

**Decision Making Under Uncertainty: Capturing
Dynamic Brand Choice Processes in Turbulent
Consumer Goods Market.**

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- In forward looking dynamic structural models, consumers may sample different brands exclusively to gather information about them.
- After some sampling, consumers may settle into a brand.
- Changes, due to the introduction of new brands, brand repositioning, price cuts of other brands may induce consumers to resample.
- Structural models of consumer choice and learning fit the data better than the standard reduced form models.

- Obtain deeper understanding of the consumer learning process, which is the source of state dependence. Policy experiments can be done.

Model

$j = 1, \dots, m$ brands, $m + 1$: no purchase option.

$E [U_{ij}(t) \mid I_i(t)]$: current period expected utility.

Bellman Equation:

$$V_j (I_j(t), t) = E [U_{ij}(t) \mid I_i(t)] + \beta E [V (I(t + 1), t + 1) \mid I(t)]$$

$$V (I(t), t) = \text{Max}_j V_j (I_j(t), t)$$

Consumer Expected Utility

A_{Eijt} : consumer i 's experience of a brand's attribute.

Utility of consumer i purchasing a brand j :

$$U_{ijt} = -w_j P_{ijt} + w_A A_{Eijt} - w_A r A_{Eijt}^2 + e_{ijt}$$

r : consumer risk component.

$r > 0$ risk averse

$r = 0$ risk neutral

$r < 0$ risk loving

$$\begin{aligned}
& E [U_{ijt} \mid I_i(t)] \\
= & -w_j P_{ijt} + w_A E [A_{Eijt} \mid I_i(t)] - w_A r E [A_{Eijt} \mid I_i(t)]^2 \\
& - w_A r E [A_{Eijt} - E [A_{Eijt}^2 \mid I_i(t)]]^2 + e_{ijt}
\end{aligned}$$

r : consumer risk component.

Other small brands:

$$E [U_{iOt}] = U_{iOt} = \Phi_O + \Phi_{Ot} + e_{iOt}$$

No purchase option:

$$E [U_{iNPt}] = U_{iNPt} = \Phi_{NP} + \Phi_{NPt} + e_{iNPt}$$

Consumer Learning about Brand Attributes:

At each purchase:

$$A_{Eijt} = A_{ijt} + \eta_{ijt}$$

$$A_{Eijt} = A_j + \delta_{ijt}, \quad \delta_{ijt} = \xi_{ijt} + \eta_{ijt}$$

ξ_{ijt} : attribute variability of the product

η_{ijt} : variability of the perception of the product.

Consumer cannot separate ξ_{ijt} and η_{ijt} from each other.

At the introduction of the brand (no past learning)

$$\delta_{ijt} \sim N(0, \sigma_\delta^2), \quad A_j \sim N(A, \sigma_A^2(0))$$

Advertising signal:

$$S_{ijt} = A_j + \varsigma_{ijt}, \quad \varsigma_{ijt} \sim N(0, \sigma_\varsigma^2)$$

Consumer update:

$$\begin{aligned} & E \left[A_{Eijt+1} \mid I_i(t) \right] \\ = & E \left[A_{Eijt} \mid I_i(t-1) \right] + D_{1ijt} \beta_{1ij}(t) \left[A_{Eijt} - E \left[A_{Eijt} \mid I_i(t-1) \right] \right] \\ & + D_{2ijt} \beta_{2ij}(t) \left[S_{Eijt} - E \left[S_{Eijt} \mid I_i(t-1) \right] \right] \end{aligned}$$

D_{1ijt} : dummy of whether consumer purchases brand j or not.

D_{2ijt} : dummy of whether consumer receives an advertising signal of brand j or not.

