



# Decision support for truckload carriers in one-shot combinatorial auctions

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## ABSTRACT

Combinatorial auctions have become popular for shippers to secure transportation services. It is, however, very difficult for truck carriers to solve bid generation and evaluation problems in combinatorial auctions. The objective of this paper is to develop a bidding advisor to help truckload (TL) carriers overcome such challenging problems in one-shot combinatorial auctions. The proposed advisor integrates the load information in e-marketplaces with carriers' current fleet management plans, and then chooses the desirable load bundles. In this paper, a TL carrier's bid generation and evaluation problems in one-shot combinatorial auctions are formulated as a *synergetic minimum cost flow problem* by estimating the average synergy values between loads through the proposed approximation. The conventional solution approaches for solving the minimum cost flow problems cannot be applied to the synergetic network flow problem. Thus, we propose a column generation approach to solve this specific network flow problem. The main contribution of this paper is that a TL carrier adopting the proposed advisor can easily determine the desirable bid packages without evaluating all  $2^n - 1$  possible bundles of loads, where  $n$  is the number of loads.

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## 1. Introduction

There are two major types of motor carriers: truckload (TL) carriers and less-than-truckload (LTL) carriers. Of particular interest in this paper are the TL carriers which operate over irregular routes and perform direct line-hauls from origins to destinations. As mentioned by Forkenbrock (1999), "The TL market is quite easy to enter because all that is needed is a driver, rolling stock, and a freight broker with whom to work." Because of very low fixed cost and sensitivity to the balance of the loads (empty repositioning is one of the major sources of cost for TL carriers), TL carriers tend to have slight diseconomies of scale and exhibit significant economies of scope (see e.g., Caplice, 1996, p. 27). Economies of scope means, in this paper, the total cost of a single carrier serving a set of loads is lower than that of multiple carriers serving the same set of loads. Since TL carriers show significant economies of scope, combinatorial auctions are effective auction mechanisms for shippers to procure TL transportation services.

Combinatorial auctions, where bidders are allowed to submit bids on combinations of products or services, can lead to more economical allocations of products or services when there exist complementarities over goods or services, and the source of the synergies varies for different bidders (see e.g., Pekeč and Rothkopf, 2003). Combinatorial auctions empowered by the exponential growth of online procurement have recently received much attention in transportation and logistics industries. For instance, Sears Logistics Services, Home Depot, Walmart Stores, Compaq Computer, and Limited Logistics Services are using combinatorial auctions to procure logistics services (Elmaghraby and Keskinocak, 2003). Sears Logistics

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Services in the early 1990s saved more than \$84 million by running six combinatorial auctions over a 3-year period (Ledyard et al., 2002). Limited Logistics Services saved \$1.24 million in 2001 compared to the previous year (Elmaghraby and Keskinocak, 2003). It is noted that the TL trucking companies in the above-mentioned practical applications bid for fixed-term contracts instead of spot-market loads. So far, we have not found spot-market combinatorial auctions in action. However, we strongly believe that they should be and will be adopted and prevail in TL procurement auctions in the future.

In spite of the above-mentioned attractive characteristics, it is known that determining the winners and constructing desired bids in combinatorial auctions pose a big burden on the auctioneers and bidders. For instance, if  $n$  products or services are posted, each bidder may submit bids for up to the theoretical  $2^n - 1$  different combinations of goods or services. That is, both the auctioneer's winner determination problem, and the bidder's bid construction and valuation problems are NP-hard (Song and Regan, 2005). To solve the winner determination problem, some researchers have tried to develop efficient heuristic algorithms (see e.g., Sandholm et al., 2005), while others have designed alternative auction mechanisms that restrict bidders to bid on the set of permitted combinatorial bids (see e.g., Rothkopf et al., 1998). These restricted approaches, although making the winner determination problem computationally manageable, might reduce the value of combinatorial auctions because the bidders cannot perfectly express their synergetic values. Compared to the winner determination problem, the bid generation and valuation problems deserve much attention, and are the focus of this paper. Since, in practice, it is not always worthwhile to prepare all possible packages in many settings (e.g., truckload transportation procurement), a bidder needs only to submit "necessary" packages. Therefore, the techniques to extract desirable ones from a potentially huge number of possible bid packages are required.

So far, there are only a few studies that focus on bid generation and evaluation problems. An et al. (2005) propose a model to assess bundle values given pairwise synergies and develop bundle creation algorithms for selecting profitable bundle bids based on the model. Their algorithms add as many profitable items as possible to a bundle given that the value of a bundle increases, on average, linearly in the bundle size. Song and Regan (2005) propose a two-phase strategy to solve a TL vehicle routing problem to generate bids. The first phase enumerates all routes satisfying routing constraints to generate candidate bid packages. The second phase associates a binary variable with each candidate bid package, and solves a set partitioning problem (bid construction problem) to determine desirable bids. Their bid construction problem assumes that all auctioned lanes must be served, and the objective is to minimize total operation empty movement cost. As pointed out by Lee et al. (2007), minimizing the total empty repositioning cost may not generate the right set of bid packages. They develop a TL vehicle routing model to maximize the profit in order to simultaneously, instead of sequentially such as that in Song and Regan (2005), solve the route (package) generation and selection problems. In addition, their model allows auctioned lanes to be uncovered. Wang and Xia (2005) define first-order synergy as the complementarity between a set of auctioned lanes and a set of booked lanes, and second-order synergy as the complementarity between a pair of sets of auctioned lanes and booked lanes. They then demonstrate that the synergy of a bid package may depend on other packages that will be won. In addition, they define the profit-based optimality criterion for a combinatorial bid, and based on some specific assumptions, change the criterion into the cost-based optimality criterion. Based on the cost-based optimality criterion, they take the winning probability into account and show that the optimal solution to a vehicle routing problem may lead to inferior bid packages. Even though they make so much effort to define different synergies and demonstrate the drawback of employing vehicle routing models to generate bids, they eventually model their bid generation problem as a generic vehicle routing problem with time windows. The routing problem assumes that all auctioned lanes must be served, and the objective is to minimize the total transportation cost. Their elaborate definitions of different synergies are, however, not implemented in their bid generation problem.

As noted earlier, only one of the above papers explicitly shows how to calculate the synergy of their suggested load bundles. Besides, the future demands with high probability of being materialized (based on historical data) are also neglected in the above literature, which may result in generating undesirable bids. Furthermore, for those who model their bid generation problems as vehicle routing problems, only one allows auctioned items to be uncovered. The assumption that all auctioned items must be covered may generate inferior bids due to neglecting the TL carrier's current fleet management plan. Note that although the aforementioned facts are neglected in the combinatorial TL auction literature, they have been considered in some single-item TL auction studies (a special case of the combinatorial TL auction; see e.g., Figliozzi et al., 2006, 2007). Moreover, all of the above studies consider only closed loop packages. Plummer (2003), after examining some real bids, however, points out that there are four common package patterns: (1) round trip/closed loop packages; (2) destination/inbound packages; (3) origin/outbound packages; and (4) disparate packages. Therefore, in truckload procurement, the bid generation problem is better formulated as time-space network based fleet management problems (see e.g., Powell et al., 1995) instead of vehicle routing problems. It is worth noting that dynamic vehicle routing problems under auction settings have gained growing attention in recent years. Figliozzi et al. (2006, 2007) model the auction problems considering optimal pricing, dynamic synergies, physical locations of loads and vehicles (current and future), and stochastic future demands. Figliozzi et al. (2007) study carrier pricing decisions for the Vehicle Routing Problem in a Competitive Environment (VRPCE) that is an extension of the Traveling Salesman Problem with Profits to a dynamic competitive auction environment. In the VRPCE, the carrier must estimate the incremental cost of servicing new service requests as they arrive dynamically. Figliozzi et al. (2006) quantify the opportunity costs in sequential transportation auctions focusing on a marketplace with time-sensitive truckload pickup-and-delivery requests.

The purpose of this paper is to design a bidding advisor to help TL carriers (bidders) make bidding decisions in one-shot combinatorial auctions for spot-market loads. The considered setting is as follows. Once learning some e-marketplace will

conduct a one-shot combinatorial auction, the TL carrier will apply the bidding advisor to generate desirable bids and their associated prices (or price ranges) by solving the embedded bid generation and evaluation problems based on the currently known and estimated information within the planning horizon (1 week in this paper). The known and estimated information includes the number of vehicles (assume that all vehicles are available for the first time at the beginning of the planning horizon), the current locations of all vehicles, and the booked, forecasted and auctioned loads. The known and estimated information corresponding to each (booked, forecasted, or auctioned) load consists of its pickup time and location, delivery time and location, and revenue. In this paper, the revenue of pulling a load is defined as follows: each booked load has a known revenue. The revenue of a forecasted load is calculated by its estimated value multiplied by its occurrence probability assuming that both estimated value and occurrence probability can be obtained from historical data. The revenue of an auctioned load is set to be its historically estimated value. Hence, the revenue of every type of load is treated as a deterministic and static number. We emphasize and will show later that it is important to take both booked and forecasted (never considered in the related literature) loads into account when constructing the bid generation and evaluation problems. The reasons are that when TL carriers are making bidding decisions, they might already have some loads contracted, and have some future demands with high probability of being materialized (based on the historical data). Thus, TL carriers' fleet management plans must incorporate both booked and forecasted loads. Consequently, new (auctioned) loads will be considered only if they can fit as seamlessly as possible into such fleet management plans. In short, our proposed bidding advisor tightly integrates the load information in e-marketplaces with TL carriers' current fleet management plans. Therefore, it can help TL carriers make effective bidding decisions.

From the above description, it is clear that the core components of the bidding advisor are the embedded bid generation and evaluation problems. The problems should consist of the following major distinct characteristics as described from other studies in the literature: (1) they do not pre-specify any aforementioned package patterns; (2) they take into account three types of loads: booked, auctioned, and forecasted loads; and (3) they explicitly consider physical locations of different types of loads, and thus different kinds of synergies between loads. In this paper, a TL carrier's bid generation and evaluation problems in one-shot combinatorial auctions are formulated as a synergetic minimum cost flow problem instead of a vehicle routing problem popular in the literature. The network flow problem takes into account all the above-mentioned crucial characteristics of the bid evaluation and generation problems. The conventional solution approaches for solving minimum cost flow problems, however, cannot be applied to the synergetic network flow problem. We therefore propose a column generation approach equipped with a synergetic shortest path algorithm to solve the specific network flow problem. The main contribution of this paper is that the TL carriers adopting the proposed advisor can easily and promptly determine their desired bid packages without evaluating all  $2^n - 1$  possible bid packages, where  $n$  is the number of loads.

The remainder of this paper is organized as follows: Section 2 describes how to transform the bid generation and evaluation problems in the one-shot combinatorial auction into a minimum cost network flow problem. Section 3 evaluates the profit contributed by the activity on each link in the network. Section 4 models the synergetic minimum cost network flow problem and proposes the solution algorithm to the problem. Section 5 first demonstrates the bidding advisor for bidding in one-shot combinatorial auctions and then presents hypothetical numerical examples to test the performance of the proposed advisor. Section 6 summarizes the major contributions and suggests extensions to this research.

## 2. Bid generation and evaluation problems

Naturally, TL carriers would only bid on the loads that are profitable under their current fleet management plans. It follows that the bid generation and evaluation problems can be considered as a fleet management optimization problem incorporating the load information in e-markets and the synergy effect among loads. The fleet management optimization problem can be further approached using time–space networks. Therefore, this section shows how to formulate a TL carrier's bid generation and evaluation problems in one-shot combinatorial auctions as a time–space-based network flow problem through a fleet management optimization problem.

