terms α_{yi}^p	β_{yi}^n	Phase terms ω_{yi}	ψ_{yi}
$r_i^y = 0$	$[0, 0.25)$	$[-1, -0.75)$	$[0, 0.25\pi)$	$[-\pi, -0.75\pi)$
$r_i^y = 1$	$[0.25, 0.50)$	$[-0.75, -0.50)$	$[0.25\pi, 0.50\pi)$	$[-0.75\pi, -0.50\pi)$
$r_i^y = 2$	$[0.50, 0.75)$	$[-0.50, -0.25)$	$[0.50\pi, 0.75\pi)$	$[-0.50\pi, -0.25\pi)$
$r_i^y = 3$	$[0.75, 1.00]$	$[-0.25, -0.00]$	$[0.75\pi, \pi]$	$[-0.25\pi, -0.00]$

Table 5: Tabular representation of the $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$.

$(\Upsilon_\Phi, \mathbb{T}, 4)$	l_1	l_2	l_3	l_4
v_1	$\langle 2, (0.55e^{i0.62\pi}, -0.33e^{-i0.29\pi})\rangle$	$\langle 2, (0.71e^{i0.63\pi}, -0.44e^{-i0.38\pi})\rangle$	$\langle 1, (0.42e^{i0.38\pi}, -0.57e^{-i0.65\pi})\rangle$	$\langle 2, (0.69e^{i0.65\pi}, -0.45e^{-i0.42\pi})\rangle$
v_2	$\langle 3, (0.81e^{i0.89\pi}, -0.15e^{-i0.12\pi})\rangle$	$\langle 3, (0.89e^{i0.81\pi}, -0.13e^{-i0.07\pi})\rangle$	$\langle 3, (0.86e^{i0.93\pi}, -0.08e^{-i0.17\pi})\rangle$	$\langle 3, (0.79e^{i0.82\pi}, -0.05e^{-i0.21\pi})\rangle$
v_3	$\langle 1, (0.46e^{i0.43\pi}, -0.52e^{-i0.62\pi})\rangle$	$\langle 2, (0.68e^{i0.66\pi}, -0.31e^{-i0.37\pi})\rangle$	$\langle 1, (0.39e^{i0.34\pi}, -0.64e^{-i0.57\pi})\rangle$	$\langle 1, (0.31e^{i0.29\pi}, -0.59e^{-i0.72\pi})\rangle$
v_4	$\langle 2, (0.65e^{i0.58\pi}, -0.29e^{-i0.34\pi})\rangle$	$\langle 3, (0.91e^{i0.78\pi}, -0.17e^{-i0.11\pi})\rangle$	$\langle 0, (0.15e^{i0.21\pi}, -0.89e^{-i0.92\pi})\rangle$	$\langle 1, (0.27e^{i0.31\pi}, -0.68e^{-i0.61\pi})\rangle$

(c) If $A(\chi_{yi}) = A(\chi_{xi})$, then $\chi_{yi} \sim \chi_{xi}$ (χ_{yi} is equivalent to χ_{xi})

Remark. It can be observed that

(1) For $N = 2$, $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ becomes $\mathscr{CBF}\mathscr{S}_f\mathscr{S}$.

(2) When $N = 2$ and $|\mathbb{L}| = 1$, $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ becomes $\mathscr{CBF}\mathscr{S}$.

Now, we will investigate the notions of complementarities of $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$.

Definition 3.6. Let $(\Upsilon_\Phi, \mathbb{T}, N)$ be a $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ over nonempty set V, where $\mathbb{T} = (\Phi, \mathbb{L}, N)$ is an $N\mathscr{S}_f\mathscr{S}$, then $(\Upsilon_\Phi, \mathbb{T}^c, N)$ is called *weak complement* if $\mathbb{T}^c = (\Phi^c, \mathbb{L}, N)$ is a weak complement of $\mathbb{T} = (\Phi, \mathbb{L}, N)$. In consequence, $\Phi^c(l_i) \cap \Phi(l_i) = \varnothing$ for all $l_i \in \mathbb{L}$.

Remember that a weak compliment is not peculiar.

Definition 3.7. Let $(\Upsilon_\Phi, \mathbb{T}, N)$ be a $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ over V, such that $\mathbb{T} = (\Phi, \mathbb{L}, N)$ is an $N\mathscr{S}_f\mathscr{S}$, then a *complex bipolar fuzzy complement* is represented by $(\Upsilon_\Phi^c, \mathbb{T}, N)$, such that $\Upsilon_\Phi^c\colon \mathbb{L} \longrightarrow \mathscr{CBFV}^{(V\times\mathscr{R})}$, which is conveyed by the following:

$$\Upsilon_\Phi^c(l_i) = \left\langle \left(v_y, r_i^y\right), \left(1 - \alpha_{yi}^p\right) e^{i\left(1-\left(\omega_{yi}/\pi\right)\right)\pi}, \left(-1 - \beta_{yi}^n\right) e^{i\left(-1-\left(\psi_{yi}/\pi\right)\right)\pi} \right\rangle. \tag{17}$$

Definition 3.8. Let $(\Upsilon_\Phi, \mathbb{T}, N)$ be a $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ over V, such that $\mathbb{T} = (\Phi, \mathbb{L}, N)$ is an $N\mathscr{S}_f\mathscr{S}$, then $(\Upsilon_\Phi^c, \mathbb{T}^c, N)$ is stated as *weak complex bipolar fuzzy complement* $\Leftrightarrow$ $(\Upsilon_\Phi, \mathbb{T}^c, N)$ is a weak complement and $(\Upsilon_\Phi^c, \mathbb{T}, N)$ is a complex bipolar fuzzy complement.

Example 3.9. Consider a $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$ $(\Upsilon_\Phi, \mathbb{T}, 4)$ as arranged in Example 3.2. A weak complement $(\Upsilon_\Phi, \mathbb{T}^c, 4)$, complex bipolar fuzzy complement $(\Upsilon_\Phi^c, \mathbb{T}, 4)$, and weak complex bipolar fuzzy complement $(\Upsilon_\Phi^c, \mathbb{T}^c, 4)$ are calculated and organized by Tables 6–8, respectively.

Definition 3.10. For a $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ $(\Upsilon_\Phi, \mathbb{T}, N)$, where $\mathbb{T} = (\Phi, \mathbb{L}, N)$ is an $N\mathscr{S}_f\mathscr{S}$, the *top weak complement* of $(\Upsilon_\Phi, \mathbb{T}, N)$ is $(\Upsilon_\Phi, \mathbb{T}^t, N)$, also the *top weak complex bipolar fuzzy complement* of $(\Upsilon_\Phi, \mathbb{T}, N)$ is $(\Upsilon_\Phi^c, \mathbb{T}^t, N)$ and defined as follows:

$$\left(\Upsilon_\Phi, \mathbb{T}^t, N\right) = \begin{cases} \Upsilon_\Phi(l_i) = \left(\left(v_y, N-1\right), \alpha_{yi}^p e^{i\omega_{yi}}, \beta_{yi}^n e^{i\psi_{yi}}\right), & \text{if } r_i^y < N-1, \\ \Upsilon_\Phi(l_i) = \left\langle\left(v_y, 0\right), \alpha_{yi}^p e^{i\omega_{yi}}, \beta_{yi}^n e^{i\psi_{yi}}\right\rangle, & \text{if } r_i^y = N-1. \end{cases}$$
$$\left(\Upsilon_\Phi^c, \mathbb{T}^t, N\right) = \begin{cases} \Upsilon_\Phi(l_i) = \left\langle\left(v_y, N-1\right), \left(1-\alpha_{yi}^p\right) e^{i\left(1-\omega_{yi}/\pi\right)\pi}, \left(-1-\beta_{yi}^n\right) e^{i\left(-1-\psi_{yi}/\pi\right)\pi}\right\rangle, & \text{if } r_i^y < N-1, \\ \Upsilon_\Phi(l_i) = \left\langle\left(v_y, 0\right), \left(1-\alpha_{yi}^p\right) e^{i\left(1-\omega_{yi}/\pi\right)\pi}, \left(-1-\beta_{yi}^n\right) e^{i\left(-1-\psi_{yi}/\pi\right)\pi}\right\rangle, & \text{if } r_i^y = N-1. \end{cases} \tag{18}$$

Example 3.11. The top weak complement and the top weak complex bipolar fuzzy complement of the $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$, defined by Table 5 in Example 3.2, are shown in Tables 9 and 10, respectively.

Definition 3.12. For a $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ $(\Upsilon_\Phi, \mathbb{T}, N)$, where $\mathbb{T} = (\Phi, \mathbb{L}, N)$ is an $N\mathscr{S}_f\mathscr{S}$, the *bottom weak complement* of $(\Upsilon_\Phi, \mathbb{T}, N)$ is $(\Upsilon_\Phi, \mathbb{T}^b, N)$ also the *bottom weak complex bipolar fuzzy complement* of $(\Upsilon_\Phi, \mathbb{T}, N)$ is $(\Upsilon_\Phi^c, \mathbb{T}^b, N)$ and defined as follows:

TABLE 6: A weak complement of. $(\Upsilon_{\Phi}, \mathbb{T}, 4)$.

$(\Upsilon_{\Phi}, \mathbb{T}^c, 4)$	l_1	l_2	l_3	l_4
v_1	$\langle 3, (0.55e^{i0.62\pi}, -0.33e^{-i0.29\pi})\rangle$	$\langle 3, (0.71e^{i0.63\pi}, -0.44e^{-i0.38\pi})\rangle$	$\langle 2, (0.42e^{i0.38\pi}, -0.57e^{-i0.65\pi})\rangle$	$\langle 1, (0.69e^{i0.65\pi}, -0.45e^{-i0.42\pi})\rangle$
v_2	$(0, (0.81e^{i0.89\pi}, -0.15e^{-i0.12\pi}))$	$\langle 1, (0.89e^{i0.81\pi}, -0.13e^{-i0.07\pi})\rangle$	$\langle 1, (0.86e^{i0.93\pi}, -0.08e^{-i0.17\pi})\rangle$	$\langle 0, (0.79e^{i0.82\pi}, -0.05e^{-i0.21\pi})\rangle$
v_3	$\langle 2, (0.46e^{i0.43\pi}, -0.52e^{-i0.62\pi})\rangle$	$\langle 0, (0.68e^{i0.66\pi}, -0.31e^{-i0.37\pi})\rangle$	$\langle 0, (0.39e^{i0.34\pi}, -0.64e^{-i0.57\pi})\rangle$	$\langle 3, (0.31e^{i0.29\pi}, -0.59e^{-i0.72\pi})\rangle$
v_4	$\langle 1, (0.65e^{i0.58\pi}, -0.29e^{-i0.34\pi})\rangle$	$\langle 2, (0.91e^{i0.78\pi}, -0.17e^{-i0.11\pi})\rangle$	$\langle 3, (0.15e^{i0.21\pi}, -0.89e^{-i0.92\pi})\rangle$	$\langle 2, (0.27e^{i0.31\pi}, -0.68e^{-i0.61\pi})\rangle$

TABLE 7: Complex bipolar fuzzy complement.

$(\Upsilon^c_{\Phi}, \mathbb{T}, 4)$	l_1	l_2	l_3	l_4
v_1	$\langle 2, (0.45e^{i0.38\pi}, -0.67e^{-i0.71\pi})\rangle$	$\langle 2, (0.29e^{i0.37\pi}, -0.56e^{-i0.62\pi})\rangle$	$\langle 1, (0.58e^{i0.62\pi}, -0.43e^{-i0.35\pi})\rangle$	$\langle 2, (0.31e^{i0.35\pi}, -0.55e^{-i0.58\pi})\rangle$
v_2	$\langle 3, (0.19e^{i0.11\pi}, -0.85e^{-i0.88\pi})\rangle$	$\langle 3, (0.11e^{i0.19\pi}, -0.87e^{-i0.93\pi})\rangle$	$\langle 3, (0.14e^{i0.07\pi}, -0.92e^{-i0.83\pi})\rangle$	$\langle 3, (0.21e^{i0.18\pi}, -0.95e^{-i0.79\pi})\rangle$
v_3	$\langle 1, (0.54e^{i0.57\pi}, -0.48e^{-i0.38\pi})\rangle$	$\langle 2, (0.32e^{i0.34\pi}, -0.69e^{-i0.63\pi})\rangle$	$\langle 1, (0.61e^{i0.66\pi}, -0.36e^{-i0.43\pi})\rangle$	$\langle 1, (0.69e^{i0.71\pi}, -0.41e^{-i0.28\pi})\rangle$
v_4	$\langle 2, (0.35e^{i0.42\pi}, -0.71e^{-i0.66\pi})\rangle$	$\langle 3, (0.09e^{i0.22\pi}, -0.83e^{-i0.89\pi})\rangle$	$\langle 0, (0.85e^{i0.79\pi}, -0.11e^{-i0.08\pi})\rangle$	$\langle 1, (0.73e^{i0.69\pi}, -0.32e^{-i0.39\pi})\rangle$

TABLE 8: Weak complex bipolar fuzzy complement.

