

DECISION OPTIMIZATION OF MACHINE SETS TAKING INTO CONSIDERATION LOGICAL TREE MINIMIZATION OF DESIGN GUIDELINES

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The method of minimization of complex partial multi-valued logical functions determines the degree of importance of construction and exploitation parameters playing the role of logical decision variables. Logical functions are taken into consideration in the issues of modelling machine sets. In multi-valued logical functions with weighting products, it is possible to use a modified Quine - McCluskey algorithm of multi-valued functions minimization. Taking into account weighting coefficients in the logical tree minimization reflects a physical model of the object being analysed much better.

Key words: optimization, logical tree minimization, modified Quine- McCluskey algorithm, proportional valve.

1. Introduction

In process planning it is of importance to elaborate the concept, models and their influence (by means of controlling, directing, obtaining) to get the maximum satisfaction out of them. It is, thus, stated that planning has an interdisciplinary character, which is similar to cybernetics. Three elements of knowledge are essential to design a system, that is knowledge in the scope of physical processes, regularity of influencing these processes and improving the optimization processes which take place. That why, an optimal designing or re-designing in general is regarded as a dynamic process which includes interaction and creation of new values. An introduction of appropriate formal formulas in the process of problem structuration combines complex features of quantity and quality of different levels of detail according to rules of the multi-dimensional morphological table (Deptuła and Partyka, 2010; Partyka, 1984). Decision and morphological tables can be analytically and numerically coded in accordance with the definition and logic of multi-valued decision processes what makes it possible to apply variant way of identifying and classifying information while looking for solutions in the designing process (Filla and Palmberg, 2003; Francis and Betts, 1997; Giergiel, 1990; Kurowski, 2001; Żak and Stefanowski, 1994). The method of minimizing complex partial multi-valued logical functions indicates the degree of importance of construction and exploitation parameters playing the role of logical decision variables (Osiński *et al.*, 2013). Introducing weighting sets of logical equations as design guidelines with the possibility of a separate or joint minimization with keeping the logic equivalence becomes important. Even in the Boolean case, the joint minimization is not worse, from the point of view of the number of literals, than the separate minimization. A modified Quine - McCluskey algorithm of minimizing multi-valued functions has been presented in the paper. Taking into consideration weighting coefficients in the logical tree minimization reflects more precisely the physical model of the object which is analysed.

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2. Logical tree minimization of design guidelines

A logical tree is a structural presentation of a logical function, written in the form of a sum of products, where every element is the realisation of one solution and each component in the product is a logic variable (Partyka, 1984). Complex multi-valued logical functions state the degree of importance of logic variables, by means of changing the logical tree levels, from the most important ones (near the root) to the least important (in the upper part) because there is a generalisation of a bivalent indicator of quality into a multi-valued one; $(C_k - k_i m_i) + (k_i + K_i)$, where C_k – the number of branches k -th level, k_i – times simplification on the k -th level m_i - valued variable, K_i – number of branches of $(k-1)$ -th level, from which branches of k -th level which cannot be simplified were created. All transformations are described by the so-called Quine – Mc Cluskey algorithm of minimization of individual partial multi-valued logical functions.

Example 1.

For the multi-valued logical function $f(x_1, x_2, x_3)$, where $x_1, x_2, x_3 = 0, 1, 2$, written in the following form KAPN: 100, 010, 002, 020, 101, 110, 021, 102, 210, 111, 201, 120, 022, 112, 211, 121, 212, 221, 122, there is one MZAPN after applying the Quine – Mc Cluskey algorithm of minimizing individual partial multi-valued logical functions which has 13 literals (Partyka, 1984)

$$f(x_1, x_2, x_3) = j_0(x_1)(j_0(x_2)j_2(x_3) + j_1(x_2)j_0(x_3) + j_2(x_2)) + j_1(x_1) + j_2(x_1)(j_0(x_2)j_1(x_3) + j_1(x_2) + j_2(x_2)j_1(x_3))).$$

MAPN of a given multi-valued logical function has been shown in Fig.2.

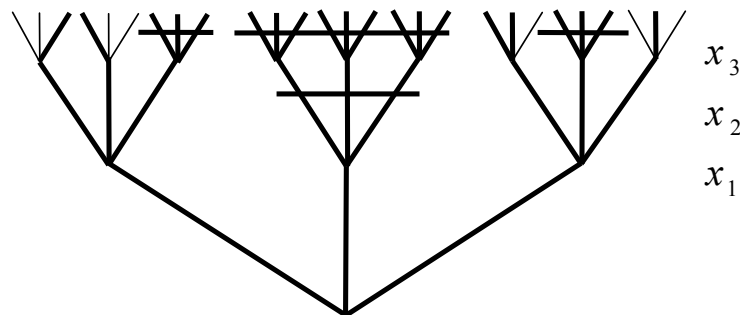


Fig.1. MAPN of a given multi-valued logical function.

2. Logical tree minimization with weighting coefficients

2.1. Weighting coefficients

In given partial multi-valued logical functions $f_i(x_1, \dots, x_n)$ n variables (m_1, \dots, m_n) - valued, it is necessary to take weighting coefficients $(w_n, w_{n-1}, w_{n-2}, \dots, w_1)$ into consideration in clustering and pseudo-clustering, assigned to appropriate multi-valued logical products.

