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Decision Making on Pension Schemes Under Rational Expectations

By

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In this paper the formation of expectations during decision making processes on pension schemes is the main focus. An overlapping generations model is used where politicians control the tax-transfer system and the young determine savings. No generation of decision makers is committed to previous decisions. It appears that the outcome in the stationary state depends on the efficiency of the tax-transfer system compared with savings and on the preferences of politicians relative to young individuals with respect to the division of endowments between young-age and old-age consumption. One of the main conclusions is that if the parameters of the system are constant the stationary state enters within a finite time interval. So, if the system is initially outside the stationary state, the decision makers can calculate the path of taxes and savings towards the stationary state. This feature is also used to determine the effects of a demographic change.

1. Introduction

In most developed countries, a substantial part of the elderly's income is provided for by means of public and private pension schemes. Generally, public pension plans are financed by a Pay-As-You-Go (or PAYG) scheme where current pension payments are financed out of current tax payments. Fund raising as in a Capital Reserve (or CR) pension plan, in principle, does not occur. Private pension schemes, on the other hand,

* Comments from Dieter Bös, Fons Groot, Lex Meijdam, Willem Thorbecke, and David Wildasin are gratefully acknowledged. All remaining errors are, of course, ours.

are typically financed by a CR-scheme. These pension plans are generally decided upon during decentralized wage negotiations. In this paper it is assumed that no coordination between the different CR-schemes is possible. Within the context of our model that implies that the functioning of private pension schemes can be identified with that of individual savings for old age.

The operation of PAYG-schemes has drawn a great deal of attention recently due to the aging of the population which has become apparent since the 1970s. Because of this aging, the long-run effects of PAYG-schemes compared with CR-schemes may be to reduce the lifetime utility of future generations. In view of this it is natural to ask in what sense public pension schemes and private pension schemes interact in the face of demographic change.

The interaction between private and public pension schemes has already been studied to a certain extent, for example in a paper by Boadway and Wildasin (1989). They used a median-voter framework in a three-overlapping-generations model, for the first time introduced in the literature by Samuelson (1958). In the Boadway and Wildasin paper a cycle of alternating over- and undershooting tax rates evolves that gradually damps down. The occurrence of this cycle is closely related to the way expectations are treated. Individuals are assumed to believe that the tax rate that holds at the time of planning will hold for the rest of their lives. Another interesting paper that deals with the interaction between public pensions and savings is the paper by Hansson and Stuart (1989). They use a two-overlapping-generations model to consider the evolution of public pension schemes. In their paper transfers are motivated by altruism. It is assumed that decisions on public pension schemes are inherently constitutional, which in the two-generations model implies that any living generation can block legislation that would leave it off worse. At the time of the introduction of the public pension scheme the living generations, who have perfect foresight, decide on the value of the transfer to the old for the current time period and all future time periods. It is shown that a scheme is chosen that will not be amended by any future pair of generations. However, public pension schemes are in actual fact seldom part of the constitution. At any time the decision makers are free to change the scheme to their own discretion and, indeed, public pension schemes are frequently altered. If such changes are possible the Hansson–Stuart framework does not necessarily give a clue to what the future development of public pension schemes will be.

The treatment of expectations in the papers by Boadway and Wildasin, and by Hansson and Stuart is typical for the literature on public pensions in general; either it is assumed that expectations are not rational or decision making can somehow be precommitted.[1] Therefore, the role of expectations will be the main focus of this paper. We start from the assumption that in every period a decision is made on the size of the intergenerational transfer scheme. Let us assume, with an eye turned to reality, that decisions on transfers to the old are coordinated by a group of individuals (to be called politicians) recruited from the old and young alive at that period of time. In this respect our approach differs from most literature to date that, starting with Browning (1975), relies on the median-voter framework. The outcome of the political decision making process depends on the pressure exercised by the old and young individuals alive at that time period. The young decide each for themselves on the size of savings. The decision makers have perfect foresight over an infinitely long horizon. Because it is assumed that no generation of decision makers is committed to previous decisions, every generation has to build an expectation of the outcome of the future decision making process for itself. As this outcome is in turn determined by the expectations held by the next generation of decision makers, an in principle endless sequence of expectations has to be determined before a generation can take a decision. However, given constant values of the system's parameters the emergence of a stationary state within a finite interval can be proved. So, a calculation of finite length suffices to generate the path of expected taxes and savings towards the stationary state. This enables decision makers in the framework presented here to base their decisions on correct beliefs about the future development of the system, without having to impose *ad hoc* restrictions on the possible outcomes of the system. Moreover, the effects of temporary deviations of the parameters from their constant values, such as a baby boom, can be calculated as well.

The set-up of this paper is as follows. In Sect. 2 the basic model is introduced and the stationary states are derived. In Sect. 3 the effects of a temporary demographic shock are calculated. Section 4 contains some remarks on the economic interpretation of the model and concluding comments.

[1] An exception is Veall (1986) where rational expectations and uncommitted decision making are combined. Contrary to the analysis of this paper, however, Veall does not consider dynamics.

2. The Model

2.1 The Individuals

The starting point of our analysis is the well known two-overlapping-generations model where every individual lives for two periods and where everyone is identical except for age differences. In order to concentrate on decision making aspects, the economic characteristics of the model are kept very simple. Since we assume the young's saving decisions to be decentralized and uncoordinated, the influence of each young decision maker on the behavior of other agents is negligibly small. In other words, we use the Nash assumption that the young have to take current, past, and future tax rates, as well as past and future saving rates as given when deciding on their savings. If the politicians in turn take the saving rates and tax rates, as far as not under their direct control, as given, a non-cooperative Nash equilibrium can be determined.[2]

It is assumed that every individual born at time t receives an endowment of one unit at the start of life. Part of this endowment is taxed away by the government to be transferred to the old (τ_t). The remainder is used for savings for old age (s_t) and immediate consumption (c_t^y). So we get

$$c_t^y = 1 - \tau_t - s_t \; . \tag{1}$$

When old, the individual born at t consumes the return on his savings [the rate of return is assumed to be fixed at $r - 1$, $r \in [0, \infty)$] and the transfer payment from the government (η_{t+1})

$$c_t^r = r s_t + \eta_{t+1} \; . \tag{2}$$

The size of the generation born at time t is denoted by N_t. Therefore at time t the number of young individuals per old individual is equal to $N_t / N_{t-1} \equiv n_t$, where $n_t \in [0, \infty)$. It is assumed that n_t is exogenously determined. Abstracting from administrative costs the budget restriction of the PAYG-scheme run by the government reads $\eta_t N_{t-1} = \tau_t N_t$, which implies

$$\eta_t = n_t \tau_t \; . \tag{3}$$

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[2] Alternatively, it could be assumed that the politicians are the Stackelberg leaders with the young of that period or with future generations of decision makers as followers. For the former case it can be demonstrated that the transfers to the old will be the same as when the politicians take the share of endowment currently saved by the young as given. See Veall (1986) for the case where the politicians are the Stackelberg leaders with respect to future generations. We will abstain from the Stackelberg case in the following.

Lifetime utility of an individual born at time t can be represented by the following utility function

$$U_t^y = U(c_t^y, c_t^r) , \tag{4}$$

which is twice differentiable, strictly increasing and strictly concave in both arguments as long as $c_t^y > 0$ and $c_t^r > 0$. Furthermore,[3] $U_t^{y.1} = \infty$ if $c_t^y = 0$ and $U_t^{y.2} = \infty$ if $c_t^r = 0$. It is not possible to have negative values of young and old-age consumption. After one period what was future consumption for the individual when he was young, is current consumption now. So, if a young individual discounts marginal utility of consumption after retirement relative to current consumption and if preferences do not change over time, an old individual's marginal utility is equal to the marginal utility of consumption of future consumption when he was young where the discount factor is eliminated. This is analogous to

$$U_t^{r.2} = \delta U_t^{y.2} , \tag{5}$$

where δ is assumed to be a constant and can be interpreted as one plus the rate of time preference.

Equations (4) and (5) indicate that individuals are not altruistic; they do not care about the levels of consumption or utility of individuals born in another time period. The young maximize their lifetime utility using the saving rate as an instrumental variable. Obviously, they cannot borrow from the old so that savings cannot be negative. Assuming non-negative taxes (current as well as future, see below), this assures that old-age consumption is not negative. Another restriction that has to be taken into account is that consumption in the first period cannot be negative, or, from Eq. (1), $s_t \leq 1 - \tau_t$, implying that $\tau_t \leq 1$, since $s_t \geq 0$. Solving the restricted optimization problem of the young individual, the first-order condition determining savings turns out to be[4]

$$\frac{U_t^{y.1}}{U_t^{y.2}} \geq r , \tag{6}$$

where the equality sign holds if $s_t > 0$. Notice that the non-negativity requirement on consumption does not effectively restrict the optimal

[3] $U_t^{y,i}$ denotes marginal utility with respect to the i'th argument in the utility function. $U_t^{y,i,i}$ denotes the second derivative.