Each day, a TL carrier must assign vehicles to booked loads; move them empty to other locations to pick up booked or expected loads; or hold them in the same locations until the next day. Obviously, a TL carrier's operation plan is an infinite horizon problem that is difficult to tackle. The most common method to approximate an infinite horizon problem is to solve a finite horizon problem. This, however, brings up the problem of selecting an approximate planning horizon (Powell et al., 1995). Several methods have been developed to cope with this problem. One popular approach proposed by Grinold and Hopkins (1973) is to use salvage values. Based on this approach, TL carriers' fleet management problems are solved using finite horizon dynamic (time–space) networks (Powell, 1991). Here, we first summarize and briefly describe the procedures as proposed in Powell (1991) for constructing a time–space network (called standard time–space network hereafter) to model a TL carrier's fleet management problem. We then show how to build a new type of time–space network in order to solve our proposed bid generation and evaluation problems based on the standard network.

A TL carrier's operation area is divided into a set of regions, and the planning horizon  $H$  is divided into discrete intervals. Note that the planning horizon must allow the TL carrier to take into account forecasted events for a reasonable period of time into the future. Each time interval is generally set to 1 day. However, it is noted that, as mentioned in Frantzeskakis and Powell (1990), such a setting can be easily relaxed, and is made only to simplify the presentation of the problem. Then, to form a time–space network, let a node represent a particular region at a point in time, and a link connect one node to

another accessible node. Different types of links, characterized by costs and flow upper bounds, represent different activities. Since a load is characterized by its pickup location and time and its delivery location and time, it can be naturally represented as a link in the time–space network. In addition, the locations of a vehicle at different points in time can be depicted by the nodes in the time–space network. All vehicles depart from the network over the salvage links and out of the supersink. The supersink acts as a convergent point since we do not know *a priori* where the vehicles will end up. The salvage links perform the function of moving all the flows out of the supersink, and provide an opportunity to place costs and bounds on the flows that end in each region.

Based on the above-described standard time–space network, we construct a new type of time–space network to solve the bid generation and evaluation problems. Let  $R$  be the set of regions with cardinality  $|R|$ , and  $s_i$  and  $\hat{s}_i$  represent, respectively, the supply of vehicles initially at region  $i$  and the forecasted supply of vehicles at region  $i$  at the end of planning horizon  $H$ . Assume that all vehicles are available at the beginning of the first time period (we refer to the beginning of the time period when a time period is mentioned); thus,  $\sum_{i=1}^{|R|} s_i = \sum_{i=1}^{|R|} \hat{s}_i$ . Obviously, a link cannot represent more than one activity. For instance, once a link represents a load serviced by a vehicle, it cannot simultaneously represent empty movements used by other vehicles. Thus, it is necessary to create multiple links for different purposes; from here, our proposed time–space network significantly deviates from the standard one. Here, each link is characterized by triplet parameters (flow lower bound, flow upper bound, and unit net cost (negative profit)) and represents one of the seven different activities: booked load, forecasted load, auctioned load, empty movement, holding, salvage, and dummy.

To handle the problem of multiple activities on the same link, we let the loads across the link with the same type and the same profit be represented by the same link. For example, if there are three types of loads (auctioned, booked, and forecasted loads) across link  $i, j$ , and the loads belonging to the same type have the same profit, then we duplicate three nodes, say  $j^1, j^2$  and  $j^3$ , and six links  $(i, j^1), (j^1, j), (i, j^2), (j^2, j), (i, j^3),$  and  $(j^3, j)$ . Let link  $(i, j^1)$  represent the activity of pulling the auctioned loads, and set the triplet parameters to  $(0, a_{ij}, -u_{ij}^a)$ , where  $a_{ij}$  represents the number of auctioned loads, and  $u_{ij}^a$  represents the profit of pulling an auctioned load across link  $(i, j)$ . The lower bound is set to 0 and upper bound is set to  $a_{ij}$  because some auctioned loads may be unprofitable under current fleet deployment. Likewise, let link  $(i, j^2)$  represent the activity of pulling the booked loads, and set the triplet parameters to  $(b_{ij}, b_{ij}, -u_{ij}^b)$  where  $b_{ij}$  denotes the number of booked loads, and  $u_{ij}^b$  represents the profit of pulling a booked load across link  $(i, j)$ . Since all booked loads must be served, both the lower and upper bounds are set to  $b_{ij}$ . Finally, let link  $(i, j^3)$  represent the activity of pulling the forecasted loads, and set the triplet parameters to  $(f_{ij}, f_{ij}, -u_{ij}^f)$  where  $f_{ij}$  denotes the number of forecasted loads, and  $u_{ij}^f$  represents the profit of pulling a forecasted load across link  $(i, j)$ . All forecasted loads (implying future booked loads) must be served; thus, both the lower and upper bounds are set to  $f_{ij}$ . The function of links  $(j^1, j), (j^2, j),$  and  $(j^3, j)$  is simply to connect nodes  $j^1, j^2,$  and  $j^3$  to node  $j$ . Thus, we call those links dummy links and set the triplet parameters  $(0, \infty, 0)$  to all dummy links. Clearly, if the load of a certain type has a different profit from other loads sharing the same link and the same type, we can simply perform the above duplication procedures to handle it. The original link  $(i, j)$  is used to represent repositioning movements, and the triplet parameters are set to  $(0, \infty, -u_{ij}^e)$ , where  $u_{ij}^e$  denotes the profit of repositioning an empty vehicle across link  $(i, j)$ . Each holding link  $(i, j)$  is associated with triplet  $(0, \infty, -u_{ij}^h)$  where  $u_{ij}^h$  represents the negative overnight cost. Finally, let  $v$  represent the supersink, and  $u_t^s$  denote the salvage value of a vehicle at region  $t$  at the end of planning horizon  $H$ . Then, each salvage link  $(t, v)$  is associated with triplet  $(\hat{s}_t, \hat{s}_t, -u_t^s)$  where both the lower and upper bounds are set to be equal to  $\hat{s}_t$ , the forecasted supply of vehicles in region  $t$  at the end of planning horizon. The salvage link records the expected profit incurred by a vehicle once in region  $t$  at the end of planning horizon  $H$ ; such a setting implies a constant marginal profit of a vehicle, and also no gain for excess vehicles. Yet to be defined is the profit of performing the activity on each type of link; that is, the values of  $u_{ij}^a, u_{ij}^b, u_{ij}^f, u_{ij}^e, u_{ij}^h,$  and  $u_t^s$ . We will elaborate how to evaluate the profits corresponding to different types of links in the next section. An example of the above-defined time–space network is shown in Fig. 1.

From the network shown in Fig. 1, it is easy to see that the bid generation and evaluation problems can be embedded into the fleet management problem solved using the constructed time–space network. That is, the TL carriers will only bid for the auctioned loads that optimize their fleet management plans. The process also demonstrates the possibility of using the constructed network to model the fleet management problem as a network flow problem. To do so, however, we must first develop the techniques to calculate the synergy values between loads over the network. Hence, in the next section, we will introduce an approach of applying the “average synergy” concept to determine the synergy values. Then, we show how to model the fleet management problem as a network flow problem by way of the “average synergy” concept and describe the corresponding solution algorithm in Section 4.

### 3. Evaluation of the profit contributed by the activity on each link

As we know, the connection to follow-on loads is one of the major sources of cost for TL carriers, and there may exist synergy between consecutive loads. Therefore, to evaluate the profit (net cost) contributed by the activity on a link, we must take the potential synergy effect between the current activity and its follow-on activity into account; the synergy effect is ignored in most, if not all, fleet management literature. It is, however, not trivial to estimate the synergy value of a load bundle with arbitrary combination of auctioned loads. Theoretically, the value should be a function of all “environmental variables” (other loads (inside and outside the bundle), all vehicles, and so on). Obviously, it is very difficult, if not impossible, for a TL carrier to know the functional form. Therefore, we solve the problem from another angle. The underlying ideas are that

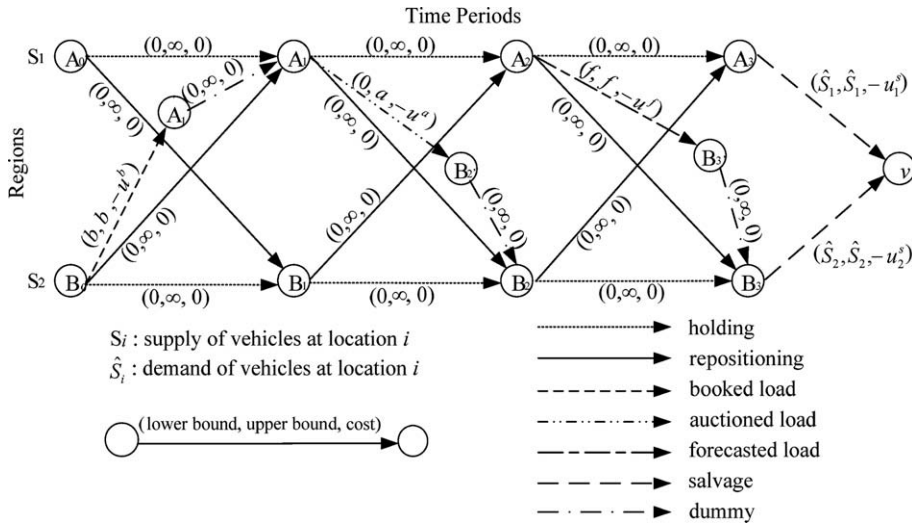


Fig. 1. An example network.

the TL carrier’s optimal fleet management plan will only choose the profitable auctioned loads, and will lead to the desirable load bundles by grouping the auctioned loads assigned to the same vehicle; that is, the selected load bundles are mutually exclusively embedded in different vehicle tours. Thus, it is clear that if we can find a way to compute the synergy values between loads during the course of solving the TL carrier’s fleet management problem, we can avoid evaluating the entire  $2^n - 1$  potential bundles with arbitrary load combinations to generate desirable load bundles.

An et al. (2005) propose a generic synergy model to generate bundle values. Their model actually calculates the average pairwise synergy and assumes that the synergy of a bundle increases linearly in the bundle size. The model is as follows:

$$V_B = |B| \times AC_B$$

where  $V_B$  represents the value of bundle  $B$ ,  $|B|$  is the size of  $B$ , and  $AC_B$  is the average unit contribution of  $B$  (i.e., the average item value in  $B$  plus the average pairwise synergy value in  $B$ ). They admit that they do not test and validate the model, since real data from combinatorial auctions are generally not publicly available. The average pairwise synergy concept and the linear assumption may oversimplify the estimation of synergy values, but they make it possible to model the fleet management problem (thus, also the bid generation and evaluation problems) as a minimum cost flow problem. The solution algorithm to the network flow problem can easily identify the desirable bid packages without evaluating the entire  $2^n - 1$  potential bundles with arbitrary load combinations. This argument will be clearly shown in the next section. By applying the idea of average pairwise synergy, the pairwise synergy between two graphically consecutive loads is defined as  $S$ , the one between an empty move and a follow-on load is defined by  $\beta S$  ( $0 < \beta < 1$ ), the one between a load and a follow-on empty move is set to zero, and the one between two consecutive empty moves is also set to zero. The rationale behind the definitions of the pairwise synergy under different scenarios is elaborated as follows. Due to the economies of scope, there exists synergy represented by  $S$  between two consecutive loads. In addition, a follow-on load can avoid a deadhead trip and thus create some degree of synergy for a vehicle currently moving empty. The value of the synergy is obviously less than  $S$ ; we use parameter  $\beta$  to restrict the magnitude of the value that is specified by carriers. The value of  $\beta S$  is called *partial* pairwise synergy hereafter. Moreover, a follow-on empty move creates no synergy whether the vehicle is currently loaded or empty moved. In the rest of this section, we describe the formulas to estimate transportation costs and discuss the estimates of transportation revenues. Finally, based on the estimated costs and revenues, we demonstrate the transportation profits (revenue minus cost) with respect to different activities.