It is possible to use the Quine - McCluskey algorithm of minimizing multi-valued functions in the case of multi-valued logical functions with weighting products (Deptuła and Partyka, 2012; Partyka, 1984).

2.2. The Quine-McCluskey algorithm of minimizing multi-valued logical functions with weighting coefficients

There is an isomorphic interpretation of logical transformations, therefore the Quine - McCluskey algorithm of the minimization of individual partial multivalued logical functions can be analysed with taking into consideration weighting coefficients, what is important in describing the degree of importance of design guidelines (Deptuła and Partyka, 2012; Partyka, 1984).

The Quine - McCluskey algorithm of minimizing multi-valued logical functions is built of n -th columns of weighting coefficients (w_1, \dots, w_n) (Tab.1).

Table 1. The column form of weighting coefficients of elementary coefficients in the (m_1, \dots, m_n) positional system. w_{n-2} .

	x_1, \dots, x_n					w_n					w_{n-2}	...	w_1	
(w_1, \dots, w_n)	0	0	...	0	0				
(w_1, \dots, w_n)	0	0	...	0	1				
(w_1, \dots, w_n)	0	0	...	1	$m_n - 2$				
							
							
(w_1, \dots, w_n)	$m_1 - 1$	$m_2 - 1$...	$m_{n-1} - 1$	$m_n - 2$				
(w_1, \dots, w_n)	$m_1 - 1$	$m_2 - 1$...	$m_{n-1} - 1$	$m_n - 1$				
	p_i					1	2	3	4	n...	$i = 1, \dots, n$	$i = 1, \dots, n$	$i = 1, \dots, n$	

Symbols, signifying pseudo-clustering (\mathbf{V}) and clustering (\mathbf{v}) subsequently in relation to groups of indices which differ by 1, are placed in columns corresponding to weighting coefficient values for appropriate logical products in columns with (w_1, \dots, w_n) weighting coefficients (Tab.1) position numbers p_i are introduced, where $i=1, \dots, n$, what is important in calculating the quality of minimization in further steps.

In the case of clustering of individual partial multi-valued logical functions with weighting coefficients, the definitions of “**clear**” and “**unclear**” clustering are introduced:

The process of **clear clustering** is the following transformation

$$w_i A_{j_0}(x_r) + \dots + w_i A_{j_{m_r-1}}(x_r) = w_i A. \tag{2.1}$$

The process of **unclear clustering** is the following transformation

$$\begin{aligned}
 & w_0 A j_0(x_r) + \dots + w_{m_r-1} A j_{m_r-1}(x_r) = \\
 & = \left(\min \{w_0, \dots, w_{m_r-1}\} \right) \cdot A + \sum_{s=i_0, \dots, i_{m_r-2}} w_s \cdot A \cdot j_s(x_r)
 \end{aligned} \tag{2.2}$$

where: $r = 1, \dots, n$, $w_s > \min \{w_0, \dots, w_{m_r-1}\}$ and A - partial elementary product, variables of which belong to the set $\{x_1, \dots, x_{r-1}, x_{r+1}, \dots, x_n\}$. In n variables (m_1, \dots, m_n) -valued, the weighting coefficient w_i before

the partial canonical product adopts values from the range $\langle w_1, \dots, w_n \rangle$, if $w_j = w_{j-1} + w_{j-2} + \dots + w_1$ where $j = 2, \dots, n$ (in the case of z signs, an exemplary form can be as follows: $1 \cdot (000) + 2 \cdot (010) + 2 \cdot (020) = 1 \cdot (0-0) + 2(010) + 2(020)$).

In the case of unclear clustering, signs (V) of elementary products with (w_1, \dots, w_n) coefficients are put in one column related to the smallest coefficient value $\min \{w_1, \dots, w_n\}$ on the position p_i . Signs (\underline{V}) are put in the case of the remaining clustering products of weighting coefficients $w_s > \min \{w_0, \dots, w_{m_r-1}\}$, in columns with an appropriate coefficient (w_1, \dots, w_n) . However, they are placed on the same position p_i , on which V signs are placed in the column with the coefficient $\min \{w_1, \dots, w_n\}$.

The minimization quality can be calculated according to the following formula

$$C_k - \sum_p \left(k_{m_{p_i}} - k_{i_{p_i}} \right)_{p_i} + n_{pk_m} + n_{pk_i} + K_{i_n} \tag{2.3}$$

where:

$k_{m_{p_i}}$ - multivalency of i -th variable subject to clustering of products on the p_i position in the case of clear clustering of products with the m_i coefficient or in the case of unclear clustering with the coefficient (m_1, \dots, m_n) ,

$k_{i_{p_i}}$ - the number of literals representing products of the coefficient bigger than the minimum coefficient,

n_{pk_m} - multifactor occurrence of clear clustering,

n_{pk_i} - multifactor occurrence of non-clustering of literals $k_{i_{p_i}}$ in the process of unclear clustering,

K_{i_n} - times of pseudo-clustering of i -th variable that is arbitrary clustering on the basis of a smaller number of components than m_i , used only for the needs of calculating literals for $(k-1)$ -th level.