From Kalman Filter

$$\beta_{1ijt} = \frac{\sigma_{vij}^2(t)}{\sigma_{vij}^2(t) + \sigma_{\delta}^2}, \quad \beta_{2ijt} = \frac{\sigma_{vij}^2(t)}{\sigma_{vij}^2(t) + \sigma_{\varsigma}^2}$$

$$v_{ij} = E \left[A_j \mid I_{ij}(t) \right] - A_j$$

Then, because

$$A_j = E \left[A_j \mid I_{ij}(t) \right] + v_{ij}(t)$$

and

$$A_{Eijt} = A_j + \delta_{ijt}, \quad S_{ijt} = A_j + \varsigma_{ijt}$$

$$\begin{aligned} & v_{ij}(t) \\ = & v_{ij}(t-1) + D_{1ijt}\beta_{1ij}(t) \left[-v_{ij}(t-1) + \delta_{ijt} \right] \\ & + D_{2ijt}\beta_{2ij}(t) \left[-v_{ij}(t-1) + \varsigma_{ijt} \right] \end{aligned}$$

$$\sigma_{vij}^2(t) = \frac{1}{\frac{1}{\sigma_v^2(0)} + \frac{\sum_{s=0}^t D_{1ijs}}{\sigma_\delta^2} + \frac{\sum_{s=0}^t D_{2ijs}}{\sigma_\zeta^2}}$$

Hence,

$$\begin{aligned} E \left[U_{ij} \mid I_i(t) \right] &= w_A A_j - w_A r A_j^2 - w_A r \sigma_\delta^2 - w_P P_{ij} \\ &\quad - w_A r \sigma_{vij}^2(t) - w_A r v_{ij}(t)^2 - w_A v_{ij}(t) - 2w_A r A_j v_{ij}(t) \\ &\quad + e_{ijt} \end{aligned}$$

Static brand choice probability:

$$P_i(I_i(t), t) = \int \frac{\exp \left\{ E \left[U_{ij} \mid I_i(t) \right] \right\}}{\sum_k \exp \left\{ E \left[U_{ik} \mid I_i(t) \right] \right\}} f(v) dv$$

Dynamic brand choice probability:

$$P_i(I_i(t), t) = \int \frac{\exp \left\{ E \left[V_{ij} \mid I_i(t) \right] \right\}}{\sum_k \exp \left\{ E \left[V_{ik} \mid I_i(t) \right] \right\}} f(v) dv$$

where

$$E \left[V_{ij} \mid I_i(t) \right] = E \left[U_{ij} \mid I_i(t) \right] + \beta E \left[V_{ij} \mid I_i(t+1) \mid d_{ijt} = 1, I_i(t) \right]$$

Data:

- Scanner panel data for laundry detergent, 3,000 households from year 1986 to 1988.
- New brands were introduced
- Firms heavily advertise
- Low in variety seeking.
- Only liquid detergents.

Panel Member Selection Criteria:

- a** Telemeter attached to TV
- b** More than 80% of the total detergent purchase is liquid
- c** At least 20 purchases
- d** At least 7, at most 24 liquid purchases in the last 51 weeks.

Assume that consumer knows the mean price level.

$$P_{ijt} = P_j + w_{ijt}, \quad w_{ijt} \sim N(0, \sigma_{w_j}^2)$$

Advertising exposure data:

- commercial viewing files. Household was exposed to the commercial at least once during that given week.
- Advertising frequency: percentage of weeks the household was exposed to the ad for brand j .

Model Estimation and Validation

- Choice of planning horizon: $T = 100$
- Initial Conditions problem. Use first two years of Nielsen data to impute past consumption and advertising experience at the start of year 3.

$$\begin{aligned} E[U_{ij} | I_i(t_0)] &= w_A A_j - w_A r A_j^2 - w_A r \sigma_\delta^2 - w_P P_{ij} \\ &\quad - w_A r \sigma_{v_{ij}}^2(t_0) - w_A r v_{ij}(t_0)^2 - w_A v_{ij}(t_0) \\ &\quad - 2w_A r A_j v_{ij}(t_0) + e_{ijt} \end{aligned}$$

But because there is no data before initial period t_0 , we cannot calculate $\sigma_{v_{ij}}^2(t_0)$, $v_{ij}(t_0)$.