$(\Upsilon^c_{\Phi}, \mathbb{T}^c, 4)$	l_1	l_2	l_3	l_4
v_1	$\langle 3, (0.45e^{i0.38\pi}, -0.67e^{-i0.71\pi})\rangle$	$\langle 3, (0.29e^{i0.37\pi}, -0.56e^{-i0.62\pi})\rangle$	$\langle 2, (0.58e^{i0.62\pi}, -0.43e^{-i0.35\pi})\rangle$	$\langle 1, (0.31e^{i0.35\pi}, -0.55e^{-i0.58\pi})\rangle$
v_2	$\langle 0, (0.19e^{i0.11\pi}, -0.85e^{-i0.88\pi})\rangle$	$\langle 1, (0.11e^{i0.19\pi}, -0.87e^{-i0.93\pi})\rangle$	$\langle 1, (0.14e^{i0.07\pi}, -0.92e^{-i0.83\pi})\rangle$	$\langle 0, (0.21e^{i0.18\pi}, -0.95e^{-i0.79\pi})\rangle$
v_3	$\langle 2, (0.54e^{i0.57\pi}, -0.48e^{-i0.38\pi})\rangle$	$\langle 0, (0.32e^{i0.34\pi}, -0.69e^{-i0.63\pi})\rangle$	$\langle 0, (0.61e^{i0.66\pi}, -0.36e^{-i0.43\pi})\rangle$	$\langle 3, (0.69e^{i0.71\pi}, -0.41e^{-i0.28\pi})\rangle$
v_4	$\langle 1, (0.35e^{i0.42\pi}, -0.71e^{-i0.66\pi})\rangle$	$\langle 2, (0.09e^{i0.22\pi}, -0.83e^{-i0.89\pi})\rangle$	$\langle 3, (0.85e^{i0.79\pi}, -0.11e^{-i0.08\pi})\rangle$	$\langle 2, (0.73e^{i0.69\pi}, -0.32e^{-i0.39\pi})\rangle$

$$\left(\Upsilon_{\Phi}, \mathbb{T}^b, N\right) = \begin{cases} \Upsilon_{\Phi}(l_i) = \left\langle \left(v_y, 0\right), \alpha^p_{yi} e^{i\omega_{yi}}, \beta^n_{yi} e^{i\psi_{yi}} \right\rangle, & \text{if } r^y_i > 0, \\ \Upsilon_{\Phi}(l_i) = \left\langle \left(v_y, N-1\right), \alpha^p_{yi} e^{i\omega_{yi}}, \beta^n_{yi} e^{i\psi_{yi}} \right\rangle, & \text{if } r^y_i = 0. \end{cases}$$
$$\left(\Upsilon^c_{\Phi}, \mathbb{T}^b, N\right) = \begin{cases} \Upsilon_{\Phi}(l_i) = \left\langle \left(v_y, 0\right), \left(1-\alpha^p_{yi}\right) e^{i\left(1-\omega_{yi}/\pi\right)\pi}, \left(-1-\beta^n_{yi}\right) e^{i\left(-1-\left(\psi_{yi}/\pi\right)\right)\pi} \right\rangle, & \text{if } r^y_i > 0, \\ \Upsilon_{\Phi}(l_i) = \left\langle \left(v_y, N-1\right), \left(1-\alpha^p_{yi}\right) e^{i\left(1-\omega_{yi}/\pi\right)\pi}, \left(-1-\beta^n_{yi}\right) e^{i\left(-1-\psi_{yi}/\pi\right)\pi} \right\rangle & \text{if } r^y_i = 0. \end{cases} \quad (19)$$

Example 3.13. The bottom weak complement and the bottom weak complex bipolar fuzzy complement of the $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$, defined by Table 5 in Example 3.2, are given by Tables 11 and 12, respectively.

Definition 3.14. Let V be a nonempty set and $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$ and $(\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)$ be two $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}s$ over V, where $\mathbb{T}_1 = (\Phi_1, \mathbb{L}_1, N_1)$ and $\mathbb{T}_2 = (\Phi_2, \mathbb{L}_2, N_2)$ are $N\mathscr{S}_f\mathscr{S}s$ over V, then their *restricted intersection* is defined as follows:

$$\left(\varrho, \mathbb{T}_1 \cap_{\mathfrak{r}} \mathbb{T}_2, N_3\right) = \left(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1\right) \cap_{\mathfrak{R}} \left(\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2\right), \quad (20)$$

where, $N_3 = \min(N_1, N_2)$ and $\mathbb{T}_1 \cap_{\mathfrak{r}} \mathbb{T}_2 = (\mathfrak{A}, \mathbb{L}_1 \cap \mathbb{L}_2, N_3)$ i.e. $\forall l_i \in \mathbb{L}_1 \cap \mathbb{L}_2$, $v_y \in V$, $\langle (v_y, r^y_i), \quad a, b\rangle \in \varrho(l_i) \Leftrightarrow r^y_i = \min(r^{v^1}_i, r^{v^2}_i)$, $a = \min(\alpha^p_{\mathscr{E}}, \quad \alpha^p_{\mathscr{F}})\, e^{i\min(\omega_{\mathscr{E}}, \omega_{\mathscr{F}})}$, $b = \max(\beta^n_{\mathscr{E}}, \beta^n_{\mathscr{F}}) e^{i\max(\psi_{\mathscr{E}}, \psi_{\mathscr{F}})}$, if $\left\langle (v_y, r^{v^1}_i), \alpha^p_{\mathscr{E}} e^{i\omega_{\mathscr{E}}}, \beta^n_{\mathscr{E}} e^{i\psi_{\mathscr{E}}} \right\rangle \in \mathbb{L}_1(l_i)$ and $\left\langle (v_y, r^{v^2}_i), \alpha^p_{\mathscr{F}} e^{i\omega_{\mathscr{F}}}, \beta^n_{\mathscr{F}} e^{i\psi_{\mathscr{F}}} \right\rangle \in \mathbb{L}_2(l_i)$, $\mathscr{E}$ and $\mathscr{F}$ are complex bipolar fuzzy sets on $\Phi_1(l_i)$ and $\Phi_2(l_i)$, respectively.

Example 3.15. Let $(\Upsilon_{\Phi_1}, \mathbb{T}_1, 5)$ be a $\mathscr{CBF}5\mathscr{S}_f\mathscr{S}$ and $(\Upsilon_{\Phi_2}, \mathbb{T}_2, 4)$ be a $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$ given by Tables 13 and 14, respectively, then their restricted intersection $(\varrho, \mathbb{T}_1 \cap_{\mathfrak{r}} \mathbb{T}_2, 4) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, 5) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, 4)$, is presented in Table 15.

Definition 3.16. Let V be a universe of discourse and $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$ and $(\Upsilon_{\Phi_2}, \mathbb{T}_2, N_1)$ be two $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}s$ over V, where $\mathbb{T}_1 = (\Phi_1, \mathbb{L}_1, N_1)$ and $\mathbb{T}_2 = (\Phi_2, \mathbb{L}_2, N_2)$ are $N\mathscr{S}_f\mathscr{S}s$ on V; then their *extended intersection* is given as $(\eta, \mathbb{T}_1 \cap_{\mathfrak{e}} \mathbb{T}_2, N_4) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)$, where $N_4 = \max(N_1, N_2)$, $\mathbb{T}_1 \cap_{\mathfrak{e}} \mathbb{T}_2 = (\mathfrak{B}, \mathbb{L}_1 \cup \mathbb{L}_2, N_4)$ defined by the following:

Table 9: Top weak complement of the $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$.

$(\Upsilon_\Phi, \mathbb{T}^t, 4)$	l_1	l_2	l_3	l_4
v_1	$\langle 3, (0.55e^{i0.62\pi}, -0.33e^{-i0.29\pi})\rangle$	$\langle 3, (0.71e^{i0.63\pi}, -0.44e^{-i0.38\pi})\rangle$	$\langle 3, (0.42e^{i0.38\pi}, -0.57e^{-i0.65\pi})\rangle$	$\langle 3, (0.69e^{i0.65\pi}, -0.45e^{-i0.42\pi})\rangle$
v_2	$\langle 0, (0.81e^{i0.89\pi}, -0.15e^{-i0.12\pi})\rangle$	$\langle 0, (0.89e^{i0.81\pi}, -0.13e^{-i0.07\pi})\rangle$	$\langle 0, (0.86e^{i0.93\pi}, -0.08e^{-i0.17\pi})\rangle$	$\langle 0, (0.79e^{i0.82\pi}, -0.05e^{-i0.21\pi})\rangle$
v_3	$\langle 3, (0.46e^{i0.43\pi}, -0.52e^{-i0.62\pi})\rangle$	$\langle 3, (0.68e^{i0.66\pi}, -0.31e^{-i0.37\pi})\rangle$	$\langle 3, (0.39e^{i0.34\pi}, -0.64e^{-i0.57\pi})\rangle$	$\langle 3, (0.31e^{i0.29\pi}, -0.59e^{-i0.72\pi})\rangle$
v_4	$\langle 3, (0.65e^{i0.58\pi}, -0.29e^{-i0.34\pi})\rangle$	$\langle 0, (0.91e^{i0.78\pi}, -0.17e^{-i0.11\pi})\rangle$	$\langle 3, (0.15e^{i0.21\pi}, -0.89e^{-i0.92\pi})\rangle$	$\langle 3, (0.27e^{i0.31\pi}, -0.68e^{-i0.61\pi})\rangle$

Table 10: Top weak complex bipolar fuzzy complement.

$(\Upsilon^c_\Phi, \mathbb{T}^t, 4)$	l_1	l_2	l_3	l_4
v_1	$\langle 3, (0.45e^{i0.38\pi}, -0.67e^{-i0.71\pi})\rangle$	$\langle 3, (0.29e^{i0.37\pi}, -0.56e^{-i0.62\pi})\rangle$	$\langle 3, (0.58e^{i0.62\pi}, -0.43e^{-i0.35\pi})\rangle$	$\langle 3, (0.31e^{i0.35\pi}, -0.55e^{-i0.58\pi})\rangle$
v_2	$\langle 0, (0.19e^{i0.11\pi}, -0.85e^{-i0.88\pi})\rangle$	$\langle 0, (0.11e^{i0.19\pi}, -0.87e^{-i0.93\pi})\rangle$	$\langle 0, (0.14e^{i0.07\pi}, -0.92e^{-i0.83\pi})\rangle$	$\langle 0, (0.21e^{i0.18\pi}, -0.95e^{-i0.79\pi})\rangle$
v_3	$\langle 3, (0.54e^{i0.57\pi}, -0.48e^{-i0.38\pi})\rangle$	$\langle 3, (0.32e^{i0.34\pi}, -0.69e^{-i0.63\pi})\rangle$	$\langle 3, (0.61e^{i0.66\pi}, -0.36e^{-i0.43\pi})\rangle$	$\langle 3, (0.69e^{i0.71\pi}, -0.41e^{-i0.28\pi})\rangle$
v_4	$\langle 3, (0.35e^{i0.42\pi}, -0.71e^{-i0.66\pi})\rangle$	$\langle 0, (0.09e^{i0.22\pi}, -0.83e^{-i0.89\pi})\rangle$	$\langle 3, (0.85e^{i0.79\pi}, -0.11e^{-i0.08\pi})\rangle$	$\langle 3, (0.73e^{i0.69\pi}, -0.32e^{-i0.39\pi})\rangle$

Table 11: Bottom weak complement of the $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$.

$(\Upsilon_\Phi, \mathbb{T}^b, 4)$	l_1	l_2	l_3	l_4
v_1	$\langle 0, (0.55e^{i0.62\pi}, -0.33e^{-i0.29\pi})\rangle$	$\langle 0, (0.71e^{i0.63\pi}, -0.44e^{-i0.38\pi})\rangle$	$\langle 0, (0.42e^{i0.38\pi}, -0.57e^{-i0.65\pi})\rangle$	$\langle 0, (0.69e^{i0.65\pi}, -0.45e^{-i0.42\pi})\rangle$
v_2	$\langle 0, (0.81e^{i0.89\pi}, -0.15e^{-i0.12\pi})\rangle$	$\langle 0, (0.89e^{i0.81\pi}, -0.13e^{-i0.07\pi})\rangle$	$\langle 0, (0.86e^{i0.93\pi}, -0.08e^{-i0.17\pi})\rangle$	$\langle 0, (0.79e^{i0.82\pi}, -0.05e^{-i0.21\pi})\rangle$
v_3	$\langle 0, (0.46e^{i0.43\pi}, -0.52e^{-i0.62\pi})\rangle$	$\langle 0, (0.68e^{i0.66\pi}, -0.31e^{-i0.37\pi})\rangle$	$\langle 0, (0.39e^{i0.34\pi}, -0.64e^{-i0.57\pi})\rangle$	$\langle 0, (0.31e^{i0.29\pi}, -0.59e^{-i0.72\pi})\rangle$
v_4	$\langle 0, (0.65e^{i0.58\pi}, -0.29e^{-i0.34\pi})\rangle$	$\langle 0, (0.91e^{i0.78\pi}, -0.17e^{-i0.11\pi})\rangle$	$\langle 3, (0.15e^{i0.21\pi}, -0.89e^{-i0.92\pi})\rangle$	$\langle 0, (0.27e^{i0.31\pi}, -0.68e^{-i0.61\pi})\rangle$

Table 12: Bottom weak complex bipolar fuzzy complement.

$(\Upsilon^c_\Phi, \mathbb{T}^b, 4)$	l_1	l_2	l_3	l_4
v_1	$\langle 0, (0.45e^{i0.38\pi}, -0.67e^{-i0.71\pi})\rangle$	$\langle 0, (0.29e^{i0.37\pi}, -0.56e^{-i0.62\pi})\rangle$	$\langle 0, (0.58e^{i0.62\pi}, -0.43e^{-i0.35\pi})\rangle$	$\langle 0, (0.31e^{i0.35\pi}, -0.55e^{-i0.58\pi})\rangle$
v_2	$\langle 0, (0.19e^{i0.11\pi}, -0.85e^{-i0.88\pi})\rangle$	$\langle 0, (0.11e^{i0.19\pi}, -0.87e^{-i0.93\pi})\rangle$	$\langle 0, (0.14e^{i0.07\pi}, -0.92e^{-i0.83\pi})\rangle$	$\langle 0, (0.21e^{i0.18\pi}, -0.95e^{-i0.79\pi})\rangle$
v_3	$\langle 0, (0.54e^{i0.57\pi}, -0.48e^{-i0.38\pi})\rangle$	$\langle 0, (0.32e^{i0.34\pi}, -0.69e^{-i0.63\pi})\rangle$	$\langle 0, (0.61e^{i0.66\pi}, -0.36e^{-i0.43\pi})\rangle$	$\langle 0, (0.69e^{i0.71\pi}, -0.41e^{-i0.28\pi})\rangle$
v_4	$\langle 0, (0.35e^{i0.42\pi}, -0.71e^{-i0.66\pi})\rangle$	$\langle 0, (0.09e^{i0.22\pi}, -0.83e^{-i0.89\pi})\rangle$	$\langle 3, (0.85e^{i0.79\pi}, -0.11e^{-i0.08\pi})\rangle$	$\langle 0, (0.73e^{i0.69\pi}, -0.32e^{-i0.39\pi})\rangle$

Table 13: Tabular representation of $\mathscr{CBF}5\mathscr{S}_f\mathscr{S}$ $(\Upsilon_{\Phi_1}, \mathbb{T}_1, 5)$.