To construct the cost models, let  $c_{ij}^a$ ,  $c_{ij}^b$  and  $c_{ij}^f$  denote the costs of pulling an auctioned, a booked, and a forecasted load, respectively, across link  $(i, j)$ ; let  $c_{ij}^e$  represent the cost of empty repositioning a vehicle across link  $(i, j)$ ; and let  $c_{ij}^h$  denote the overnight holding cost. In addition, to record the synergy effect between the activities on two consecutive links performed by the same vehicle, we let link  $(j, k)$  indicate the next link traversed after link  $(i, j)$ , and use an indicator variable  $\alpha_j$  to track the activity on link  $(j, k)$ . That is,  $\alpha_j$  (the subscript  $j$  denotes the head node of link  $(i, j)$  and also the tail node of link  $(j, k)$ ) takes the value of 1 if link  $(j, k)$  is associated with a loaded movement, and 0 if link  $(j, k)$  is associated with an empty movement or if node  $j$  is the eventual destination of the vehicle at the end of planning horizon  $H$ . This variable plays an important role in the synergetic shortest path algorithm detailed in Section 4. The formulas for estimating the costs of different activities on link  $(i, j)$ , considering the synergy effect, are shown as follows:

$$c_{ij}^e = d_{ij}C_D - \alpha_j\beta S \tag{1}$$

$$c_{ij}^a = c_{ij}^b = c_{ij}^f = d_{ij}C_D + 2TC_T - \alpha_j S \tag{2}$$

Where  $d_{ij}$  is the travel time from node  $i$  to node  $j$ ,  $C_D$  the operation cost per unit time,  $C_T$  the loading/unloading cost per unit time,  $T$  the time needed to load/unload a load,  $\alpha_j \begin{cases} 1 & \text{if link } (j, k) \text{ is associated with a loaded movement} \\ 0 & \text{otherwise;} \end{cases}$  or node  $j$  is the destination of the vehicle,  $S$  the pairwise synergy, and  $\beta$  is a TL carrier specified constant ( $0 < \beta < 1$ ).

The estimation of transportation costs is definitely not trivial (see, e.g., [Higginson, 1993](#)). Since accurately estimating the costs is not the major concern of this paper, the goal of formulas (1) and (2) is to capture the main structures of the costs and to show the different cost structures between pulling a load and empty repositioning a vehicle. Accordingly,  $c_{ij}^a$ ,  $c_{ij}^b$ ,  $c_{ij}^c$ , and  $c_{ij}^e$  are considered as *generalized costs*. For expression convenience, we assume that loading time is equal to unloading time, and that both are independent of load size. Here,  $c_{ij}^e$  in Eq. (1) is defined as the hauling cost (without loading/unloading cost) minus the potential partial pairwise synergy of connecting to a follow-on load. The term “potential” reflects the unknown status of the follow-on demand that is controlled by the indicator variable  $\alpha_j$ . In addition,  $\alpha_j$  is paired with partial synergy  $\beta S$  in Eq. (1) since link  $(i, j)$  denotes an empty movement. On the other hand,  $c_{ij}^a (= c_{ij}^b = c_{ij}^c)$  is defined as the hauling cost plus the loading and unloading costs minus the potential pairwise synergy of connecting to a follow-on load. Likewise,  $\alpha_j$  is paired with  $S$  in Eq. (2) since link  $(i, j)$  denotes a loaded movement. As for  $c_{ij}^b$ , the cost of holding a vehicle overnight in the same location, we treat the drivers' salaries of the TL carrier as a sunk cost and set the holding cost to zero, which is also the treatment in most of the TL carrier operation literature (see, e.g., [Frantzeskakis and Powell, 1990](#)). By combining the parameter  $\beta$  and the indicator variable  $\alpha_j$  with the pairwise synergy  $S$ , the two cost formulas (1) and (2) can explicitly demonstrate the effect of economies of scope. Eqs. (1) and (2) also reflect that the values of  $c_{ij}^a$ ,  $c_{ij}^b$ ,  $c_{ij}^c$ , and  $c_{ij}^e$  cannot be individually identified, but depend on the activity on its follow-on link, say link  $(j, k)$ ; a transportation network with this characteristic is called a *synergetic network* in this paper. Thus, we call the minimum cost flow problem constructed in the preceding section the *synergetic minimum cost flow problem*.

Eqs. (1) and (2), dealing with the single-link case, can be easily extended to handle multiple consecutive-link scenarios. Consider that a vehicle sequentially services two demand links associated with loaded or empty movements. Since  $c_{ij}^a = c_{ij}^b = c_{ij}^c$ , for convenience, let  $c_{ij}^l$  represent them. If two loads, one from node 1 to node 2 and the other from node 2 to node 3, are transported sequentially by the same vehicle, then the cost of servicing these two loads is equal to  $(c_{12}^l + c_{23}^l) = (d_{12} + d_{23})C_D + 4TC_T - (1 + \alpha_3)S$ . If the vehicle first services a load across link (1, 2) and moves empty through link (2, 3), then the cost of servicing these two demand links is equal to  $(c_{12}^l + c_{23}^e) = (d_{12} + d_{23})C_D + 2TC_T - \alpha_3\beta S$ . On the other hand, if the vehicle moves empty through link (1, 2), and then services a follow-on load across link (2, 3), then the cost is equal to  $(c_{12}^e + c_{23}^l) = (d_{12} + d_{23})C_D + 2TC_T - (\beta + \alpha_3)S$ . Finally, if the vehicle moves empty through two consecutive links (1, 2) and (2, 3), then the cost is equal to  $(c_{12}^e + c_{23}^e) = (d_{12} + d_{23})C_D - \alpha_3\beta S$ . The calculation of the costs can be easily extended to the cases where a vehicle continuously serves more than two demand links.

To accurately calculate the revenues of pulling a booked, a forecasted, and an auctioned load is a challenging and complicated task, particularly in a competitive and dynamic environment ([Figliozzi et al., 2007](#)). Since it is not the focus of this paper, we briefly describe the characteristics of the revenues associated with different loads. The revenues of pulling an auctioned, a booked, and a forecasted load from node  $i$  to node  $j$  are denoted as  $\gamma_{ij}^a$ ,  $\gamma_{ij}^b$  and  $\gamma_{ij}^f$ , respectively, and are treated as deterministic and static numbers; the revenues of an empty move and holding a vehicle are zero. The value of  $\gamma_{ij}^b$  is obviously a known number; the value of  $\gamma_{ij}^f$  represents an expected revenue (estimated revenue multiplied by occurrence probability); and the value of  $\gamma_{ij}^a$  represents a historically estimated value. We can obtain a reasonable estimate on the expected revenue of a forecasted load based on historical data. However, to estimate the revenue of an auctioned load is a major research issue on its own. For instance, a complete estimation of the value of an auctioned load should take both the economies of scope and the opportunity cost into consideration. This paper, however, does not explicitly consider the opportunity cost. We refer the reader to [Figliozzi \(2004\)](#), [Yan et al. \(1995\)](#), and [Powell et al. \(1988\)](#) in which the concept of the opportunity cost is applied to estimate load values. Therefore, the bid packages and their associated bid prices obtained by applying our proposed bidding advisor may not provide “optimal solutions” in complex practical situations.

Based on the above estimations of costs and revenues, we can calculate the profits (revenue minus cost) with respect to different activities; that is, the values of  $u_{ij}^a$ ,  $u_{ij}^b$ ,  $u_{ij}^f$ ,  $u_{ij}^e$ , and  $u_{ij}^h$ . The profits of pulling an auctioned, a booked, and a forecasted load on *loaded link*  $(i, j)$  are equal to  $u_{ij}^a = (\gamma_{ij}^a - c_{ij}^a)$ ,  $u_{ij}^b = (\gamma_{ij}^b - c_{ij}^b)$  and  $u_{ij}^f = (\gamma_{ij}^f - c_{ij}^f)$ , respectively. The profit of repositioning a vehicle across *repositioning link*  $(i, j)$  is equal to  $u_{ij}^e = (-c_{ij}^e)$ . In addition, the negative holding cost on *holding link*  $(i, j)$ ,  $u_{ij}^h$ , is set to zero. How to estimate the value of  $u_t^e$ , the salvage value of a vehicle at region  $t$  at the end of planning horizon  $H$ , remains to be explained. There have been a few good approaches for estimating  $u_t^e$  (see e.g., [Frantzeskakis and Powell, 1988](#)). It is known that the major difficulty of managing fleets dynamically is that the demands are uncertain and the level of uncertainty increases further into the future, which makes the estimation of  $\hat{s}_t$ , and thus  $u_t^e$ , very hard. Eventually, we apply an approach analogous in spirit to the one proposed in [Frantzeskakis and Powell \(1988\)](#) to calculate the value of  $u_t^e$ , where  $u_t^e$  is set to be a function of  $\hat{s}_t$ , the vehicle supply of region  $t$  at the end of planning horizon  $H$  (i.e.,  $u_t^e = f(\hat{s}_t)$ ). The unit profit with respect to each type of link is summarized and shown in [Table 1](#).

#### 4. The synergetic minimum cost flow problem and its solution algorithm

In this section, we first model the synergetic minimum cost network flow problem (bid generation and evaluation problems), and then propose a solution algorithm to the problem.

