# Decode-and-Forward Relaying for Cooperative NOMA Systems with Direct Links

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# Decode-and-Forward Relaying for Cooperative NOMA Systems with Direct Links

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***Abstract*—This paper investigates a cooperative non-orthogonal multiple access (NOMA) system, in which a base station communicates with two far users with the aid of a decode-and-forward (DF) relay. Three cooperative relaying schemes, namely, the fixed relaying (FR), selective DF with coordinated direct and relay transmission (SDF-CDRT), and incremental-selective DF (ISDF) relaying are proposed to enhance the outage performance for the two far users by utilizing both the direct and relay links. Taking into account the received signal-to-noise ratio (SNR) events at the relay, the SDF-CDRT scheme adaptively forms an orthogonal transmission branch with respect to the direct link or keeps silent to reduce error propagation. Besides considering the relay detection results, the ISDF scheme further exploits the limited feedback of the received SNR events from two users, so that error propagation can be avoided and unnecessary relaying can be reduced. Analytical expressions for the outage probabilities and average throughputs of the paired users are derived in the closed-form for the three cooperative relaying schemes. Asymptotic expressions for the outage probabilities are derived in the high SNR region. It is shown that the FR and SDF-CDRT schemes achieve a diversity order of one for both users, while the ISDF scheme achieves a diversity order of two for both users. The superior system performance achieved by the proposed schemes over those of the existing methods is verified by Monte Carlo simulations.**

***Index Terms*—Non-orthogonal multiple access, decode-and-forward, selective relaying, outage probability.**

## I. Introduction

Due to its superior spectral efficiency and its capability to support massive connectivity, non-orthogonal multiple access (NOMA) has been envisioned as a promising multiple access candidate for fifth generation (5G) networks [1]–[3]. In particular, power-domain NOMA can serve multiple users for demanding large-scale heterogeneous traffic by using the same time/frequency/ code resource but with different power levels [3], [4]. As a special case of NOMA, multiple user superposition transmission (MUST) has been proposed for 3GPP long term evolution (LTE) [5], [6]. To exploit channel conditions of different users opportunistically, users with poorer channel qualities are allocated more transmit power in NOMA systems, while users with better channel qualities are less allocated. In this way, both poorer-channel and better-channel users can decode their messages successfully at the cost of applying successive interference cancellation (SIC) at better-channel users. It has shown that NOMA is more power efficient than conventional orthogonal multiple access (OMA) when users are properly grouped according to their channel qualities or quality of service (QoS) requirements [7]–[10].

Recently, cooperative NOMA schemes incorporating relaying have been proposed in the literature to strengthen system performances. The first cooperative NOMA scheme was proposed in [2], where a user with strong channel conditions was used as a relay beside decoding its own message via SIC. In [11], a dedicated multi-antenna amplify-and-forward (AF) relay was introduced to aid transmissions from a base station (BS) to NOMA users. Considering perfect and imperfect knowledge of channel state information (CSI), the performances of cooperative NOMA systems with an AF relay under Nakagami-$m$ fading were studied in [12] and [13], respectively. A partial relay selection (RS) was proposed in [14], where several AF relay criteria were proposed to assist the BS-to-users transmissions. Furthermore, the authors in [15] proposed a two-stage AF RS scheme, which achieves a diversity order the same as the number of the relay nodes. The two-stage RS schemes have also been investigated for NOMA decode-and-forward (DF) systems to decrease outage probability and obtain spatial diversity in [15], [16]. To enhance cell edge coverage, multiple antennas have been employed at the DF relay and users for cooperative NOMA systems [17]. Based on the user-aided relay and dedicated relay, several full-duplex relaying schemes have been proposed for cooperative NOMA DF systems in [18]–[21]. In [22] and [23], cooperative NOMA transmissions have been combined with simultaneous wireless information and power transfer.

When direct links between the BS and users are non-negligible, it is worth pointing out that coordinated transmissions incorporating direct links can significantly enhance the performance of cooperative NOMA systems [14], [18], [19], [24], [25]. Direct links exist widely in not only the overlapping of macro and micro cells [14], [24], but also device-to-device (D2D) communications [19], [25]. Compared

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to the conventional NOMA-DF scheme adopted in [15]–[17], the coordinated direct and relay transmission (CDRT) scheme achieves the improved outage performance and increased ergodic sum rate [24]. However, in coordinated transmissions, a main challenge is to acquire side information for interference cancellation, which may result in huge overhead due to massive connectivity in cooperative NOMA systems. Another issue in the current CDRT schemes is that the users are paired based on channel quality, so that the far user always achieves a low-rate transmission. Currently, the CDRT schemes designed for aiding two far users can be recognized as the novel contribution. It is also worthwhile to point out that the existences of direct links are commonly assumed in multiple access relay channels (MARCs) for downlink and uplink transmissions [26]–[31]. In [26], the authors proposed a reverse compute-and-forward (CF) relaying scheme for the dowlink MARCs under an error-free and capacity-limited souce-to-relay channel. For the uplink MARCs with solid relay-to-destination links, the CF relaying scheme for two user access was investigated in [27] and lately extended to the scenario with multiuser and multiple relays in [28].

On the other hand, when direct links are available in cooperative systems, adaptive relaying schemes such as the selective relaying and incremental relaying can be applied to reduce error propagation resulted from the detection failure at the relay [32]–[36]. As a simple way to reduce error propagation for cooperative systems, the selective DF (SDF) relaying makes a decision whether it operates in the DF mode or keeps silent based on a received signal-to-noise ratio (SNR) threshold at the relay, so that the end-to-end performance can be improved [32], [33]. By exploiting the limited SNR feedback from the destination, the authors in [34] proposed the incremental relaying with a significant improvement of spectrum efficiency over the conventional fixed relaying (FR) and SDF relaying schemes. The outage probability and end-to-end performance of the incremental relaying have been investigated in [35] and [36], respectively. In [37], the authors proposed the incremental-selective DF (ISDF) relaying by combining incremental relaying and SDF relaying, which achieved an improved system performance over that of the simple incremental DF relaying. Several hybrid relaying schemes that combine SDF, AF, and incremental relaying have been investigated in [38]–[40]. However, to the best knowledge of the authors, selective relaying and incremental relaying have not been applied for cooperative NOMA systems taking into account the superposition in the power domain for multiple access.

In this paper, we consider a cooperative NOMA system with one BS communicating with two far receivers with the aid of an intermediate relay. Different from existing works on MARCs where the CF relaying is applied, we assume that the relay operates in the DF mode and investigate cooperative relaying taking into account both the direct and relay links. The main contributions of this paper are summarized as follows.

- In the presence of direct links, three cooperative relaying schemes, i.e., the FR, SDF with CDRT (SDF-CDRT), and ISDF schemes, are individually investigated for the cooperative NOMA system. Compared to most existing cooperative NOMA schemes incorporating a single direct link to the near user [18], [19], [24], [25], our work considers the direct links from the BS to two far users, so that two far users with different QoS requirements can be paired with each other and get benefit from cooperative relaying. In contrast to cooperative NOMA systems where the direct links from the BS to the far users are unavailable, the proposed cooperative relaying schemes significantly improve the outage performance by taking advantages of both the direct and relay links.
- To reduce error propagation resulted from the detection failures at the relay, the SDF-CDRT scheme is designed to switch the relaying on or off depending on the received SNRs at the relay. When the received SNRs at the relay are high enough to ensure the correct detection, the SDF-CDRT scheme forms an orthogonal branch with respect to the direct link transmissions, so that only linear combination is applied at each receiver and the decoding complexity can be reduced in such a case. When the received SNRs cannot ensure the correct detection for the superposition, the relay keeps silent and the BS starts a new transmission block, which takes the advantages provided by the SDF relaying scheme [32], [33]. Based on limited information feedback of the received SNR events from two users, the ISDF scheme switches the relaying on or off depending on the combinations of the received SNR events at the relay and two users. Accordingly, the unnecessary relaying can be reduced by the ISDF scheme over the SDF scheme.
- For the FR, SDF-CDRT, and ISDF schemes, the closed-form expressions for the outage probabilities are derived for both two users. To highlight the impact of the system parameters on the outage performance, asymptotic outage probabilities achieved at both two users are derived for the three proposed cooperative relaying schemes, respectively. It is shown that the FR and SDF-CDRT schemes achieve a diversity order of one for both two users, whereas the ISDF scheme provides a diversity gain order of two for the two users' case. Taking into account the additional time slots consumed for relaying, the analytical results for the average throughputs are derived for the FR, SDF-CDRT, and ISDF schemes, respectively.