[4] If $\tau_t = 1$ and $\tau_{t+1} = 0$ condition (6) does not uniquely determine the optimal saving rate. Then, savings are given by the requirement that consumption cannot be negative: $s_t = 0$.

choice of the saving rate as long as current taxes are smaller than endowments. This is due to the assumptions that marginal utility tends to infinity as consumption goes to zero and that the interest rate has a finite value.

Condition (6) states, not surprisingly, that as long as there is some freedom of choice ($\tau_t < 1$), the young use savings to equate the marginal rate of substitution of consumption in the first and the second part of their lives to the interest rate or, if that would imply negative savings, to minimize the difference. Note that in line with the Nash assumption discussed in Sect. 1, the tax rates τ_t and τ_{t+1} have been taken as given in deriving condition (6). Since all individuals are exactly alike, each individual will choose the same saving rate. Therefore, the individual decision making process as described by condition (6) can be interpreted as the decision making process of a representative individual.

2.2 *The Politicians*

The government, that is the politicians, decide on the PAYG-scheme. They are under the influence of both the old and the young. Old individuals would of course like the government to tax away all endowments possessed by the young as that would maximize their pensions and consumption. The young take the future benefit rate $\eta_{t+1} = n_{t+1}\tau_{t+1}$ as given, so they do not perceive a relation between their old-age consumption and current taxes. Consequently, the young prefer the lowest possible tax rate. Decision making by the government follows from maximizing the following continuously differentiable function with respect to the tax rate at time t [5]

$$D_t = D(U_t^y, U_{t-1}^r) \; . \tag{7}$$

where $D_t^i > 0$ and finite for $i = 1, 2$. These partial derivatives should be interpreted as the marginal power of the young and the old. The

[5] Underlying Eq. (7) is the assumption that decision making on public pension schemes does not take place in a direct democracy, as assumed for example in the seminal paper by Browning (1975). Instead, the institutional form of the political decision making process is assumed to be a representative democracy. The function D_t then represents the effective political support for the current government by the young and the old, respectively. An analogous approach to the political decision making process has been applied before by Verbon (1988) and Weizsäcker (1990).

power of the two generations depends upon the support they can provide for the government's survival. This may depend on the likelihood that members of the two groups will vote for the government in the next election, but also on lobbying activities, entrance to news media, economic power as measured by income or wealth status, etc. Obviously, according to Eq. (7) the government is assumed to be myopic as future generations' utilities are no argument in the function D_t. This can be justified by the fact that governments only take account of future generations' interests in so far as the support of current generations depends upon their offspring's utility. In that case the utility of future generations is an argument in the function determining U_t and/or U_{t-1}. In this paper intergenerational altruism is assumed to be absent, however.

We placed a non-negativity constraint on the instrument τ_t, implying there will be no transfers from the old to the young. Moreover, the requirement that young-age consumption cannot be negative, i.e. $\tau_t \leq 1 - s_t$, must be taken into account. If politicians take the (non-negative) past and current saving rate as given, the first-order condition for the tax rate is

$$\frac{D_t^1 U_t^{y,1}}{D_t^2 U_{t-1}^{r,2}} \geq n_t. \tag{8}$$

where the equality sign holds if $\tau_t > 0$. As in the cases of savings, and for similar reasons, the restriction of non-negative young-age consumption does not effectively influence the optimal choice of the tax rate. According to condition (8), at the optimum and if there is some freedom of choice for the politicians ($s_t < 1$), the marginal effect of the tax rate on the utility of the young and the old is weighted by the quotient of their respective marginal power. In other words, condition (8) implies that the subjective marginal rate of substitution of politicians between consumption of the young and of the old alive at time period t is greater than or equal to the population growth of that period, that is the rate at which consumption of the old and of the young can be traded off objectively through the PAYG-scheme [see Eq. (3)].

The relative marginal power term D_t^1/D_t^2 can be assumed to depend on the relative size of the young versus the old generation n_t.[6] This influence can be positive (a larger numerical strength implies a larger political strength) as well as negative (a smaller size implies

[6] Note that this implies that political power is an exogenous variable. It can be endogenized, for example by assuming that the distribution of political power is determined by the relative share of incomes as in Weizsäcker (1990). This would not alter our main results.

lower costs of organization). In the former case a fall in population growth implies an upward effect on the tax rate τ_t, while in the latter case τ_t will fall. In what follows, we will concentrate our attention on the case where the relative marginal power is proportional to the relative size of the generations alive at time t. This can be written as $D_t^1/D_t^2 = n_t\lambda/(1-\lambda)$, where λ is the exogenously given political power of a representative young individual. The power parameter λ measures the degree to which the value of the marginal lifetime utility of a representative young individual counts in the political decision making process. It is assumed constant and, moreover, it is normalized so that $1-\lambda$ is the political power of a representative old individual. So, according to this specification the relative political power of a generation is a linear positive function of its relative size. If this assumption is essential for the results of the next section, a more general result is indicated as well. In Appendix C the more general case is discussed at length.

2.3 Nash Equilibrium

In the above analysis we have abstracted from interaction between the PAYG- and the CR-scheme; savings were assumed to be constant. In general, interaction between the two pension plans will occur, as can be seen from condition (6) in combination with Eqs. (1)–(3). To determine the way in which the two pension plans interact with each other it can be assumed that both the young and the politicians take each other's as well as all future decisions as given. In other words, the young born at time t choose the optimal s_t^* that satisfies $U_t^{y.1}/U_t^{y.2} \geq r$, while in that same period, the politicians choose the optimal τ_t^* that satisfies $U_t^{y.1}/U_{t-1}^{r.2} = \delta^{-1}U_t^{y.1}/U_{t-1}^{y.2} \geq (1-\lambda)/\lambda$. The saving rate of the previous period and the tax rate of the next period, s_{t-1} and τ_{t+1}, are the result of similar decision making processes as the ones determining s_t and τ_t. Writing down these conditions for the tax and saving rates of all time periods we have the conditions for a Nash equilibrium. In the stationary state the Nash equilibrium can easily be established. In particular, the following Lemma holds[7]

Lemma: In Nash equilibrium, if $\lambda/(1-\lambda) < \delta/r$, a unique equilibrium exists for the stationary state where transfers from the PAYG-scheme are positive ($\tau^* > 0$) and savings are zero ($s^* = 0$). If $\lambda/(1-\lambda) > \delta/r$,

[7] If the time subscripts are omitted the stationary state value is meant.

there is also a unique equilibrium. Then, taxes and pension benefits are zero ($\tau^* = 0$) while savings are positive ($s^* > 0$). A situation where $s^* > 0$ as well as $\tau^* > 0$ is only possible when $\lambda/(1-\lambda) = \delta/r$.

Not surprisingly, if the young have great political power, i.e. if $n\lambda/(1-\lambda) > n\delta/r$, a CR-scheme will exist in the stationary state. This result is driven by the Nash assumption that the young take the future public pension benefit as given. Notice that in the stationary state a PAYG-pension scheme is intertemporally efficient if $n > r$ under the conditions of the current model.[8] Obviously, in the positive framework adopted here if $n > r$ the inefficient CR-scheme may exist nevertheless in the stationary state.

3. The Effects of a Temporary Demographic Change: Comparative Statics

In this section we study how the stationary state outcomes for the tax and saving rates established in the previous section are affected if the system is hit by a shock. In particular, the effects of a temporary demographic change are analyzed. In this way insight is gained in the question under what circumstances the system will get out of the stationary state. A basic result of this section is that if the system is outside the stationary state in some period and if the parameters remain constant, it is impossible for the system to remain outside the stationary state forever. Another basic result is that if it is known when the stationary state re-enters, any generation can calculate the paths of future tax and saving rates.[9] This implies that we can describe the evolution of tax and saving rates from any arbitrary initial value of the saving rate inherited from the previous generation towards the stationary state.

The precise method that is used here to analyze the effects of temporary shocks is the following. Suppose that at a certain time $\underline{t}$,

[8] Aaron (1966) was the first to make this condition more explicit.