The Quine - McCluskey algorithm with multi-valued weighting coefficients requires:

1. Looking for clustering products,
2. Assigning the (v) sign to the clustering products on the n position with the smallest coefficient,
3. Assigning the (v) sign to the products earlier clustered with the remaining coefficients,
4. Assigning the \underline{v} sign to the remaining products of pseudo-clustering on n positions with appropriate coefficients,

5. Calculating the minimization value: $C_k - \sum_p (k_{m_{pi}} - k_{i_{pi}})_{pi} + n_{pk_m} + n_{pk_i} + K_{i_n}$,
6. Putting the “-“ sign before products with weighting coefficients taking part in clustering in view of point 2,
7. Memorising subsequent literals which are not clustered together with the weighting coefficients which had earlier received the (V) sign in view of point 2,
8. Memorising subsequent literals which are not clustered together with the weighting coefficients which had earlier received the (V) sign in view of point 3.

Example 2.

In the partial logical function $f(x_1, x_2, x_3)$, written in the following form in KAPN: 010, 100, 002, 011, 110, 012, 112, the Quine - Mc Cluskey algorithm of minimizing logical functions with multi-valued weighting coefficients gives one MZAPN, which has 11 literals $f(x_1, x_3, x_2)$, that is

$$f(x_1, x_3, x_2) = j_0(x_1)(Ij_0(x_3)j_1(x_2)) + 2j_1(x_3)2j_1(x_2) + 2j_2(x_3) + j_1(x_1)(Ij_2(x_3)j_1(x_2)) + 2j_0(x_3)j_1(x_2)$$

whereas the remaining ZAPN $f(x_1, x_2, x_3)$, $f(x_2, x_1, x_3)$, $f(x_2, x_3, x_1)$, $f(x_3, x_1, x_2)$ of a given logical function have respectively 12 and $f(x_3, x_2, x_1)$ 13 literals

$$f(x_2, x_3, x_1) = j_0(x_2)(Ij_0(x_3)j_1(x_1) + 2j_2(x_3)j_0(x_1)) + j_1(x_2)(2j_0(x_3)j_1(x_1) + 2j_1(x_3)j_0(x_1) + 2j_1(x_3)j_0(x_1)),$$

$$f(x_2, x_1, x_3) = j_0(x_2)(2j_0(x_1)j_2(x_3) + Ij_1(x_1)j_0(x_3)) + j_1(x_2)(2j_0(x_1)(j_1(x_3) + j_2(x_3)) + j_1(x_1)(2j_0(x_3) + Ij_2(x_3))),$$

$$f(x_1, x_2, x_3) = j_0(x_1)(2j_0(x_2)j_2(x_3) + 2j_1(x_2)(j_1(x_3) + j_2(x_3))) + j_1(x_1)(Ij_0(x_2)j_0(x_3) + j_1(x_2)(2j_0(x_3) + Ij_2(x_3))),$$

$$f(x_3, x_1, x_2) = j_0(x_3)(Ij_0(x_1)j_1(x_2) + 2j_1(x_1)j_1(x_2)) + 2j_1(x_3)j_0(x_1)j_1(x_2) + j_2(x_3)(2j_0(x_1) + Ij_1(x_1)j_1(x_2)),$$

$$f(x_3, x_2, x_1) = j_0(x_3)(Ij_0(x_2)j_1(x_1) + 2j_1(x_2)j_1(x_1)) + 2j_1(x_3)j_1(x_2)j_0(x_1) + j_2(x_3)(2j_0(x_2)j_0(x_1) + 2j_1(x_2)j_0(x_1)).$$