The rest of this paper is organized as follows: Section II presents the system model, the FR scheme, and the SDF-CDRT scheme; Section III presents the ISDF scheme. Section IV derives the outage probabilities for the three proposed cooperative relaying schemes and conducts asymptotic outage performance analyses; Section V gives simulation results to verify the superior performance achieved by the proposed cooperative relaying schemes and Section VI summarizes the paper.

## II. System Model and Simple Relaying Schemes

We consider a downlink cooperative NOMA system consisting of a BS, a DF relay, and two far users. The system model is depicted in Fig. 1, in which the two far users are

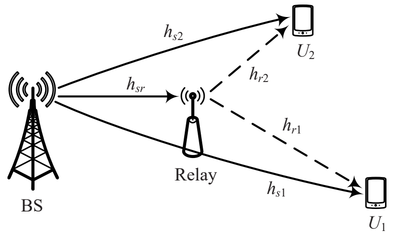



Fig. 1. System model of the cooperative NOMA system with a DF relay.

denoted by $U_1$ and $U_2$, respectively. We assume that all the nodes are equipped with a single antenna and work in the half-duplex mode. In the considered system, the BS and relay can be the transmitters of a macro cell and a micro cell, respectively, and we assume that the direct links from the BS to two users are non-negligible. The considered system model can also be applied for D2D communications [25], in which the relay can be a device in the proximity of the two users and we also assume that the direct links from the BS to the two users exist. The channels from the BS to the relay, from the BS to $U_i$, and from the relay to $U_i$ are denoted by $h_{sr}$, $h_{si}$, and $h_{ri}$, respectively, with $i = 1$ and 2. All the channels are modeled as independent but non-identically distributed (i.n.i.d.) complex Gaussian random variables (RVs) with zero means and variances $\beta_{sr}$, $\beta_{si}$, and $\beta_{ri}$ for $h_{sr}$, $h_{si}$, and $h_{ri}$, respectively. Moreover, all the channels are assumed to be block fading, i.e., a channel keeps constant during one transmission block and can vary from one transmission block to another. The additive white Gaussian noises (AWGNs) at all the receivers are assumed to have zero mean and variance $\sigma^2$, while the information signal for $U_i$ is denoted by $x_i$ satisfying $\mathbb{E}\{x_i\} = 0$ and $\mathbb{E}\{|x_i|^2\} = 1$.

In cooperative NOMA systems, users can be ordered by their channel qualities, which in turn determines the decoding order at the receiver side and results in a huge overhead for the case with a large number of users. Another user pairing criterion is to order users according to their QoS requirements [15], [16], which enables grouping users even if they have statistically the same channel qualities. Moreover, the CSI acquirements at the transmitters can be significantly reduced if QoS-based user pairing is applied [15]. In this paper, we also apply QoS-based user pairing and assume that user $U_1$ is a low target rate device but requiring a timely service, while user $U_2$ is more delay-tolerated than user $U_1$ but needs a higher throughput.

In the considered system, each downlink transmission block can be accomplished within two time slots. In the first time slot, the BS broadcasts the superposed signal $x_b \triangleq \sqrt{\alpha_1}x_1 + \sqrt{\alpha_2}x_2$ to all the other nodes, where $\alpha_i$ is the power allocation coefficient, which satisfies $\sum_{i=1}^{2}\alpha_i = 1$ and $\alpha_i > 0$. The received signals at the relay and $U_i$ are respectively given by

$$y_r = h_{sr}\big(\sqrt{\alpha_1 P_s}x_1 + \sqrt{\alpha_2 P_s}x_2\big) + n_r \quad \text{and} \tag{1}$$

$$y_i^{(1)} = h_{si}\big(\sqrt{\alpha_1 P_s}x_1 + \sqrt{\alpha_2 P_s}x_2\big) + n_i^{(1)}, \tag{2}$$

where $P_s$ is the transmit power, $n_r$ and $n_i^{(1)}$ are the AWGNs at the relay and $U_i$, respectively. Since we assume that the QoS requirements of $U_1$ are much less demanding than those of $U_2$, the decoding order at the relay is always from $U_1$ to $U_2$ [15]. Therefore, the received SNRs at the relay for decoding $x_1$ and $x_2$ can be respectively expressed as

$$\gamma_r^{x_1} = \frac{\alpha_1\rho|h_{sr}|^2}{\alpha_2\rho|h_{sr}|^2 + 1} \quad \text{and} \quad \gamma_r^{x_2} = \alpha_2\rho|h_{sr}|^2, \tag{3}$$

where $\rho \triangleq P_s/\sigma^2$ is the transmit SNR.

TABLE I
RECEIVED SNR EVENTS AND DETECTION RESULTS AT THE RELAY

| Received SNR event | Detection result |
|---|---|
| $\varepsilon_1 := (\gamma_r^{x_1} \geq \gamma_{\text{th},1}) \cap (\gamma_r^{x_2} \geq \gamma_{\text{th},2})$ | Correct $x_1$ and $x_2$ |
| $\varepsilon_2 := (\gamma_r^{x_1} \geq \gamma_{\text{th},1}) \cap (\gamma_r^{x_2} < \gamma_{\text{th},2})$ | Correct $x_1$ and incorrect $x_2$ |
| $\varepsilon_3 := \gamma_r^{x_1} < \gamma_{\text{th},1}$ | Incorrect $x_1$ and $x_2$ |

### *A. FR Scheme*

In this subsection, we investigate the conventional FR scheme which utilizes both the direct and relay link transmissions to enhance the detection performance at the receiver side. After correct detecting $x_1$ and $x_2$, the relay regenerates and forwards the superposition $x_b$, so that the received signal at $U_i$ in the second time slot can be expressed as

$$y_i^{(2)} = h_{ri}\big(\sqrt{\alpha_1 P_r}x_1 + \sqrt{\alpha_2 P_r}x_2\big) + n_i^{(2)}, \tag{4}$$

where $P_r$ is the relaying transmit power and $n_i^{(2)}$ is the AWGN at $U_i$ in the second time slot. It is worthwhile to point out that the above signal model strictly relies on the correct detection at the relay. When the source-to-relay link suffers deep fading, the relay may not always detect $x_1$ and $x_2$ correctly. In Table I, three detection results and corresponding received SNR events at the relay are presented, where $\gamma_{\text{th},i} \triangleq 2^{R_i} - 1$ is the required SNR threshold for correct detecting $x_i$ with $R_i$ denoting the target transmission rate of $U_i$. In the FR scheme, each user applies maximum ratio combining (MRC) to recover its desired signal. The MRC processing and the end-to-end received SNRs at $U_1$ and $U_2$ are discussed as follows.