[9] Notice, however, that these results do not necessarily imply that a path of tax and saving rates all obeying the first-order conditions can always be found. As is often the case in games with infinitely many players, it cannot be excluded that a Nash equilibrium in this model does not exist or that it is not unique. However, if utility is intertemporally separable and instantaneous utility is of the constant relative risk aversion type it can be shown that there always exists a Nash equilibrium (see Meijdam et al., 1992). In the following it is assumed that a Nash equilibrium exists.

when the system is in the stationary state, it becomes known for the first time that in some future time period, say $\underline{t} + j$, an exogenous shock will occur. Let us assume that this new information concerns a population growth shock of size $dn_{\underline{t}+j} \equiv n_{\underline{t}+j} - n$. Given perfect foresight over an infinitely long horizon, rational decision makers can calculate, using comparative statics, whether this change will lead to a disturbance of the stationary state at times $\underline{t} + j$ or $\underline{t} + j - 1$. If not, the demographic change will not have any effect at all ($n_{\underline{t}+j}$ only directly influences the first-order conditions for $s_{\underline{t}+j-1}$ and $\tau_{\underline{t}+j}$). If the stationary state is disturbed at times $\underline{t} + j$ and/or $\underline{t} + j - 1$ this can in turn lead to further perturbations after and before these time periods. By taking into account the feedback effects of these indirect changes on the initial disturbance of $s_{\underline{t}+j-1}$ and $\tau_{\underline{t}+j}$, the decision makers at time $\underline{t}$ can calculate the final effects of the population growth change on their own and future decision making. The perturbations at and after time $\underline{t} + j$ then show whether a movement towards the stationary state exists and if so, whether the stationary state will be re-entered within a limited period of time.

In order to obtain explicit results, the above described procedure will be applied to the case where the intertemporal utility functions are additively separable:

$$U_t^y = u(c_t^y) + \delta^{-1} u(c_t^r) \ . \tag{4'}$$

For reasons of convenience it is furthermore assumed that $r = \delta$. Then the first-order condition (6) determining savings at time t reads

$$s_t^* = \max\left(0, \frac{1 - \tau_t^* - n_{t+1}\tau_{t+1}^*}{1 + r}\right) \ . \tag{6'}$$

Consequently, if the tax rate is zero in the stationary state the saving rate will be equal to $1/(1 + r)$. The optimal tax rate can be calculated by solving

$$\frac{u^1(c_t^y)}{u^1(c_{t-1}^r)} = \delta^{-1} \frac{u^1(c_t^y)}{u^1(c_{t-1}^y)} \geq \frac{1 - \lambda}{\lambda} \ . \tag{8'}$$

where, as in (8), the equality sign holds when $\tau_t > 0$. It should be remembered that attention is focussed here on the case where the relative marginal power of the young with respect to the old is proportional to the rate of population growth. Results for the more general case where the functional form of this relation is not specified can be found in Appendix C.

The following can be shown to be true:

Proposition 1: Suppose at some time $\tilde{t}$ the system is out of the stationary state. If the system's parameters remain constant in every future time period $t > \tilde{t}$ then a positive and finite number k exists such that $s_t^* = s^*$ and $\tau_t^* = \tau^*$ for all $t > \tilde{t} + k$.

For a proof of proposition 1 we refer to Appendix A. Proposition 1 implies that if the system starts from an initial situation out of the stationary state and the system's parameters remain constant, the stationary state will be reached within a finite time interval. It is also implied by proposition 1 that if the system is hit by a temporary shock that disturbs the stationary state, this deviation from the stationary state can only last for a limited number of periods after the occurrence of the shock. The argument also goes the other way around; a change in the parameter values can only disturb the outcomes of a limited number of time periods before the shock. This implies that if it is foreseen in period $\underline{t}$ that a shock will occur in period $\underline{t} + j$, the stationary state may not be disturbed before $\underline{t} + i$, where $1 \leq i \leq j$.

Let us consider the consequences of a temporary demographic shock in more detail and let us start with the situation where there is only a CR-scheme in the stationary state. According to the Lemma, this will be the case if $\lambda > 1/2$ $(\delta = r)$. Then it can be shown that the following must hold:

Proposition 2: In the case that $D_t^1/D_t^2 = n_t\lambda/(1-\lambda)$ and $\lambda > 1/2$, the stationary state [where $s^* = 1/(1+r)$ and $\tau^* = 0$, see the Lemma and Eq. (6′)] cannot be disturbed by a temporary change in population growth at time $\underline{t} + j$. If, however, $D_t^1/D_t^2 = n_t\lambda_t/(1-\lambda_t)$, where λ_t is some function of n_t such that as a result of the shock $\lambda_{\underline{t}+j} < 1/2$, then $d\tau^*_{\underline{t}+j+k} > 0$ and $ds^*_{\underline{t}+j+k} < 0$, respectively, for $k = 0, 1, \ldots, \bar{\kappa}$, and $d\tau^*_{\underline{t}+j-k+1} > 0$ and $ds^*_{\underline{t}+j-k} < 0$, respectively, for $k = 0, 1, \ldots, \underline{\kappa}$, with positive and finite $\bar{\kappa}$ and $\underline{\kappa}$.

What this proposition says is that a demographic shock may affect the outcomes before and after the period of the shock only if the relative power of a representative young individual at the time of the shock falls, i.e., $\lambda_{\underline{t}+j} < 1/2$. In particular, if this power would decrease more than proportionally with size, then a drop in the population growth rate $(dn_{\underline{t}+j} < 0)$ could lead to a temporary increase in the tax rate. In a world where the young have the larger political weight $(\lambda > 1/2)$, the existence of a PAYG-scheme can be triggered by an expected decrease

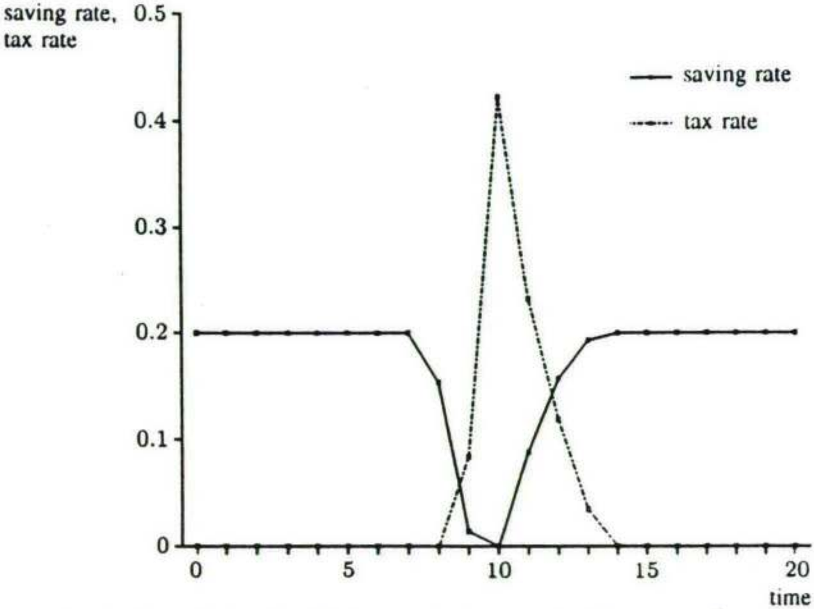

Fig. 1: The effects of a fall in population growth if $\lambda_t = \frac{1}{3-0.38n_t}$
$[n = 2.75,\ n_{\underline{t}+j} = n_{10} = 2,\ r = \delta = 4,\ u(c) = 2c^{.5}]$

in the rate of population growth. The establishment of a PAYG-scheme can then be interpreted as an anticipation to the temporary decline of the relative power of the young. As an illustration of this case, in Fig. 1 the dynamics of tax rates and saving rates is produced when the representative young individual's power falls as a consequence of the decline in population growth. Due to the incidently-lower weight of the young in the political decision making process, at the time of the shock the tax rate is set at a positive value. This incites a sequence of, relative to the stationary state, low saving rates and high tax rates, both before and after the time period of the shock. In accordance with proposition 1, these effects can be seen to last only for a limited number of periods.