Subsequent stages of the algorithm

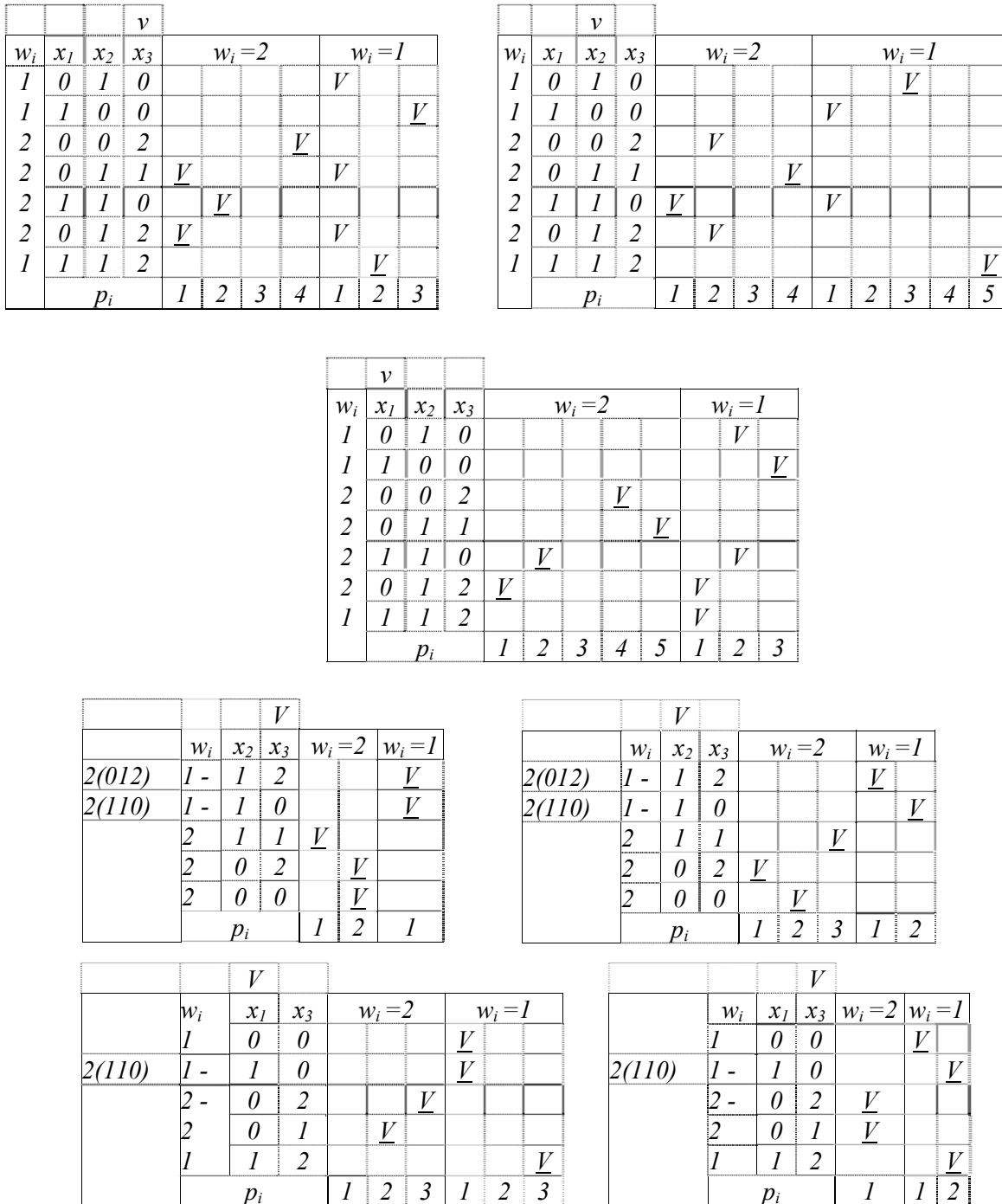


Figure 2 shows MZAPN and ZAPN of a given logical function from example 2.

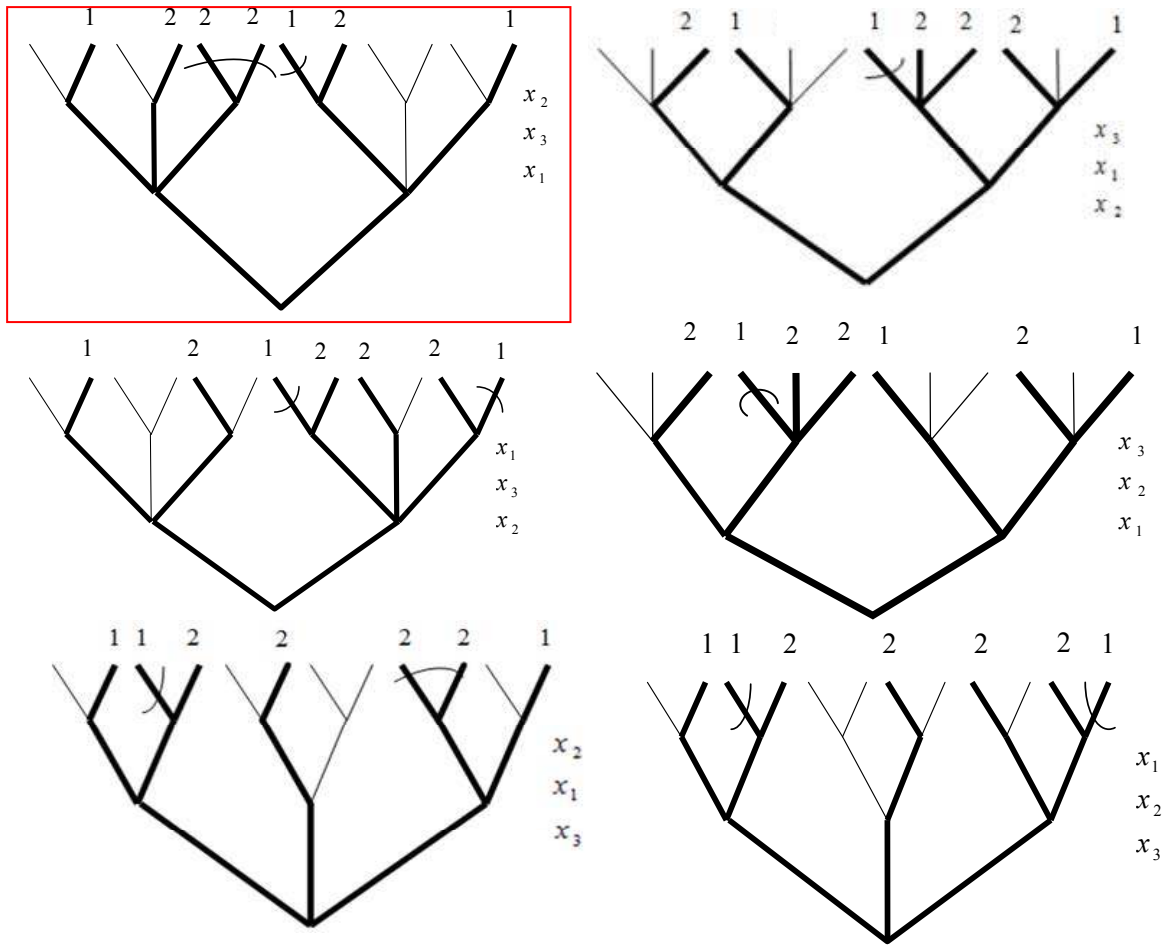


Fig.2. Logical trees MZAPN and ZAPN of a multi-valued logical function $f(x_1, x_3, x_2)$ from example 3.