*1) Event $\varepsilon_1$*: For each user, MRC is applied to (2) and (4) at the receiver side to recover its desired signal. When the event $\varepsilon_1$ occurs, the relay can correctly forward $x_b$, so that the end-to-end received SNR at $U_1$ for detecting $x_1$ is given by

$$\gamma_{1,\,\text{fr}}^{x_1}(\varepsilon_1) = \frac{\alpha_1\rho(|h_{s1}|^2 + |\tilde{h}_{r1}|^2)}{\alpha_2\rho(|h_{s1}|^2 + |\tilde{h}_{r1}|^2) + 1}, \tag{5}$$

where $\tilde{h}_{r1} \triangleq \sqrt{\chi}h_{r1}$ with $\chi \triangleq P_r/P_s$. At $U_2$, MRC is also applied to (2) and (4) for detecting $x_1$. Then, SIC is performed for detecting $x_2$. The end-to-end received SNRs at $U_2$ for detecting $x_1$ and $x_2$ can be respectively expressed as

$$\gamma_{2,\,\text{fr}}^{x_1}(\varepsilon_1) = \frac{\alpha_1\rho(|h_{s2}|^2 + |\tilde{h}_{r2}|^2)}{\alpha_2\rho(|h_{s2}|^2 + |\tilde{h}_{r2}|^2) + 1} \quad \text{and} \tag{6}$$

$$\gamma_{2,\,\text{fr}}^{x_2}(\varepsilon_1) = \alpha_2\rho(|h_{s2}|^2 + |\tilde{h}_{r2}|^2). \tag{7}$$

*2) Event $\varepsilon_2$*: When the event $\varepsilon_2$ occurs, we have $\gamma_{1,\mathrm{fr}}^{x_1}(\varepsilon_2) = \gamma_{1,\mathrm{fr}}^{x_1}(\varepsilon_1)$ and $\gamma_{2,\mathrm{fr}}^{x_1}(\varepsilon_2) = \gamma_{2,\mathrm{fr}}^{x_1}(\varepsilon_1)$ due to the correct forwarding of $\sqrt{\alpha_1}x_1$ by the relay. However, the relay forwards the incorrect $\sqrt{\alpha_2}x_2$ in this case, so that the end-to-end received SNR for $U_2$ to detect $x_2$ becomes

$$\gamma_{2,\mathrm{fr}}^{x_2}(\varepsilon_2) = \frac{\alpha_2\rho|h_{s2}|^2}{\alpha_2\rho|\tilde{h}_{r2}|^2 + 1}. \quad (8)$$

Compared to (7), we can see that the end-to-end received SNR in (8) becomes smaller.

*3) Event $\varepsilon_3$*: When the event $\varepsilon_3$ occurs, neither $x_1$ nor $x_2$ is correctly forwarded by the relay, so that the end-to-end received SNRs at $U_1$ and $U_2$ degrade to

$$\gamma_{1,\mathrm{fr}}^{x_1}(\varepsilon_3) = \frac{\alpha_1\rho|h_{s1}|^2}{\alpha_2\rho|h_{s1}|^2 + \rho|\tilde{h}_{r1}|^2 + 1}, \quad (9)$$

$$\gamma_{2,\mathrm{fr}}^{x_1}(\varepsilon_3) = \frac{\alpha_1\rho|h_{s2}|^2}{\alpha_2\rho|h_{s2}|^2 + \rho|\tilde{h}_{r2}|^2 + 1}, \quad \text{and} \quad (10)$$

$$\gamma_{2,\mathrm{fr}}^{x_2}(\varepsilon_3) = \frac{\alpha_2\rho|h_{s2}|^2}{\rho|\tilde{h}_{r2}|^2 + 1}. \quad (11)$$

The expressions in (5)-(11) show that the end-to-end received SNRs are heavily affected by the detection results at the relay. When the events $\varepsilon_2$ and $\varepsilon_3$ occur, error propagation is unavoidable, which decreases the end-to-end received SNRs at $U_1$ and $U_2$. In order to reduce error propagation when the events $\varepsilon_2$ and $\varepsilon_3$ occur and to avoid SIC when $\varepsilon_1$ occurs, we propose the SDF-CDRT scheme in the next subsection.

### *B. SDF-CDRT Scheme*

Based on a certain received SNR threshold at the relay, the SDF relaying is a simple way to reduce error propagation for cooperative relaying systems [33]. In this section, we propose an SDF-CDRT scheme for the considered cooperative NOMA systems. In the SDF-CDRT scheme, when the event $\varepsilon_1$ occurs, the relay generates a new superposition such that an orthogonal structure is formed with respect to the direct and relay transmissions. Consequently, linear combining is applied at each receiver to recover the desired signal and SIC can be avoided in such a case. Moreover, when $\varepsilon_2$ or $\varepsilon_3$ occurs, the relay keeps silent, so that error propagation resulted from the failure detection at the relay can be avoided and spectral efficiency can be improved by a new block transmission via the BS [33].

In the SDF-CDRT scheme, the BS broadcasts $x_b$ to the relay and two users in the first time slot. The operation procedure of the SDF-CDRT scheme in the second time slot and the end-to-end received SNRs at $U_1$ and $U_2$ are described as follows.

*1) Event $\varepsilon_1$*: When the event $\varepsilon_1$ occurs, the relay generates a new superposition

$$x_r = \sqrt{\alpha_2}x_1 - \sqrt{\alpha_1}x_2 \quad (12)$$

for its transmission in the second time slot. Then, the received signal at $U_i$ in the second time slot is given by

$$y_i^{(2)} = h_{ri}\left(\sqrt{\alpha_2 P_r}x_1 - \sqrt{\alpha_1 P_r}x_2\right) + n_i^{(2)}. \quad (13)$$

For each user, the received signals from both the first and second time slots can be rewritten in a matrix form as

$$\begin{bmatrix} \frac{y_i^{(1)}}{h_{si}\sqrt{P_s}} \\ \frac{y_i^{(2)}}{h_{ri}\sqrt{P_r}} \end{bmatrix} = \underbrace{\begin{bmatrix} \sqrt{\alpha_1} & \sqrt{\alpha_2} \\ \sqrt{\alpha_2} & -\sqrt{\alpha_1} \end{bmatrix}}_{\mathcal{H}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{n_i^{(1)}}{h_{si}\sqrt{P_s}} \\ \frac{n_i^{(2)}}{h_{ri}\sqrt{P_r}} \end{bmatrix}, \quad (14)$$

where $\mathcal{H}$ is the equivalent power allocation matrix. Different from the $2 \times 2$ orthogonal design for space-time block coding (STBC) [41], where $\mathcal{H}$ contains the complex space-time channel coefficients weighted by $\pm 1$, the proposed power allocation matrix is a $2 \times 2$ real matrix, which avoids the conjugate of the signal in the power-domain. In general, $\mathcal{H}$ can be in any form of the $2 \times 2$ rotation or reflection, i.e.,

$$\mathcal{H} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \text{ or } \mathcal{H} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}, \quad (15)$$

where $\theta \in (0, \pi/2)$. Thus, the power allocation can be interpreted as a rotation or reflection of $[x_1, x_2]^T$. It can be shown that $\mathcal{H}$ is an orthogonal matrix with a unit norm for each of its row, so that it not only satisfies the power constraint $\alpha_1^2 + \alpha_2^2 = 1$, but also provides the orthogonality for the proposed SDF-CDRT scheme. Without loss of generality, we adopt the reflection form (14) in the sequential. To recover the desired singal, each user applies linear combining to its received signals from both the first and second time slots. Specifically, $U_1$ and $U_2$ apply the linear combinations as

$$\begin{aligned} \breve{x}_1 &= y_1^{(1)}\sqrt{\alpha_1}\tilde{h}_{r1} + y_1^{(2)}\sqrt{\alpha_2}h_{s1} \\ &= h_{s1}\tilde{h}_{r1}\sqrt{P}x_1 + \sqrt{\alpha_1}\tilde{h}_{r1}n_1^{(1)} + \sqrt{\alpha_2}h_{s1}n_1^{(2)} \quad \text{and} \end{aligned} \quad (16)$$

$$\begin{aligned} \breve{x}_2 &= y_2^{(1)}\sqrt{\alpha_2}\tilde{h}_{r2} - y_2^{(2)}\sqrt{\alpha_1}h_{s2} \\ &= h_{s2}\tilde{h}_{r2}\sqrt{P}x_2 + \sqrt{\alpha_2}\tilde{h}_{r2}n_2^{(1)} - \sqrt{\alpha_1}h_{s2}n_2^{(2)} \end{aligned} \quad (17)$$

for detecting $x_1$ and $x_2$, respectively. Based on (16) and (17), the end-to-end received SNRs at $U_1$ and $U_2$ for detecting $x_1$ and $x_2$ can be respectively expressed as

$$\gamma_{1,\mathrm{sdf-cdrt}}^{x_1}(\varepsilon_1) = \frac{\rho|h_{s1}|^2|\tilde{h}_{r1}|^2}{\alpha_1|\tilde{h}_{r1}|^2 + \alpha_2|h_{s1}|^2} \quad \text{and} \quad (18)$$

$$\gamma_{2,\mathrm{sdf-cdrt}}^{x_2}(\varepsilon_1) = \frac{\rho|h_{s2}|^2|\tilde{h}_{r2}|^2}{\alpha_1|h_{s2}|^2 + \alpha_2|\tilde{h}_{r2}|^2}. \quad (19)$$