Let us now turn to the case where only a PAYG-scheme exists in the stationary state so that $s^* = 0$ and $\tau^* > 0$. According to the Lemma this will be the case if $\lambda < 1/2$ ($\delta = r$). Before formulating the proposition, let us first indicate in what direction the tax and saving rates may change as a consequence of a demographic shock. The total effect of the population growth change on the tax rate in period $\underline{t} + j$ can be calculated by differentiating condition (8′) with respect to the population growth rate, the tax rate, and the saving rate. Using Eqs. (1) through (3) we get

$$\mathrm{d}\tau^*_{\underline{t}+j} = -\frac{\tau^*\,\mathrm{d}n_{\underline{t}+j} + V_{\underline{t}+j}\,\mathrm{d}s^*_{\underline{t}+j} + r\,\mathrm{d}s^*_{\underline{t}+j-1}}{n + V_{\underline{t}+j}}\,, \tag{9}$$

where $V_{\underline{t}+j} \equiv \frac{\lambda}{1-\lambda} \cdot \frac{u^{1,1}(c^y_{\underline{t}+j})}{u^{1,1}(c^r_{\underline{t}+j-1})}$. From condition (6′), the effect on savings in the period before the shock can be derived as:

$$\begin{aligned} &\mathrm{d}s^*_{\underline{t}+j-1} = \\ &= \max\left(0, \frac{1-\tau^*-n\tau^*-\mathrm{d}\tau^*_{\underline{t}+j-1}-\tau^*\,\mathrm{d}n_{\underline{t}+j}-n\,\mathrm{d}\tau^*_{\underline{t}+j}}{1+r}\right). \end{aligned} \tag{10}$$

Following the procedure outlined at the beginning of this section, first the initial effects of the population growth change on $\tau^*_{\underline{t}+j}$ and $s^*_{\underline{t}+j-1}$ should be determined. These initial effects can be derived by inserting (9) and (10) into each other, while abstaining from the feedback effects $\mathrm{d}\tau^*_{\underline{t}+j-1}$ and $\mathrm{d}s^*_{\underline{t}+j}$. It follows that the saving rate $s_{\underline{t}+j-1}$ will become positive as a direct consequence of the shock only if $\mathrm{d}n_{\underline{t}+j} < \frac{n+V_{\underline{t}+j}}{V_{\underline{t}+j}} \cdot \frac{1-\tau^*-n\tau^*}{\tau^*} < 0$. Furthermore, $\mathrm{d}\tau_{\underline{t}+j} < 0$ if $\mathrm{d}n_{\underline{t}+j} > 0$, while $\mathrm{d}\tau_{\underline{t}+j} > 0$ if $\mathrm{d}n_{\underline{t}+j} < 0$. In the case of a negative population growth shock public pension benefits will fall since the increase in the tax rate does not compensate for the decrease in the rate of population growth. Therefore, the generation born in the period before the shock occurs may start to save. As a result $\tau_{\underline{t}+j-1}$ will be below the stationary state value τ^* [see condition (8′)]. Then $s_{\underline{t}+j-2}$ may also become positive, and therefore $\tau_{\underline{t}+j-2} < \tau^*$, etc. In this way the effect of a negative population growth shock may stretch out backwards until the stationary state or time period $\underline{t}$ is reached. By a similar line of reasoning, a positive population growth shock may stretch out forwards in time. Then $\mathrm{d}\tau_{\underline{t}+j} < 0$ as a direct consequence of the shock, and therefore $s_{\underline{t}+j}$ may become positive, implying that $\mathrm{d}\tau_{\underline{t}+j+1} < 0$, etc. By proposition 1, for some finite $t > \underline{t}+j$ the stationary state will be re-established, however. In Appendix C the feedback effects are considered in more detail. There it is established that these feedback effects do not influence the result that a negative population growth cannot have any effects after the time period of the shock, and that a positive shock cannot have effects before the period of the shock, as long as relative marginal power D^1_t/D^2_t is proportional to the relative size of the generations n_t. On the basis of the above arguments and Appendix C the following proposition can be established:

Proposition 3: If $\lambda < 1/2$ and $D_t^1/D_t^2 = n_t(1-\lambda)/\lambda$, the effects of a temporary demographic shock will spread out a finite (not necessarily non-zero) number $\underline{\kappa} \geq 0$ of periods before or a finite number $\bar{\kappa} \geq 0$ periods after the shock, depending on the direction and size of the shock. In particular, if $\mathrm{d}n_{\underline{t}+j} > 0$ then $\mathrm{d}\tau^*_{\underline{t}+j+k} < 0$ for $k = 0, 1, \ldots, \bar{\kappa}$, while $\mathrm{d}\tau_{\underline{t}+j-k} = 0$ for $k \geq 1$. If $\mathrm{d}n_{\underline{t}+j} < 0$, then $\mathrm{d}\tau^*_{\underline{t}+j} > 0$ and $\mathrm{d}\tau^*_{\underline{t}+j+k} = 0$ for $k \geq 1$. Moreover, if $\mathrm{d}n_{\underline{t}+j} < \frac{n+V_{\underline{t}+j}}{V_{\underline{t}+j}} \cdot \frac{1-\tau^*-n\tau^*}{\tau^*} < 0$ then $\mathrm{d}\tau^*_{\underline{t}+j-k} < 0$ for $k = 1, 2, \ldots, \underline{\kappa}$. If the marginal power of the young is no constant, but depends on the rate of population growth a demographic shock may affect the outcomes both before and after the time period of the shock.

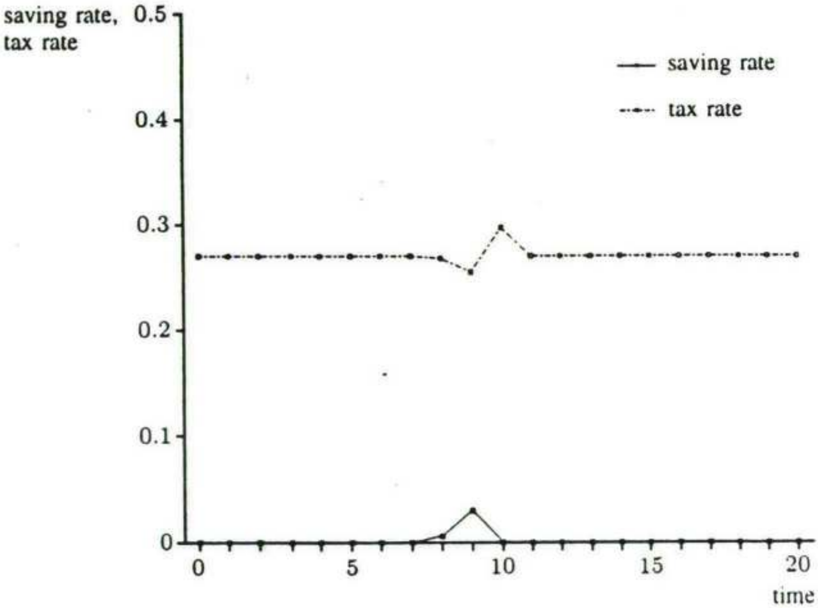

Fig. 2: The effects of a fall in population growth if $\lambda_t = .498$
[All other parameters have the same value as in Fig. 1.]

For the case where relative power is proportional to n_t, Fig. 2 illustrates how the system reacts to a negative population growth shock if it is in a stationary state with positive taxes and transfers and zero savings and if $dn_{\underline{t}+j} < \frac{n+V_{\underline{t}+j}}{V_{\underline{t}+j}} \cdot \frac{1-\tau^{*}-n\tau^{*}}{\tau^{*}}$. Then the shock will only have backward effects, softening the initial negative effect of the drop in population growth on the old-age consumption of the generation born in the period before the shock occurs. The increased consumption possibilities of the young born at the time of the shock due to the decreased tax rate does not have any forward influence; saving less than in the stationary state is impossible since that would imply negative savings.

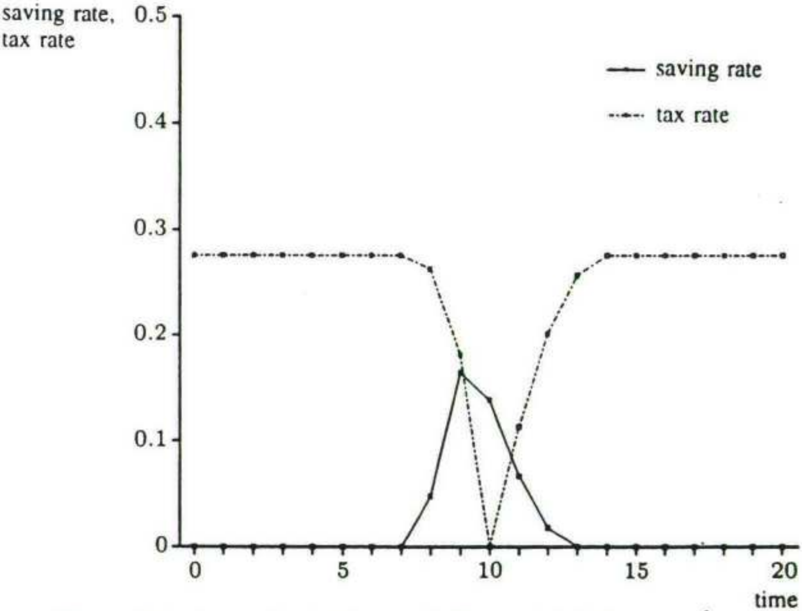

Fig. 3: The effects of a fall in population growth if $\lambda_t = \frac{1}{.65+.5n_t}$
[All other parameters have the same value as in Figs. 1 and 2.]