Guardani-Little Model

$$E \left[U_{ij} \mid I_i(t) \right] = a_j - w_p P_j + w_E \sum_{s=t_0}^t D_{1ijs} + w_{Ad} \sum_{s=t_0}^t D_{2ijs}$$

parameter	estimate	t-stat
Price (w_p)	-1.077	-18.10
"brand loyalty" (w_E)	3.363	53.18
Advertising (w_{Ad})	0.144	0.31
Brand intercepts		
dash	0.000	
cheer	1.115	8.87
solo	0.917	7.22
surf	1.382	14.43
era	1.601	11.03
wisk	1.102	6.78
ride	1.700	12.29

parameter	estimate	t-stat
Other brands intercept	-0.633	-2.98
Other brands trend	0.011	4.87
No purchase intercept	1.636	8.02
No purchase trend	0.005	1.35
Brand loyalty smoothing	0.770	50.62
Advertising smoothing	0.788	2.95

Advertising coefficient has positive sign but not significant.

Smoothing: total past purchases or advertising?

Structural Model Estimates

	$\beta = 0$		$\beta = 0.995$	
parameter	estimate	t-stat	estimate	t-stat
Price	-0.790	-12.26	-0.795	-12.31
Utility weight	28.356	1.73	34.785	1.84
Risk coefficient	3.625	2.08	4.171	2.25
Initial variance	0.053	4.64	0.040	4.21
Mean attribute levels				
dash	0.049	0.74	0.040	0.74
cheer	0.019	0.27	0.012	0.21
solo	0.056	0.84	0.047	0.87
surf	0.105	1.65	0.089	1.77
era	0.137	2.41	0.120	2.64
wisk	0.040	0.59	0.029	0.53
ride	0.138	-	0.120	-

	$\beta = 0$		$\beta = 0.995$	
parameter	estimate	t-stat	estimate	t-stat
Other brands intercept	-17.657	-7.98	-17.267	-7.59
Other brands trend	0.018	8.53	0.018	8.91
No purchase intercept	-15.408	-6.99	-19.537	-8.55
No purchase trend	0.011	3.17	0.012	3.42
Experience variability	0.374	9.17	0.33	8.37
Advertising variability	3.418	6.29	3.08	5.57

- Fixed one attribute level (tide) to a value such that the utility is increasing in the attribute level.
- Parameter estimates of the two models are similar.
- Price coefficients are negative and significant
- High utility weight on latent attribute: cleansing power.
- Attribute levels are not significant, but the differences are.

- Positive risk coefficient: risk averse consumers.
- Advertising variability higher than experience variability: experience a better signal than advertising.
- Small initial variance: consumers' prior quality for a new product has small variance.

Goodness of Fit

Within Sample:

	GL	myopic	Forward looking
-LL	7463.23	7312.09	7306.05
AIC	7478.23	7324.09	7322.05
BIC	7531.10	7384.49	7378.45

Out of sample

	GL	myopic	Forward looking
-LL	2000.69	1951.38	1952.98
AIC	2015.69	1967.38	1968.98
BIC	2059.21	2013.80	2015.40

- Both in sample and out of sample, structural models predict better than the reduced form models.
- There is not much difference in predictive performance between the myopic and forward looking models.
- Perhaps because the product is mature, the value associated with experimentation is small.

Scenario Evaluations:

- 1** The higher the advertising frequency, the higher the brand choice probability. This effect is more for new product than mature product

- 2 Lowering advertising variability increases choice probability.
- 3 In forward looking model, consumers are more willing to try new products than myopic model.