*2) Event $\varepsilon_2$ or $\varepsilon_3$*: When the event $\varepsilon_2$ or $\varepsilon_3$ occurs, the relay cannot correctly regenerate $x_r$. In such a case, the relay keeps silent and the BS needs to start a new superposition transmission [33]. At each receiver, the detection is based on the received signal in the first time slot. Thus, when $\varepsilon_i$ ($i = 2, 3$) occurs, the end-to-end received SNRs at $U_1$ and $U_2$ are respectively given by

$$\gamma_{1,\mathrm{sdf-cdrt}}^{x_1}(\varepsilon_i) = \frac{\alpha_1\rho|h_{s1}|^2}{\alpha_2\rho|h_{s1}|^2 + 1}, \quad (20)$$

$$\gamma_{2,\mathrm{sdf-cdrt}}^{x_1}(\varepsilon_i) = \frac{\alpha_1\rho|h_{s2}|^2}{\alpha_2\rho|h_{s2}|^2 + 1}, \quad \text{and} \quad (21)$$

$$\gamma_{2,\mathrm{sdf-cdrt}}^{x_2}(\varepsilon_i) = \frac{\alpha_1\rho|h_{s1}|^2}{\alpha_2\rho|h_{s1}|^2 + 1}. \quad (22)$$

TABLE II
PRELIMINARY DECISION RULE FOR THE ISDF RELAYING

| Decision $U_2$ / $U_1$ | $\tilde{\varepsilon}_1$ | $\tilde{\varepsilon}_2$ | $\tilde{\varepsilon}_3$ |
|---|---|---|---|
| $\hat{\varepsilon}_1$ | Silent | $x_2$ | $x_b$ |
| $\hat{\varepsilon}_2$ | $x_1$ | $x_b$ | $x_b$ |

Compared to the conventional FR scheme, the SDF-CDRT scheme forms an orthogonal transmission branch with respect to the direct link when $\varepsilon_1$ occurs. When either $\varepsilon_2$ or $\varepsilon_3$ occurs, the SDF-CDRT scheme stops relaying, so that error propagation can be avoided. However, when $\varepsilon_1$ occurs, the SDF-CDRT scheme always operates in the DF mode no matter what the direct links' qualities are, which results in unnecessary relaying if $U_i$ can recover its desired signal via the direct link transmission. Motivated by this, we propose the ISDF relaying scheme in the next section.

## III. ISDF RELAYING

By exploiting the limited SNR feedback from the receiver to the BS and relay, the incremental relaying can significantly improve reception reliability over the fixed and selective relaying schemes for point-to-point communication systems [34]–[36]. However, due to the superposition in the power-domain for multiple access, the feedback of the received SNRs in the considered system is much more complicated than that of a point-to-point communication system. In the case that two users are grouped for multiple access, there are six combinations of the received SNR events with respect to the detection results at $U_1$ and $U_2$ in delay-limited transmissions based on the proposed ISDF relaying scheme.

In the ISDF scheme, the BS broadcasts a superposition $x_b$ in the first time slot. Then, the received SNR events at $U_1$ and $U_2$ are fed back to the relay, based on which the relay makes a preliminary decision on whether it forwards a signal/superposition or not. If the preliminary decision is to forward, the relay further checks whether its received SNRs are high enough to correctly regenerate the required signal. Only when the required signal/superposition can be regenerated correctly, the relay forwards the signal/superposition; otherwise, the relay keeps silent and the BS starts a new transmission [34]–[36].

The preliminary decision rule at the relay is presented in Table II, which is designed to forward a signal to either $U_1$, $U_2$, or both if it is necessary. In Table II, the received SNR events at $U_1$ and $U_2$ are defined as

$$\begin{aligned} \hat{\varepsilon}_1 &:= \gamma_1^{x_1} \geq \gamma_{\mathrm{th},1}, \\ \hat{\varepsilon}_2 &:= \gamma_1^{x_1} < \gamma_{\mathrm{th},1}, \\ \tilde{\varepsilon}_1 &:= (\gamma_2^{x_1} \geq \gamma_{\mathrm{th},1}) \cap (\gamma_2^{x_2} \geq \gamma_{\mathrm{th},2}), \\ \tilde{\varepsilon}_2 &:= (\gamma_2^{x_1} \geq \gamma_{\mathrm{th},1}) \cap (\gamma_2^{x_2} < \gamma_{\mathrm{th},2}), \quad \text{and} \\ \tilde{\varepsilon}_3 &:= \gamma_2^{x_1} < \gamma_{\mathrm{th},1}, \end{aligned} \tag{23}$$

where $\gamma_1^{x_1}$ is the received SNR at $U_1$ in the first time slot for detecting $x_1$, which is given by

$$\gamma_1^{x_1} = \frac{\alpha_1 \rho |h_{s1}|^2}{\alpha_2 \rho |h_{s1}|^2 + 1}. \tag{24}$$

Moreover, $\gamma_2^{x_1}$ and $\gamma_2^{x_2}$ are the received SNRs at $U_2$ in the first time slot for detecting $x_1$ and $x_2$, respectively, which are respectively given by

$$\gamma_2^{x_1} = \frac{\alpha_1 \rho |h_{s2}|^2}{\alpha_2 \rho |h_{s2}|^2 + 1} \quad \text{and} \tag{25}$$

$$\gamma_2^{x_2} = \alpha_2 \rho |h_{s2}|^2. \tag{26}$$

After a preliminary decision is made, the relay further evaluates its received SNRs and makes a formal decision on whether it operates in the DF relaying mode or keeps silent according to the formal decision rule provided in Table III. Note that Table III is extended from Table II by taking into account the received SNR events at the relay. For example, when the events $\{\hat{\varepsilon}_2, \tilde{\varepsilon}_2\}$ occur, the preliminary rule is to make the relay forward $x_b$ according to Table II. In such a case, the relay further checks which one of $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ occurs to make the formal decision. If $\varepsilon_1$ occurs, the relay can forward $x_b$. However, if $\varepsilon_2$ occurs, the relay can only forward $x_1$ due to its failure detection of $x_2$. As such, when $\varepsilon_3$ occurs, the relay keeps silent even if the events $\{\hat{\varepsilon}_2, \tilde{\varepsilon}_2\}$ occur. At the receiver side, we assume that each user can acquire the indication knowledge of the forwarded signal, i.e., which signal of $\{x_1, x_2, x_b\}$ is forwarded, so that MRC can be performed adaptively at each receiver. The end-to-end received SNRs achieved by the ISDF scheme are discussed as follows.