**Table 1**  
Unit link profit

Type of links	Unit profit
Holding link $(i, j)$	$u_{ij}^h = 0$
Repositioning link $(i, j)$	$u_{ij}^e = (-c_{ij}^e)$
Loaded (booked) link $(i, j)$	$u_{ij}^b = (\gamma_{ij}^b - c_{ij}^b)$
Loaded (forecasted) link $(i, j)$	$u_{ij}^f = (\gamma_{ij}^f - c_{ij}^f)$
Loaded (auctioned) link $(i, j)$	$u_{ij}^a = (\gamma_{ij}^a - c_{ij}^a)$
Salvage link $(t, v)$	$u_t^s = f(\hat{s}_t)$

4.1. The path-based formulation of the network flow problem

Let  $G = (V, E)$  be a directed synergetic network defined by a set  $V$  of nodes and a set  $E$  of directed links. Let  $v \in V$  denote the supersink. Furthermore, let  $A, B, C \subset E$  be the set of auctioned links, booked and forecasted links, and salvage links, respectively. Note that since all booked and forecasted loads have the same structure of triplet parameters (flow lower bound, flow upper bound, and unit net cost (negative profit)), they are considered as a whole in the synergetic network. Each link  $(i, j) \in E$  has an associated capacity denoting the maximum amount that can flow on the link, and a lower bound denoting the minimum amount that must flow on the link. Each node  $i \in V$  is associated with an integer number  $b(i)$  representing its supply/demand of vehicles. If  $b(i) > 0$ , node  $i$  is a supply node with a supply of  $b(i)$ ; if  $b(i) < 0$ , node  $i$  is a demand node with a demand of  $-b(i)$ ; if  $b(i) = 0$ , node  $i$  is a transshipment node. Let  $x_{ij}$  represent the flow on link  $(i, j) \in E$ , and  $-u_{ij}$  denote the net cost (negative profit) per unit flow on link  $(i, j)$ ; that is, depending on the activity corresponding to link  $(i, j)$ ,  $u_{ij}$  may take the value of  $u_{ij}^a, u_{ij}^b, u_{ij}^f, u_{ij}^e, u_{ij}^h$  or  $u_t^s$  (if  $j = v$ ; that is, if  $j$  is the supersink). The proposed synergetic minimum cost flow problem is formulated as follows:

$$\text{Minimize } \sum_{(i,j) \in E} (-u_{ij})x_{ij} \tag{3}$$

$$\text{Subject to } \sum_{\{j:(i,j) \in E\}} x_{ij} - \sum_{\{j:(j,i) \in E\}} x_{ji} = b(i) \quad \forall i \in V \tag{4}$$

$$0 \leq x_{ij} \leq a_{ij} \quad \forall (i, j) \in A \tag{5}$$

$$x_{ij} = b_{ij} \quad \forall (i, j) \in B \tag{6}$$

$$x_{iv} = \hat{s}_i \quad \forall (i, v) \in C \tag{7}$$

$$0 \leq x_{ij} \leq U \quad \forall (i, j) \in E \setminus (A \cup B \cup C) \tag{8}$$

The objective is to minimize net total cost (negative total profit). Constraints (4) are mass balance constraints. Constraints (5)–(7) are flow bound constraints. Constraints (4) ensure that only the profitable auctioned loads will be picked. Constraints (6) ensure that all booked and forecasted loads are serviced. Note that for conciseness,  $b_{ij}$  is used to represent both  $f_{ij}$  and  $b_{ij}$ . Constraints (7) ensure that the expected vehicle supply at the end of planning horizon is met. Note that constraints (8) are actually redundant because  $U$  represents a “very large” positive number; they are included here for completeness.

The consideration of potential pairwise synergy between two consecutive vehicle operations in network flow problem (3)–(8) results in that the problem violates both Bellman’s Principle of Optimality (Bellman, 1958) and direction reversibility of link cost that is the basis to build a residual network (Ahuja et al., 1993). Most, if not all, solution techniques to the link flow-based minimum cost flow problems build on these two standard assumptions, and thus they cannot be applied to problem (3)–(8). Therefore, we reformulate the minimum cost flow problem as a path-based network flow problem based on the flow decomposition theorem (see e.g., Ahuja et al., 1993). That is, each link flow is represented as path and cycle flows; however, since the synergetic network is acyclic due to the specific properties of time–space networks, each link flow can be simply represented as path flows.

To construct the path-based minimum cost flow problem, let  $\delta_{ij}(P)$  equal 1 if link  $(i, j)$  is contained in path  $P$ , and be 0 otherwise; let  $O$  denote the set of supply nodes; let  $D$  denote the set of demand nodes; and let  $P_{mn}$  denote the set of paths connecting node  $m \in O$  to node  $n \in D$ . Obviously, the flow  $x_{ij}$  on link  $(i, j)$  equals the sum of flows  $f(P)$  for all paths  $P$  that contain this link. That is,

$$x_{ij} = \sum_{m \in O} \sum_{n \in D} \sum_{P \in P_{mn}} \delta_{ij}(P)f(P)$$

Let  $c(P) = \sum_{(i,j) \in E} (-u_{ij})\delta_{ij}(P) = \sum_{(i,j) \in P} (-u_{ij})$  represent the per unit cost of flow on path  $P$ , let  $s_m$  be the supply of vehicles at node  $m$ , and let  $d_n$  be the demand of vehicles at node  $n$ . Then, the path flow formulation of the proposed synergetic minimum cost flow problem can be represented as follows:

$$\text{Minimize } \sum_{m \in O} \sum_{n \in D} \sum_{P \in P_{mn}} c(P)f(P) \tag{9}$$

$$\text{Subject to } \sum_{n \in D} \sum_{P \in P_{mn}} f(P) = s_m \quad \forall m \in O \tag{10}$$

$$\sum_{m \in O} \sum_{P \in P_{mn}} f(P) = d_n \quad \forall n \in D \tag{11}$$

$$\sum_{m \in O} \sum_{n \in D} \sum_{P \in P_{mn}} \delta_{ij}(P) f(P) \leq a_{ij} \quad \forall (i, j) \in A \tag{12}$$

$$\sum_{m \in O} \sum_{n \in D} \sum_{P \in P_{mn}} \delta_{ij}(P) f(P) = b_{ij} \quad \forall (i, j) \in B \tag{13}$$

$$\sum_{m \in O} \sum_{n \in D} \sum_{P \in P_{mn}} \delta_{iv}(P) f(P) = \hat{s}_i \quad \forall (i, v) \in C \tag{14}$$

$$f(P) \geq 0 \quad \forall m \in O; \quad n \in D; \quad P \in P_{mn} \tag{15}$$

Constraints (10) and (11) are mass balance constraints. Constraints (12)–(14) are flow bound constraints. Constraints (12) indicate that only those profitable auctioned loads are selected. Constraints (13) ensure that all booked and forecasted loads are serviced. Constraints (14) ensure that the expected vehicle supply at the end of the planning horizon is met.

Problem (9)–(15) has a single constraint for each link  $(i, j) \in A \cup B \cup C$  stating that the sum of the path flows passing through the link must satisfy the lower bound and capacity constraint. Moreover, the problem has a single constraint for each origin  $m$  and destination  $n$  indicating that the total flow on all the paths originating from the source node  $m$  must equal the supply  $s_m$ , and that the total flow on all the paths entering the sink node  $n$  must equal the demand  $d_n$ . Therefore, problem (9)–(15) contains  $|O| + |D| + |A| + |B| + |C|$  constraints (in addition to the nonnegativity constraints imposed on the path flow values), where  $|\cdot|$  represent the cardinality of set  $(\cdot)$ .

#### 4.2. Solution techniques

Most solution techniques in the literature for minimum cost network flow problems rely on the concept of residual networks (see, e.g., Ahuja et al., 1993). These algorithms share a commonality of repeatedly solving shortest path problems in residual networks. However, the building of a residual network is based on the assumption of direction reversibility of link cost that is violated in the synergetic network. Therefore, these residual-network-based algorithms cannot be applied to the proposed synergetic minimum cost network flow problem. The network simplex algorithm, another main type of solution technique, does not rely on the concept of residual networks but on the concept of spanning tree solutions. However, the network simplex algorithm still cannot be applied to the proposed problem because the link-based procedure of updating the tree also cannot cope with the synergetic property.

Problem (9)–(15) has a variable for each path connecting a source node to a destination node; there may exist a huge number of paths between an origin–destination pair. The problem consists of  $|O| \times |D|$  origin–destination pairs, where  $|O|$  and  $|D|$  represent the number of supply nodes and demand nodes, respectively. Therefore, the problem typically consists of a large number of variables, and the number of variables grows exponentially with the size of the network. For such a large-scale linear program with a large number of variables, an efficient solution procedure called *column generation* can be applied to solve the problem (see, e.g., Barnhart et al., 1998). The key idea in column generation method is to generate columns (paths) only as needed, and thus only a small fraction of all feasible paths are generated instead of explicitly enumerating all feasible paths. Before going into the procedures of column generation, we will first introduce the preparatory work needed to apply the technique.

Let  $\alpha^m (m \in O)$ ,  $\beta^n (n \in D)$ ,  $\theta_{ij} ((i, j) \in A)$ ,  $\omega_{ij} ((i, j) \in B)$  and  $\rho_{iv} ((i, v) \in C)$  be the dual variables associated with constraints (10)–(14), respectively. The dual of problem (9)–(15) is then as follows:

$$\text{Maximize} \quad \sum_{m \in O} \alpha^m s_m + \sum_{n \in D} \beta^n d_n - \sum_{(i,j) \in A} \theta_{ij} a_{ij} + \sum_{(i,j) \in B} \omega_{ij} b_{ij} + \sum_{(i,v) \in C} \rho_{iv} \hat{s}_i \tag{16}$$

$$\text{Subject to} \quad \alpha^m + \beta^n + \sum_{(i,v) \in C \cap P} \rho_{iv} + \sum_{(i,j) \in B \cap P} \omega_{ij} - \sum_{(i,j) \in A \cap P} \theta_{ij} \leq c(P) \quad \forall m \in O; \quad n \in D; \quad P \in P_{mn} \tag{17}$$

$$\alpha^m \text{ free} \quad \forall m \in O \tag{18}$$

$$\beta^n \text{ free} \quad \forall n \in D \tag{19}$$

$$\theta_{ij} \geq 0 \quad \forall (i, j) \in A \tag{20}$$

$$\omega_{ij} \text{ free} \quad \forall (i, j) \in B \tag{21}$$

$$\rho_{iv} \text{ free} \quad \forall (i, v) \in C \tag{22}$$

Assume that path flow vector  $F(P)$  is a feasible solution to problem (9)–(15), and let  $(\alpha, \beta, \theta, \omega, \rho)$  be the associated dual solution to problem (16)–(22). Based on linear programming duality, the primal solution  $F(P)$  is optimal if and only if the reduced cost  $\bar{c}_p$  is nonnegative; that is,

$$\bar{c}_p = c(P) + \sum_{(i,j) \in P \cap A} \theta_{ij} - \sum_{(i,j) \in P \cap B} \omega_{ij} - \sum_{(i,v) \in P \cap C} \rho_{iv} - \alpha^m - \beta^n \geq 0 \quad \forall m \in O; \quad n \in D; \quad P \in P_{mn}$$

Thus, to check the optimality of  $F(P)$ , we can solve the following pricing problem:



$$z = \min_{m \in O, n \in D, P \in P_{mn}} \left( c(P) + \sum_{(i,j) \in P \cap A} \theta_{ij} - \sum_{(i,j) \in P \cap B} \omega_{ij} - \sum_{(i,v) \in P \cap C} \rho_{iv} - \alpha^m - \beta^n \right) \quad (23)$$

If  $z \geq 0$ , then  $F(P)$  is an optimal solution; otherwise, if any reduced cost is negative, the approach will introduce the variable (column) associated with most negative reduced cost into the basis in place of one of the current basic variables. The pricing problem can be actually solved by a synergetic shortest path algorithm on the *modified network* defined later. The explanation is as follows. The value of  $[c(P) + \sum_{(i,j) \in P \cap A} \theta_{ij} - \sum_{(i,j) \in P \cap B} \omega_{ij} - \sum_{(i,v) \in P \cap C} \rho_{iv}]$  denoted as  $\hat{c}(P)$  in the right-hand side of Eq. (23) can be considered as the per unit cost of flow on the path  $P$  with respect to the modified network. In the modified network, we impose extra cost  $\theta_{ij}$  on each link  $(i, j) \in A$ ,  $-\omega_{ij}$  on each link  $(i, j) \in B$ , and  $-\rho_{iv}$  on each link  $(i, v) \in C$ . It is noted that the modified network is still a synergetic network. Consequently,

$$\hat{c}(P) = \min_{m \in O, n \in D, P \in P_{mn}} \left( c(P) + \sum_{(i,j) \in P \cap A} \theta_{ij} - \sum_{(i,j) \in P \cap B} \omega_{ij} - \sum_{(i,v) \in P \cap C} \rho_{iv} \right)$$

is the length of the shortest path connecting the source node  $m$  to the sink node  $n$ , with respect to the modified costs. Therefore, we can use a synergetic shortest path algorithm detailed below to easily solve the pricing problem (23) and locate paths that price out.