Figure 1 Surf Increases Ad Frequency from 0.234 to 0.700

(I. Dynamic/New Product)

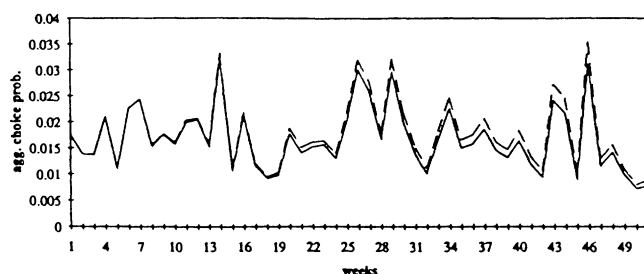


Figure 2 Surf Increases Ad Frequency from 0.234 to 0.700

(F. Dynamic/New Product)

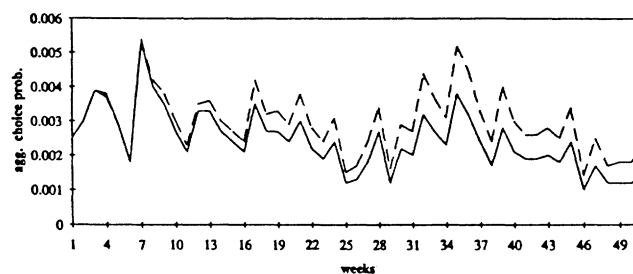


Figure 3 Surf Increases Ad Frequency from 0.234 to 0.700

(I. Dynamic/Mature Product)

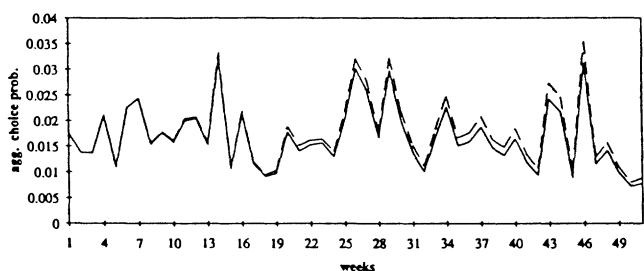
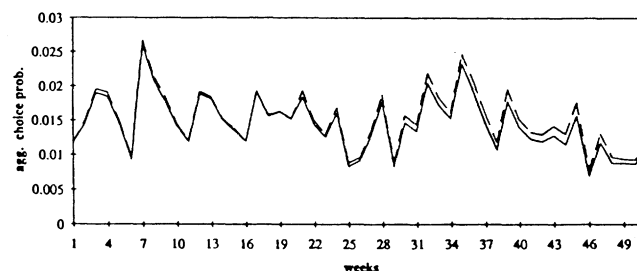


Figure 4 Surf Increases Ad Frequency from 0.234 to 0.700

(F. Dynamic/Mature Product)



marketing strategy. The results can be summarized as follows:

(1) The higher the advertising frequency, the higher the brand choice probability, and this holds more for a "new product" than a "mature product." Furthermore, although the short run effect of advertising is not large, advertising has a strong cumulative effect on choice over time as it gradually reduces the perceived riskiness of a brand. This can be seen in the results for both structural models (Figures 1 through 4).

(2) Figure 5 and 6 show that lowering advertising variability increases choice probabilities. Thus, precise advertising messages have a major positive impact on new product sales growth. In the "mature product" case the positive impact is very slight (Figures 7 and 8), but the forward-looking dynamic structural model suggests that there is more impact than the dynamic structural model with immediate utility maximization.

(3) The forward-looking dynamic structural model suggests that consumers are more willing to try new brands than suggested by the dynamic structural model

with immediate utility maximization (see the baseline for the two models in the "New Product" condition). This is because of the information-gathering motive for trying new brands that is captured by the forward-looking dynamic model.

These results are straightforward. They reveal that the two structural models produce very similar results when applied to a product category like laundry detergent, for which the additional value gained by sampling different brands is not large because consumers already know quite a lot about the products. Thus, in the case of mature product categories, it may well be that a structural model that assumes immediate utility maximization can capture most of what a more complicated "dynamic" structural model captures.