$(\Upsilon_{\Phi_1}, \mathbb{T}_1, 5)$	l_1	l_2	l_3
v_1	$\langle 3, (0.65e^{i0.72\pi}, -0.33e^{-i0.29\pi})\rangle$	$\langle 4, (0.92e^{i0.83\pi}, -0.03e^{-i0.15\pi})\rangle$	$\langle 1, (0.25e^{i0.32\pi}, -0.63e^{-i0.72\pi})\rangle$
v_2	$\langle 2, (0.43e^{i0.52\pi}, -0.52e^{-i0.43\pi})\rangle$	$\langle 4, (0.87e^{i0.93\pi}, -0.13e^{-i0.09\pi})\rangle$	$\langle 2, (0.57e^{i0.45\pi}, -0.45e^{-i0.51\pi})\rangle$
v_3	$\langle 1, (0.27e^{i0.21\pi}, -0.65e^{-i0.75\pi})\rangle$	$\langle 3, (0.75e^{i0.69\pi}, -0.25e^{-i0.33\pi})\rangle$	$\langle 0, (0.12e^{i0.07\pi}, -0.92e^{-i0.83\pi})\rangle$

Table 14: Tabular representation of $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$ $(\Upsilon_{\Phi_2}, \mathbb{T}_2, 4)$.

$(\Upsilon_{\Phi_2}, \mathbb{T}_2, 4)$	l_1	l_2	l_4
v_1	$\langle 3, (0.81e^{i0.92\pi}, -0.17e^{-i0.09\pi})\rangle$	$\langle 2, (0.56e^{i0.67\pi}, -0.29e^{-i0.32\pi})\rangle$	$\langle 1, (0.31e^{i0.37\pi}, -0.65e^{-i0.59\pi})\rangle$
v_2	$\langle 2, (0.61e^{i0.57\pi}, -0.35e^{-i0.41\pi})\rangle$	$\langle 3, (0.89e^{i0.95\pi}, -0.13e^{-i0.19\pi})\rangle$	$\langle 2, (0.71e^{i0.63\pi}, -0.38e^{-i0.42\pi})\rangle$
v_3	$\langle 0, (0.09e^{i0.21\pi}, -0.89e^{-i0.91\pi})\rangle$	$\langle 1, (0.43e^{i0.47\pi}, -0.58e^{-i0.61\pi})\rangle$	$\langle 0, (0.12e^{i0.17\pi}, -0.86e^{-i0.79\pi})\rangle$

Table 15: Restricted intersection.

$(\varrho, \mathbb{T}_1 \cap_{\mathfrak{r}} \mathbb{T}_2, 4)$	l_1	l_2
v_1	$\langle 3, (0.65e^{i0.72\pi}, -0.17e^{-i0.09\pi})\rangle$	$\langle 2, (0.56e^{i0.67\pi}, -0.03e^{-i0.15\pi})\rangle$
v_2	$\langle 2, (0.43e^{i0.52\pi}, -0.35e^{-i0.41\pi})\rangle$	$\langle 3, (0.87e^{i0.93\pi}, -0.13e^{-i0.09\pi})\rangle$
v_3	$\langle 0, (0.09e^{i0.21\pi}, -0.65e^{-i0.75\pi})\rangle$	$\langle 1, (0.43e^{i0.47\pi}, -0.25e^{-i0.33\pi})\rangle$

$$\eta(l_i) = \begin{cases} \Upsilon_{\Phi_1}(l_i), & \text{if } l_i \in \mathbb{L}_1 - \mathbb{L}_2, \\ \Upsilon_{\Phi_2}(l_i), & \text{if } l_i \in \mathbb{L}_2 - \mathbb{L}_1, \\ \left\langle \left(v_y, r_i^y\right), a, b \right\rangle, & \text{such that } r_i^y = \min\left(r_1^{v^1}, r_1^{v^2}\right), \\ a = \min\left(\alpha_{\mathscr{E}}^p, \alpha_{\mathscr{F}}^p\right)e^{i\min(\omega_{\mathscr{E}},\omega_{\mathscr{F}})}, & b = \max\left(\beta_{\mathscr{E}}^n, \beta_{\mathscr{F}}^n\right)e^{i\max(\psi_{\mathscr{E}},\psi_{\mathscr{F}})}, \\ \text{where } \left\langle \left(v_y, r_1^{v^1}\right), \alpha_{\mathscr{E}}^p e^{i\omega_{\mathscr{E}}}, \beta_{\mathscr{E}}^n e^{i\psi_{\mathscr{E}}} \right\rangle \in \mathbb{L}_1(l_i), \\ \text{and } \left\langle \left(v_y, r_1^{v^2}\right), \alpha_{\mathscr{F}}^p e^{i\omega_{\mathscr{F}}}, \beta_{\mathscr{F}}^n e^{i\psi_{\mathscr{F}}} \right\rangle \in \mathbb{L}_2(l_i), \\ \mathscr{E} \text{ and } \mathscr{F} \text{ are complex bipolar fuzzy sets on } \Phi_1(l_i) \text{ and } \Phi_2(l_i), \\ \text{respectively.} \end{cases} \tag{21}$$

Example 3.17. Let $(\Upsilon_{\Phi_1}, \mathbb{T}_1, 5)$ and $(\Upsilon_{\Phi_2}, \mathbb{T}_2, 4)$ be two $\mathscr{CBF}5\mathscr{S}_f\mathscr{S}$ and $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$, respectively, arranged by Tables 13 and 14, respectively. Then their extended intersection $(\eta, \mathbb{T}_1 \cap_{\mathfrak{e}} \mathbb{T}_2, 5) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, 5) \cap_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, 4)$ is given in Table 16.

Definition 3.18. Let V be a universal set and $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$ and $(\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)$ be two $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}s$ over V, where $\mathbb{T}_1 = (\Phi_1, \mathbb{L}_1, N_1)$ and $\mathbb{T}_2 = (\Phi_2, \mathbb{L}_2, N_2)$ are $N\mathscr{S}_f\mathscr{S}s$ on V; then their *restricted union* is defined as follows:

$$\left(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1\right) \cup_{\mathfrak{R}} \left(\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2\right) = (\sigma, \mathbb{T}_1 \cup_{\mathfrak{r}} \mathbb{T}_2, N_4), \tag{22}$$

where $N_4 = \max(N_1, N_2)$, $\mathbb{T}_1 \cup_{\mathfrak{r}} \mathbb{T}_2 = (\mathfrak{E}, \mathbb{L}_1 \cap \mathbb{L}_2, N_4)$, i.e. $\forall l_i \in \mathbb{L}_1 \cap \mathbb{L}_2$, $v_y \in V$, $\langle (v_y, r_i^y), a, \quad b\rangle \in \sigma(l_i) \Leftrightarrow r_i^y = \max(r_1^{v^1}, r_1^{v^2})$, $a = \max(\alpha_{\mathscr{E}}^p, \quad \alpha_{\mathscr{F}}^p)e^{i\max(\omega_{\mathscr{E}},\omega_{\mathscr{F}})}$, $b = \min(\beta_{\mathscr{E}}^n, \beta_{\mathscr{F}}^n)e^{i\min(\omega_{\mathscr{E}},\omega_{\mathscr{F}})}$, if $\langle (v_y, r_i^{v^1}), \ \alpha_{\mathscr{E}}^p e^{i\omega_{\mathscr{E}}}, \ \beta_{\mathscr{E}}^n e^{i\psi_{\mathscr{E}}}\rangle \in \mathbb{L}_1(l_i)$ and $\langle (v_y, r_i^{v^2}), \alpha_{\mathscr{F}}^p e^{i\omega_{\mathscr{F}}}, \beta_{\mathscr{F}}^n e^{i\psi_{\mathscr{F}}}\rangle \in \mathbb{L}_2(l_i)$, $\mathscr{E}$ and $\mathscr{F}$ are complex bipolar fuzzy sets on $\Phi_1(l_i)$ and $\Phi_2(l_i)$, respectively.

Example 3.19. Let $(\Upsilon_{\Phi_1}, \mathbb{T}_1, 5)$ and $(\Upsilon_{\Phi_2}, \mathbb{T}_2, 4)$ be two $\mathscr{CBF}5\mathscr{S}_f\mathscr{S}$ and $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$, respectively, defined by Tables 13 and 14, respectively; then their restricted union $(\Upsilon_{\Phi_1}, \mathbb{T}_1, 5) \cup_{\mathfrak{R}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, 4) = (\sigma, \mathbb{T}_1 \cup_{\mathfrak{r}} \mathbb{T}_2, 5)$ is defined in Table 17.

Definition 3.20. Let V be a nonempty and $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$ and $(\Upsilon_{\Phi_2}, \mathbb{T}_2, N_1)$ be two $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}s$ on V, where $\mathbb{T}_1 = (\Phi_1, \mathbb{L}_1, N_1)$ and $\mathbb{T}_2 = (\Phi_2, \mathbb{L}_2, N_2)$ are $N\mathscr{S}_f\mathscr{S}s$ over V; then their *extended union* is described as $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) = (\vartheta, \mathbb{T}_1 \cup_{\epsilon} \mathbb{T}_2, N_4)$, where $N_4 = \max(N_1, N_2)$, $\mathbb{T}_1 \cup_{\varepsilon} \mathbb{T}_2 = (\mathfrak{F}, \mathbb{L}_1 \cup \mathbb{L}_2, N_4)$, and $\vartheta(l_i)$ is given by the following:

$$\vartheta(l_i) = \begin{cases} \Upsilon_{\Phi_1}(l_i), & \text{if } l_i \in \mathbb{L}_1 - \mathbb{L}_2, \\ \Upsilon_{\Phi_2}(l_i), & \text{if } l_i \in \mathbb{L}_2 - \mathbb{L}_1, \\ \left\langle \left(v_y, r_i^y\right), a, b \right\rangle, & \text{such that } r_i^y = \max\left(r_1^{v^1}, r_1^{v^2}\right), \\ a = \max\left(\alpha_{\mathscr{E}}^p, \alpha_{\mathscr{F}}^p\right)e^{i\max(\omega_{\mathscr{E}},\omega_{\mathscr{F}})}, & b = \min\left(\beta_{\mathscr{E}}^n, \beta_{\mathscr{F}}^n\right)e^{i\min(\psi_{\mathscr{E}},\psi_{\mathscr{F}})}, \\ \text{where } \left\langle \left(v_y, r_1^{v^1}\right), \alpha_{\mathscr{E}}^p e^{i\omega_{\mathscr{E}}}, \beta_{\mathscr{E}}^n e^{i\psi_{\mathscr{E}}} \right\rangle \in \mathbb{L}_1(l_i), \\ \text{and } \left\langle \left(v_y, r_1^{v^2}\right), \alpha_{\mathscr{F}}^p e^{i\omega_{\mathscr{F}}}, \beta_{\mathscr{F}}^n e^{i\psi_{\mathscr{F}}} \right\rangle \in \mathbb{L}_2(l_i), \\ \mathscr{E} \text{ and } \mathscr{F} \text{ are complex bipolar fuzzy sets on } \Phi_1(l_i) \text{and } \Phi_2(l_i), \\ \text{respectively.} \end{cases} \tag{23}$$

Table 16: Extended intersection.

$(\eta, \mathbb{T}_1 \cap_{\mathfrak{r}} \mathbb{T}_2, 5)$	l_1	l_2
v_1	$\langle 3, (0.65e^{i0.72\pi}, -0.17e^{-i0.09\pi})\rangle$	$\langle 2, (0.56e^{i0.67\pi}, -0.03e^{-i0.15\pi})\rangle$
v_2	$\langle 2, (0.43e^{i0.52\pi}, -0.35e^{-i0.41\pi})\rangle$	$\langle 3, (0.87e^{i0.93\pi}, -0.13e^{-i0.09\pi})\rangle$
v_3	$\langle 0, (0.09e^{i0.21\pi}, -0.65e^{-i0.75\pi})\rangle$	$\langle 1, (0.43e^{i0.47\pi}, -0.25e^{-i0.33\pi})\rangle$
	l_3	l_4
v_1	$\langle 1, (0.25e^{i0.32\pi}, -0.63e^{-i0.72\pi})\rangle$	$\langle 1, (0.31e^{i0.37\pi}, -0.65e^{-i0.59\pi})\rangle$
v_2	$\langle 2, (0.57e^{i0.45\pi}, -0.45e^{-i0.51\pi})\rangle$	$\langle 2, (0.71e^{i0.63\pi}, -0.38e^{-i0.42\pi})\rangle$
v_3	$\langle 0, (0.12e^{i0.07\pi}, -0.92e^{-i0.83\pi})\rangle$	$\langle 0, (0.12e^{i0.17\pi}, -0.86e^{-i0.79\pi})\rangle$

Table 17: Restricted union.

$(\sigma, \mathbb{T}_1 \cap_{\mathfrak{r}} \mathbb{T}_2, 5)$	l_1	l_2
v_1	$\langle 3, (0.81e^{i0.92\pi}, -0.33e^{-i0.29\pi})\rangle$	$\langle 4, (0.92e^{i0.83\pi}, -0.29e^{-i0.32\pi})\rangle$
v_2	$\langle 2, (0.61e^{i0.57\pi}, -0.52e^{-i0.43\pi})\rangle$	$\langle 4, (0.89e^{i0.95\pi}, -0.13e^{-i0.19\pi})\rangle$
v_3	$\langle 1, (0.27e^{i0.21\pi}, -0.89e^{-i0.91\pi})\rangle$	$\langle 3, (0.75e^{i0.69\pi}, -0.58e^{-i0.61\pi})\rangle$

Example 3.21. Let $(\Upsilon_{\Phi_1}, \mathbb{T}_1, 5)$ and $(\Upsilon_{\Phi_2}, \mathbb{T}_2, 4)$ be two $\mathscr{CBF}5\mathscr{S}_f\mathscr{S}$ and $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$, respectively, shown by Tables 13 and 14, respectively; then their extended union $(\Upsilon_{\Phi_1}, \mathbb{T}_1, 5) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, 4) = (\vartheta, \mathbb{T}_1 \cup_{\epsilon} \mathbb{T}_2, 5)$ is given in Table 18.

Definition 3.22. Consider that $(\Upsilon_\Phi, \mathbb{T}, N)$ be a $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ over nonempty set V, where $\mathbb{T} = (\Phi, \mathbb{L}, N)$ is an $N\mathscr{S}_f\mathscr{S}$ on V. Let $0 < \mathscr{D} < N$ be a threshold. A $\mathscr{CBFS}\{\_f\}\mathscr{S}$ related with $(\Upsilon_\Phi, \mathbb{T}, N)$ and $\mathscr{D}$, denoted by $(\Upsilon_\Phi^{\mathscr{D}}, \mathbb{L})$, is defined as follows:

$$\Upsilon_\Phi^{\mathscr{D}}(l_i) = \begin{cases} \left(\alpha_{yi}^{p} e^{i\omega_{yi}}, \beta_{yi}^{n} e^{i\psi_{yi}}\right), & \text{if } \Upsilon_\Phi(l_i) = \left\langle \left(v_y, r_i^y\right), \alpha_{yi}^{p} e^{i\omega_{yi}}, \beta_{yi}^{n} e^{i\psi_{yi}} \right\rangle, \\ r_i^y \geq \mathscr{D}, & \\ \left(0.0e^{i0\pi}, -1.0e^{-i\pi}\right), & \text{otherwise}. \end{cases} \quad (24)$$

Particularly, $(\Upsilon_\Phi^1, \mathbb{L})$, and $(\Upsilon_\Phi^{N-1}, \mathbb{L})$, are known as bottom $\mathscr{CBFS}\{\_f\}\mathscr{S}$ and top $\mathscr{CBFS}\{\_f\}\mathscr{S}$, respectively.