Figure 3 illustrates what may happen if relative marginal power is not proportional to the relative size of the living generations. This figure makes clear that a temporary negative demographic shock can have both forward and backward effects if the fall in population growth is not matched by a proportional decrease in the power of the young. At the time of the shock taxes fall due to the shift in power balance towards the young, inducing increased savings in the period before the shock.

It is apt to set the results of this section in the context of the conventional dynamic rational-expectations model (for an outline of these models, see Blanchard and Kahn, 1982). In these rational-expectations models, the system typically moves towards the stationary state along a saddle path. Saddle point stability, however, is not important in the model outlined here. The return of the system to the stationary state after a shock does not depend on its stability. It should furthermore be noted that in this model decision makers do not necessarily react to new information by changing their political or economic behavior. Even if the shocks that are foreseen for the future are bound to affect the stationary state at some later date, the current state of the system may remain undisturbed (see propositions 2 and 3). In contrast, in a conventional rational-expectations model new information, that is relevant for future decision making, is always immediately reacted upon; the forward looking variables are moved to a new saddle path immediately. A final difference that we want to point out here is that in the model of this paper the stationary state is always reached within a finite time period after a temporary disturbance. In a traditional rational-expectations model, convergence to a stationary state lasts forever.

4. Interpretation and Conclusions

The model analyzed in this paper is stylized in the sense that endowments, the interest rate, and population growth are given by nature. The neglect of economic and demographic feedback effects makes it, however, possible to analyze the interaction between public pensions and private savings in an overlapping-generations model in more detail. In particular, stationary states and dynamic paths outside the stationary state can be derived under different values of the parameters of the system and given perfect foresight of the distinguished agents. An important characteristic is that decision making on savings and the tax-transfer scheme takes place in every period by young individuals and politicians, respectively, and that no generation of young individuals or politicians can in any way be committed by previous generations of decision makers. So, contrary to for example Hansson and Stuart's (1989) conceptualization of the decision making process in this model future decisions cannot be prescribed by the current generation but have to be taken as given.

Given this set-up it is possible to trace the evolution of private and public pension schemes. In particular, the effects of demographic changes on saving and tax rates can be analyzed. It appears that temporary changes can have both forward and backward effects, depending on the impact of the demographic shock on the relative political weight of young individuals. However, the stationary state will always be re-established again within a finite time-interval after the shock. This is illustrated in Figs. 1 and 3. This result can also be interpreted as implying that the stationary states derived in Sect. 2 are stable: disturbances of the stationary state can only be temporary. The results were derived under the assumption of a temporary change in one of the parameters, i.e., the population growth rate. Temporary shocks, however, are not essential to the argument of this paper. In particular, if a permanent exogenous change occurs such that the new values of the parameters correspond to another stationary state, this stationary state will gradually be reached. This switching of stationary states can be in both directions: from a state with zero tax rates and benefits to one with a positive tax rate, or the other way around.

From the above discussions we may conclude that alternating decreases and increases in tax rates can occur in a world where agents have rational expectations. This contrasts with Boadway and Wildasin (1989) where decreases and increases in the tax rate are caused by wrong expectations. Moreover, a situation where tax rates gradually increase and saving rates gradually approach the value zero need not be the result of a constitutional decision making process where the current generation precommits future generations as in Hansson and Stuart (1989).

The model presented here is not intended to give a full explanation of the history and future of public and private pension schemes. For that purpose our model is too stylized and, furthermore, it lacks the possibilities for thorough empirical testing. For instance, it is not clear whether agents take future decision making fully into account when deciding on pensions. On the other hand, the model shows that agents need not have perfect foresight over an infinitely long horizon to accurately calculate their optimal behavior.

Assuming that the model provides us with an adequate description of decision making processes, it can be used for the interpretation of actual developments. If the foundation of PAYG-financed public pension plans were in fact meant as a compensation for the decline of private inter-family intergenerational transfers in the middle of this century (or in some countries even earlier), only PAYG transfers can exist in the stationary state. The development of CR-schemes, which we have witnessed in recent decades, can then be interpreted as an anticipation to the aging of the population that is expected to reach its peak at the beginning of the next century. If this interpretation is correct Figs. 2 or 3 would give an adequate description of past and future developments. Currently, we find ourselves in the part of the figure where the tax rates go down while old-age savings are rising. This can be expected to last until the decline in population growth stops. Then taxes and savings will immediately (Fig. 2) or gradually (Fig. 3) return to their stationary state values.

Finally, there is also a normative lesson to be learnt from this model. If decision makers do not look ahead and if they cannot precommit their successors, their plans may be frustrated by unanticipated changes in the size of the public and private pension schemes. Therefore it is better to look ahead and take note of the interrelations between CR- and PAYG-financed pension schemes.

Appendix A. Return to the Stationary State After the Time Period of the Shock Within a Finite Time Interval (Proof of Proposition 1)

Suppose that at time $\tilde{t}$ the system is out of the stationary state while for all times $t > \tilde{t}$ the parameter n_t is a constant, $n_t = n$. Then we can prove for $\delta = r$ that the system will return to the stationary state in finite time. We will give the proof for $\lambda > 1/2$. For $\lambda < 1/2$ the proof is analogous. The claim that if a disturbance is foreseen at a sufficiently early stage, the current state of that period need not be disturbed, also requires a similar proof.

Since the stationary state [$\tau^* = 0$ and $s^* = 1/(1+r)$, see the Lemma and condition (6′)] is unique, it is only possible that the stationary state does not return again within a finite time interval if the tax rate follows an oscillatory path without ever actually reaching zero (if the tax rate is zero for one period the stationary state immediately re-enters), if the tax rate can rise indefinitely without converging, or if the tax rate converges to some $\bar{\tau} \geq 0$. It is shown below that neither of these cases can occur within the context of the model.

First, observe that if the system is outside the stationary state at time $t > \tilde{t}$ the tax rate must be positive. Assuming that $s_t^* > 0$ and $s_{t-1}^* > 0$ and inserting Eqs. (1) through (3) and (6′) into condition (8′) gives

$$u^1\left(\frac{r(1-\tau_{t-1}^*)+n_t\tau_t^*}{1+r}\right) = \tag{A.1}$$
$$= \frac{\lambda}{1-\lambda}\, u^1\left(\frac{r(1-\tau_t^*)+n_{t+1}\tau_{t+1}^*}{1+r}\right) .$$

Since $\lambda > 1/2$ and $n_t = n_{t+1} = n$ it follows that $r(\tau_t^* - \tau_{t-1}^*) < n(\tau_{t+1}^* - \tau_t^*)$. This inequality implies that if savings remain positive the system cannot proceed along an oscillatory path because when the tax rate starts to rise at time period t, $\tau_t^* - \tau_{t-1}^* > 0$, it cannot go down again at a later time period; if $\tau_t^* - \tau_{t-1}^* > 0$ then also $\tau_{t+1}^* - \tau_t^* > 0$ and $\tau_{t+2}^* - \tau_{t+1}^* > 0$, and so on. But what if $s_t^* = 0$? Since $\lambda > 1/2$, conditions (6′) and (8′) imply that $c_t^r > c_{t-1}^r$. Using Eqs. (2) and (3) and $s_t^* = 0$ while $s_{t-1}^* \geq 0$ it follows that $\tau_t^* < \tau_{t+1}^*$. For period $t+1$ it can be derived from condition (8′) that $c_{t+1}^y > c_t^r$, which implies $1 - s_{t+1}^* - \tau_{t+1}^* > n_{t+1}\tau_{t+1}^*$. But since $s_t^* = 0$ it must hold that $1 - \tau_t^* \leq n_{t+1}\tau_{t+1}^*$ [see condition (6′)], which implies that $n_{t+1}\tau_{t+1}^* > 1 - \tau_{t+1}^*$ as $\tau_{t+1}^* > \tau_t^*$. Consequently, condition (8′) can only hold at time $t+1$ following $s_t^* = 0$ if $s_{t+1}^* < 0$, which cannot be true. This excludes the possibility of $s_t = 0$, and for that same reason, of $s_{t-1} = 0$.