3. Application of the logical tree minimization with weighting coefficients in the analysis of the degree of importance of construction parameters of the proportional valve

Figure 3 shows the drive system with a proportional valve with a valve receiver.

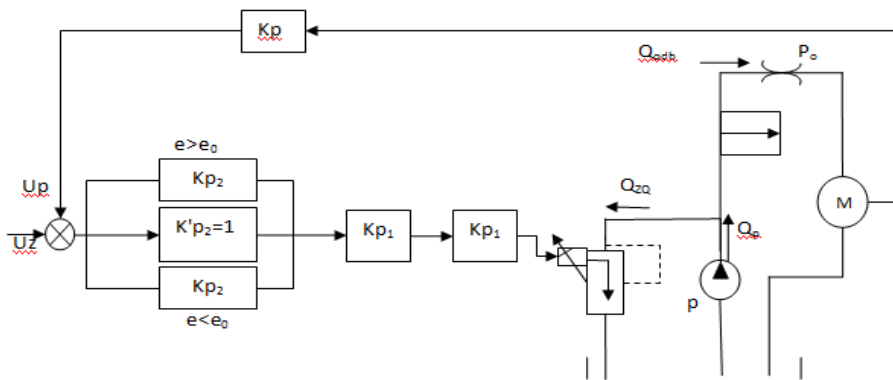


Fig.3. Drive system scheme.

$$\left\{ \begin{array}{l} 1: \frac{dx_1}{dt} = x_2, \\ 2: \frac{dx_2}{dt} = -14846.301x_2 - 801.2102 \cdot 10^{-3} (k_{vx}x_1)x_3 - 147224.3x_1 - 1925.135 + 5.3792244 \cdot 10^{-3} \left[(1 - 10^3 x_1)x_3 - x_6 \right], \\ 3: \frac{dx_3}{dt} = 0.2851216 \cdot 10^9 (1 - 1.32 \cdot 10^{-9} x_3) - 0.5279061 \cdot 10^9 (k_{vx}x_1)\sqrt{x_3} - 0.1226361 \cdot 10^9 x_2 - 7.65(x_3 - x_6) + \\ \quad - 0.3227777 \cdot 10^{12} Q_{odb}, \\ 4: \frac{dx_4}{dt} = x_5, \\ 5: \frac{dx_5}{dt} = -5.5688865 \cdot 10^3 x_5 - 0.840264 \cdot 10^6 x_5^2 \text{sign}x_5 + 0.7123874 \cdot 10^{-4} x_7 + 418.87733 (k_{vy}x_4)^2 x_7 + \\ \quad - 2.616 \text{sign}x_5 - 33.33333 F_m, \\ 6: \frac{dx_6}{dt} = 0.276556 \cdot 10^5 (x_3 - x_6) - 0.312234 \cdot 10^{12} (k_{vy}x_4)\sqrt{x_6} + 0.4432633 \cdot 10^{12} x_3 - 2.060625 \cdot 10^9 x_5, \\ 7: x_7 = x_6 - 0.2025169 \cdot 10^6 (k_{vy}x_4)\sqrt{x_6} - 1328.096 x_5. \end{array} \right.$$

The initial conditions of differential equations are set by introducing $\frac{dx_i}{dt} = 0$.

3.1. The degree of importance of construction and exploitation parameters of the hydraulic proportional valve

In the optimization process, the changeable parameters of the proportional valve are as follows: regulator strengthening $K_{p1} \cdot K_{p2}$ (as a complex variable), flow intensity Q_{odb} (depending on the enforcement of step changes in supply control voltage U_z) and magnetic force F_m - when the flow intensity Q and pressure p are being watched.

A simulation was made in the Matlab/Simulink

$$\left\{ \begin{array}{l} -801.2102 \cdot 10^{-3} (k_{vx}x_1)x_3 - 147224.3x_1 - 1925.135 + 5.3792244 \cdot 10^{-3} \left[(1 - 10^3 x_1)x_3 - x_6 \right] = 0, \\ 0.2851216 \cdot 10^9 (1 - 1.32 \cdot 10^{-9} x_3) - 0.5279061 \cdot 10^9 (k_{vx}x_1)\sqrt{x_3} - 7.65(x_3 - x_6) - 0.3227777 \cdot 10^{12} Q_{odb} = 0, \\ 0.7123874 \cdot 10^{-4} x_7 + 418.87733 (k_{vy}x_4)^2 x_7 - 33.33333 F_m = 0, \\ 0,276556 \cdot 10^5 (x_3 - x_6) - 0.312234 \cdot 10^{12} (k_{vy}x_4)\sqrt{x_6} = 0, \\ x_7 = x_6 - 0.2025169 \cdot 10^6 (k_{vy}x_4)\sqrt{x_6}. \end{array} \right.$$