Definition 3.23. Let $0 < \mathscr{D} < N$ and $\rho \in [0, 2]$ be the threshold. Then the $\mathscr{S}_f\mathscr{S}$ on V associated with $(\Upsilon_\Phi, \mathbb{T}, N)$ and $(\mathscr{D}, \rho)$ denoted by $(\Upsilon_\Phi^{(\mathscr{D},\rho)}, \mathbb{L})$ is defined as follows:

$$\Upsilon_\Phi^{(\mathscr{D},\rho)}(l_i) = \left\{ y \in V \colon S\left(\Upsilon_\Phi^{\mathscr{D}}(l_i)\right) > \rho, \forall l_i \in \mathbb{L} \right\}, \quad (25)$$

where, $S(\Upsilon_\Phi^{\mathscr{D}}(l_i))$ is represented as the score value of $\Upsilon_\Phi^{\mathscr{D}}(l_i) = (\alpha_{yi}^{p(\mathscr{D})} e^{i\omega_{yi}^{\mathscr{D}}}, \beta_{yi}^{n(\mathscr{D})} e^{i\psi_{yi}^{\mathscr{D}}})$.

Example 3.24. Consider the $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$ defined in Example 3.2. By employing Definition 3.22, the associated $\mathscr{CBFS}_f\mathscr{S}s$ with $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$ can be found out. Let $0 < \mathscr{D} < 4$ be the thresholds. The possible associated $\mathscr{CBFS}_f\mathscr{S}s$ with threshold values 1, 2 and 3 are given by Tables 19–21. Moreover, by taking $(\mathscr{D}, \rho) = (2, 0.9)$, associated $\mathscr{S}_f\mathscr{S}$ $(\Upsilon_\Phi^{(2,0.9)}, \mathbb{L})$ is arranged by Table 22.

It is clear from above analysis that $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ can be converted into $\mathscr{CBFS}_f\mathscr{S}$ and $\mathscr{S}_f\mathscr{S}$. Hence, it is a generalization of both these models. The following properties are stated without proofs.

Theorem 3.25. *Let $(\Upsilon_\Phi, \mathbb{T}, N)$ be a $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ over V. Then:*

(1) $(\Upsilon_\Phi, \mathbb{T}, N) \cap_{\mathfrak{R}} (\Upsilon_\Phi, \mathbb{T}, N) = (\Upsilon_\Phi, \mathbb{T}, N)$

(2) $(\Upsilon_\Phi, \mathbb{T}, N) \cap_{\mathfrak{E}} (\Upsilon_\Phi, \mathbb{T}, N) = (\Upsilon_\Phi, \mathbb{T}, N)$

(3) $(\Upsilon_\Phi, \mathbb{T}, N) \cup_{\mathfrak{R}} (\Upsilon_\Phi, \mathbb{T}, N) = (\Upsilon_\Phi, \mathbb{T}, N)$

(4) $(\Upsilon_\Phi, \mathbb{T}, N) \cup_{\mathfrak{E}} (\Upsilon_\Phi, \mathbb{T}, N) = (\Upsilon_\Phi, \mathbb{T}, N)$

Theorem 3.26. *Let $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$ and $(\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)$ be two $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}s$ over V. Then the absorption properties are preserved:*

(1) $((\Upsilon_{\Phi_1}, T_1, N_1) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$

(2) $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{E}} ((\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$

(3) $((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$

(4) $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{R}} ((\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$

Theorem 3.27. *Let $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$, $(\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)$ and $(\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3)$ be three $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}s$ over V, then the following properties hold:*

(1) $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{R}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) = (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cup_{\mathfrak{R}} (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$

(2) $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) = (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$

Table 18: Extended union.

$(\vartheta, \mathbb{T}_1 \cap \ \mathfrak{r}\, \mathbb{T}_2, 5)$	l_1	l_2
v_1	$\langle 3, (0.81e^{i0.92\pi}, -0.33e^{-i0.29\pi})\rangle$	$\langle 4, (0.92e^{i0.83\pi}, -0.29e^{-i0.32\pi})\rangle$
v_2	$\langle 2, (0.61e^{i0.57\pi}, -0.52e^{-i0.43\pi})\rangle$	$\langle 4, (0.89e^{i0.95\pi}, -0.13e^{-i0.19\pi})\rangle$
v_3	$\langle 1, (0.27e^{i0.21\pi}, -0.89e^{-i0.91\pi})\rangle$	$\langle 3, (0.75e^{i0.69\pi}, -0.58e^{-i0.61\pi})\rangle$
	l_3	l_4
v_1	$\langle 1, (0.25e^{i0.32\pi}, -0.63e^{-i0.72\pi})\rangle$	$\langle 1, (0.31e^{i0.37\pi}, -0.65e^{-i0.59\pi})\rangle$
v_2	$\langle 2, (0.57e^{i0.45\pi}, -0.45e^{-i0.51\pi})\rangle$	$\langle 2, (0.71e^{i0.63\pi}, -0.38e^{-i0.42\pi})\rangle$
v_3	$\langle 0, (0.12e^{i0.07\pi}, -0.92e^{-i0.83\pi})\rangle$	$\langle 0, (0.12e^{i0.17\pi}, -0.86e^{-i0.79\pi})\rangle$

Table 19: $\mathscr{CBFS}_f\mathscr{S}$ associated with $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$ and threshold $\mathscr{D} = 1$.

$(\Upsilon^1_\Phi, \mathbb{L})$,	l_1	l_2	l_3	l_4
v_1	$(0.55e^{i0.62\pi}, -0.33e^{-i0.29\pi})$	$(0.71e^{i0.63\pi}, -0.44e^{-i0.38\pi})$	$(0.42e^{i0.38\pi}, -0.57e^{-i0.65\pi})$	$(0.69e^{i0.65\pi}, -0.45e^{-i0.42\pi})$
v_2	$(0.81e^{i0.89\pi}, -0.15e^{-i0.12\pi})$	$(0.89e^{i0.81\pi}, -0.13e^{-i0.07\pi})$	$(0.86e^{i0.93\pi}, -0.08e^{-i0.17\pi})$	$(0.79e^{i0.82\pi}, -0.05e^{-i0.21\pi})$
v_3	$(0.46e^{i0.43\pi}, -0.52e^{-i0.62\pi})$	$(0.68e^{i0.66\pi}, -0.31e^{-i0.37\pi})$	$(0.39e^{i0.34\pi}, -0.64e^{-i0.57\pi})$	$(0.31e^{i0.29\pi}, -0.59e^{-i0.72\pi})$
v_4	$(0.65e^{i0.58\pi}, -0.29e^{-i0.34\pi})$	$(0.91e^{i0.78\pi}, -0.17e^{-i0.11\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0.27e^{i0.31\pi}, -0.68e^{-i0.61\pi})$

Table 20: $\mathscr{CBFS}_f\mathscr{S}$ associated with $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$ and threshold $\mathscr{D} = 2$.

$(\Upsilon^2_\Phi, \mathbb{L})$,	l_1	l_2	l_3	l_4
v_1	$(0.55e^{i0.62\pi}, -0.33e^{-i0.29\pi})$	$(0.71e^{i0.63\pi}, -0.44e^{-i0.38\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0.69e^{i0.65\pi}, -0.45e^{-i0.42\pi})$
v_2	$(0.81e^{i0.89\pi}, -0.15e^{-i0.12\pi})$	$(0.89e^{i0.81\pi}, -0.13e^{-i0.07\pi})$	$(0.86e^{i0.93\pi}, -0.08e^{-i0.17\pi})$	$(0.79e^{i0.82\pi}, -0.05e^{-i0.21\pi})$
v_3	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0.68e^{i0.66\pi}, -0.31e^{-i0.37\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$
v_4	$(0.65e^{i0.58\pi}, -0.29e^{-i0.34\pi})$	$(0.91e^{i0.78\pi}, -0.17e^{-i0.11\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$

Table 21: $\mathscr{CBFS}_f\mathscr{S}$ associated with $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$ and threshold $\mathscr{D} = 3$.

$(\Upsilon^3_\Phi, \mathbb{L})$,	l_1	l_2	l_3	l_4
v_1	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$
v_2	$(0.81e^{i0.89\pi}, -0.15e^{-i0.12\pi})$	$(0.89e^{i0.81\pi}, -0.13e^{-i0.07\pi})$	$(0.86e^{i0.93\pi}, -0.08e^{-i0.17\pi})$	$(0.79e^{i0.82\pi}, -0.05e^{-i0.21\pi})$
v_3	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$
v_4	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0.91e^{i0.78\pi}, -0.17e^{-i0.11\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$	$(0e^{i0\pi}, -1e^{-i\pi})$

Table 22: $\mathscr{S}_f\mathscr{S}$ associated with $\mathscr{CBF}4\mathscr{S}_f\mathscr{S}$ and thresholds $\mathscr{D} = 2$, $\rho = 0.9$.

$(\Upsilon^2_\Phi, \mathbb{L})$,	l_1	l_2	l_3	l_4
v_1	1	1	0	1
v_2	1	1	1	1
v_3	0	1	0	0
v_4	1	1	0	0

(3) $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) = (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$

(4) $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) = (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cap_{\mathfrak{E}} (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1)$

(5) $((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{R}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)) \cup_{\mathfrak{R}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{R}} ((\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cup_{\mathfrak{R}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3))$

(6) $((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{E}} ((\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3))$

(7) $((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{R}} ((\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3))$

(8) $((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)) \cap_{\mathfrak{E}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3) = (\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{E}} ((\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cap_{\mathfrak{E}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3))$

(9) $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{E}} ((\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3)) = ((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)) \cap_{\mathfrak{R}} ((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3))$

(10) $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{E}} ((\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cup_{\mathfrak{R}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3)) = ((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{E}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)) \cup_{\mathfrak{R}} ((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{E}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3))$

(11) $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{R}} ((\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cap_{\mathfrak{E}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3)) = ((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{R}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)) \cap_{\mathfrak{E}} ((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cup_{\mathfrak{R}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3))$

(12) $(\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{R}} ((\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2) \cup_{\mathfrak{E}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3)) = ((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_2}, \mathbb{T}_2, N_2)) \cup_{\mathfrak{E}} ((\Upsilon_{\Phi_1}, \mathbb{T}_1, N_1) \cap_{\mathfrak{R}} (\Upsilon_{\Phi_3}, \mathbb{T}_3, N_3))$

Now, we will define some fundamental operations on complex bipolar fuzzy N-soft numbers.

Definition 3.28. Let $\chi_{yi} = \langle r_i^y, (\alpha_{yi}^p e^{i\omega_{yi}}, \beta_{yi}^n e^{\psi_{yi}}) \rangle$ and $\chi_{xi} = \langle r_i^x, (\alpha_{xi}^p, \beta_{xi}^n) \rangle$ be two $\mathscr{CBFNS}_f\mathscr{N}s$ and $\tau > 0$. Then, some operations for $\mathscr{CBFNS}_f\mathscr{N}s$ are as follows:

$$
\begin{aligned}
\tau\chi_{yi} &= \left\langle r_i^y, \left(\left(1-\left(1-\alpha_{yi}^p\right)^\tau\right) e^{i\left(1-\left(1-\left(\omega_{yi}/\pi\right)\right)^\tau\right)\pi}, -\left|\beta_{yi}^n\right|^\tau e^{-i\left|\psi_{yi}/\pi\right|^\tau \pi}\right)\right\rangle, \\
\left(\chi_{yi}\right)^\tau &= \left\langle r_i^y, \left(\left(\alpha_{yi}^p\right)^\tau e^{i\left(\omega_{yi}/\pi\right)^\tau \pi}, \left(-1+\left|1+\beta_{yi}^n\right|^\tau\right) e^{-i\left(-1+\left|1+\psi_{yi}/\pi\right|^\tau\right)\pi}\right)\right\rangle, \\
\chi_{yi} \otimes \chi_{xi} &= \left\langle \max\left(r_i^y, r_i^x\right), \left(\left(\alpha_{yi}^p + \alpha_{xi}^p - \alpha_{yi}^p\alpha_{xi}^p\right) e^{i\left(\left(\omega_{yi}/\pi\right)+\left(\omega_{xi}/\pi\right)-\left(\omega_{yi}/\pi\right)\left(\omega_{xi}/\pi\right)\right)\pi}\right\rangle, \\
&\quad \left\langle \left(-\left|\beta_{yi}^n\right|\left|\beta_{xi}^n\right|\right) e^{-i\left(\left|\psi_{yi}/\pi\right|\left|\psi_{xi}/\pi\right|\right)\pi}\right), \\
&\quad \min\left(r_i^y, r_i^x\right), \left(\left(\alpha_{yi}^p\alpha_{xi}^p\right) e^{i\left(\omega_{yi}/\pi\right)\left(\omega_{xi}/\pi\right)\pi}, \left(-\left|\beta_{yi}^n\right| - \left|\beta_{xi}^n\right| + \left|\beta_{yi}^n\right|\left|\beta_{xi}^n\right|\right)\right\rangle \\
&\quad \left\langle e^{i\left(-\left|\psi_{yi}/\pi\right|-\left|\psi_{xi}/\pi\right|+\left|\psi_{yi}/\pi\right|\left|\psi_{xi}/\pi\right|\right)\pi}\right).
\end{aligned}
\tag{26}
$$

Theorem 3.29. *Let* $\chi = \langle g, (\alpha^p, \beta^n) \rangle$, $\chi_{yi} = \langle r_i^y, (\alpha_{yi}^p e^{i\omega_{yi}}, \beta_{yi}^n e^{\psi_{yi}}) \rangle$ *and* $\chi_{xi} = \langle r_i^x, (\alpha_{xi}^p, \beta_{xi}^n) \rangle$ *be three* $\mathscr{CBFNS}_f\mathscr{N}s$ *and* $\tau, \tau_y, \tau_x > 0$ *be any real numbers; then*

(1) $\chi_{yi} \oplus \chi_{xi} = \chi_{xi} \oplus \chi_{yi}$

(2) $\chi_{yi} \otimes \chi_{xi} = \chi_{xi} \otimes \chi_{yi}$

(3) $\tau\chi_{yi} \oplus \tau\chi_{xi} = \tau(\chi_{yi} \oplus \chi_{xi})$

(4) $\chi_{yi}^\tau \otimes \chi_{xi}^\tau = (\chi_{yi} \otimes \chi_{xi})^\tau$

(5) $\tau_y\chi \oplus \tau_x\chi = (\tau_y + \tau_x)\chi$

(6) $\chi^{\tau_y} \otimes \chi^{\tau_x} = \chi^{\tau_y + \tau_x}$

4. Formation of Decision-Making Algorithms under $\mathscr{CBFNS}_f\mathscr{S}$ Framework

In this section, we will present the three Algorithms 1, 2, and 3 in order to deal with MADM problems in the framework of $\mathscr{CBFNS}_f$ model that will help to choose the best opt. Consider $V = \{v_1, v_2, v_3, \ldots, v_k\}$ to be the universal set of alternatives and $\mathbb{L} = \{l_1, l_2, l_3, \ldots, l_r\}$ the set of criteria that will be used to solve the decision-making problems. Let $\tau = \{\tau_1, \tau_2, \tau_3, \ldots, \tau_r\}$ be the weight vector of criteria representing the importance of the parameters in the MADM problem, where $\sum_{i=1}^r \tau_i = 1$ and $\tau_i \in [0, 1]$.