This also proves that the tax rate cannot rise indefinitely without converging; then it would sooner or later hold that $s_t^* \leq 0$, which cannot be true.

The tax rate might also converge towards some $\bar{\tau} \geq 0$. But in that case $c_{t-1}^r = \left(r(1 - \tau_{t-1}^*) + n_t\tau_t^*\right)/(1+r)$ would converge towards $c_t^y = \left(r(1 - \tau_t^*) + n_{t+1}\tau_{t+1}^*\right)/(1+r)$. Then, for some finite $\bar{t} > \tilde{t}$ it must hold that $c_{\bar{t}}^r > c_{\bar{t}}^y - \epsilon$ where ϵ satisfies $u^1(c_{\bar{t}-1}^r) < u^1(c_{\bar{t}}^y - \epsilon) \leq \lambda/(1-\lambda)u^1(c_{\bar{t}}^y)$. The latter expression implies that $\tau_{\bar{t}}^* = 0$, contradicting the assumption of convergence towards $\bar{\tau}$.

Since none of the alternatives is consistent with the model, the system has to return to the stationary state with $\tau^* = 0$ and $s^* = 1/(1+r)$ after a disturbance (this is provided, of course, that such a path exists).

Q.E.D.

Appendix B. Disturbance of the Stationary State by a Demographic Shock in the Case that $\lambda < 1/2$ (Proof of Proposition 2)

First take the case where relative marginal power is proportional to the relative size of generations $[D_t^1/D_t^2 = n_t\lambda/(1-\lambda)]$. Note that the change $dn_{\underline{t}+j}$ only has a direct effect on $s_{\underline{t}+j-1}^*$ and $\tau_{\underline{t}+j}^*$. Using condition (8′) for $t = \underline{t} + j$ and condition (6′) for $t = \underline{t} + j - 1$, it can be proved by contradiction that $\tau_{\underline{t}+j}^*$ cannot become positive and $s_{\underline{t}+j-1}^*$ cannot deviate from $1/(1+r)$ given that all other variables apart from $n_{\underline{t}+j}$ initially remain unchanged. Suppose the tax rate at time $\underline{t} + j$ is larger than zero. Then condition (8′) would hold with an equality sign. Since $(1 - \lambda_{\underline{t}+j})/\lambda_{\underline{t}+j} > n_{\underline{t}+j}$, it must be true that $c^y_{\underline{t}+j} > c^r_{\underline{t}+j-1}$. Using Eqs. (1)–(3) and (6′) and $s^*_{\underline{t}+j} = 1/(1+r)$ while $\tau^*_{\underline{t}+j+1} = 0$, it can be derived that $\tau^*_{\underline{t}+j} < 0$, which implies a contradiction. Furthermore, as can be immediately seen from condition (6′), if the tax rate at time $\underline{t} + j$ does not deviate from the stationary state value, then the saving rate of the previous period cannot differ from its stationary state value $1/(1 + r)$ either.

Now consider the case where relative marginal power is not proportional to relative size $[D^1_t/D^2_t = n_t\lambda_t/(1-\lambda_t)$ where $\lambda_t = \lambda(n_t)]$. The proposition states that if the demographic change leads to $\lambda_{\underline{t}+j} < 1/2$, then the stationary state cannot be maintained anymore. This can be shown as follows. If it is assumed that the stationary state will be left undisturbed it must hold that $s^*_{\underline{t}+j-1} = s^*_{\underline{t}+j} = s^* = 1/(1 + r)$ and $\tau^*_{\underline{t}+j} = \tau^* = 0$. The first-order condition for the tax rate at time $\underline{t} + j$ would therefore read $\frac{u^1(r/(1+r))}{u^1(r/(1+r))} \geq \frac{1-\lambda_{\underline{t}+j}}{\lambda_{\underline{t}+j}}$, clearly contradicting $\lambda_{\underline{t}+j} < 1/2$. Therefore, $\tau^*_{\underline{t}+j} > 0$. If the tax rate of period $\underline{t} + j$ becomes positive then the saving rates at times $\underline{t} + j$ and $\underline{t} + j - 1$ will drop below their stationary state values, i.e., $s^*_{\underline{t}+j} < 1/(1+r)$ and $s^*_{\underline{t}+j-1} < 1/(1+r)$ [see condition (6′)]. Due to the decrease in savings at time $\underline{t}+j-1$ consumption by the young born at that time rises, while consumption by the old living in that period is not affected initially. As a consequence, marginal utility of the young relative to marginal utility of the old $u^1(c^y_{\underline{t}+j-1})/u^1(c^r_{\underline{t}+j-2})$ will fall. If this term decreases to a large enough extent, $\tau^*_{\underline{t}+j-1}$ will turn out to be positive due to the shock [see condition (8′)]. The disturbance of the stationary state in period $\underline{t}+j-1$ can lead to the same kind of deviation in period $\underline{t}+j-2$, and so on. This effect stretches out backwards until $\tau^*_{\underline{t}+j-\underline{\kappa}} = \tau^* = 0$, where $0 < \underline{\kappa} \leq j$, implying that $\tau^*_{\underline{t}+j-i} > 0$ for $0 \leq i < \underline{\kappa}$. Then it will also hold that $s^*_{\underline{t}+j-i} < s^*$, $s^*_{\underline{t}+j-\underline{\kappa}} < s^*$, and $s^*_{\underline{t}+j-\underline{\kappa}-1} = s^*$. Besides the backward effects described above, there may also be a forward effect of the disturbance at time $\underline{t} + j$. Since $s^*_{\underline{t}+j} < s^*$ the tax rate in period $\underline{t} + j + 1$ can become positive $[u^1(c^r_{\underline{t}+j})$ rises relative to $u^1(c^y_{\underline{t}+j+1})$, *ceteris paribus*], which in turn implies that $s^*_{\underline{t}+j+1} < s^*$. This effect continues in forward direction until $\tau^*_{\underline{t}+j+\bar{\kappa}} = \tau^* = 0$ for some $\bar{\kappa} > 0$, implying $s^*_{\underline{t}+j+\bar{\kappa}} = s^*$. From period $\underline{t} + j + \bar{\kappa}$ on the stationary state with zero taxes and positive savings will be restored again.

Q.E.D.

Appendix C. Derivation of Dynamic Paths Outside the Stationary State (Proof of Proposition 3)

In this Appendix it is assumed that $\lambda_t = \lambda(n_t)$, where λ can be any differentiable function. In other words, the assumption that relative marginal power is proportional to the relative size of the generations, that underlies most calculations in Sect. 3, is relaxed.

Case 1. $\lambda > 1/2$ in the stationary state and $\mathrm{d}n_{\underline{t}+j} \neq 0$.
Let us suppose that as a direct consequence of the demographic shock $\mathrm{d}\tau_{\underline{t}+j} > 0$. If it is assumed that after the period of the shock there are no effects of the shock it follows that [by differentiating condition (6′)]

$$\mathrm{d}s^*_{\underline{t}+j} = -\frac{\mathrm{d}\tau^*_{\underline{t}+j}}{1+r} \tag{C.1}$$

and

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j+1}}{\mathrm{d}\tau^*_{\underline{t}+j}} = 0 \tag{C.2}$$

by assumption. From condition (8′) it can be derived, however, that $\tau^*_{\underline{t}+j+1}$ may become larger than zero as a consequence of $\mathrm{d}s^*_{\underline{t}+j} < 0$. That would imply (assuming that $\mathrm{d}\tau^*_{\underline{t}+j+2} = 0$)

$$\mathrm{d}s^*_{\underline{t}+j} = -\frac{\mathrm{d}\tau^*_{\underline{t}+j} + n\mathrm{d}\tau^*_{\underline{t}+j+1}}{1+r}\,. \tag{C.3}$$

$$\mathrm{d}s^*_{\underline{t}+j+1} = -\frac{\mathrm{d}\tau^*_{\underline{t}+j+1}}{1+r}\,. \tag{C.4}$$