*1) Events* $\{\varepsilon_1, \hat{\varepsilon}_1, \tilde{\varepsilon}_2\}$: According to Table III, when $\{\varepsilon_1, \hat{\varepsilon}_1, \tilde{\varepsilon}_2\}$ occurs, the relay only forwards $x_2$ with its full transmit power to help $U_2$. For $U_1$, its end-to-end received SNR is determined by its received signal in the first time slot, which is characterized by

$$\gamma_{1,\mathrm{isdf}}^{x_1}(\varepsilon_1, \hat{\varepsilon}_1, \tilde{\varepsilon}_2) \geq \gamma_{\mathrm{th},1}. \tag{27}$$

At $U_2$, $x_1$ is detected based on the received signal in the first time slot. Since $U_2$ can detect $x_1$ successfully when $\tilde{\varepsilon}_2$ occurs, we have

$$\gamma_{2,\mathrm{isdf}}^{x_1}(\varepsilon_1, \hat{\varepsilon}_1, \tilde{\varepsilon}_2) \geq \gamma_{\mathrm{th},1}. \tag{28}$$

After subtracting the detected $x_1$ from the received signal of the first time slot, MRC is applied to the remaind signal and the received signal in the second time slot for recovering $x_2$. Thus, the end-to-end received SNR for detecting $x_2$ at $U_2$ is given by

$$\gamma_{2,\mathrm{isdf}}^{x_2}(\varepsilon_1, \hat{\varepsilon}_1, \tilde{\varepsilon}_2) = \rho(\alpha_2 |h_{s2}|^2 + |\tilde{h}_{r2}|^2). \tag{29}$$

*2) Events* $\{\varepsilon_1, \hat{\varepsilon}_1, \tilde{\varepsilon}_3\}$: When $\{\varepsilon_1, \hat{\varepsilon}_1, \tilde{\varepsilon}_3\}$ occurs, the relay forwards $x_b$ in the second time slot. Since $\hat{\varepsilon}_1$ occurs in this case, $U_1$ detects $x_1$ based on the received signal in the first time slot. Thus, the end-to-end received SNR at $U_1$ for detecting $x_1$ satisfies

$$\gamma_{1,\mathrm{isdf}}^{x_1}(\varepsilon_1, \hat{\varepsilon}_1, \tilde{\varepsilon}_3) \geq \gamma_{\mathrm{th},1}. \tag{30}$$

At $U_2$, MRC is applied to detect $x_1$ based on the received signals from the two time slots. After then, SIC is performed for detecting $x_2$. The end-to-end received SNRs at $U_2$ for detecting $x_1$ and $x_2$ can be respectively expressed as

$$\gamma_{2,\mathrm{isdf}}^{x_1}(\varepsilon_1, \hat{\varepsilon}_1, \tilde{\varepsilon}_3) = \frac{\alpha_1 \rho (|h_{s2}|^2 + |\tilde{h}_{r2}|^2)}{\alpha_2 \rho (|h_{s2}|^2 + |\tilde{h}_{r2}|^2) + 1} \quad \text{and} \tag{31}$$

TABLE III
FORMAL DECISION RULE AT THE RELAY

| Decision / Feedback / Relay's SNR | $\{\hat{\varepsilon}_1,\tilde{\varepsilon}_1\}$ | $\{\hat{\varepsilon}_1,\tilde{\varepsilon}_2\}$ | $\{\hat{\varepsilon}_1,\tilde{\varepsilon}_3\}$ | $\{\hat{\varepsilon}_2,\tilde{\varepsilon}_1\}$ | $\{\hat{\varepsilon}_2,\tilde{\varepsilon}_2\}$ | $\{\hat{\varepsilon}_2,\tilde{\varepsilon}_3\}$ |
|---|---|---|---|---|---|---|
| $\varepsilon_1$ | Silent | $x_2$ | $x_b$ | $x_1$ | $x_b$ | $x_b$ |
| $\varepsilon_2$ | Silent | Silent | $x_1$ | $x_1$ | $x_1$ | $x_1$ |
| $\varepsilon_3$ | Silent | Silent | Silent | Silent | Silent | Silent |

TABLE IV
FEEDBACK AND SIGNALING REQUIRED BY THE RELAYING SCHEMES

| Relaying scheme | Received SNR event feedback | Signaling for detecting |
|---|---|---|
| FR | Null | One bit for indicating either the BS or the relay is transmitting |
| SDF-CDRT | Relay: one bit for indicating $\varepsilon_1$ or non-$\varepsilon_1$ event | One bit for indicating either $x_b$ or $x_r$ is transmitted |
| ISDF | $U_1$: one bit for indicating $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$; $U_2$: two bits for indicating $\tilde{\varepsilon}_1$, $\tilde{\varepsilon}_2$, and $\tilde{\varepsilon}_3$ | Two bits for indicating which one of $\{x_1, x_2, x_b\}$ is transmitted |

$$\gamma_{2,\text{isdf}}^{x_2}(\varepsilon_1,\hat{\varepsilon}_1,\tilde{\varepsilon}_3) = \alpha_2\rho(|h_{s2}|^2 + |\tilde{h}_{r2}|^2). \tag{32}$$

*3) Events* $\{\varepsilon_1,\hat{\varepsilon}_2,\tilde{\varepsilon}_1\}$: In this case, the relay forwards $x_1$ with its full transmit power to aid $U_1$. At $U_1$, MRC is applied for detecting $x_1$ with the end-to-end received SNR given by

$$\gamma_{1,\text{isdf}}^{x_1}(\varepsilon_1,\hat{\varepsilon}_2,\tilde{\varepsilon}_1) = \frac{\rho(\alpha_1|h_{s1}|^2 + |\tilde{h}_{r1}|^2)}{\alpha_2\rho|h_{s1}|^2+1}. \tag{33}$$

Due to $\tilde{\varepsilon}_1$, $U_2$ detects its desired messages based on its received signal in the first time slot. The end-to-end received SNRs at $U_2$ satisfy

$$\gamma_{2,\text{isdf}}^{x_1}(\varepsilon_1,\hat{\varepsilon}_2,\tilde{\varepsilon}_1) \geq \gamma_{\text{th},1} \quad \text{and} \tag{34}$$

$$\gamma_{2,\text{isdf}}^{x_2}(\varepsilon_1,\hat{\varepsilon}_2,\tilde{\varepsilon}_1) \geq \gamma_{\text{th},2}. \tag{35}$$

*4) Events* $\{\varepsilon_1,\hat{\varepsilon}_2,\tilde{\varepsilon}_2\}$: When $\{\varepsilon_1,\hat{\varepsilon}_2,\tilde{\varepsilon}_2\}$ occurs, $x_b$ is forwarded by the relay and MRC is applied at both users. The end-to-end received SNR at $U_1$ for detecting $x_1$ is given by

$$\gamma_{1,\text{isdf}}^{x_1}(\varepsilon_1,\hat{\varepsilon}_2,\tilde{\varepsilon}_1) = \frac{\alpha_1\rho(|h_{s1}|^2 + |\tilde{h}_{r1}|^2)}{\alpha_2\rho(|h_{s1}|^2 + |\tilde{h}_{r1}|^2)+1}. \tag{36}$$

Since that $\tilde{\varepsilon}_2$ already occurs in the first time slot, the end-to-end received SNR for $U_2$ to detect $x_1$ with the aid of MRC also satisfies

$$\gamma_{2,\text{isdf}}^{x_1}(\varepsilon_1,\hat{\varepsilon}_2,\tilde{\varepsilon}_2) \geq \gamma_{\text{th},1}. \tag{37}$$

Moreover, the end-to-end received SNR for $U_2$ to detect $x_2$ is the same as that of (32).

*5) Events* $\{\varepsilon_1,\hat{\varepsilon}_2,\tilde{\varepsilon}_3\}$: Since $x_b$ is forwarded by the relay, the end-to-end received SNR at $U_1$ is the same as that of (36). At $U_2$, the end-to-end received SNRs for detecting $x_1$ and $x_2$ are the same as those of (31) and (32), respectively.

*6) Events* $\{\varepsilon_2,\hat{\varepsilon}_1,\tilde{\varepsilon}_3\}$: In this case, the relay can only forward $x_1$ taking into account $\varepsilon_2$. Due to the event $\hat{\varepsilon}_1$, we have $\gamma_{1,\text{isdf}}^{x_1}(\varepsilon_2,\hat{\varepsilon}_1,\tilde{\varepsilon}_3) \geq \gamma_{\text{th},1}$. At $U_2$, the end-to-end received SNRs for detecting $x_1$ and $x_2$ can be respectively derived as

$$\gamma_{2,\text{isdf}}^{x_1}(\varepsilon_2,\hat{\varepsilon}_1,\tilde{\varepsilon}_3) = \frac{\rho(\alpha_1|h_{s2}|^2 + |\tilde{h}_{r2}|^2)}{\alpha_2\rho|h_{s2}|^2+1} \quad \text{and} \tag{38}$$

$$\gamma_{2,\text{isdf}}^{x_2}(\varepsilon_2,\hat{\varepsilon}_1,\tilde{\varepsilon}_3) = \alpha_2\rho|h_{s2}|^2. \tag{39}$$

*7) Events* $\{\varepsilon_2,\hat{\varepsilon}_2,\tilde{\varepsilon}_1\}$: Only $x_1$ is forwarded in this case. The end-to-end received SNR at $U_1$ is the same as that of (33). Moreover, the end-to-end received SNRs at $U_2$ are characterized by $\gamma_{2,\text{isdf}}^{x_1}(\varepsilon_2,\hat{\varepsilon}_2,\tilde{\varepsilon}_1) \geq \gamma_{\text{th},1}$ and $\gamma_{2,\text{isdf}}^{x_2}(\varepsilon_2,\hat{\varepsilon}_2,\tilde{\varepsilon}_1) \geq \gamma_{\text{th},2}$.