5. Selection of the Best COVID-19 Vaccine

COVID-19 is a contagious disease that has heavily influenced globally, leading to the pandemic. COVID-19 vaccine is a vaccine designed to provide acquired immunity against severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), the virus that causes COVID-19. COVID-19 vaccine is being formulated in various advanced countries, but underdeveloped countries have not been able to invent their own vaccine. So they have to import it from the developed countries. Suppose that a developing country wants to import the vaccine with the highest urgency for the sake of saving the lives of citizens, but due to the low GDP and budget, the government can import only one vaccine at a time. The following are the available options of the vaccines:

v_1: Sinopharm,

v_2: AstraZeneca,

v_3: Sinovac,

v_4: Novavax,

v_5: SANOFI.

The attributes on the basis of which the decision-maker will assess the alternatives and assign them grades are as follows:

l_1: age factor

l_2: cost

l_3: efficacy

l_4: manufacturer,

l_5: administration

Each expert will assign the $8\mathscr{S}_f$ grade in order to find out the best alternative with respect to the parameters. The rating and associated $8\mathscr{S}_f\mathscr{S}$ of alternatives are given in Table 23, where

Seven $\star$ symbolize $<$ Exceptional'

Six $\star$ symbolize $<$ Superb'

Five $\star$ symbolize $<$ Impressive'

Four $\star$ symbolize $<$ Excellent'

Three $\star$ symbolize $<$ Good'

Input:$V = \{v_1, v_2, \ldots, v_k\}$: Universal set of objects,
$\mathbb{L} = \{l_1, l_2, \ldots, l_r\}$: Set of parameters,
$(\Phi, \mathbb{L}, N)$:$N\mathcal{S}_f\mathcal{S}$ with $\mathcal{R} = \{0, 1, 2, \ldots, N-1\}$ where $N \in \{2, 3, 4, \ldots\}$,
$(\Upsilon_\Phi, \mathbb{T}, N)$:$\mathcal{CBF}N\mathcal{S}_f\mathcal{S}$, where $\mathbb{T} = (\Phi, \mathbb{L}, N)$.
Compute $\vartheta_y = \oplus_{i=1}^{r}\chi_{yi}$ where $\chi_{yi} = \left\langle r_y^i, (\alpha_{yi}^p e^{i\omega_{yi}}, \beta_{yi}^n e^{i\psi_{yi}})\right\rangle$, and sum of two $\mathcal{CBF}N\mathcal{S}_f\mathcal{N}s$ χ_{yi} and χ_{yg} is calculated as follows:

$$\chi_{yi} \oplus \chi_{yg} = \max(r_i^y, r_g^y), ((\alpha_{yi}^p + \alpha_{yg}^p - \alpha_{yi}^p\alpha_{yg}^p)e^{i((\omega_{yi}/\pi)+t(\omega_{yg}/\pi)n-q(\omega_{yi}/\pi)h(\omega_{yg}/\pi))\pi}, \rangle \left\langle (-|\beta_{yi}^n||\beta_{yg}^n|)e^{-i(|\psi_{yi}/\pi||\psi_{yg}/\pi|)\pi}\right).$$

Compute the choice values of each $v_y \in V$ by employing the S_{ϑ_y}, using (15), $\forall y = \{1, 2, \ldots, k\}$.
Reckon all the indices y for which $S_y = \max_y S_{\vartheta_y}$.
if $S_y = S_x$, *for arbitrary* $y, x \in \{1, 2, 3, \ldots, k\}$, **then**
apply accuracy function given in (16) and find out the alternative that has the highest accuracy value;
else
Determine the alternative with the highest score value.
output: The alternative having highest score or accuracy value will be the optimal solution.

ALGORITHM 1: The algorithm of choice values of $\mathcal{CBF}N\mathcal{S}_f\mathcal{S}s$.

Input:$V = \{v_1, v_2, \ldots, v_k\}$: Universal set of objects,
$\mathbb{L} = \{l_1, l_2, \ldots, l_r\}$: Set of parameters,
$(\Phi, \mathbb{L}, N)$:$N\mathcal{S}_f\mathcal{S}$ with $\mathcal{R} = \{0, 1, 2, \ldots, N-1\}$ where $N \in \{2, 3, 4, \ldots\}$,
$(\Upsilon_\Phi, \mathbb{T}, N)$: $\mathcal{CBF}N\mathcal{S}_f\mathcal{S}$, where $\mathbb{T} = (\Phi, \mathbb{L}, N)$,
$\tau = (\tau_1, \tau_2, \tau_3, \ldots, \tau_r)$: weight vector for attributes.
Compute $\vartheta_y^\tau = \oplus_{i=1}^{r}\tau_i\chi_{yi}$ where$\tau\chi_{yi} = \left\langle r_i^y, ((1-(1-\alpha_{yi}^p)^\tau)e^{i(1-(1-\omega_{yi}/\pi)^\tau)\pi}, -|\beta_{yi}^n|^\tau e^{-i|\psi_{yi}/\pi|^\tau\pi})\right\rangle$.
Sum of two weighted $\mathcal{CBF}N\mathcal{S}_f\mathcal{N}s$ χ_{yi}^τ and χ_{yg}^τ is calculated as follows:

$$\tau\chi_{yi} \oplus \tau\chi_{yg} = \max(r_i^{y(\tau)}, r_g^{y(\tau)}), ((\alpha_{yi}^{p(\tau)} + \alpha_{yg}^{p(\tau)} - \alpha_{yi}^{p(\tau)}\alpha_{yg}^{p(\tau)})e^{i((\omega_{yi}^\tau/\pi)+t(\omega_{yg}^\tau/\pi)n-q(\omega_{yi}^\tau/\pi)h(\omega_{yg}^\tau/\pi))\pi}, \rangle \left\langle (-|\beta_{yi}^{n(\tau)}||\beta_{yg}^{n(\tau)}|)e^{-i(|\psi_{yi}^\tau/\pi||\psi_{yg}^\tau/\pi|)\pi}\right).$$

Compute the weighted choice values of each $v_y \in V$ by employing the $S_{\vartheta_y^\tau}$, using (15), $\forall y = \{1, 2, \ldots, k\}$.
Determine all indices y for which $S_y = \max_y S_{\vartheta_y^\tau}$.
if $S_y^{(w)} = S_x^{(w)}$, *for some* $y, x \in \{1, 2, 3, \ldots, k\}$, **then**
utilize accuracy function defined in (16) and find out the alternative that has the extreme accuracy value;
else
Specify the alternative with the greatest score value.
output: The alternative having greatest score or accuracy value will be the optimum solution.

ALGORITHM 2: The algorithm of weighted choice values of $\mathcal{CBF}N\mathcal{S}_f\mathcal{S}s$.

Two ⋆ symbolize < Average'
One ⋆ symbolizes < Subpar',2009
◇ symbolizes < Substandard'

The corresponding $8\mathcal{S}_f\mathcal{S}$ can be associated as follows:

<◇' refers to 0
<⋆' refers to 1
<⋆⋆' refers to 2
<⋆⋆⋆' refers to 3
<⋆⋆⋆⋆' refers to 4
<⋆⋆⋆⋆⋆' refers to 5
<⋆⋆⋆⋆⋆⋆' refers to 6
<⋆⋆⋆⋆⋆⋆⋆' refers to 7

Corresponding to the grades, the $\mathcal{CBF}8\mathcal{S}_f\mathcal{N}s$ are assigned to the criteria of vaccines by utilizing the following grading criteria:

$$\begin{aligned}
0.0 &\leq S(\mathcal{B}_{yi}) < 0.25 \quad \text{when grade } 0,\\
0.25 &\leq S(\mathcal{B}_{yi}) < 0.50 \quad \text{when grade } 1,\\
0.50 &\leq S(\mathcal{B}_{yi}) < 0.75 \quad \text{when grade } 2,\\
0.75 &\leq S(\mathcal{B}_{yi}) < 1.00 \quad \text{when grade } 3,\\
1.00 &\leq S(\mathcal{B}_{yi}) < 1.25 \quad \text{when grade } 4,\\
1.25 &\leq S(\mathcal{B}_{yi}) < 1.50 \quad \text{when grade } 5,\\
1.50 &\leq S(\mathcal{B}_{yi}) < 1.75 \quad \text{when grade } 6,\\
1.75 &\leq S(\mathcal{B}_{yi}) \leq 2.00 \quad \text{when grade } 7.
\end{aligned} \tag{27}$$

Input: $V = \{v_1, v_2, \ldots, v_k\}$: Universal set of objects,
$\mathbb{L} = \{l_1, l_2, \ldots, l_r\}$: Set of parameters,
$(\Phi, \mathbb{L}, N)$: $N\mathcal{S}_f\mathcal{S}$ with $\mathcal{R} = \{0, 1, 2, \ldots, N-1\}$ where $N \in \{2, 3, 4, \ldots\}$,
$(\Upsilon_\Phi, \mathbb{T}, N)$: $\mathcal{CBF}N\mathcal{S}_f\mathcal{S}$, where
$\mathbb{T} = (\Phi, \mathbb{L}, N)$, $0 < \mathcal{D} < N$: threshold value.

Compute $\Upsilon_\Phi^{\mathcal{D}}(l_i) = \begin{cases} \langle /\text{span}\rangle \langle /p\rangle (\alpha_{yi}^{p} e^{i\omega_{yi}}, \beta_{yi}^{n} e^{i\psi_{yi}}), & \text{if } \Upsilon_\Phi(l_i) = \langle (v_y, r_y^i), \alpha_{yi}^{p} e^{i\omega_{yi}}, \beta_{yi}^{n} e^{i\psi_{yi}} \rangle, \text{ and } r_y^i \geq \mathcal{D}, \\ (0.0 e^{i0\pi}, -1.0 e^{-i\pi}), & \text{otherwise.} \end{cases}$

Compute $\vartheta_y^{\mathcal{D}} = \oplus_{y=1}^{k} \chi_{yi}^{\mathcal{D}}$, where $\chi_{yi}^{\mathcal{D}} = (\alpha_{yi}^{p(\mathcal{D})} e^{i\omega_{yi}^{\mathcal{D}}}, \beta_{yi}^{n(\mathcal{D})} e^{i\psi_{yi}^{\mathcal{D}}})$, and sum of two $\mathcal{CBF}N\mathcal{S}_f\mathcal{N}s$ $\chi_{yi}^{\mathcal{D}}$ and $\chi_{yg}^{\mathcal{D}}$ is calculated as follows:
$\chi_{yi}^{\mathcal{D}} \oplus \chi_{yg}^{\mathcal{D}} = ((\alpha_{yi}^{p(\mathcal{D})} + \alpha_{yg}^{p(\mathcal{D})} - \alpha_{yi}^{p(\mathcal{D})} \alpha_{yg}^{p(\mathcal{D})}) e^{i(\omega_{yi}^{\mathcal{D}}/\pi + \omega_{yg}^{\mathcal{D}}/\pi - (\omega_{yi}^{\mathcal{D}}/\pi)(\omega_{yg}^{\mathcal{D}}/\pi))\pi}, (-|\beta_{yi}^{n(\mathcal{D})}||\beta_{yg}^{n(\mathcal{D})}|) e^{-i(|\psi_{yi}^{\mathcal{D}}/\pi||\psi_{yg}^{\mathcal{D}}/\pi|)\pi})$.
Compute the $\mathcal{D}$-choice values of each $v_y \in V$ by employing the $S_{\vartheta_y^{\mathcal{D}}}$, $\forall y = \{1, 2, \ldots, k\}$. Here,
$S_{\vartheta_y^{\mathcal{D}}} = \alpha_{yi}^{p(\mathcal{D})} + \beta_{yi}^{n(\mathcal{D})} + 1/2 + 1/2(\omega_{yi}^{\mathcal{D}}/\pi + \omega_{yi}^{\mathcal{D}}/\pi + 1)$
Determine all indices y for which $S_y = \max_y S_{\vartheta_y^{\mathcal{D}}}$.
if $S_y^{\mathcal{D}} = S_x^{\mathcal{D}}$, *for some* $y, x \in \{1, 2, 3, \ldots, k\}$, **then**
utilize accuracy function and find out the alternative that has the extreme accuracy value;
else
Find out the alternative with maximum score value.
output: The alternative having maximum accuracy or score value will be the best solution.

Algorithm 3: The algorithm of $\mathcal{D}$-choice values of $\mathcal{CBF}N\mathcal{S}_f\mathcal{S}s$.

where $\mathcal{B}_{yi} = (\alpha_{yi}^{p} e^{i\omega_{yi}}, \beta_{yi}^{n} e^{i\psi_{yi}})$, $S(\mathcal{B}_{yi}) = \alpha_{yi}^{p} + \beta_{yi}^{n} + 1/2 + 1/2(\omega_{yi}/\pi + \psi_{yi}/\pi + 1)$ and $y = 1, 2, 3, 4, 5$; $i = 1, 2, 3, 4, 5$.