Bellman's Principle of Optimality (Bellman, 1958) may not hold for path-finding problems in the synergetic network. The standard shortest path algorithms (see, e.g., Ahuja et al., 1993) build on the principle, and thus cannot be applied to locate shortest paths in such a network. Hence, we propose a synergetic shortest path algorithm. The input to the algorithm is a synergetic network  $G$ , a specified origin node  $s$ , and a specified destination node  $r$ . Let  $w_{ij}$  be an indicator variable such that  $w_{ij} = 1$  if there exists a load across link  $(i, j)$ ; otherwise,  $w_{ij} = 0$ . The algorithm can be summarized as follows:

- (1) Create an empty priority queue (heap).
- (2) Insert the trivial path to node  $s$  into the priority queue with  $key = 0$ . Store this path in the table under node  $s$ .
- (3) Remove the highest priority path from the priority queue. Assume node  $i$  is the destination node of the path, and let  $pred(i)$  represent the predecessor of node  $i$ . Denote this path as  $q$ .
- (4) Extend path  $q$  from  $i$  to all other nodes,  $j$ , that are adjacent to it. This creates a set of trial paths whose elements we will denote by  $q_j$  because they are extensions of path  $q$  and they terminate at node  $j$ .
- (5) For each trial path  $q_j$  associated with the last link  $(i, j)$ , compare it to all other paths in the table that terminate at node  $j$  denoted as path  $q'_j$  associated with the last link  $(pred(j), j)$ .
  - (a) If  $w_{ij} \leq w_{pred(j)j}$  and path  $q_j$  is dominated (using the values of *keys*), discard path  $q_j$ .
  - (b) If  $w_{ij} \geq w_{pred(j)j}$  and path  $q_j$  dominates path  $q'_j$ , delete path  $q'_j$  from the priority queue and from the table. Also delete from the priority queue and the table any path that is an extension of  $q'_j$  to some other nodes.
  - (c) If path  $q_j$  dominates another path, or if no comparisons are possible, add path  $q_j$  to the priority queue, with a *key* equal to  $\hat{c}(q_j)$ .
- (6) If the priority queue is not empty, return to step 2; otherwise, terminate. The paths in the table under node  $r$  are the shortest paths.

**Proposition.** *The synergetic shortest path algorithm generates all optimal paths between a specified origin node  $s$ , and a specified destination node  $r$  on a synergetic network  $G$ .*

**Proof.** The algorithm is implemented as a generation of best-first search (or priority-first search). Let link  $(i, j)$  with indicator variable  $w_{ij}$ , and link  $(pred(j), j)$  with indicator variable  $w_{pred(j)j}$  be the last links constituting partial path  $q_j$ , and partial path  $q'_j$ , respectively. Since the Bellman's Principle of Optimality may not hold in the synergetic network, the algorithm does not prune the partial path, say  $q'_j$ , that has larger travel cost unless either of the following two conditions is met: (1) the endpoint  $j$  is the destination  $r$ ; (2)  $w_{ij} \geq w_{pred(j)j}$ . Condition (1) is obvious and does not need further explanation. Based on Eqs. (1) and (2), once  $w_{ij} \geq w_{pred(j)j}$  and the travel cost of path  $q_j$  is smaller than that of path  $q'_j$ , the travel cost of any path extending from path  $q'_j$ , say  $(q'_j \cup q_{jv})$ , will be always larger than that of path  $(q_j \cup q_{jv})$ , where  $q_{jv}$  denotes a partial path from node  $j$  to node  $v$ . Therefore, partial path  $q'_j$  can be discarded from further consideration. Obviously, the synergetic shortest path algorithm equipped with the aforementioned pruning mechanism will not pre-prune any optimal path, and thus will generate all optimal paths.  $\square$

To implement the column generation approach, we call problem (9)–(15) the master problem (MP), and when it includes only partial columns it is called restricted master problem (RMP). The column generation method used to solve problem (9)–(15) is summarized as follows:

- Step 1. Determine an initial feasible RMP.
- Step 2. Solve the current RMP.

Step 3. Solve the pricing problem. If  $z \geq 0$  then stop; otherwise, add the column with the most negative reduced cost to the RMP, form a new current RMP, and go to Step 2.

Quite often, determining an initial feasible RMP in Step 1 is not trivial. However, as mentioned in Barnhart et al. (1998), Savelsbergh and Sol (1998), if such an initial restricted master problem exists, it can always be found using a two-phase approach similar in spirit to the two-phase method embodied in simplex methods to find an initial basic feasible solution.

Let  $M = \sum_{m \in O} s_m = \sum_{n \in D} d_n$  (the number of vehicles). Then, after solving problem (9)–(15), the output will contain  $M$  paths. Each path corresponds to a vehicle, and may form a load package if the path contains at least one auctioned load. Note that the loads may connect or disconnect to each other. It is easy to check that all load packages are mutually exclusive. Consequently, applying the proposed network flow technique to prepare the bids, the TL carrier, instead of valuing up to  $(2^n - 1)$  packages, only needs to consider submitting at most  $k = \min\{M, n\}$  mutually exclusive bid packages, where  $M$  denotes the number of vehicles and  $n$  is the number of auctioned loads.

So far, we have not yet explicitly explained how to determine the bidding prices of suggested load packages. Here, we show that the result of implementing the column generation method to solve problem (9)–(15) can be further used to generate bid price ranges for load packages. The idea is as follows. Let  $RMP^*$  represent the last RMP generated by implementing the column generation approach, and let  $F(P^*)$  be the corresponding optimal path flow vector associated with the net path cost (negative profit) vector  $C(P^*)$ . Denote  $SP^* = \{P_1, \dots, P_k\} \subseteq P^*$  as the set of paths containing load bundles, and let  $SC(P^*) = \{c(P_1), \dots, c(P_k)\} \subseteq C(P^*)$  be the corresponding set of net path costs. In addition, let  $\gamma(B_i)$  represent the revenue of the load bundle  $B_i$  embedded in  $P_i \in SP^*$ ; that is,  $\gamma(B_i) = \sum_{l \in B_i} \gamma_l$  where  $l$  represents an auctioned load and  $\gamma_l$  denotes its associated revenue (recall that, in this paper, the revenues of auctioned loads are treated as known). We can first conduct sensitivity analysis on  $RMP^*$  to obtain the range of  $c(P_i)$  in the objective function such that  $F(P^*)$  remains optimal. Let the range be  $c_l^l \leq c(P_i) \leq c_l^u$ . Then, the bid price range for load bundle  $B_i$  can be derived from the values of  $\gamma(B_i)$ ,  $c(P_i)$ ,  $c_l^l$  and  $c_l^u$ . Note that elementary sensitivity analysis holds only for a single coefficient change, with all other data held constant. To simultaneously obtain the bid price ranges for more than one load bundle, we can conduct the 100% rule or parametric multiple-change analysis to track the effect when multiple objective function coefficients are changing. Since the above-mentioned analysis techniques are discussed in most OR textbooks, they are skipped here.

## 5. The bidding advisor and empirical analysis

In this section, we first detail the proposed bidding advisor for assisting TL carriers in preparing bids when they participate in one-shot combinatorial auctions. Then, we conduct experiments to analyze the performance of the advisor. The developed bidding advisor has two major advantages: (1) it tightly integrates the load information in the e-markets with TL carriers' current fleet management plans, and thus makes the proposed bidding strategies very effective; and (2) it removes the great burden of evaluating the huge number of possible bid packages from the TL carriers, and thus makes the TL carriers capable of promptly making bidding decisions.

The bidding advisor for one-shot combinatorial auctions is shown in Fig. 2. The core design ideas behind the advisor have been described in Section 3. Fig. 2 applies a flow chart to delineate the whole framework of the advisor. First, the input data to the advisor includes the costs of empty repositioning vehicles and pulling loads, the known revenues of pulling booked loads, the expected revenues of transporting auctioned and forecasted loads, the pairwise synergy value, and the marginal values of vehicles in different regions at the end of planning horizon. In addition, the link and node data constituting a physical network are also required. Based on the above information, the advisor creates a synergetic network (see Section 3 for details). The bid generation and evaluation problems are then formulated as a synergetic minimum cost flow problem embedded in the synergetic network. The problem is solved by the column generation approach equipped with the synergetic shortest path algorithm. The result of implementing the column generation method is then used to generate bid prices (or price ranges) corresponding to suggested load bundles (see Section 4 for details). The output shows  $M$  (the number of vehicles available at the beginning of the planning horizon) paths. The auctioned loads on the same path constitute a bid package; therefore, the output provides up to  $k = \min\{M, n\}$  bid packages, where  $n$  denotes the number of auctioned loads. Finally, submit optimal bids to the target one-shot combinatorial auction. The proposed bidding advisor can substantially remove the burden of making bidding decisions from those TL carriers who participate in one-shot combinatorial auctions. In the remainder of this section, we present the results of experiments to analyze the performance of the proposed bidding advisor.

It has been pointed out that real data from combinatorial auctions are generally not publicly available (An et al., 2005). Thus, to test the proposed bidding advisor, we construct several hypothetical data sets. We first consider a case that a carrier (bidder) owns a fleet of 20 vehicles serving 48 loads in 15 regions within one week (planning horizon). Then, to evaluate the computational limitations of the column generation solution approach, we consider larger-size problems in which a TL carrier may own a fleet of 100, 250, 500, 750, or 1000 vehicles serving loads with a double size of corresponding fleet between 20, 40, or 60 regions within 1 week (planning horizon). The determination of the ratio of loads per vehicle per week is based on the information that the typical ratio in practice ranges from 2 to 2.5 (Powell, 1996); thus, we set the ratio in the small-size problem to 2.4, and in every large-size problem to 2.0. In each case, the number of vehicles available in each region at the beginning of the planning horizon is randomly generated. Assume that the carrier's

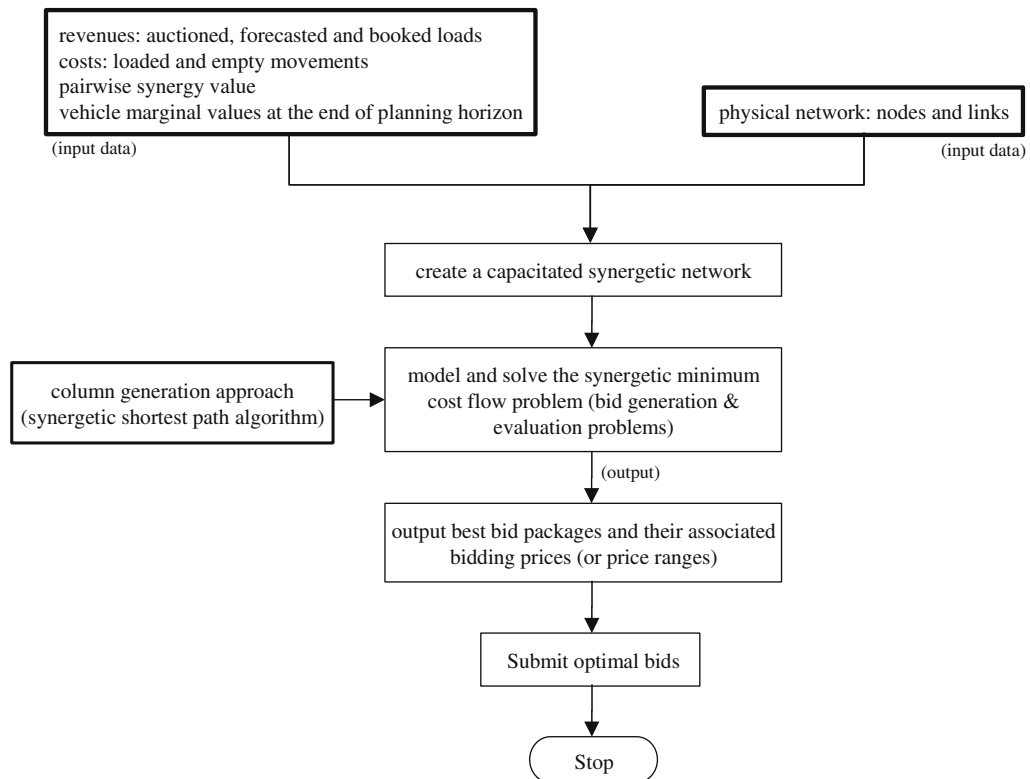


Fig. 2. The structure of the bidding advisor for one-shot combinatorial auctions.