6. Discussion and Conclusions

Using Nielsen liquid detergent data, we estimated two structural models of the brand choice behavior of Bayesian consumers in an environment with uncertainty about brand attributes. We also estimate an

Figure 5 Surf Reduces Ad Variability by 50%

(I. Dynamic/New Product)

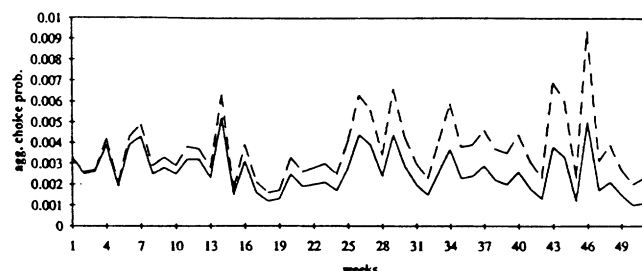


Figure 6 Surf Reduces Ad Variability by 50%

(F. Dynamic/New Product)

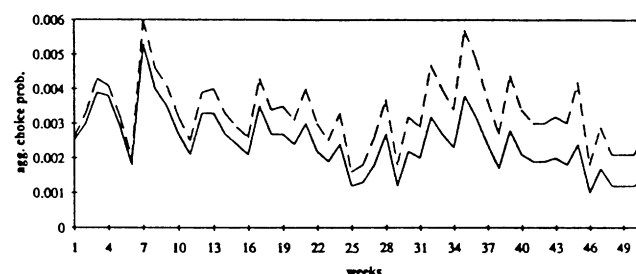


Figure 7 Surf Reduces Ad Variability by 50%

(I. Dynamic/Mature Product)

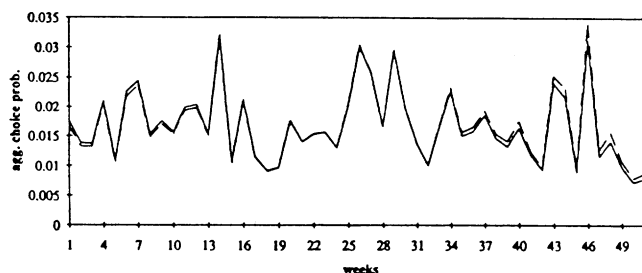
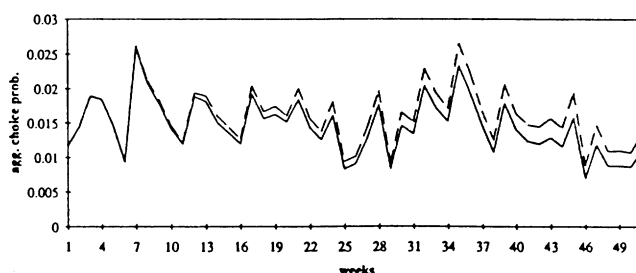


Figure 8 Surf Reduces Ad Variability by 50%

(F. Dynamic/Mature Product)



approximation to the reduced form decision rule that emerges from these models. We refer to the approximation to the reduced form decision rule as the GL model because of its similarity to the Guadagni and Little model (1983).

The main contribution of the paper is to introduce the structural approach to econometric modeling, which has gained wide acceptance in economics during the past decade, into the marketing literature. Within the structural econometrics paradigm, the paper makes two important contributions: (1) it is the first application of the Keane and Wolpin (1994) approximate solution method for DP problems; and (2) it is the first application of structural modelling in a case in which serially correlated error processes must be integrated over in order to solve the consumers' problem. (Such serial correlation arises naturally in Bayesian learning models.)

The proposed structural models incorporate purchase feedback, cumulative advertising effects and consumer expectations about brand attributes as determinants of brand choice. Our structural models provide a theoretic

cal explanation for behavioral variation across households and within households over time. Specifically, we model both usage experience and advertising as sources of information regarding uncertain brand attributes. Over time, households have different experiences when they try brands, and they also may receive different advertising signals. Although consumers have the same priors about brands, their perception errors about mean brand attribute levels diverge over time as they receive different signals. Hence, unobserved heterogeneity of consumers arises endogenously over time. We view this as a great theoretical strength of our models because a behavioral explanation of heterogeneity is provided rather than simply assuming a parametric heterogeneity distribution a priori.

Because heterogeneity arises endogenously over time, our structural models provide a theoretical explanation for brand loyalty formation. Brand loyalty occurs because of the low riskiness of familiar brands. Given a good fit between attributes of a familiar brand and individual tastes, a risk-averse consumer will, *ceteris par-*