Arithmetic values of the parameters which had been coded by means of decision variables were chosen to be analysed

$$(K_{p1} \cdot K_{p2}) = 30 \sim 0; \quad (K_{p1} \cdot K_{p2}) = 40 \sim 1; \quad (K_{p1} \cdot K_{p2}) = 50 \sim 2; \quad (K_{p1} \cdot K_{p2}) = 60 \sim 3.$$

$$F_m = 1.96 [N] \sim 0; \quad F_m = 2.96 [N] \sim 1; \quad F_m = 3.96 [N] \sim 2; \quad F_m = 4.96 [N] \sim 3;$$

$$Q_{rz} = 36 \rightarrow 24 [dm^3 / min] \sim 0; Q_{rz} = 24 \rightarrow 12 [dm^3 / min] \sim 1; Q_{rz} = 36 \rightarrow 12 [dm^3 / min] \sim 2.$$

In the limitation $t_w < 0.48t_o$ the following values of weighting coefficients were adopted

$$w_i=3, \quad t_w \leq 0.16t_o; \quad w_i=2, \quad 0.16t_o < t_w \leq 0.32t_o; \quad w_i=1, \quad 0.32t_o < t_w \leq 0.48t_o.$$

Table 2 shows changes in coding of construction parameters K_{p1} , K_{p2} , Q_{rz} , F_m taking into consideration multi-valued weighting coefficients and the limitation $t_w < 0.48t_o$.

Table 2. KAPN of the code data of parameters $K_{p1}K_{p2}$, Q_{rz} , F_m taking into consideration weighting coefficients w_i .

w_i	F_m	$K_{p1}K_{p2}$	Q_{rz}	w_i	F_m	$K_{p1}K_{p2}$	Q_{rz}
2	2	1	2	3	0	1	1
2	2	3	2	1	0	1	2
2	2	2	1	3	0	1	0
2	2	2	2	2	0	0	1
2	1	2	1	1	0	0	2
3	3	0	2	1	1	2	2
1	1	0	2	2	1	1	2
2	0	2	1	1	1	3	2
1	0	2	2	1	3	2	2
2	0	2	0	1	3	1	2
3	0	3	1	3	3	3	2
2	0	3	2				

Exemplary time spans of the functions Q and p have been shown in Fig.5 with indicating ranges of weighting coefficients w_i : p ■ and Q ■.

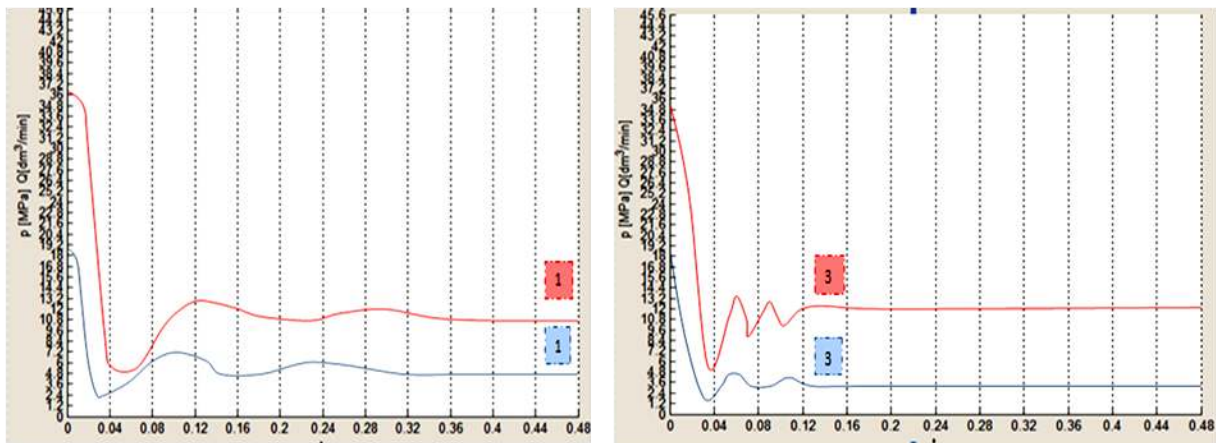


Fig.5. Timing Q and p for code changes parameter $F_m, K_{p1} \cdot K_{p2}, Q_{rz} : 1(102), 3(010)$.

Figure 6 shows the optimal multivalued logic tree with weighting factors in Tab.2. Other multivalued logic trees of weighting factors are shown in Fig.7.