*8) Events* $\{\varepsilon_2,\hat{\varepsilon}_2,\tilde{\varepsilon}_2\}$: In this case, the relay can forward only $x_1$ due to $\varepsilon_2$, so that the end-to-end received SNR at $U_1$ can be enhanced by using MRC as that of (33). At $U_2$, the end-to-end received SNRs are based on its received signal in the first time slot, which are characterized by $\gamma_{2,\text{isdf}}^{x_1}(\varepsilon_2,\hat{\varepsilon}_2,\tilde{\varepsilon}_2) \geq \gamma_{\text{th},1}$ and $\gamma_{2,\text{isdf}}^{x_2}(\varepsilon_2,\hat{\varepsilon}_2,\tilde{\varepsilon}_2) < \gamma_{\text{th},2}$.

*9) Events* $\{\varepsilon_2,\hat{\varepsilon}_2,\tilde{\varepsilon}_3\}$: In this case, the relay forwards $x_1$ with its full transmit power. Both $U_1$ and $U_2$ apply MRC to enhance the received SNRs. Thus, the end-to-end received SNR at $U_1$ for detecting $x_1$ has the same form as that of (33). At $U_2$, the end-to-end received SNRs for detecting $x_1$ and $x_2$ are the same as those of (38) and (39), respectively.

*10) Events for relay to keep silent*: For all the scenarios in which the relay keeps silent regarding Table III, the end-to-end received SNRs at $U_1$ and $U_2$ are the same as those achieved by the direct link transmissions in the first time slot, which are characterized by the corresponding events $\{\hat{\varepsilon}_j,\tilde{\varepsilon}_k\}$.

It is worthwhile to point out that all the 18 combinations of the received SNR event $\{\varepsilon_i,\hat{\varepsilon}_j,\tilde{\varepsilon}_k\}$ belong to the above ten cases. Moreover, these events can be classified into three types, i.e., outage event, non-outage event, and undetermined event as summarized in Table VII in Appendix B, which will be used to facilitate the outage probability analysis for the ISDF scheme.

For the proposed FR, SDF-CDRT, and ISDF schemes, the required feedback for the received SNR and control signaling for detection processing are summarized in Table IV. Especially, we can see that the required feedback and signaling for the FR and SDF-CDRF schemes are small and realistic. Although the received SNR events have 18 combinations in the ISDF scheme, the required control signaling at each receiver is still small, i.e., only the indicating information for the transmitted $\{x_1, x_2, x_b\}$ is needed for the SIC/MRC processing.

## IV. Outage Probability Analyses

In this section, we perform the outage probability analyses for the FR, SDF-CDRT, and ISDF schemes, respectively. The closed-form expressions are derived for the outage probabilities achieved by the FR, SDF-CDRT, and ISDF schemes, respectively. Then, asymptotic outage performance analyses for the FR, SDF-CDRT, and ISDF schemes are respectively derived in the high transmit SNR region.

### A. FR Scheme

For the conventional FR scheme, each receiver always combines the received signals from the two time slots for detecting its desired message irrespective of the decision results made at the relay. Accordingly, error propagation due to the incorrect detections at the relay cannot be avoided. With respect to the three types of the received SNR events at the relay, the outage probabilities achieved by the FR scheme at $U_1$ and $U_2$ can be respectively expressed as

$$P_{\text{out},1}^{\text{fr}} = \sum_{i=1}^{3} \Pr(\varepsilon_i)\mathcal{A}_1(\varepsilon_i) \quad \text{and} \tag{40}$$

$$P_{\text{out},2}^{\text{fr}} = \sum_{i=1}^{3} \Pr(\varepsilon_i)(\mathcal{A}_2(\varepsilon_i) + \mathcal{A}_3(\varepsilon_i)), \tag{41}$$

where $\mathcal{A}_1(\varepsilon_i) \triangleq \Pr\left\{\gamma_{1,\text{fr}}^{x_1}(\varepsilon_i) < \gamma_{\text{th},1}\right\}$, $\mathcal{A}_2(\varepsilon_i) \triangleq \Pr\left\{\gamma_{2,\text{fr}}^{x_1}(\varepsilon_i) < \gamma_{\text{th},1}\right\}$, and $\mathcal{A}_3(\varepsilon_i) \triangleq \Pr\left\{\gamma_{2,\text{fr}}^{x_1}(\varepsilon_i) \geq \gamma_{\text{th},1}, \gamma_{2,\text{fr}}^{x_2}(\varepsilon_i) < \gamma_{\text{th},2}\right\}$.

Since we assumed that all the channel gains follow i.n.i.d. exponential distributions, the probabilities of the received SNR events at the relay can be obtained as

$$\Pr\{\varepsilon_1\} = e^{-\frac{\xi_{\max}}{\beta_{sr}}}, \tag{42}$$

$$\Pr\{\varepsilon_2\} = \left[e^{-\frac{\xi_1}{\beta_{sr}}} - e^{-\frac{\xi_2}{\beta_{sr}}}\right]^+, \quad \text{and} \tag{43}$$

$$\Pr\{\varepsilon_3\} = 1 - e^{-\frac{\xi_1}{\beta_{sr}}}, \tag{44}$$

where $\xi_1 = \frac{\gamma_{\text{th},1}}{\rho(\alpha_1 - \alpha_2\gamma_{\text{th},1})}$, $\xi_2 = \frac{\gamma_{\text{th},2}}{\alpha_2\rho}$, $\xi_{\max} \triangleq \max(\xi_1, \xi_2)$, and $[x]^+ \triangleq \max(x, 0)$. After some mathematical manipulations, the terms $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ are derived in Table V, where $\tilde{\beta}_i \triangleq \chi\beta_i$ for $i = 1$ and 2. Based on the result in Table V and (42)-(44), the closed-form evaluations for the outage probabilities of $U_1$ and $U_2$ can be readily conducted via (40) and (41).

**Theorem 1.** *For the FR scheme, as $\rho \to \infty$, asymptotic outage probabilities at $U_1$ and $U_2$ are respectively given by*

$$P_{\text{out},1}^{\text{fr},\infty} = \frac{\xi_1}{\beta_{sr}} \frac{\chi\beta_{r1}\gamma_{\text{th},1}}{\beta_{s1}(\alpha_1 - \alpha_2\gamma_{\text{th},1}) + \chi\beta_{r1}\gamma_{\text{th},1}} \quad \textit{and} \tag{45}$$

$$P_{\text{out},2}^{\text{fr},\infty} = \frac{\xi_1}{\beta_{sr}} \frac{\chi\beta_{r2}\gamma_{\text{th},1}}{\beta_{s2}(\alpha_1 - \alpha_2\gamma_{\text{th},1}) + \chi\beta_{r2}\gamma_{\text{th},1}} + \frac{[\xi_2 - \xi_1]^+}{\beta_{sr}} \frac{\chi\beta_{r2}\gamma_{\text{th},2}}{\chi\beta_{r2}\gamma_{\text{th},2} + \beta_{s2}}. \tag{46}$$

*Proof:* Applying the fact that $e^{-x} \to 1 - x$ as $x \to 0$ to (42)-(44) and the terms in Table IV, the asymptotic expressions for (40) and (41) can be readily derived as (45) and (46), respectively. ■

Defining the diversity order achieved by $U_i$ by

$$d^i = -\lim_{\rho\to\infty} \frac{\log(P_{\text{out},i}^{\text{fr}})}{\log(\rho)} \tag{47}$$

and based on (45) and (46), it can be shown that both $U_1$ and $U_2$ achieve a diversity order of one when the FR scheme is applied. Considering that relaying occurs in each transmission block, the average throughput achieved by the FR scheme for $U_i$ $(i = 1, 2)$ can be expressed as follows:

$$R_i^{\text{fr}} = \frac{1}{2} R_i \big(1 - P_{\text{out},i}^{\text{fr}}\big), \tag{48}$$

where the factor $\frac{1}{2}$ is resulted from half-duplex relaying.