operation cycle is one week starting from Monday and ending Sunday. Thus, the vehicle distribution at the end of the planning horizon is set to be equal to that at the beginning. The salvage value,  $u_t^s$ , at region  $t$  at the end of the planning horizon is estimated roughly by using the procedure proposed in Frantzeskakis and Powell (1988). The procedure is briefly described in Appendix A. The day of week load distribution refers to the day of week booking profile and the day of week call-in distribution in Powell (1996). The cost of moving empty across link  $(i, j)$ ,  $c_{ij}^e$ , is uniformly distributed in [400, 600]. The profit of pulling a booked load across link  $(i, j)$ ,  $u_{ij}^b$ , is set to be equal to  $c_{ij}^e/1.5$ . However, the profit of a forecasted load,  $u_{ij}^f$ , is set to be equal to  $\theta_{ij}(c_{ij}^e/1.5)$ , where  $\theta_{ij}$  represents the realization probability of the load. The profits of serving auctioned loads are decided according to those of booked loads. Finally, An et al. (2005) using the data from Plummer (2003) propose that  $\frac{\text{average pairwise synergy value}}{\text{average item value}} = 0.18$ . Thus, in this paper,  $S$ , the pairwise synergy is set to be equal to 0.18 multiplied by the average profit of pulling a load across a link; that is,  $S = 60$ . In addition, the value of parameter  $\beta$  is set to 0.5. The solution algorithm is implemented in Java with an interface to the CPLEX 11.0 linear programming solver on a desktop PC with a 2.40 GHz Core2 Quad Q6600 processor and 1 GB RAM.

### 5.1. A small-size problem

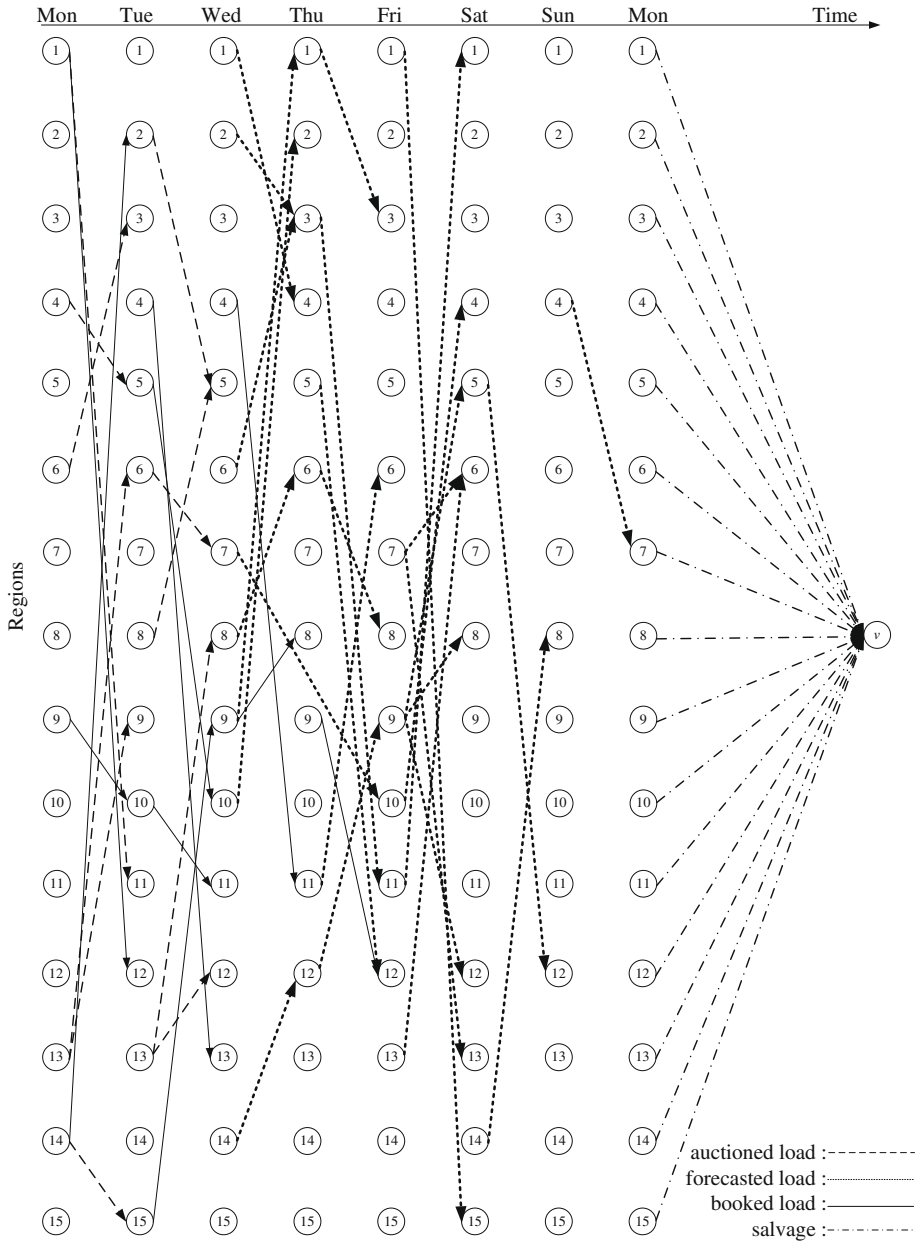
Consider the case that a carrier owns a fleet of 20 vehicles serving 48 loads in 15 regions within one week. The 48 loads constitute 10 auctioned, 12 booked, and 26 forecasted loads. The resulting distributions of the number of vehicles available in each region at the beginning and the end of the planning horizon are shown in Table 2. The unit salvage value corresponding to each region is also shown in Table 2. The day of week load distribution is shown in Table 3.

Table 2  
Vehicle amount distribution and unit salvage value

Region	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Number of vehicles (beginning)	4	1	0	1	1	1	0	0	2	1	0	1	3	2	3	20
Number of vehicles (end)	4	1	0	1	1	1	0	0	2	1	0	1	3	2	3	20
Unit salvage value	232	190	188	199	164	187	227	173	189	217	229	183	204	171	194	–

**Table 3**  
Day of week load distribution

	Pickup day						
	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Auctioned loads	3	4	2	1			
Booked loads	7	5					
Forecasted loads			7	7	9	2	1



**Fig. 3.** Relative positions of all considered loads.

The resulting synergetic time–space network includes 169 nodes and 1686 links. Let the 10 auctioned loads be labeled 1, 2, ..., 10 associated with triplet (origin, destination, pickup-day), (9, 10, Monday), (14, 2, Monday), (10, 11, Tuesday),

(5, 10, Tuesday), (1, 12, Monday), (9, 8, Wednesday), (15, 9, Tuesday), (4, 11, Wednesday), (4, 13, Tuesday), and (9, 12, Thursday), respectively. The costs of moving empty across links are shown in Appendix B. The information (origin, destination, pickup date and profit) corresponding to each load is shown in Appendix C, where  $\theta_{ij}$  is set to 0.8. The relative positions of all considered loads are graphically shown in Fig. 3. Note that we only demonstrate the loaded and salvage links for clarity purpose. We will show later that it is very easy to trace the bid-package patterns via the network in Fig. 3.

The solution algorithm to the synergetic minimum cost (maximum profit) flow problem is implemented in Java with an interface to the CPLEX 11.0 linear programming solver. The computations to most instances require less than one minute, and only few instances take a little more than one minute. The output consists of 20 paths (one vehicle on one path), and five profitable bid packages: {1, 3}, {2}, {4}, {5}, and {7}. Four auctioned loads, 6, 8, 9, and 10, are not considered because they are not profitable. The five paths embedded with the five bid packages are graphically shown in Fig. 4. Each of the five paths gains both *explicit synergy* (i.e.,  $S$  and/or  $\beta S$ ) and *implicit synergy* (i.e., the sum of the profits of