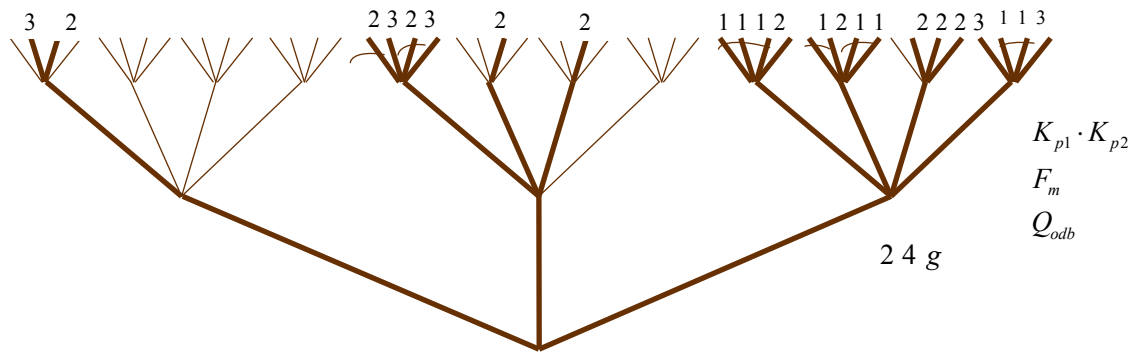
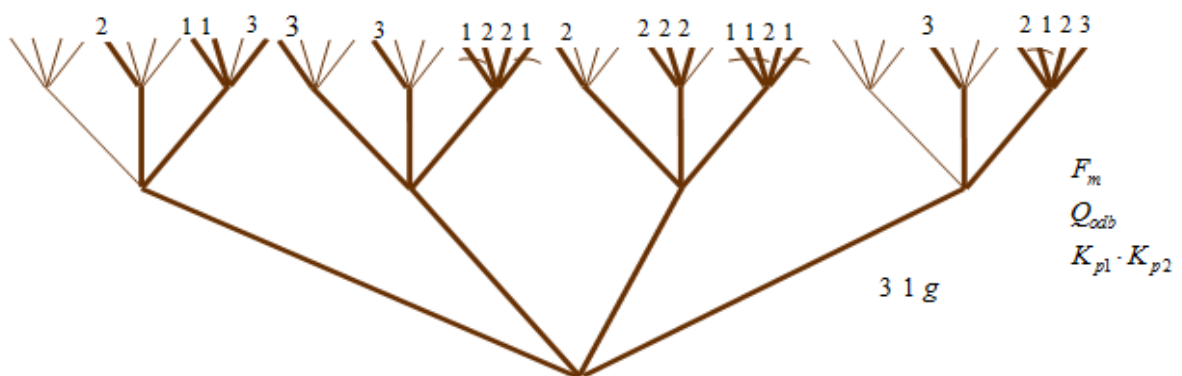
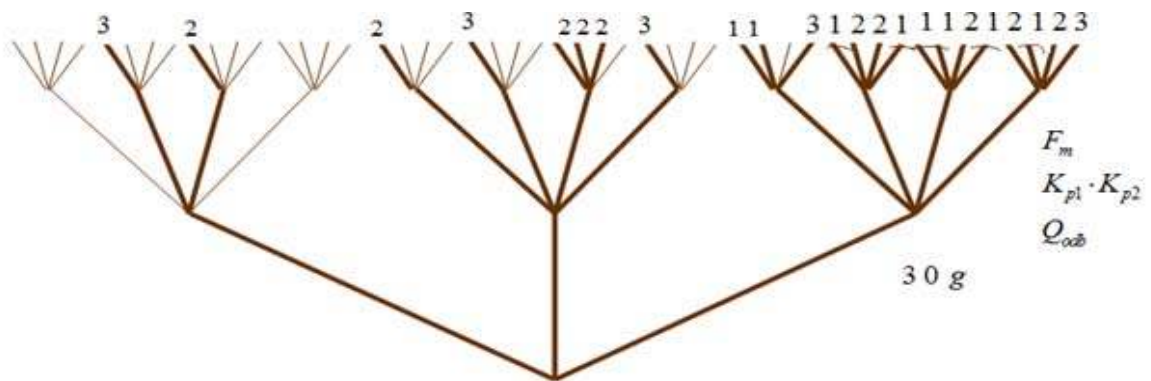
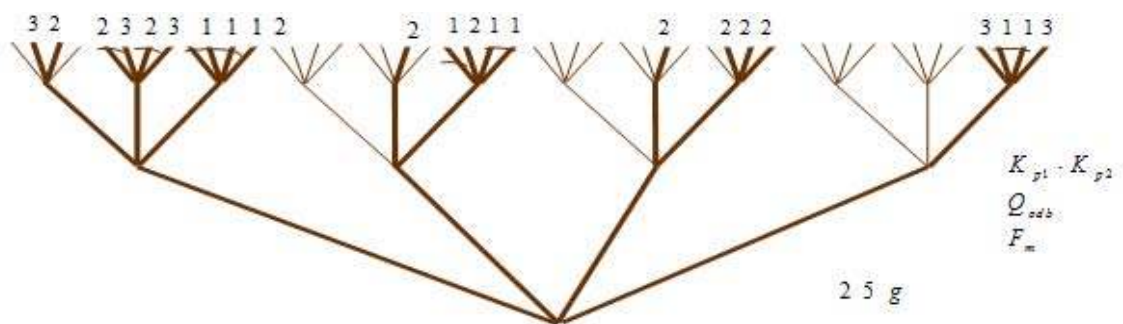


Fig.6. Optimal multivalued logic tree with weighting factors from Tab.2 for the parameters: Q_{rz} , F_m , $K_{p1} \cdot K_{p2}$.