### B. SDF-CDRT scheme

In the SDF-CDRT scheme, the operation at the relay depends on its own received SNR events, which also results in different end-to-end received SNRs at $U_1$ and $U_2$. Based on the derived end-to-end received SNRs at $U_1$ and $U_2$ and the received SNR events at the relay, the outage probabilities at $U_1$ and $U_2$ can be respectively expressed as

$$P_{\text{out},1}^{\text{sdf}-\text{cdrt}} = \sum_{i=1}^{3} \Pr\{\varepsilon_i\}\mathcal{B}_1(\varepsilon_i) \quad \text{and} \tag{49}$$

$$P_{\text{out},2}^{\text{sdf}-\text{cdft}} = \sum_{i=1}^{3} \Pr\{\varepsilon_i\}\mathcal{B}_2(\varepsilon_i), \tag{50}$$

where $\mathcal{B}_1(\varepsilon_i) \triangleq \Pr\left\{\gamma_{1,\text{sdf}-\text{cdrt}}^{x_1}(\varepsilon_i) < \gamma_{\text{th},1}\right\}$, $\mathcal{B}_2(\varepsilon_1) \triangleq \Pr\left\{\gamma_{2,\text{sdf}-\text{cdrt}}^{x_2}(\varepsilon_1) < \gamma_{\text{th},2}\right\}$, and $\mathcal{B}_2(\varepsilon_i) \triangleq \Pr\left\{\gamma_{2,\text{sdf}-\text{cdrt}}^{x_1}(\varepsilon_i) < \gamma_{\text{th},1}\right\} + \Pr\left\{\gamma_{2,\text{sdf}-\text{cdrt}}^{x_1}(\varepsilon_i) \geq \gamma_{\text{th},1}, \gamma_{2,\text{idf}}^{x_2}(\varepsilon_i) < \gamma_{\text{th},2}\right\}$ for $i = 2$ and 3. After some mathematical manipulations (see Appendix A), $\mathcal{B}_1$ and $\mathcal{B}_2$ are derived as follows:

$$\mathcal{B}_1(\varepsilon_1) = 1 - \frac{2\gamma_{\text{th},1}}{\rho}\sqrt{\frac{\alpha_1\alpha_2}{\beta_{s1}\tilde{\beta}_{r1}}} e^{-\left(\frac{\alpha_2}{\tilde{\beta}_{r1}} + \frac{\alpha_1}{\beta_{s1}}\right)\frac{\gamma_{\text{th},1}}{\rho}} \times K_1\left(\frac{2\gamma_{\text{th},1}}{\rho}\sqrt{\frac{\alpha_1\alpha_2}{\beta_{s1}\tilde{\beta}_{r1}}}\right), \tag{51}$$

$$\mathcal{B}_1(\varepsilon_2) = \mathcal{B}_1(\varepsilon_3) = 1 - e^{-\frac{\xi_1}{\beta_{s1}}}, \tag{52}$$

TABLE V
$\mathcal{A}_i$ FOR THE OUTAGE PROBABILITY EXPRESSION OF THE FR SCHEME

$$\mathcal{A}_1(\varepsilon_1)=\begin{cases} 1+\frac{\beta_{s1}e^{-\frac{\xi_1}{\beta_{s1}}}}{\tilde{\beta}_{r1}-\beta_{s1}}-\frac{\tilde{\beta}_{r1}e^{-\frac{\xi_1}{\tilde{\beta}_{r1}}}}{\tilde{\beta}_{r1}-\beta_{s1}}, & \tilde{\beta}_{r1}\neq\beta_{s1} \\ 1-e^{-\frac{\xi_1}{\beta_{s1}}}\left(1+\frac{\xi_1}{\beta_{s1}}\right), & \tilde{\beta}_{r1}=\beta_{s1} \end{cases}$$

$$\mathcal{A}_1(\varepsilon_2)=\mathcal{A}_1(\varepsilon_1)$$

$$\mathcal{A}_1(\varepsilon_3)=1-\frac{\beta_{s1}e^{-\frac{\xi_1}{\beta_{s1}}}}{\rho\tilde{\beta}_{r1}\xi_1+\beta_{s1}}$$

$$\mathcal{A}_2(\varepsilon_1)=\begin{cases} 1+\frac{\beta_{s2}e^{-\frac{\xi_1}{\beta_{s2}}}}{\tilde{\beta}_{r2}-\beta_{s2}}-\frac{\tilde{\beta}_{r2}e^{-\frac{\xi_1}{\tilde{\beta}_{r2}}}}{\tilde{\beta}_{r2}-\beta_{s2}}, & \tilde{\beta}_{r2}\neq\beta_{s2} \\ 1-e^{-\frac{\xi_1}{\beta_{s2}}}\left(1+\frac{\xi_1}{\beta_{s2}}\right), & \tilde{\beta}_{r2}=\beta_{s2} \end{cases}$$

$$\mathcal{A}_2(\varepsilon_2)=\mathcal{A}_2(\varepsilon_1)$$

$$\mathcal{A}_2(\varepsilon_3)=1-\frac{\beta_{s2}e^{-\frac{\xi_1}{\beta_{s2}}}}{\rho\tilde{\beta}_{r2}\xi_1+\beta_{s2}}$$

$$\mathcal{A}_3(\varepsilon_1)=\begin{cases} \left[\frac{\tilde{\beta}_{r2}\left(e^{-\frac{\xi_1}{\tilde{\beta}_{r2}}}-e^{-\frac{\xi_2}{\tilde{\beta}_{r2}}}\right)}{\tilde{\beta}_{r2}-\beta_{s2}}+\frac{\beta_{s2}\left(e^{-\frac{\xi_2}{\beta_{s2}}}-e^{-\frac{\xi_1}{\beta_{s2}}}\right)}{\tilde{\beta}_{r2}-\beta_{s2}}\right]^+, & \tilde{\beta}_{r2}\neq\beta_{s2} \\ \left[e^{-\frac{\xi_1}{\beta_{s2}}}\left(1+\frac{\xi_1}{\beta_{s2}}\right)-e^{-\frac{\xi_2}{\beta_{s2}}}\left(1+\frac{\xi_2}{\beta_{s2}}\right)\right]^+, & \tilde{\beta}_{r2}=\beta_{s2} \end{cases}$$