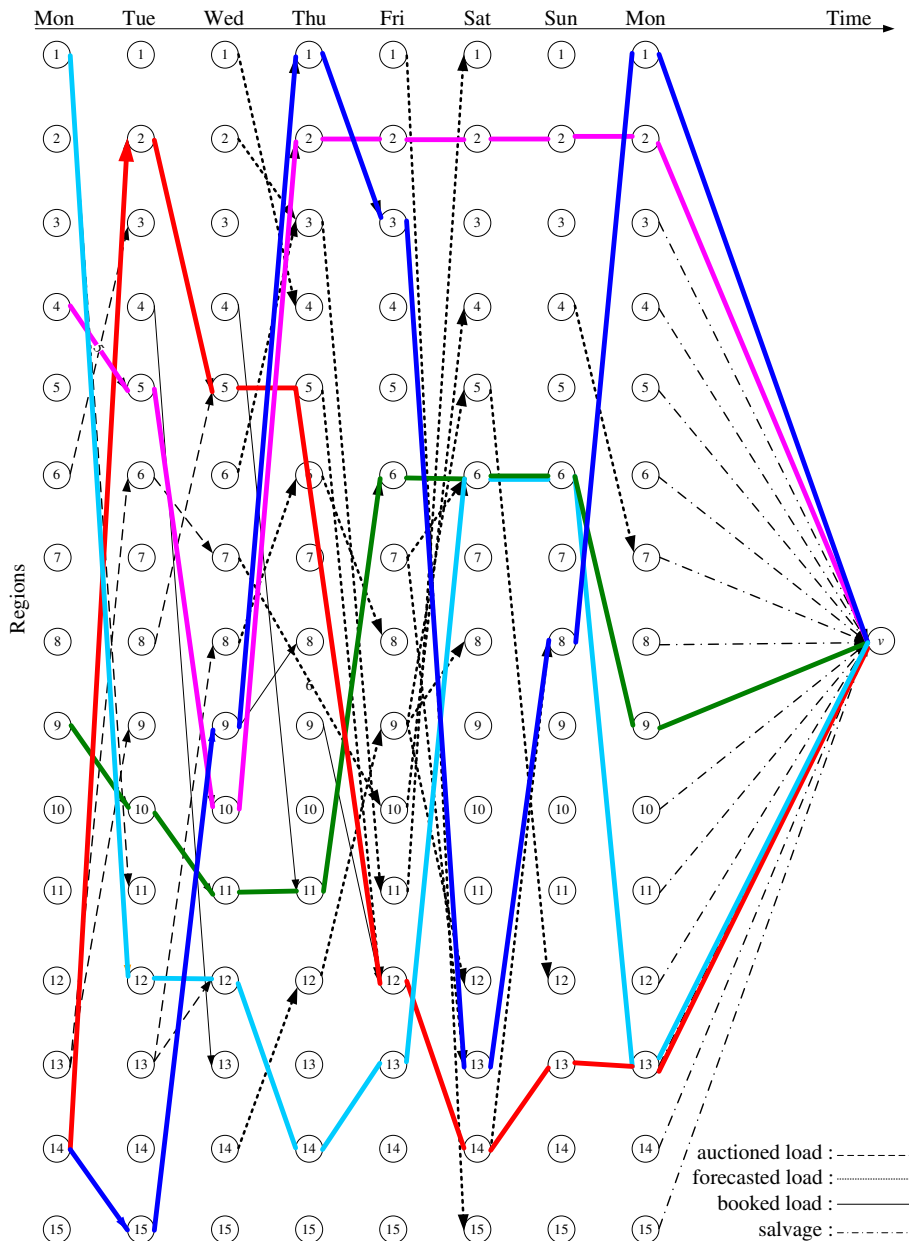
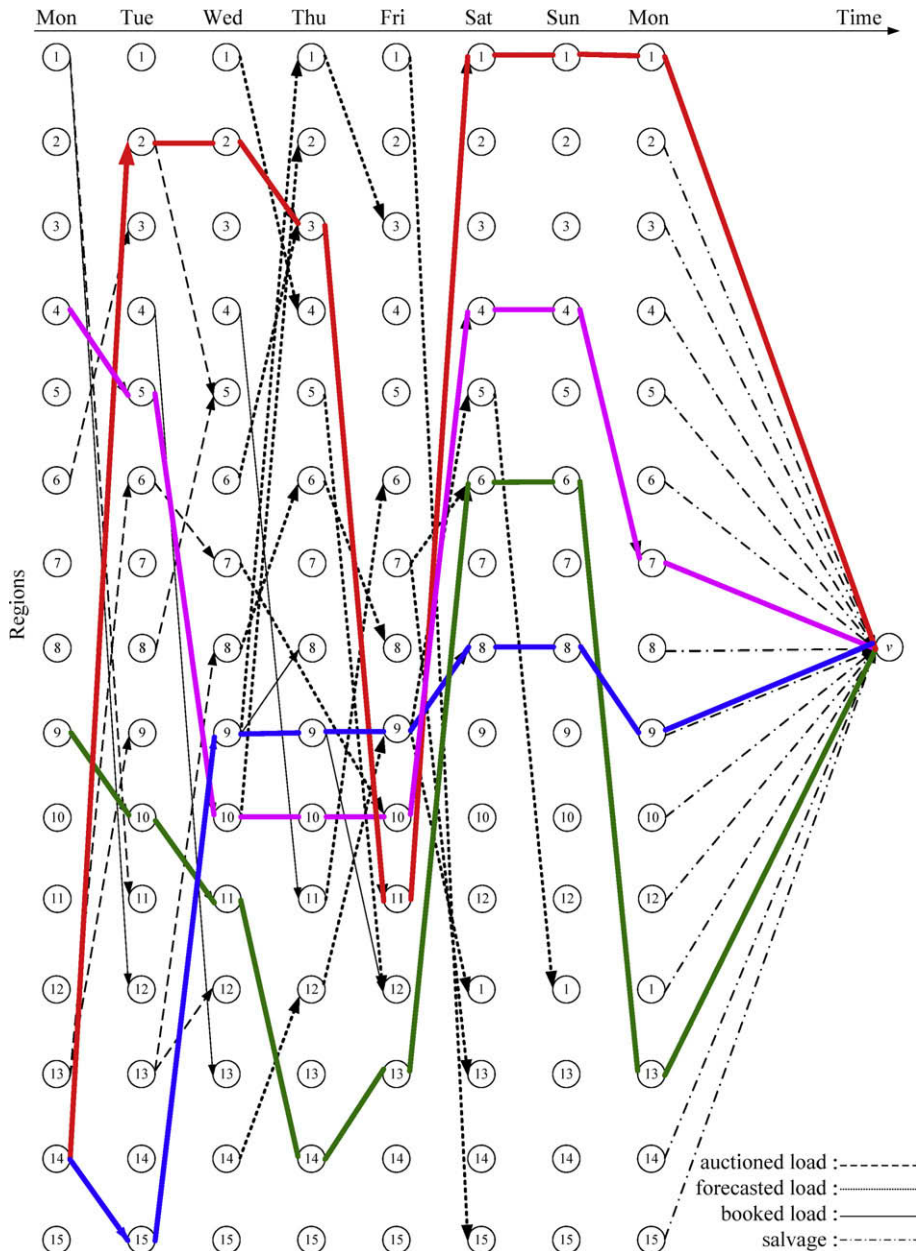


Fig. 4. Output paths and corresponding bid packages.

**Table 4**  
Sources of explicit and implicit synergies

Bid package	Number of forecasted loads	Number of booked loads
{1, 3}	1	
{2}	1	1
{4}	1	1
{5}	2	
{7}	3	1

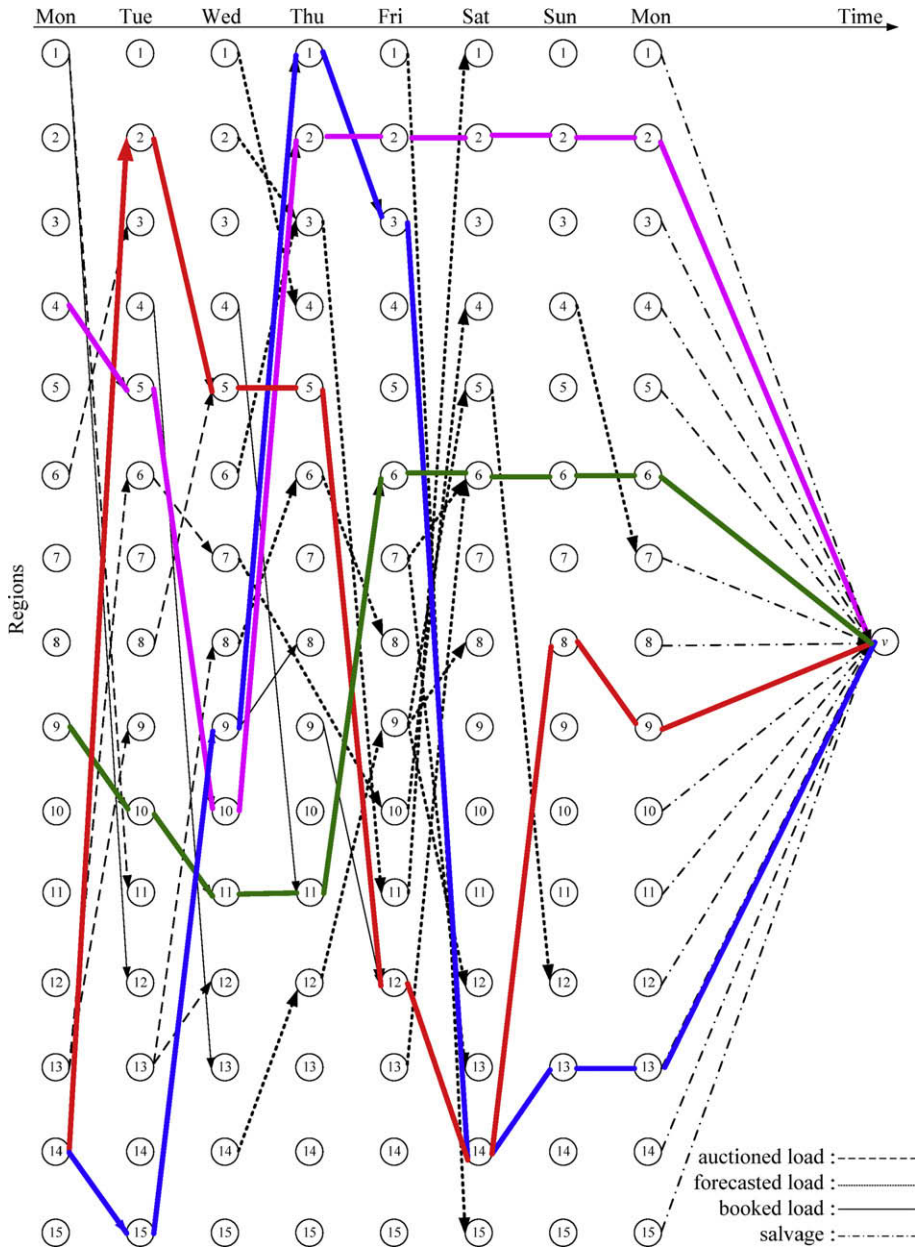


**Fig. 5.** Output paths and corresponding bid packages ( $S = 0$ ).

all embedded loads) from the (auctioned, booked, and forecasted) loads embedded in the path to increase its profitability (see Table 4). The path with load package {1, 3} gains synergy from one forecasted load, the path with load package {2}

**Table 5**  
Sources of implicit synergy

Bid package	Number of forecasted loads	Number of booked loads
{1, 3}	2	
{2}	2	
{4}	2	1
{7}	1	1



**Fig. 6.** Output paths and corresponding bid packages ( $\theta = 0.6$ ).

from one forecasted and one booked loads, the path with load package {4} from one forecasted and one booked loads, the path with load package {5} from two forecasted loads, and the path with package {7} from one booked and three fore-

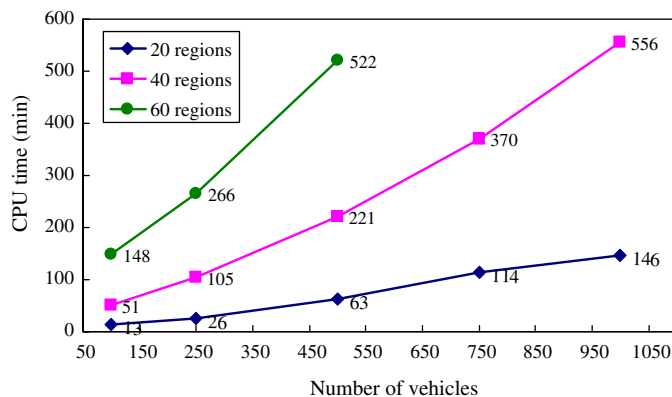
**Table 6**  
Sources of explicit and implicit synergies ( $\theta = 0.6$ )

Bid package	Number of forecasted loads	Number of booked loads
{1, 3}	1	
{2}	2	1
{4}	1	1
{7}	2	1

casted loads. Such results indicate that it is crucial to consider both booked and forecasted loads in the combinatorial bid generation problem for transportation procurement. Fig. 4 shows that there is only one round-trip load package, and three paths contain sub-tours, which demonstrates the insufficiency of considering only round-trip load-package patterns in combinatorial auctions for TL service procurement. Consequently, the bidding advisor, instead of evaluating  $2^{10}-1$  (i.e., 1023) possible bid packages, only suggests submitting the five profitable bids. This fact justifies the value of the proposed bidding advisor.