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j+1}}{\mathrm{d}\tau^*_{\underline{t}+j}} \leq \frac{r}{n+rV_{\underline{t}+j+1}}\,, \tag{C.5}$$

and

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j+2}}{\mathrm{d}\tau^*_{\underline{t}+j+1}} = 0 \tag{C.6}$$

by assumption. Equation (C.5) can be calculated by differentiating condition (8′). In deriving (C.1) and (C.2) it was assumed that the stationary state returned in period $\underline{t}+j+1$ or, in other words, that $\bar{\kappa}$, the number of periods after the shock that are disturbed, is equal to one. Equations (C.3) through (C.6) were derived under the assumption that $\bar{\kappa} = 2$. Analogously, expressions can be derived that describe the evolution of tax rates and saving rates after a disturbance under the assumption that $\bar{\kappa} = 3, 4, \ldots$ or, equivalently, that $\mathrm{d}\tau^*_{\underline{t}+j+3}/\mathrm{d}\tau^*_{\underline{t}+j+2} = 0$, $\mathrm{d}\tau^*_{\underline{t}+j+4}/\mathrm{d}\tau^*_{\underline{t}+j+3} = 0$, etc. It can be checked which of these assumptions is true for a certain situation by calculating $\mathrm{d}\tau^*_{\underline{t}+j+\bar{\kappa}}/\mathrm{d}\tau^*_{\underline{t}+j+\bar{\kappa}-1}$ for $\bar{\kappa} = 1, 2, \ldots$ using condition (8′). The $\bar{\kappa}$ for which this expression is equal to zero is the correct $\bar{\kappa}$ since $\mathrm{d}\tau^*_{\underline{t}+j+\bar{\kappa}}/\mathrm{d}\tau^*_{\underline{t}+j+\bar{\kappa}-1} = \frac{\mathrm{d}\tau^*_{\underline{t}+j+\bar{\kappa}}}{\mathrm{d}\tau^*_{\underline{t}+j}} \Big/ \frac{\mathrm{d}\tau^*_{\underline{t}+j+\bar{\kappa}-1}}{\mathrm{d}\tau^*_{\underline{t}+j}} = 0$ only if $\mathrm{d}\tau^*_{\underline{t}+j+\bar{\kappa}}/\mathrm{d}\tau^*_{\underline{t}+j} = 0$ and $\mathrm{d}\tau^*_{\underline{t}+j+\bar{\kappa}-1}/\mathrm{d}\tau^*_{\underline{t}+j} \neq 0$ implying that the stationary state will be re-established from time $\underline{t} + j + \bar{\kappa}$ on. The exact value of $\bar{\kappa}$ will of course depend on the actual parameters of the system, the instantaneous utility function, the function determining λ_t, and the size and the sign of the shock.

The expressions for the tax rates that can be found by extending the above analysis to $\bar{\kappa} = 3, 4, \ldots$ can be generalized by induction

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j+\bar{\kappa}-i}}{\mathrm{d}\tau^*_{\underline{t}+j}} \leq$$

$$\leq \frac{r^{\bar{\kappa}-i} n^{i-1} + r^{\bar{\kappa}-i} \sum_{h=1}^{i-1} \left(n^{i-1-h} r^h \prod_{p=0}^{h-1} V_{\underline{t}+j+\bar{\kappa}-1-p} \right)}{n^{\bar{\kappa}-1} + \sum_{h=0}^{\bar{\kappa}-2} \left(n^{\bar{\kappa}-2-h} r^{h+1} \prod_{p=0}^{h} V_{\underline{t}+j+\bar{\kappa}-1-p} \right)} \tag{C.7}$$

for $0 < i < \bar{\kappa}$. The expressions for the saving rate follow automatically from (6′) and (C.7).

There can also be a backward effect if $\mathrm{d}\tau_{\underline{t}+j-1} > 0$ since then also $\mathrm{d}s_{\underline{t}+j-1} < 0$. By assuming that the effect only stretches out backwards for one period it can be derived that

$$\mathrm{d}s^*_{\underline{t}+j-1} = -\frac{n\,\mathrm{d}\tau^*_{\underline{t}+j} + \tau^*\,\mathrm{d}n_{\underline{t}+j} + \mathrm{d}\tau^*_{\underline{t}+j-1}}{1+r}\,, \tag{C.8}$$

$$\mathrm{d}s^*_{\underline{t}+j-2} = -\frac{n\,\mathrm{d}\tau^*_{\underline{t}+j-1}}{1+r}\,, \tag{C.9}$$

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j-1}}{\mathrm{d}\tau^*_{\underline{t}+j}} \leq \frac{nV_{\underline{t}+j-1}}{n + rV_{\underline{t}+j-1}} \cdot \left(1 + \frac{\tau^*}{n} \cdot \frac{\mathrm{d}n_{\underline{t}+j}}{\mathrm{d}\tau^*_{\underline{t}+j}}\right)\,. \tag{C.10}$$

Analogously to the calculations for the forward effects it can be derived that generally

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j-\underline{\kappa}+i}}{\mathrm{d}\tau^*_{\underline{t}+j}} \leq \tag{C.11}$$

$$\leq \frac{\sum_{h=0}^{i-1}\left(n^{\underline{\kappa}-1-h}r^h \prod_{p=1}^{\underline{\kappa}-i+h} V_{\underline{t}+j-p}\right)}{n^{\underline{\kappa}-1}+\sum_{h=1}^{\underline{\kappa}-1}\left(n^{\underline{\kappa}-1-h}r^h \prod_{p=1}^{h} V_{\underline{t}+j-p}\right)} \cdot \left(1+\frac{\tau^*}{n}\cdot\frac{\mathrm{d}n_{\underline{t}+j}}{\mathrm{d}\tau^*_{\underline{t}+j}}\right)$$

for $0 < i < \underline{\kappa}$. The expression $\mathrm{d}\tau^*_{\underline{t}+j}/\mathrm{d}n_{\underline{t}+j}$ in Eq. (C.11) can be derived to be [differentiate condition (8′)]

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j}}{\mathrm{d}n_{\underline{t}+j}} \leq \frac{(1+r)\cdot\frac{\lambda^1(n_{\underline{t}+j})}{(1-\lambda_{\underline{t}+j})^2}\cdot\frac{u^1(c^y_{\underline{t}+j})}{u^{1,1}(c^r_{\underline{t}+j-1})}-\tau^*}{n+rV_{\underline{t}+j}-rA-nV_{\underline{t}+j}B}+$$
$$+\frac{\frac{r}{n}\tau^* A-(r+V_{\underline{t}+j})\cdot\frac{1-\tau^*-n\tau^*}{\mathrm{d}n_{\underline{t}-j}}}{n+rV_{\underline{t}+j}-rA-nV_{\underline{t}+j}B}, \tag{C.12a}$$

if $\mathrm{d}n_{\underline{t}+j} > 0$, and

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j}}{\mathrm{d}n_{\underline{t}+j}} \geq \frac{(1+r)\cdot\frac{\lambda^1(n_{\underline{t}+j})}{(1-\lambda_{\underline{t}+j})^2}\cdot\frac{u^1(c^y_{\underline{t}+j})}{u^{1,1}(c^r_{\underline{t}+j-1})}-\tau^*}{n+rV_{\underline{t}+j}-rA-nV_{\underline{t}+j}B}+$$
$$+\frac{\frac{r}{n}\tau^* A-(r+V_{\underline{t}+j})\cdot\frac{1-\tau^*-n\tau^*}{\mathrm{d}n_{\underline{t}-j}}}{n+rV_{\underline{t}+j}-rA-nV_{\underline{t}+j}B}, \tag{C.12b}$$

if $\mathrm{d}n_{\underline{t}+j} < 0$, where

$$A \equiv \frac{\sum_{h=0}^{\underline{\kappa}-2}\left(n^{\underline{\kappa}-1-h}r^h \prod_{p=1}^{h+1} V_{\underline{t}+j-p}\right)}{n^{\underline{\kappa}-1}+\sum_{h=1}^{\underline{\kappa}-1}\left(n^{\underline{\kappa}-1-h}r^h \prod_{p=1}^{h} V_{\underline{t}+j-p}\right)} \tag{C.13}$$

and

$$B \equiv \frac{rn^{\bar{\kappa}-2} + \sum_{h=1}^{\bar{\kappa}-2} \left(n^{\bar{\kappa}-2-h} r^{h+1} \prod_{p=0}^{h-1} V_{\underline{t}+j+\bar{\kappa}-1-p} \right)}{n^{\bar{\kappa}-1} + \sum_{h=0}^{\bar{\kappa}-2} \left(n^{\bar{\kappa}-2-h} r^{h+1} \prod_{p=0}^{h} V_{\underline{t}+j+\bar{\kappa}-1-p} \right)} . \tag{C.14}$$