According to the above-mentioned conditions, the corresponding grading criteria are represented in Table 24.

Thereby, the $\mathcal{CBF}8\mathcal{S}_f\mathcal{S}$ can be determined by using Definition 3.1 as follows:

$$\begin{aligned}
\Upsilon_\Phi(l_1) = \{ & \langle (v_1, 6), 0.77e^{i0.82\pi}, -0.13e^{-i0.17\pi} \rangle, \langle (v_2, 5), 0.71e^{i0.64\pi}, -0.35e^{-i0.32\pi} \rangle, \langle (v_3, 5), 0.69e^{i0.72\pi}, -0.32e^{-i0.26\pi} \rangle, \\
& \langle (v_4, 4), 0.52e^{i0.54\pi}, -0.42e^{-i0.48\pi} \rangle, \langle (v_5, 3), 0.38e^{i0.48\pi}, -0.53e^{-i0.55\pi} \rangle \}, \\
\Upsilon_\Phi(l_2) = \{ & \langle (v_1, 6), 0.79e^{i0.84\pi}, -0.14e^{-i0.19\pi} \rangle, \langle (v_2, 4), 0.59e^{i0.55\pi}, -0.43e^{-i0.47\pi} \rangle, \langle (v_3, 5), 0.67e^{i0.66\pi}, -0.31e^{-i0.29\pi} \rangle, \\
& \langle (v_4, 3), 0.39e^{i0.44\pi}, -0.55e^{-i0.61\pi} \rangle, \langle (v_5, 2), 0.26e^{i0.31\pi}, -0.71e^{-i0.65\pi} \rangle \}, \\
\Upsilon_\Phi(l_3) = \{ & \langle (v_1, 7), 0.92e^{i0.95\pi}, -0.07e^{-i0.05\pi} \rangle, \langle (v_2, 6), 0.81e^{i0.77\pi}, -0.18e^{-i0.14\pi} \rangle, \langle (v_3, 5), 0.65e^{i0.71\pi}, -0.34e^{-i0.27\pi} \rangle, \\
& \langle (v_4, 3), 0.45e^{i0.41\pi}, -0.52e^{-i0.58\pi} \rangle, \langle (v_5, 1), 0.15e^{i0.21\pi}, -0.79e^{-i0.81\pi} \rangle \}, \\
\Upsilon_\Phi(l_4) = \{ & \langle (v_1, 6), 0.82e^{i0.85\pi}, -0.15e^{-i0.18\pi} \rangle, \langle (v_2, 5), 0.63e^{i0.68\pi}, -0.35e^{-i0.33\pi} \rangle, \langle (v_3, 3), 0.43e^{i0.39\pi}, -0.57e^{-i0.55\pi} \rangle, \\
& \langle (v_4, 5), 0.64e^{i0.69\pi}, -0.29e^{-i0.28\pi} \rangle, \langle (v_5, 4), 0.53e^{i0.61\pi}, -0.39e^{-i0.43\pi} \rangle \}, \\
\Upsilon_\Phi(l_5) = \{ & \langle (v_1, 5), 0.74e^{i0.72\pi}, -0.28e^{-i0.31\pi} \rangle, \langle (v_2, 4), 0.61e^{i0.55\pi}, -0.39e^{-i0.43\pi} \rangle, \langle (v_3, 3), 0.41e^{i0.39\pi}, -0.61e^{-i0.59\pi} \rangle, \\
& \langle (v_4, 4), 0.54e^{i0.58\pi}, -0.44e^{-i0.49\pi} \rangle, \langle (v_5, 2), 0.32e^{i0.33\pi}, -0.71e^{-i0.68\pi} \rangle \}.
\end{aligned} \tag{28}$$

The $\mathcal{CBF}8\mathcal{S}_f$ decision matrix is organized in Table 25.

5.1. Choice Values of $\mathcal{CBF}8\mathcal{S}_f\mathcal{S}$. By employing Algorithm 1, choice values of $\mathcal{CBF}8\mathcal{S}_f$ decision matrix are computed for the purpose of choosing the best vaccine to import, and the results are arranged in Table 26.

According to the results of Table 26, the vaccines of COVID-19 are ranked as follows:

$$v_1 > v_2 > v_3 > v_4 > v_5. \tag{29}$$

Hence, the government will import the v_1 = Sinopharm on the urgent basis.

5.2. Weighted Choice Values of $\mathcal{CBF}8\mathcal{S}_f\mathcal{S}$. In order to apply Algorithm 2 on the $\mathcal{CBF}8\mathcal{S}_f$ decision matrix, the expert will assign the weight vector

$$\tau = (0.17\ 0.23\ 0.25\ 0.15\ 0.20)^T, \tag{30}$$

to the attributes. The results are accumulated in Table 27.

According to the results of Table 27, the descending order ranking of COVID-19 vaccines is given as follows:

$$v_1 > v_2 > v_3 > v_4 > v_5. \tag{31}$$

Hence, it is concluded that the government will import the v_1 = Sinopharm.

Table 23: Grading of vaccinations and associated $8\mathcal{S}_f\mathcal{S}$.

$V/\mathbb{L}$	l_1	l_2	l_3	l_4	l_5
v_1	6 = ⋆⋆⋆⋆⋆⋆	6 = ⋆⋆⋆⋆⋆⋆	7 = ⋆⋆⋆⋆⋆⋆⋆	6 = ⋆⋆⋆⋆⋆⋆	5 = ⋆⋆⋆⋆⋆
v_2	5 = ⋆⋆⋆⋆⋆	4 = ⋆⋆⋆⋆	6 = ⋆⋆⋆⋆⋆⋆	5 = ⋆⋆⋆⋆⋆	4 = ⋆⋆⋆⋆
v_3	5 = ⋆⋆⋆⋆⋆	5 = ⋆⋆⋆⋆⋆	5 = ⋆⋆⋆⋆⋆	3 = ⋆⋆⋆	3 = ⋆⋆⋆
v_4	4 = ⋆⋆⋆⋆	3 = ⋆⋆⋆	3 = ⋆⋆⋆	5 = ⋆⋆⋆⋆⋆	4 = ⋆⋆⋆⋆
v_5	3 = ⋆⋆⋆	2 = ⋆⋆	..	4 = ⋆⋆⋆⋆	2 = ⋆⋆

Table 24: Grading criteria.

Grades	Amplitude terms		Phase terms	
r_i^y	α_{yi}^p	β_{yi}^n	ω_{yi}	ψ_{yi}
$r_i^y = 0$	$[0,\ 0.125)$	$[-1, -0.875)$	$[0, 0.125\pi)$	$[-\pi, -0.875\pi)$
$r_i^y = 1$	$[0.125,\ 0.250)$	$[-0.875, -0.750)$	$[0.125\pi, 0.250\pi)$	$[-0.875\pi, -0.750\pi)$
$r_i^y = 2$	$[0.250,\ 0.375)$	$[-0.750, -0.625)$	$[0.250\pi, 0.375\pi)$	$[-0.750\pi, -0.625\pi)$
$r_i^y = 3$	$[0.375,\ 0.500)$	$[-0.625, -0.500)$	$[0.375\pi, 0.500\pi)$	$[-0.625\pi, -0.500\pi)$
$r_i^y = 3$	$[0.500,\ 0.625)$	$[-0.500, -0.375)$	$[0.500\pi, 0.625\pi)$	$[-0.500\pi, -0.375\pi)$
$r_i^y = 3$	$[0.625,\ 0.750)$	$[-0.375, -0.250)$	$[0.625\pi, 0.750\pi)$	$[-0.375\pi, -0.250\pi)$
$r_i^y = 3$	$[0.750,\ 0.875)$	$[-0.250, -0.125)$	$[0.750\pi, 0.875\pi)$	$[-0.250\pi, -0.125\pi)$
$r_i^y = 3$	$[0.875, 1.00]$	$[-0.125, 0]$	$[0.875\pi, \pi]$	$[-0.125\pi, 0]$

Table 25: $\mathcal{CBF}8\mathcal{S}_f$ decision matrix.

$(\Upsilon_\Phi, \mathbb{T}, 8)$	l_1	l_2	l_3
v_1	$\langle 6, (0.77e^{i0.82\pi}, -0.13e^{-i0.17\pi})\rangle$	$\langle 6, (0.79e^{i0.84\pi}, -0.14e^{-i0.19\pi})\rangle$	$\langle 7, (0.92e^{i0.95\pi}, -0.07e^{-i0.05\pi})\rangle$
v_2	$\langle 5, (0.71e^{i0.64\pi}, -0.35e^{-i0.32\pi})\rangle$	$\langle 4, (0.59e^{i0.55\pi}, -0.43e^{-i0.47\pi})\rangle$	$\langle 6, (0.81e^{i0.77\pi}, -0.18e^{-i0.14\pi})\rangle$
v_3	$\langle 5, (0.69e^{i0.72\pi}, -0.32e^{-i0.26\pi})\rangle$	$\langle 5, (0.67e^{i0.66\pi}, -0.31e^{-i0.29\pi})\rangle$	$\langle 5, (0.65e^{i0.71\pi}, -0.34e^{-i0.27\pi})\rangle$
v_4	$\langle 4, (0.52e^{i0.54\pi}, -0.42e^{-i0.48\pi})\rangle$	$\langle 3, (0.39e^{i0.44\pi}, -0.55e^{-i0.61\pi})\rangle$	$\langle 3, (0.45e^{i0.41\pi}, -0.52e^{-i0.58\pi})\rangle$
v_5	$\langle 3, (0.38e^{i0.48\pi}, -0.53e^{-i0.55\pi})\rangle$	$\langle 2, (0.26e^{i0.31\pi}, -0.71e^{-i0.65\pi})\rangle$	$\langle 1, (0.15e^{i0.21\pi}, -0.79e^{-i0.81\pi})\rangle$
	l_4	l_5	
v_1	$\langle 6, (0.82e^{i0.85\pi}, -0.15e^{-i0.18\pi})\rangle$	$\langle 5, (0.74e^{i0.72\pi}, -0.28e^{-i0.31\pi})\rangle$	
v_2	$\langle 5, (0.63e^{i0.68\pi}, -0.35e^{-i0.33\pi})\rangle$	$\langle 4, (0.61e^{i0.55\pi}, -0.39e^{-i0.43\pi})\rangle$	
v_3	$\langle 3, (0.43e^{i0.39\pi}, -0.57e^{-i0.55\pi})\rangle$	$\langle 3, (0.41e^{i0.39\pi}, -0.61e^{-i0.59\pi})\rangle$	
v_4	$\langle 5, (0.64e^{i0.69\pi}, -0.29e^{-i0.28\pi})\rangle$	$\langle 4, (0.54e^{i0.58\pi}, -0.44e^{-i0.49\pi})\rangle$	
v_5	$\langle 4, (0.53e^{i0.61\pi}, -0.39e^{-i0.43\pi})\rangle$	$\langle 2, (0.32e^{i0.33\pi}, -0.71e^{-i0.68\pi})\rangle$	

5.3. $\mathcal{D}$-Choice Values of $\mathcal{CBF}8\mathcal{S}_f\mathcal{S}$. The $\mathcal{D}$-choice values of $\mathcal{CBF}8\mathcal{S}_f$ decision matrix are evaluated by utilizing Algorithm 3, where $\mathcal{D} = 5$. The results are shown in Table 28.

From Table 28, it is summarized that the ranking of COVID-19 vaccines is as follows:

$$v_1 > v_2 > v_3 > v_4 > v_5. \tag{32}$$

Hence, the government will import v_1 = Sinopharm without further delay.

6. Comparison

In this section, a comparison of the proposed MADM techniques with previous methodologies, namely, choice values, weighted choices values, and $\mathcal{D}$-choice values of $\mathcal{BF}N\mathcal{S}_f\mathcal{S}$ is demonstrated, which is presented by Akram et al. [22]. This comparison will demonstrate the proficiency and authenticity of our proposed techniques by examining the numerical application of "Selection of the best COVID-19 vaccine" in the environment of $\mathcal{BF}N\mathcal{S}_f\mathcal{S}$.

6.1. Choice Value of $\mathcal{BF}8\mathcal{S}_f\mathcal{S}$. In order to apply the Choice value of $\mathcal{BF}N\mathcal{S}_f\mathcal{S}$, firstly, we arranged the $\mathcal{BF}8\mathcal{S}_f\mathcal{S}$ from Table 23 by taking all phase terms of $\mathcal{CBF}N\mathcal{S}_f\mathcal{N}s$ equal to zero. The grading criteria and the grades given by the expert will remain the same. The $\mathcal{BF}8\mathcal{S}_f$ decision matrix is arranged in Table 29.

Moreover, compute the δ_y as follows:

$$\delta_y = \oplus_{i=1}^{r}\chi_{yi}, \tag{33}$$

where $\chi_{yi} = \left\langle r_y^i, (\alpha_{yi}^p, \beta_{yi}^n)\right\rangle$, and sum of two $\mathcal{BF}N\mathcal{S}_f\mathcal{N}s$ χ_{yi} and χ_{yg} is calculated as follows:

$$\chi_{yi} \oplus \chi_{yg} = \left\langle \max\left(r_i^y, r_g^y\right), \left(\alpha_{yi}^p + \alpha_{yg}^p - \alpha_{yi}^p\alpha_{yg}^p, -\left|\beta_{yi}^n\right|\left|\beta_{yg}^n\right|\right)\right\rangle. \tag{34}$$

Further, the choice value of $\mathcal{BF}N\mathcal{S}_f\mathcal{N}s$ is calculated by using the score function as follows:

$$f_{\delta_y} = \frac{r_i^y}{N-1} + \frac{\alpha_{yi}^p + \beta_{yi}^n + 1}{2}. \tag{35}$$

Table 26: Choice values of $\mathscr{CBF}8\mathcal{S}_f\mathcal{S}$.