To know how the consideration of synergy influences the selection of bid packages, we reset the pairwise synergy,  $S$ , to 0. The output includes 20 paths (one vehicle takes one path), and four profitable bid packages: {1, 3}, {2}, {4}, and {7}. Five auctioned loads, 5, 6, 8, 9, and 10, are considered as profitless. Package {5} disappears from the set of bid packages under the scenario of  $S = 60$  due to the absence of explicit synergy. The four paths embedded with the four bid packages are graphically shown in Fig. 5. There is no round-trip package, but three paths contain sub-tours. Table 5 demonstrates the sources of implicit synergies corresponding to each bid package; the path with load package {1, 3} gains synergy from two forecasted loads; the path with load package {2} from two forecasted loads; the path with load package {4} from two forecasted and one booked loads; and the path with load package {7} from one booked and one forecasted loads. Comparing the information in Table 4 with that in Table 5, we can see that the value of synergy,  $S$ , significantly influences the combination of booked and forecasted loads (i.e., implicit synergy) corresponding to each bid package, and thus the TL carrier's fleet management plan, even though the value does not much change the set of bid packages.

Furthermore, to see how  $\theta_{ij}$ , the realization probability of the forecasted load on link  $(i, j)$ , affects the choice of bid packages, we set the realization probability to 0.6, instead of 0.8, for all forecasted loads. The output consists of 19 paths (two vehicles take the same path, and the others take one path), and four profitable bid packages: {1, 3}, {2}, {4}, and {7}. Five auctioned loads, 5, 6, 8, 9, and 10, are considered as profitless. The four paths embedded with the four bid packages are graphically shown in Fig. 6. As shown in Fig. 6, there is no round-trip package, but two paths contain sub-tours. Package {5} disappears from the set of bid packages under the scenario of  $S = 60$  due to the decrease of implicit synergy. Table 6 demonstrates the sources of explicit and implicit synergies corresponding to each bid package; the path with load package {1, 3} gains synergy from one forecasted load; the path with load package {2} from one booked and two forecasted loads; the path with load package {4} from one forecasted and one booked loads; and the path with load package {7} from one booked and two forecasted loads. Comparing the information in Table 6 with that in Table 4, we can see that the value of realization probability significant influences the combination of booked and forecasted loads (i.e., implicit synergy) corresponding to each bid package. The sets of bid packages under the scenarios of  $S = 0$  and  $\theta_{ij} = 0.6$  turn out the same. However, the sources of explicit and implicit synergies corresponding to each bid package are quite different. That is, the TL carrier will have very different fleet deployments.



**Fig. 7.** Computational results for larger-size test problems.



The above experimental results show that the bid generation and evaluation problems could be substantially affected by the synergy values, the estimated profits of forecasted and auctioned loads. It follows that TL carriers' fleet management plans should not neglect the above influence factors. In addition, the numerical results demonstrate the efficiency of the proposed solution algorithm to the synergetic minimum cost flow problem.

## 5.2. Large-size problems

To understand the computational limitations in terms of CPU time of the proposed solution algorithm, we test it on large-size instances. The cases assume that a TL carrier may own a fleet of 100, 250, 500, 750, or 1000 vehicles serving loads with a double size of corresponding fleets within one week between 20, 40, or 60 regions (the largest case we have found so far in the literature). Each synergetic network, therefore, consists of at most  $8n + m + 1$  nodes and at most  $71n + 2m$  links, where  $n$  represents the number of regions and  $m$  represents the number of loads (also the twice number of vehicles). The day of week load distribution of each case is proportionally similar to the one shown in Table 3. Following Cheung and Powell (1996), we assume that the synergetic networks are not completely dense; rather, loaded vehicles from a region can be moved to ten other regions. The travel time between each pair of connected regions is set to one day (i.e., one time period in the synergetic time–space networks). The value of  $\theta_{ij}$  is set to 0.8. On the other hand, the distribution of the number of vehicles available in each region at the beginning and thus the end of the planning horizon, and the load information (origin, destination, and profit corresponding to each load of whichever type) are randomly generated. The computational CPU time corresponding to each case is shown in Fig. 7. Approximately, the CPU time is increasing linearly with the number of vehicles (or loads), but is increasing with the square of the number of regions. We eventually omit the cases in which a TL carrier owns a fleet of 750 or 1000 vehicles serving loads between 60 regions because of long computational times; their computational CPU times can be, however, easily estimated by linear extrapolation. Fig. 7 shows that we may need to develop heuristic approaches to the synergetic minimum cost flow problem to help a big TL carrier owning a large number of vehicles and operating over a huge area make quick bidding decisions.

## 6. Conclusion and future research

This paper develops a bidding advisor to help the TL carriers who participate in one-shot combinatorial auctions make bidding decisions. The bidding advisor has two major advantages: (1) it tightly integrates load information in the e-markets with bidders' current fleet management plans, and, therefore, can provide TL carriers effective bidding strategies; and 2) it prevents TL carriers from evaluating the potentially huge number of possible bid packages, and thus makes TL carriers capable of promptly making bidding decisions. The core of the bidding advisor is the technique for converting the bid generation and evaluation problems into a synergetic minimum cost network flow problem. Due to the special structure of the synergetic network, conventional solution methods for minimum cost flow problems cannot be applied to the synergetic network flow problem. A column generation technique equipped with a synergetic shortest path algorithm is developed to solve the problem. The empirical analysis shows that a TL carrier adopting the proposed advisor can easily determine the desirable bid packages without evaluating all possible bid packages.

As mentioned above, the proposed bidding advisor suggests bid packages by solving the synergetic minimum cost network flow problem. This specific network problem builds on the "average synergy" techniques of estimating the synergy values between loads. The pros and cons of the estimation approaches are detailed in Section 3. We, however, emphasize that it is important and worthwhile to develop more sophisticated methods to accurately compute the synergy values between loads. The more accuracy of the estimated synergy values, the more efficacy of the bidding advisor.

There are two natural extensions to this research. First, this research focuses on one-shot combinatorial auctions for transportation service procurement. Multi-round combinatorial auctions have, however, received much attention recently in spite of their complexities; thus, it is also important to develop bidding advisors for those bidders who participate in multi-round combinatorial auctions. The advisor developed in Section 5 can definitely be the building blocks for developing the bidding advisors for complex multi-round combinatorial auctions. The second extension of this research is to develop bidding advisors for the other major type of freight carriers: less-than-truckload (LTL) carriers. The characteristics of TL and LTL carriers are actually quite different and should have a significant impact on their bidding strategies and thus on their needs and requirements with respect to bidding advisors.

## Acknowledgements

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**Appendix A. Calculation of the salvage values**

Appendix A briefly describes the procedure proposed in Frantzeskakis and Powell (1988) to calculate  $Q_t(H)$ , the salvage value of a vehicle at region  $t$  at the end of planning horizon  $H$ . To mitigate truncation effects, they consider a planning horizon  $H'$  that is substantially longer than  $H$ . Then, given the loaded and empty activities, they apply a backward recursion method to estimate  $Q_t(H)$ . Let

- $f_{ti}(d)$ : forecasted number of vehicles moving loaded from  $t$  to  $i$  at time  $d$ ,
- $e_{ti}(d)$ : forecasted number of vehicles moving empty from  $t$  to  $i$  at time  $d$ ,
- $\zeta_{ti}(d)$ : fraction of vehicles moving loaded from  $t$  to  $i$  at time  $d$ ,
- $\tau_{ti}(d)$ : fraction of vehicles moving empty from  $t$  to  $i$  at time  $d$ ,
- $r_{ti}$ : average profit for pulling a load from  $t$  to  $i$ ,
- $c_{ti}$ : cost of moving empty from  $t$  to  $i$ , and
- $R$ : set of regions.

The backward recursion method is described as follows: let

$$Q_t(H') = 0 \quad \forall t \in R$$

Then, beginning with  $d = H' - 1$  and working backward in time, let

$$Q_t(d) = \sum_{i \in R} [\zeta_{ti}(d) \times w_{ti}(d) - \tau_{ti}(d) \times \bar{w}_{ti}(d)]$$

where

$$w_{ti}(d) = \begin{cases} r_{ti}(H-d) & \text{if } H-d \leq 1 \\ r_{ti} + Q_i(d+1) & \text{otherwise} \end{cases}$$

$$\bar{w}_{ti}(d) = \begin{cases} c_{ti}(H-d) & \text{if } H-d \leq 1 \\ c_{ti} + Q_i(d+1) & \text{otherwise} \end{cases}$$

$$\zeta_{ti}(d) = \frac{f_{ti}(d)}{\sum_{j \in R} (f_{tj}(d) + e_{tj}(d))}$$

$$\tau_{ti}(d) = \frac{e_{ti}(d)}{\sum_{j \in R} (f_{tj}(d) + e_{tj}(d))}$$

Note that in this paper, we do not make much effort to collect the required data to the backward recursion method, and do not precisely follow the procedure. Rather, we roughly estimate the relevant information and use it to calculate  $Q_t(H)$ .

**Appendix B. Empty repositioning costs**

To	From														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	447	536	484	400	418	510	403	485	448	593	569	473	538	469
2	411	0	464	503	481	451	452	450	522	517	419	525	481	562	423
3	558	563	0	525	521	484	460	467	585	557	456	563	525	523	584
4	454	447	524	0	595	444	491	509	551	533	504	455	412	413	423
5	447	455	416	409	0	538	565	526	438	412	452	507	496	541	536
6	577	597	453	501	589	0	569	457	483	465	546	497	524	468	436
7	501	424	430	552	572	530	0	404	501	459	557	507	469	565	470
8	491	432	430	413	577	462	540	0	548	428	524	470	586	420	441
9	594	470	400	463	418	431	451	417	0	534	532	450	416	571	424
10	587	588	580	562	483	506	548	453	588	0	522	561	438	425	438
11	520	428	402	495	437	562	425	557	555	547	0	444	413	421	436
12	467	573	575	535	538	493	521	570	421	517	450	0	580	521	564
13	553	548	541	438	424	490	497	577	442	417	518	482	0	421	433
14	417	526	408	587	412	573	581	526	578	434	519	432	406	0	565
15	522	469	532	578	560	480	495	454	425	545	454	582	584	571	0

### Appendix C. Load information

Number	Auctioned loads				Booked loads				Forecasted loads			
	Origin	Destination	Pickup date	Unit profit	Origin	Destination	Pickup date	Unit profit	Origin	Destination	Pickup date	Unit profit
1	9	10	Monday	392	1	11	Monday	347	11	1	Friday	316
2	14	2	Monday	375	13	8	Tuesday	391	7	6	Friday	303
3	10	11	Tuesday	365	13	12	Tuesday	387	2	3	Wednesday	300
4	5	10	Tuesday	322	14	15	Monday	381	1	3	Thursday	298
5	1	12	Monday	311	6	7	Tuesday	353	4	7	Sunday	294
6	9	8	Wednesday	292	8	5	Tuesday	351	7	10	Wednesday	292
7	15	9	Tuesday	283	13	6	Monday	349	9	8	Friday	292
8	4	11	Wednesday	264	1	11	Monday	347	11	6	Thursday	291
9	4	13	Tuesday	234	6	3	Monday	323	5	12	Thursday	287
10	9	12	Thursday	225	2	5	Tuesday	303	5	12	Saturday	287
11					13	9	Monday	277	10	4	Friday	284
12					4	5	Monday	273	13	6	Friday	279
13									1	15	Friday	278
14									10	2	Wednesday	276
15									7	13	Friday	265
16									6	3	Wednesday	258
17									9	1	Wednesday	258
18									6	8	Thursday	246
19									8	6	Wednesday	244
20									1	4	Wednesday	242
21									12	9	Thursday	240
22									9	5	Friday	234
23									14	13	Thursday	225
24									9	12	Friday	225
25									14	8	Saturday	224
26									3	11	Thursday	214

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