Case 2. $\lambda < 1/2$ in the stationary state and $\mathrm{d}n_{\underline{t}+j} \neq 0$.
First suppose that $s^*_{\underline{t}+j} = 0$ and $s^*_{\underline{t}+j-1} = 0$. Then, of course, there is no effect of the shock before and after time period $\underline{t} + j$. If it appears, however, after checking the saving rates, that $\mathrm{d}s^*_{\underline{t}+j} > 0$ must hold, then the effect of the shock can extend itself after the time period shock. Under the assumption that $s^*_{\underline{t}+j-1} = s^* = 0$, the evolution of the tax rate after time $\underline{t} + j$ can be calculated to be

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j+\bar{\kappa}-i}}{\mathrm{d}\tau^*_{\underline{t}+j}} = \tag{C.15}$$

$$= \frac{r^{\bar{\kappa}-i} n^{i-1} + (1+r) r^{\bar{\kappa}-i} \sum_{h=1}^{i-1} \left(n^{i-1-h} r^{h-1} \prod_{p=0}^{h-1} V_{\underline{t}+j+\bar{\kappa}-1-p} \right)}{n^{\bar{\kappa}-1} + (1+r) \sum_{h=0}^{\bar{\kappa}-2} \left(n^{\bar{\kappa}-2-h} r^{h} \prod_{p=0}^{h} V_{\underline{t}+j+\bar{\kappa}-1-p} \right)}$$

and

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j}}{\mathrm{d}n_{\underline{t}+j}} = \tag{C.16}$$

$$= \frac{(1+r) \cdot \left(\frac{\lambda^1(n_{\underline{t}+j})}{(1-\lambda_{\underline{t}+j})^2} \cdot \frac{u^1(c^y_{\underline{t}+j})}{u^{1.1}(c^r_{\underline{t}+j-1})} - \tau^* \right) - V_{\underline{t}+j} \cdot \frac{1-\tau^*-n\tau^*}{\mathrm{d}n_{\underline{t}+j}}}{(1+r)n + rV_{\underline{t}+j} - nV_{\underline{t}+j} C} ,$$

where

$$C \equiv \frac{rn^{\bar{\kappa}-2} + (1+r) \sum_{h=1}^{\bar{\kappa}-2} \left(n^{\bar{\kappa}-2-h} r^{h} \prod_{p=0}^{h-1} V_{\underline{t}+j+\bar{\kappa}-1-p} \right)}{n^{\bar{\kappa}-1} + (1+r) \sum_{h=0}^{\bar{\kappa}-2} \left(n^{\bar{\kappa}-2-h} r^{h} \prod_{p=0}^{h} V_{\underline{t}+j+\bar{\kappa}-1-p} \right)} . \tag{C.17}$$

If it, however, appears that $s_{\underline{t}+j-1} > 0$ it can be calculated, assuming that $s^*_{\underline{t}+j} = 0$,

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j-\underline{\kappa}+i}}{\mathrm{d}\tau^*_{\underline{t}+j}} = \tag{C.18}$$

$$= \frac{D + n^{\underline{\kappa}-i} r^{i-1} \prod_{p=1}^{\underline{\kappa}-1} V_{\underline{t}+j-p}}{E + (1+r)n^{\underline{\kappa}-1} + r^{\underline{\kappa}-1} \prod_{p=1}^{\underline{\kappa}-1} V_{\underline{t}+j-p}} \cdot \left(1 + \frac{\tau^*}{n} \cdot \frac{\mathrm{d}n_{\underline{t}+j}}{\mathrm{d}\tau^*_{\underline{t}+j}}\right),$$

where

$$D \equiv (1+r) \sum_{h=0}^{i-2} \left(n^{\underline{\kappa}-1-h} r^h \prod_{p=1}^{\underline{\kappa}-i+h} V_{\underline{t}+j-p} \right). \tag{C.19}$$

and

$$E \equiv (1+r) \sum_{h=1}^{\underline{\kappa}-2} \left(n^{\underline{\kappa}-1-h} r^h \prod_{p=1}^{h} V_{\underline{t}+j-p} \right). \tag{C.20}$$

Furthermore

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j}}{\mathrm{d}n_{\underline{t}+j}} = \tag{C.21}$$

$$= \frac{(1+r) \cdot \frac{\lambda^1(n_{\underline{t}+j})}{(1-\lambda_{\underline{t}+j})^2} \cdot \frac{u^1(c^y_{\underline{t}+j})}{u^{1,1}(c^r_{\underline{t}+j-1})} - \tau^* + \frac{r}{n}\tau^* F - r \cdot \frac{1-\tau^*-n\tau^*}{\mathrm{d}n_{\underline{t}+j}}}{(1+r)V_{\underline{t}+j} + n - rF},$$

where

$$F \equiv \frac{(1+r) \sum_{h=0}^{\underline{\kappa}-3} \left(n^{\underline{\kappa}-1-h} r^h \prod_{p=1}^{h+1} V_{\underline{t}+j-p} \right) + nr^{\underline{\kappa}-2} \prod_{p=1}^{\underline{\kappa}-1} V_{\underline{t}+j-p}}{E + (1+r)n^{\underline{\kappa}-1} + r^{\underline{\kappa}-1} \prod_{p=1}^{\underline{\kappa}-1} V_{\underline{t}+j-p}}. \tag{C.22}$$

If it appears that $s^*_{\underline{t}+j} > 0$ as well as $s^*_{\underline{t}+j-1} > 0$ then the Eqs. (C.7) and (C.11) with an equality sign describe the evolution of the tax rates after respectively before the shock. The effect on the tax rate of the time period of the shock is

$$\frac{\mathrm{d}\tau^*_{\underline{t}+j}}{\mathrm{d}n_{\underline{t}+j}} = \frac{(1+r)\cdot\frac{\lambda^1(n_{\underline{t}+j})}{(1-\lambda_{\underline{t}+j})^2}\cdot\frac{u^1(c^y_{\underline{t}+j})}{u^{1.1}(c^r_{\underline{t}+j-1})} - \tau^*}{n + rV_{\underline{t}+j} - rA - nV_{\underline{t}+j}B} + \\ + \frac{\frac{r}{n}\tau^* A - (r + V_{\underline{t}+j})\cdot\frac{1-\tau^*-n\tau^*}{\mathrm{d}n_{\underline{t}+j}}}{n + rV_{\underline{t}+j} - rA - nV_{\underline{t}+j}B}. \tag{C.23}$$

These calculations show that, as was claimed in Proposition 3, if $\lambda^1(n_{\underline{t}+j}) \neq 0$ a population growth shock can have effects both before and after the shock. In Proposition 3 it was also claimed that if relative marginal power is proportional to the relative size [that is, $\lambda^1(n_{\underline{t}+j}) = 0$], then a negative population growth shock will only have effects in the period of the shock and the periods before that, while a positive population growth shock will only have effects in the period of the shock and later periods. This can be shown as follows. Equation (C.16) gives the total effect of a positive population growth shock ($\mathrm{d}n_{\underline{t}+j} > 0$) on the tax rate in the period of the shock, assuming that there is no change in the outcomes of periods before the stationary state. This assumption is justified if $1 - \tau^* - n\tau^* - \mathrm{d}\tau^*_{\underline{t}+j-1} - \tau^*\,\mathrm{d}n_{\underline{t}+j} - n\,\mathrm{d}\tau^*_{\underline{t}+j} \leq 0$ [see Eq. (11)]. By inserting (C.16) it can be checked, after some calculations, that this indeed is true for $\lambda^1(n_{\underline{t}+j}) = 0$ (it even holds that $1 - \tau^* - n\tau^* - \mathrm{d}\tau^*_{\underline{t}+j-1} - \tau^*\,\mathrm{d}n_{\underline{t}+j} - n\,\mathrm{d}\tau^*_{\underline{t}+j} < 0$). In the case of a negative population growth shock, Eq. (C.21) gives the effect on $\tau^*_{\underline{t}+j}$, assuming that the feedback effects do not cause any changes in the outcomes after the period of the shock. This assumption, of course, is justified if $\tau^*_{\underline{t}+j} \geq \tau^*$, since then $\mathrm{d}s_{\underline{t}+j} = 0$ and no further effects after period $t + j$ can occur. Inserting $\lambda^1(n_{\underline{t}+j}) = 0$ in Eq. (C.21) shows that $\mathrm{d}\tau_{\underline{t}+j}/\mathrm{d}n_{\underline{t}+j} > 0$, implying that indeed $\tau^*_{\underline{t}+j} > \tau^*$.

Q.E.D.

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