$(\Upsilon_\Phi, \mathbb{T}, 8)$	l_1	l_2	l_3	l_4
v_1	$\langle 6, (0.77e^{i0.82\pi}, -0.13e^{-i0.17\pi})\rangle$	$\langle 6, (0.79e^{i0.84\pi}, -0.14e^{-i0.19\pi})\rangle$	$\langle 7, (0.92e^{i0.95\pi}, -0.07e^{-i0.05\pi})\rangle$	$\langle 6, (0.82e^{i0.85\pi}, -0.15e^{-i0.18\pi})\rangle$
v_2	$\langle 5, (0.71e^{i0.64\pi}, -0.35e^{-i0.32\pi})\rangle$	$\langle 4, (0.59e^{i0.55\pi}, -0.43e^{-i0.47\pi})\rangle$	$\langle 6, (0.81e^{i0.77\pi}, -0.18e^{-i0.14\pi})\rangle$	$\langle 5, (0.63e^{i0.68\pi}, -0.35e^{-i0.33\pi})\rangle$
v_3	$\langle 5, (0.69e^{i0.72\pi}, -0.32e^{-i0.26\pi})\rangle$	$\langle 5, (0.67e^{i0.66\pi}, -0.31e^{-i0.29\pi})\rangle$	$\langle 5, (0.65e^{i0.71\pi}, -0.34e^{-i0.27\pi})\rangle$	$\langle 3, (0.43e^{i0.39\pi}, -0.57e^{-i0.55\pi})\rangle$
v_4	$\langle 4, (0.52e^{i0.54\pi}, -0.42e^{-i0.48\pi})\rangle$	$\langle 3, (0.39e^{i0.44\pi}, -0.55e^{-i0.61\pi})\rangle$	$\langle 3, (0.45e^{i0.41\pi}, -0.52e^{-i0.58\pi})\rangle$	$\langle 5, (0.64e^{i0.69\pi}, -0.29e^{-i0.28\pi})\rangle$
v_5	$\langle 3, (0.38e^{i0.48\pi}, -0.53e^{-i0.55\pi})\rangle$	$\langle 2, (0.26e^{i0.31\pi}, -0.71e^{-i0.65\pi})\rangle$	$\langle 1, (0.15e^{i0.21\pi}, -0.79e^{-i0.81\pi})\rangle$	$\langle 4, (0.53e^{i0.61\pi}, -0.39e^{-i0.43\pi})\rangle$
	l_5	ϑ_y	S_{ϑ_y}	
v_1	$\langle 5, (0.74e^{i0.72\pi}, -0.28e^{-i0.31\pi})\rangle$	$\langle 7, (0.9998e^{i0.9999\pi}, -5.35 \times 10^{-5}e^{-i9.01\times 10^{-5}\pi})\rangle$	2.8748	
v_2	$\langle 4, (0.61e^{i0.55\pi}, -0.39e^{-i0.43\pi})\rangle$	$\langle 6, (0.9967e^{i0.9946\pi}, -0.0037e^{-i0.0030\pi})\rangle$	2.7423	
v_3	$\langle 3, (0.41e^{i0.39\pi}, -0.61e^{-i0.59\pi})\rangle$	$\langle 5, (0.9880e^{i0.9897\pi}, -0.0117e^{-i0.0066\pi})\rangle$	2.6047	
v_4	$\langle 4, (0.54e^{i0.58\pi}, -0.44e^{-i0.49\pi})\rangle$	$\langle 5, (0.9733e^{i0.9802\pi}, -0.0153e^{-i0.0233\pi})\rangle$	2.5825	
v_5	$\langle 2, (0.32e^{i0.33\pi}, -0.71e^{-i0.68\pi})\rangle$	$\langle 4, (0.8754e^{i0.9259\pi}, -0.0823e^{-i0.0847\pi})\rangle$	2.3172	

Table 27: Weighted choice values of $\mathscr{CBF}8\mathscr{S}_f\mathscr{S}$.

$(\Upsilon_\Phi, \mathbb{T}, 8)$	l_1	l_2	l_3	l_4
v_1	$\langle 6, (0.2211e^{i0.2529\pi}, -0.7069e^{-i0.7399\pi})\rangle$	$\langle 6, (0.3016e^{i0.3439\pi}, -0.6362e^{-i0.6825\pi})\rangle$	$\langle 7, (0.4682e^{i0.5271\pi}, -0.5144e^{-i0.4729\pi})\rangle$	$\langle 6, (0.2268e^{i0.2477\pi}, -0.7523e^{-i0.7732\pi})\rangle$
v_2	$\langle 5, (0.1898e^{i0.1594\pi}, -0.8365e^{-i0.8239\pi})\rangle$	$\langle 4, (0.1854e^{i0.1678\pi}, -0.8236e^{-i0.8406\pi})\rangle$	$\langle 6, (0.3398e^{i0.3075\pi}, -0.6514e^{-i0.6117\pi})\rangle$	$\langle 5, (0.1385e^{i0.1571\pi}, -0.8543e^{-i0.8468\pi})\rangle$
v_3	$\langle 5, (0.1805e^{i0.1946\pi}, -0.8239e^{-i0.7953\pi})\rangle$	$\langle 5, (0.2251e^{i0.2197\pi}, -0.7639e^{-i0.7522\pi})\rangle$	$\langle 5, (0.2308e^{i0.2662\pi}, -0.7636e^{-i0.7208\pi})\rangle$	$\langle 3, (0.0809e^{i0.0715\pi}, -0.9191e^{-i0.9142\pi})\rangle$
v_4	$\langle 4, (0.1173e^{i0.1237\pi}, -0.8629e^{-i0.8827\pi})\rangle$	$\langle 3, (0.1075e^{i0.1248\pi}, -0.8715e^{-i0.8925\pi})\rangle$	$\langle 3, (0.1388e^{i0.1236\pi}, -0.8492e^{-i0.8727\pi})\rangle$	$\langle 5, (0.1421e^{i0.1611\pi}, -0.8305e^{-i0.8262\pi})\rangle$
v_5	$\langle 3, (0.0781e^{i0.1052\pi}, -0.8977e^{-i0.9034\pi})\rangle$	$\langle 2, (0.0669e^{i0.0818\pi}, -0.9242e^{-i0.9057\pi})\rangle$	$\langle 1, (0.0398e^{i0.0572\pi}, -0.9428e^{-i0.9487\pi})\rangle$	$\langle 4, (0.1071e^{i0.1317\pi}, -0.8683e^{-i0.8811\pi})\rangle$
	l_5	ϑ_y^τ	$S_{\vartheta_y^\tau}$	
v_1	$\langle 5, (0.2362e^{i0.2248\pi}, -0.7752e^{-i0.7912\pi})\rangle$	$\langle 7, (0.8291e^{i0.8648\pi}, -0.1349e^{-i0.1461\pi})\rangle$	2.5815	
v_2	$\langle 4, (0.1717e^{i0.1476\pi}, -0.8283e^{-i0.8447\pi})\rangle$	$\langle 6, (0.6891e^{i0.6519\pi}, -0.3176e^{-i0.3030\pi})\rangle$	2.1102	
v_3	$\langle 3, (0.1001e^{i0.0941\pi}, -0.9059e^{-i0.8999\pi})\rangle$	$\langle 5, (0.5960e^{i0.6121\pi}, -0.4001e^{-i0.3548\pi})\rangle$	1.8516	
v_4	$\langle 4, (0.1438e^{i0.1593\pi}, -0.8486e^{-i0.8670\pi})\rangle$	$\langle 5, (0.5017e^{i0.5260\pi}, -0.4501e^{-i0.4925\pi})\rangle$	1.6675	
v_5	$\langle 2, (0.0742e^{i0.0770\pi}, -0.9338e^{-i0.9258\pi})\rangle$	$\langle 4, (0.3172e^{i0.3792\pi}, -0.6342e^{-i0.6331\pi})\rangle$	1.2145	

Table 28: 5-Choice values of $\mathscr{CBF8S}_f\mathscr{S}$.

$(\Upsilon_\Phi^5, \mathbb{T}, 8)$	l_1	l_2	l_3	l_4
v_1	$(0.77e^{i0.82\pi}, -0.13e^{-i0.17\pi})$	$(0.79e^{i0.84\pi}, -0.14e^{-i0.19\pi})$	$(0.92e^{i0.95\pi}, -0.07e^{-i0.05\pi})$	$(0.82e^{i0.85\pi}, -0.15e^{-i0.18\pi})$
v_2	$(0.71e^{i0.64\pi}, -0.35e^{-i0.32\pi})$	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0.81e^{i0.77\pi}, -0.18e^{-i0.14\pi})$	$(0.63e^{i0.68\pi}, -0.35e^{-i0.33\pi})$
v_3	$(0.69e^{i0.72\pi}, -0.32e^{-i0.26\pi})$	$(0.67e^{i0.66\pi}, -0.31e^{-i0.29\pi})$	$(0.65e^{i0.71\pi}, -0.34e^{-i0.27\pi})$	$(0e^{i0\pi}, -1e^{-i1\pi})$
v_4	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0.64e^{i0.69\pi}, -0.29e^{-i0.28\pi})$
v_5	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0e^{i0\pi}, -1e^{-i1\pi})$
	l_5	$\vartheta_y^{\mathscr{D}}$	$S_{\vartheta_y^5}$	
v_1	$(0.74e^{i0.72\pi}, -0.28e^{-i0.31\pi})$	$(0.9998e^{i0.9999\pi}, -5.35\times10^{-5}e^{-i9.01\times10^{-5}\pi})$	1.9998	
v_2	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0.9796e^{i0.9735\pi}, -0.0221e^{-i0.0148\pi})$	1.9581	
v_3	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0.9642e^{i0.9724\pi}, -0.0337e^{-i0.0204\pi})$	1.9413	
v_4	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0.64e^{i0.69\pi}, -0.29e^{-i0.28\pi})$	1.38	
v_5	$(0e^{i0\pi}, -1e^{-i1\pi})$	$(0e^{i0\pi}, -1e^{-i1\pi})$	0	

Table 29: $\mathscr{BF8S}_f$ decision matrix.

$(\mathfrak{I}, \mathbb{T}, 8)$	l_1	l_2	l_3	l_4
v_1	⟨6, (0.77, −0.13)⟩	⟨6, (0.79, −0.14)⟩	⟨7, (0.92, −0.07)⟩	⟨6, (0.82, −0.15)⟩
v_2	⟨5, (0.71, −0.35)⟩	⟨4, (0.59, −0.43)⟩	⟨6, (0.81, −0.18)⟩	⟨5, (0.63, −0.35)⟩
v_3	⟨5, (0.69, −0.32)⟩	⟨5, (0.67, −0.31)⟩	⟨5, (0.65, −0.34)⟩	⟨3, (0.43, −0.57)⟩
v_4	⟨4, (0.52, −0.42)⟩	⟨3, (0.39, −0.55)⟩	⟨3, (0.45, −0.52)⟩	⟨5, (0.64, −0.29)⟩
v_5	⟨3, (0.38, −0.53)⟩	⟨2, (0.26, −0.71)⟩	⟨1, (0.15, −0.79)⟩	⟨4, (0.53, −0.39)⟩
	l_5			
v_1	⟨5, (0.74, −0.28)⟩			
v_2	⟨4, (0.61, −0.39)⟩			
v_3	⟨3, (0.41, −0.61)⟩			
v_4	⟨4, (0.54, −0.44)⟩			
v_5	⟨2, (0.32, −0.71)⟩			

The results are given by Table 30.

According to the results of Table 30, the vaccines of COVID-19 are ranked as follows:

$$v_1 > v_2 > v_3 > v_4 > v_5. \tag{36}$$

Hence, the government will import the v_1 = Sinopharm on an urgent basis.

6.2. Weighted Choice Values of $\mathscr{BF8S}_f\mathscr{S}$. In order to apply the weighted choice values of $\mathscr{BFNS}_f\mathscr{S}$, we used the $\mathscr{BF8S}_f$ decision matrix given by Table 29, where phase terms are taken zero. The weight vector of criteria is given below:

$$\tau = (0.17\,0.23\,0.25\,0.15\,0.20)^T. \tag{37}$$

Further, compute the δ_y^τ as follows:

$$\delta_y^\tau = \oplus_{i=1}^r \tau\chi_{yi}, \tag{38}$$

where $\chi_{yi} = \left\langle r_y^i, (\alpha_{yi}^p, \beta_{yi}^n)\right\rangle$, $\tau\chi_{yi} = \left\langle r_i^y, (1-(1-\alpha_{yi}^p)^\tau, -|\beta_{yi}^n|^\tau)\right\rangle$., and the sum of two weighted $\mathscr{BFNS}_f\mathscr{N}s$ χ_{yi}^τ and χ_{yg}^τ is calculated as follows:

$$\tau\chi_{yi} \oplus \tau\chi_{yg} = \left\langle \max\left(r_i^{y(\tau)}, r_g^{y(\tau)}\right), \left(\alpha_{yi}^{p(\tau)} + \alpha_{yg}^{p(\tau)} - \alpha_{yi}^{p(\tau)}\alpha_{yg}^{p(\tau)}, -\left|\beta_{yi}^{n(\tau)}\right|\left|\beta_{yg}^{n(\tau)}\right|\right)\right\rangle. \tag{39}$$

Further, the weighted choice values of $\mathscr{BFNS}_f\mathscr{N}s$ is calculated by using the score function as follows:

$$f_{\delta_y^\tau} = \frac{r_i^y}{N-1} + \frac{\alpha_{yi}^p + \beta_{yi}^n + 1}{2}. \tag{40}$$

The results are arranged by Table 31.

According to the results of Table 31, the descending order ranking of COVID-19 vaccines is given as follows:

$$v_1 > v_2 > v_3 > v_4 > v_5. \tag{41}$$

Hence, it is concluded that the government will import the v_1 = Sinopharm.

6.3. $\mathscr{D}$-Choice Values of $\mathscr{BF8S}_f\mathscr{S}$. In order to apply the $\mathscr{D}$-choice values of $\mathscr{BFNS}_f\mathscr{S}$, we used the $\mathscr{BF8S}_f$ decision matrix, given by Table 29, where phase terms are taken zero. The threshold $\mathscr{D} = 5$.

Further, calculate

Table 30: Choice values of $\mathscr{BF}8\mathscr{S}_f\mathscr{S}$.