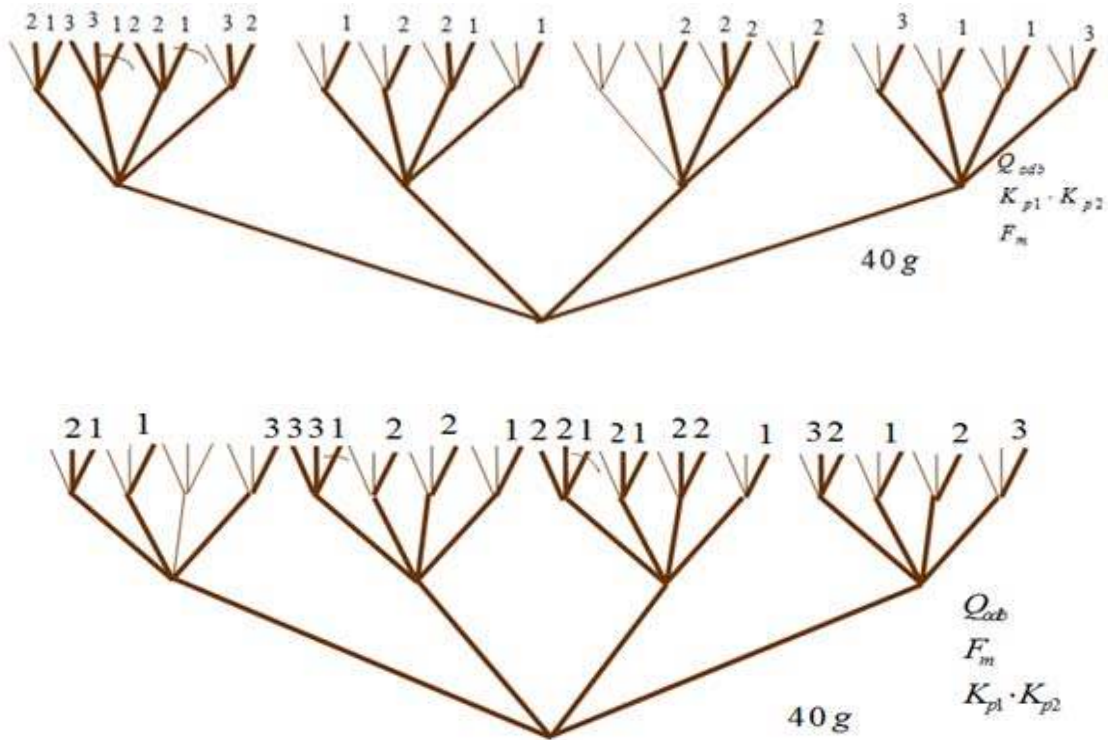


Fig.7. Multivalued logic trees with weighting factors from Tab.2.

For the condition of the limitation $t_w < 0.48t_o$ there is an optimal multi-valued logical tree presented in Fig.7 The most important parameter for the hydraulic proportional valve is the flow intensity Q_{odb} (depending on the enforcement of step changes in supply control voltage U_z).

Conclusions

In complex situations of the designing process, it is important to write the presented methods of designing graphs and game-tree structures in the form of algorithms and program them in an appropriate way in order to avoid the complexity of calculations of exponential type. Such a property is obtained among others in the issues of minimization of bivalent and multi-valued logical functions. In mathematics, a logical function grows faster than a given polynomial and that is why calculation complexity NP does not guarantee that we obtain a real-time calculation outcome of a given design-construction problem.

The minimization method of the complex, partial, multi-valued logical functions indicates the degree of importance of construction and exploitation parameters, playing the role of logical decision variables. A generalized Quine – McCluskey algorithm of minimization of multi-valued logical functions with multi-valued weighting coefficients has been presented in the paper. For an exemplary proportional valve, such actions can be undertaken separately by analysing each output parameter with an additional, weighting, logical coefficient for the stabilisation time, i.e., a shorter (better) stabilisation time has a more important (bigger) value of the weighting coefficient of an appropriate multi-valued weighting product.

Nomenclature

- C_k – the number of branches k -th level
- d – the valve diameter [m]
- F_m – electromagnetic force
- $f(x1, x2, x3)$ – partial multi-valued logical functions

- K – spring constant [N / m]
 $K_{p1} \cdot K_{p2}$ – the gain of the proportional valve
 k_{vx}, k_{vy} – degree of loss factors in the control of the hydraulic proportional valve
 MZAPN – minimum alternative complex normal form
 (m_1, \dots, m_n) – multi-valued logical function of n variables (m_1, \dots, m_n) - valued
 m – valve head mass [kg]
 n – the number of different letters in the Boolean function
 P – flow intensity [m^3/s]
 p – pressure
 Q_1 – flow rate of the pumped liquid [m^3/s]
 Q_2 – flow rate of the liquid coming out of the valve [m^3/s]
 Q_p – real pump performance
 Q_{D1}, Q_{D2}, Q_{D3} – flow rate through the nozzle D1, D2, D3 in the valve proportional
 Q_{odb} – flow proportional valve receiver
 t – time [s]
 U_z – forced displacements of the control voltage proportional valve
 V – the valve volume [m^3]
 $(\underline{V}), (V)$ – symbols signifying pseudo-clustering and clustering
 x – spring deflection [m]
 (x_1, x_2, x_3) – decision variables

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Received: May 18, 2014

Revised: June 25, 2014