$(\mathfrak{T}, \mathbb{T}, 8)$	l_1	l_2	l_3	l_4
v_1	$\langle 6, (0.77, -0.13)\rangle$	$\langle 6, (0.79, -0.14)\rangle$	$\langle 7, (0.92, -0.07)\rangle$	$\langle 6, (0.82, -0.15)\rangle$
v_2	$\langle 5, (0.71, -0.35)\rangle$	$\langle 4, (0.59, -0.43)\rangle$	$\langle 6, (0.81, -0.18)\rangle$	$\langle 5, (0.63, -0.35)\rangle$
v_3	$\langle 5, (0.69, -0.32)\rangle$	$\langle 5, (0.67, -0.31)\rangle$	$\langle 5, (0.65, -0.34)\rangle$	$\langle 3, (0.43, -0.57)\rangle$
v_4	$\langle 4, (0.52, -0.42)\rangle$	$\langle 3, (0.39, -0.55)\rangle$	$\langle 3, (0.45, -0.52)\rangle$	$\langle 5, (0.64, -0.29)\rangle$
v_5	$\langle 3, (0.38, -0.53)\rangle$	$\langle 2, (0.26, -0.71)\rangle$	$\langle 1, (0.15, -0.79)\rangle$	$\langle 4, (0.53, -0.39)\rangle$
	l_5	δ_y	f_{δ_y}	
v_1	$\langle 5, (0.74, -0.28)\rangle$	$\langle 7, (0.9998, -5.35 \times 10^{-5})\rangle$	1.8749	
v_2	$\langle 4, (0.61, -0.39)\rangle$	$\langle 6, (0.9967, -0.0037)\rangle$	1.7465	
v_3	$\langle 3, (0.41, -0.61)\rangle$	$\langle 5, (0.9880, -0.0117)\rangle$	1.6131	
v_4	$\langle 4, (0.54, -0.44)\rangle$	$\langle 5, (0.9733, -0.0153)\rangle$	1.6040	
v_5	$\langle 2, (0.32, -0.71)\rangle$	$\langle 4, (0.8754, -0.0823)\rangle$	1.3965	

Table 31: Weighted choice value of $\mathscr{BF}8\mathscr{S}_f\mathscr{S}$.

$(\mathfrak{T}, \mathbb{T}, 8)$	l_1	l_2	l_3	l_4
v_1	$\langle 6, (0.2211, -0.7069)\rangle$	$\langle 6, (0.3016, -0.6362)\rangle$	$\langle 7, (0.4682, -0.5144)\rangle$	$\langle 6, (0.2268, -0.7523)\rangle$
v_2	$\langle 5, (0.1898, -0.8365)\rangle$	$\langle 4, (0.1854, -0.8236)\rangle$	$\langle 6, (0.3398, -0.6514)\rangle$	$\langle 5, (0.1385, -0.8543)\rangle$
v_3	$\langle 5, (0.1805, -0.8239)\rangle$	$\langle 5, (0.2251, -0.7639)\rangle$	$\langle 5, (0.2308, -0.7636)\rangle$	$\langle 3, (0.0809, -0.9191)\rangle$
v_4	$\langle 4, (0.1173, -0.8629)\rangle$	$\langle 3, (0.1075, -0.8715)\rangle$	$\langle 3, (0.1388, -0.8492)\rangle$	$\langle 5, (0.1421, -0.8305)\rangle$
v_5	$\langle 3, (0.0781, -0.8977)\rangle$	$\langle 2, (0.0669, -0.9242)\rangle$	$\langle 1, (0.0398, -0.9428)\rangle$	$\langle 4, (0.1071, -0.8683)\rangle$
	l_5	δ_y^τ	$f_{\delta_y^\tau}$	
v_1	$\langle 5, (0.2362, -0.7752)\rangle$	$\langle 7, (0.8291, -0.1349)\rangle$	1.7221	
v_2	$\langle 4, (0.1717, -0.8283)\rangle$	$\langle 6, (0.6891, -0.3176)\rangle$	1.4357	
v_3	$\langle 3, (0.1001, -0.9059)\rangle$	$\langle 5, (0.5960, -0.4001)\rangle$	1.2229	
v_4	$\langle 4, (0.1438, -0.8486)\rangle$	$\langle 5, (0.5017, -0.4501)\rangle$	1.1508	
v_5	$\langle 2, (0.0742, -0.9338)\rangle$	$\langle 4, (0.3172, -0.6342)\rangle$	0.8415	

$$\mathfrak{T}^{\mathscr{D}}(l_i) = \begin{cases} \left(\alpha^p_{yi}, \beta^n_{yi}\right), & \text{if } \Upsilon_\Phi(l_i) = \left\langle \left(v_y, r^i_y\right), \alpha^p_{yi}, \beta^n_{yi}\right\rangle, \text{ and } r^i_y \geq \mathscr{D}, \\ (0.0, -1.0), & \text{otherwise.} \end{cases} \quad (42)$$

Moreover, compute the $\delta^{\mathscr{D}}_y$ as follows:

$$\delta^{\mathscr{D}}_y = \oplus^r_{i=1}\chi^{\mathscr{D}}_{yi}, \quad (43)$$

where, $\chi^{\mathscr{D}}_{yi} = (\alpha^{p(\mathscr{D})}_{yi}, \beta^{n(\mathscr{D})}_{yi}))\rangle$, and the sum of two $\mathscr{BFNS}_f\mathscr{N}s$ $\chi^{\mathscr{D}}_{yi}$ and $\chi^{\mathscr{D}}_{yg}$ is calculated as follows:

$$\chi^{\mathscr{D}}_{yi} \oplus \chi^{\mathscr{D}}_{yg} = \left(\alpha^{p(\mathscr{D})}_{yi} + \alpha^{p(\mathscr{D})}_{yg} - \alpha^{p(\mathscr{D})}_{yi}\alpha^{p(\mathscr{D})}_{yg}, -\left|\beta^{n(\mathscr{D})}_{yi}\right|\left|\beta^{n(\mathscr{D})}_{yg}\right|\right)\rangle. \quad (44)$$

Further, the 5-choice values of $\mathscr{BFNS}_f\mathscr{N}s$ is calculated by using the score function as follows:

$$f_{\delta^{\mathscr{D}}_y} = \frac{\alpha^p_{yi} + \beta^n_{yi} + 1}{2}. \quad (45)$$

The results are arranged by Table 32.

From Table 32, it is summarized that the ranking of COVID-19 vaccines is as follows:

$$v_1 > v_2 > v_3 > v_4 > v_5. \quad (46)$$

Hence, the government will import v_1 = Sinopharm without further delay.

6.4. Discussion

(1) Now, the results of the presented MADM methodologies in the framework of $\mathscr{CBFNS}_f\mathscr{S}$ are compared with MADM methods, namely, choice values of $\mathscr{BFNS}_f\mathscr{S}$, weighted choice values of $\mathscr{BFNS}_f\mathscr{S}$ and $\mathscr{D}$-choice values of $\mathscr{BFNS}_f\mathscr{S}$, presented by Akram et al. [22], to show the authenticity and veracity of the proposed decision-making algorithms.

(2) Moreover, we have also applied the decision-making methods of FNS_f S [17] in order to find the best available COVID-19 vaccine to import. The results of the proposed and existing methods, including the final ranking and best alternatives, are arranged in Table 33 as follows:

(3) It is clear from Table 33 that the v_1 is the best vaccine in all environments. Moreover, the ranking of vaccines is also similar in all methodologies, which illustrates the reliability and accuracy of the presented MADM techniques.

(4) The comparative analysis of the results is also demonstrated in Figure 3 through the bar chart

Table 32: 5-choice value of $\mathscr{B}\mathscr{F}8\mathscr{S}_f\mathscr{S}$.

$(\mathfrak{I}, \mathbb{T}, 8)$	l_1	l_2	l_3	l_4
v_1	(0.77, −0.13)	(0.79, −0.14)	(0.92, −0.07)	(0.82, −0.15)
v_2	(0.71, −0.35)	(0, −1)	(0.81, −0.18)	(0.63, −0.35)
v_3	(0.69, −0.32)	(0.67, −0.31)	(0.65, −0.34)	(0, −1)
v_4	(0, −1)	(0, −1)	(0, −1)	(0.64, −0.29)
v_5	(0, −1)	(0, −1)	(0, −1)	(0, −1)
	l_5	$\delta_y^{\mathscr{D}}$	$f_{\delta_y^{\mathscr{D}}}$	
v_1	(0.74, −0.28)	$(0.9998, -5.35 \times 10^{-5})$	0.9999	
v_2	(0, −1)	(0.9796, −0.0221)	0.9788	
v_3	(0, −1)	(0.9642, −0.0337)	0.9652	
v_4	(0, −1)	(0.64, −0.29)	0.6750	
v_5	(0, −1)	(0, −1)	0	

Table 33: Comparative analysis.

Methods	Ranking of COVID-19 vaccines	Best vaccine
Choice value of $\mathscr{C}\mathscr{B}\mathscr{F}N\mathscr{S}_f\mathscr{S}$ (proposed)	$v_1 \succ v_2 \succ v_3 \succ v_4 \succ v_5$	v_1
Weighted choice value of $\mathscr{C}\mathscr{B}\mathscr{F}N\mathscr{S}_f\mathscr{S}$ (proposed)	$v_1 \succ v_2 \succ v_3 \succ v_4 \succ v_5$	v_1
5-Choice value of $\mathscr{C}\mathscr{B}\mathscr{F}N\mathscr{S}_f\mathscr{S}$ (proposed)	$v_1 \succ v_2 \succ v_3 \succ v_4 \succ v_5$	v_1
Choice value of $\mathscr{B}\mathscr{F}N\mathscr{S}_f\mathscr{S}$ [22]	$v_1 \succ v_2 \succ v_3 \succ v_4 \succ v_5$	v_1
Weighted choice value of $\mathscr{B}\mathscr{F}N\mathscr{S}_f\mathscr{S}$ [22]	$v_1 \succ v_2 \succ v_3 \succ v_4 \succ v_5$	v_1
5-Choice value of $\mathscr{B}\mathscr{F}N\mathscr{S}_f\mathscr{S}$ [22]	$v_1 \succ v_2 \succ v_3 \succ v_4 \succ v_5$	v_1
Choice value of FNS_f S [17]	$v_1 \succ v_2 \succ v_3 \succ v_4 \succ v_5$	v_1
3-Choice value of FNS_f S [17]	$v_1 \succ v_2 \succ v_3 \succ v_4 \succ v_5$	v_1

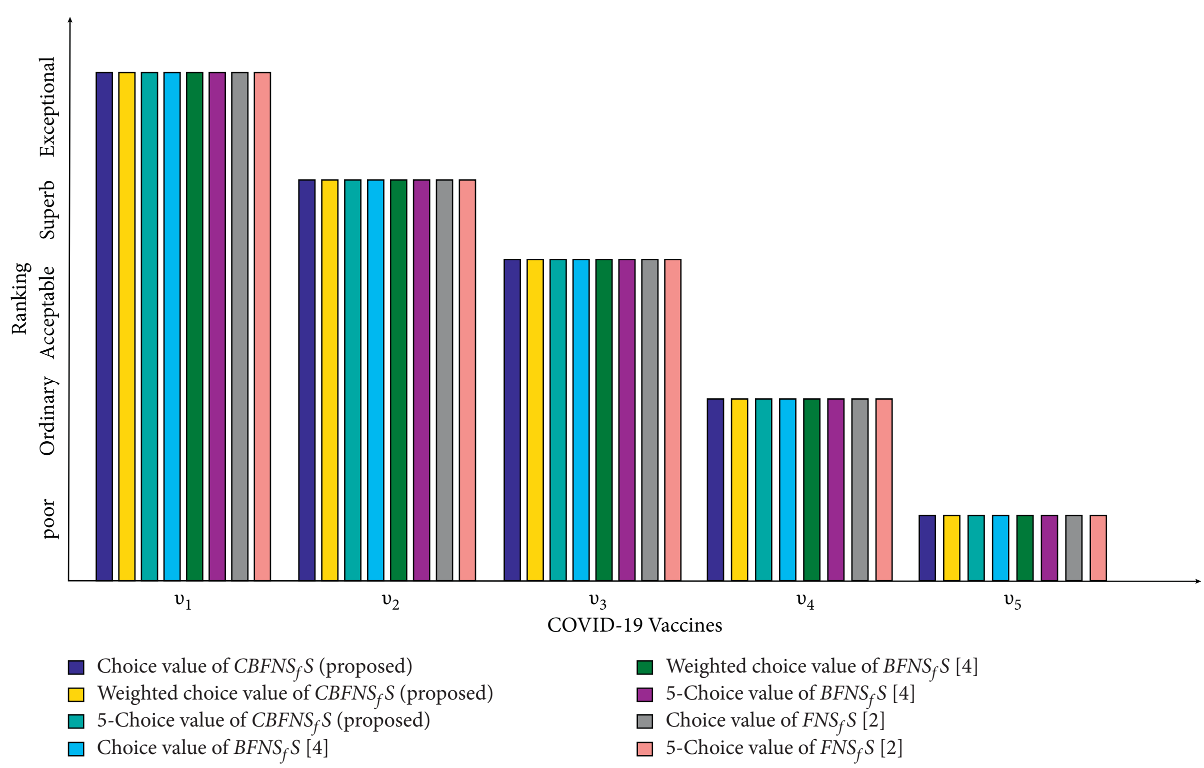

Figure 3: Comparative analysis.

between the ranking and COVID-19 vaccines, which shows the proficiency of the proposed decision-making techniques.

(5) The proposed hybrid model of $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ can effectively handle the information in the environment of $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$, $\mathscr{BFS}_f\mathscr{S}$ and $\mathscr{BFS}$ by taking $N = 2$ and $|\mathbb{L}| = 1$ respectively.

(6) The proposed decision-making methods of $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ can handle the environment of FNS_f S by taking negative membership values and phase terms equal to zero.

(7) The presented $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$ is a parameterized framework that is used to deal with the bipolar fuzziness of the information. It has the capability to handle two-dimensional vague information; i.e., it can deal with the periodicity involved in the bipolar information. Particularly, it can efficiently deal with the ranking based assessment of imprecise information that involves effects along with side effects.

7. Conclusion

In this paper, an innovative hybrid model, namely, complex bipolar fuzzy N-soft sets has been set up by integrating the $\mathscr{CBFS}s$ with $N\mathscr{S}_f\mathscr{S}s$. The generalized, more efficient theory is useful to handle the two-dimensional bipolar fuzzy information. It is superior to the existing $\mathscr{BF}N\mathscr{S}_f\mathscr{S}$ as it deals with periodic information. Firstly, we have presented the conventional definition of $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}s$ in addition to its fundamental operations and related results. We have defined elementary algebraic operations for $\mathscr{CBF}N\mathscr{S}_f\mathscr{N}s$. We have also developed three algorithms for MADM problems in order to choose the favorable parameter under the environment of $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$. We have illustrated the significance of proposed algorithms by applying them to practical applications. Finally, to demonstrate their validity and applicability, we have shown a comparison with existing MADM approaches. In the future, we intend to establish more decision-making approaches, including ELECTRE I, ELECTRE II, and TOPSIS methods under the aforementioned abundant and multifaceted model of $\mathscr{CBF}N\mathscr{S}_f\mathscr{S}$.

Data Availability

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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