$$\mathcal{A}_3(\varepsilon_2)=\begin{cases} e^{-\frac{\xi_1}{\tilde{\beta}_{r2}}}+\frac{\beta_{s2}\left(e^{-\frac{\xi_1}{\tilde{\beta}_{r2}}}-e^{-\frac{\xi_1}{\beta_{s2}}}\right)}{\tilde{\beta}_{r2}-\beta_{s2}}-\frac{\beta_{s2}e^{-\frac{\xi_2}{\beta_{s2}}}}{\alpha_2\tilde{\beta}_{r2}\rho\xi_2+\beta_{s2}}, & \tilde{\beta}_{r2}\neq\beta_{s2},\xi_2>\xi_1 \\ \frac{\tilde{\beta}_{r2}e^{-\frac{\xi_1}{\tilde{\beta}_{r2}}}}{\tilde{\beta}_{r2}-\beta_{s2}}-\frac{\tilde{\beta}_{r2}\beta_{s2}(1+\alpha_2\rho\xi_2)e^{-\frac{\xi_1(\alpha_2\tilde{\beta}_{r2}\rho\xi_2+\beta_{s2})+\xi_2(\tilde{\beta}_{r2}-\beta_{s2})}{\tilde{\beta}_{r2}\beta_{s2}(1+\alpha_2\rho\xi_2)}}}{(\tilde{\beta}_{r2}-\beta_{s2})(\alpha_2\tilde{\beta}_{r2}\rho\xi_2+\beta_{s2})}, & \tilde{\beta}_{r2}\neq\beta_{s2},\xi_2\leq\xi_1 \\ \frac{(\beta_{s2}+\xi_1)e^{-\frac{\xi_1}{\beta_{s2}}}}{\beta_{s2}}-\frac{e^{-\frac{\xi_2}{\beta_{s2}}}}{1+\alpha_2\rho\xi_2}, & \tilde{\beta}_{r2}=\beta_{s2},\xi_2>\xi_1 \\ \frac{\xi_2(1+\alpha_2\rho(\beta_{s2}+\xi_1))e^{-\frac{\xi_1}{\beta_{s2}}}}{\beta_{s2}(1+\alpha_2\rho\xi_2)}, & \tilde{\beta}_{r2}=\beta_{s2},\xi_2\leq\xi_1 \end{cases}$$

$$\mathcal{A}_3(\varepsilon_3)=\frac{\beta_{s2}e^{-\frac{\xi_1}{\beta_{s2}}}}{\tilde{\beta}_{r2}\rho\xi_1+\beta_{s2}}-\frac{\beta_{s2}e^{-\frac{\xi_2}{\beta_{s2}}}}{\tilde{\beta}_{r2}\rho\xi_2+\beta_{s2}}$$

$$\mathcal{B}_2(\varepsilon_1)=1-\frac{2\gamma_{\text{th},2}}{\rho}\sqrt{\frac{\alpha_1\alpha_2}{\beta_{s2}\tilde{\beta}_{r2}}}e^{-\left(\frac{\alpha_2}{\tilde{\beta}_{r2}}+\frac{\alpha_1}{\beta_{s2}}\right)\frac{\gamma_{\text{th},2}}{\rho}} \times K_1\left(\frac{2\gamma_{\text{th},2}}{\rho}\sqrt{\frac{\alpha_1\alpha_2}{\beta_{s2}\tilde{\beta}_{r2}}}\right), \quad \text{and} \tag{53}$$

$$\mathcal{B}_2(\varepsilon_2)=\mathcal{B}_2(\varepsilon_3)=1-e^{-\frac{\xi_1}{\beta_{s2}}}+\left[e^{-\frac{\xi_1}{\beta_{s2}}}-e^{-\frac{\xi_2}{\beta_{s2}}}\right]^+. \tag{54}$$

**Theorem 2.** *For the SDF-CDRT scheme, asymptotic outage probabilities at $U_1$ and $U_2$ are respectively given by*

$$P_{\text{out},1}^{\text{sdf}-\text{cdrt},\infty}=\frac{\gamma_{\text{th},1}}{\rho}\left(\frac{\alpha_2}{\chi\beta_{r1}}+\frac{\alpha_1}{\beta_{s1}}\right)+\frac{\xi_1([\xi_2-\xi_1]^++\xi_1)}{\beta_{sr}\beta_{s1}} \tag{55}$$

*and*

$$P_{\text{out},2}^{\text{sdf}-\text{cdrt},\infty}=\frac{\gamma_{\text{th},2}}{\rho}\left(\frac{\alpha_2}{\chi\beta_{r2}}+\frac{\alpha_1}{\beta_{s2}}\right)+\frac{([\xi_2-\xi_1]^++\xi_1)^2}{\beta_{sr}\beta_{s2}}. \tag{56}$$

*Proof:* As $x\to 0$, applying the fact that $e^{-x}\to 1-x$ and $xK_1(x)\to 1$ to (42)-(44) and $\mathcal{B}_1$ and $\mathcal{B}_2$, the asymptotic expressions for (49) and (50) can be readily derived as (55) and (56), respectively. ∎

With the obtained asymptotic expressions for the outage probabilities, it can be shown that both $U_1$ and $U_2$ achieve a diversity order of one when the SDF-CDRT scheme is applied. Since a relaying transmission occurs only when $\varepsilon_1$ happens in the SDF-CDRT scheme, whereas a new transmission block starts when $\varepsilon_2$ or $\varepsilon_3$ occurs, the average throughput achieved by the SDF-CDRT scheme for $U_i$ $(i=1,2)$ can be expressed as follows:

$$R_i^{\text{sdf}-\text{cdrt}}=\frac{1}{2}R_i\big(1-\Pr\{\varepsilon_1\}\mathcal{B}_i(\varepsilon_1)\big)+R_i\left(1-\sum_{k=2}^{3}\Pr\{\varepsilon_k\}\mathcal{B}_i(\varepsilon_k)\right). \tag{57}$$

## C. ISDF scheme

Since the relaying is switched on or off depending on not only the received SNR event feedback from two users, but also

TABLE VI
$\widehat{\mathcal{P}}_i$ AND $\widetilde{\mathcal{P}}_j$ FOR THE OUTAGE PROBABILITY EXPRESSIONS OF THE ISDF SCHEME

$$\widehat{\mathcal{P}}_1=\begin{cases}\frac{\beta_{s1}\gamma_{\text{th},1}\left(1-e^{-\frac{\xi_1}{\beta_{s1}}}\right)-\tilde{\beta}_{r1}\rho\xi_1\left(1-e^{-\frac{\gamma_{\text{th},1}}{\tilde{\beta}_{r1}\rho}}\right)}{\beta_{s1}\gamma_{\text{th},1}-\tilde{\beta}_{r1}\rho\xi_1}, & \beta_{s1}\neq\frac{\rho\tilde{\beta}_{r1}\xi_1}{\gamma_{\text{th},1}}\\ 1-e^{-\frac{\xi_1}{\beta_{s1}}}\left(1+\frac{\xi_1}{\beta_{s1}}\right), & \beta_{s1}=\frac{\rho\tilde{\beta}_{r1}\xi_1}{\gamma_{\text{th},1}}\end{cases}$$

$$\widehat{\mathcal{P}}_2=\begin{cases}\frac{\tilde{\beta}_{r1}\left(1-e^{-\frac{\xi_1}{\tilde{\beta}_{r1}}}\right)-\beta_{s1}\left(1-e^{-\frac{\xi_1}{\beta_{s1}}}\right)}{\tilde{\beta}_{r1}-\beta_{s1}}, & \beta_{s1}\neq\tilde{\beta}_{r1}\\ 1-e^{-\frac{\xi_1}{\beta_{s1}}}\left(1+\frac{\xi_1}{\beta_{s1}}\right), & \beta_{s1}=\tilde{\beta}_{r1}.\end{cases}$$

$$\widetilde{\mathcal{P}}_1=\begin{cases}\frac{\tilde{\beta}_{r2}e^{-\frac{\xi_1}{\beta_{s2}}}\left(1-e^{\frac{\alpha_2(\xi_1-\xi_2)}{\tilde{\beta}_{r2}}}\right)}{\tilde{\beta}_{r2}-\alpha_2\beta_{s2}}-\frac{\alpha_2\beta_{s2}\left(e^{-\frac{\xi_1}{\beta_{s2}}}-e^{-\frac{\xi_2}{\beta_{s2}}}\right)}{\tilde{\beta}_{r2}-\alpha_2\beta_{s2}}, & \tilde{\beta}_{r2}\neq\alpha_2\beta_{s2}\\ e^{-\frac{\xi_1}{\beta_{s2}}}-e^{-\frac{\xi_2}{\beta_{s2}}}\left(1+\frac{\xi_2-\xi_1}{\beta_{s2}}\right), & \tilde{\beta}_{r2}=\alpha_2\beta_{s2},\end{cases}$$

$$\widetilde{\mathcal{P}}_2=\begin{cases}1-e^{-\frac{\xi_1}{\tilde{\beta}_{r2}}}+\frac{\beta_{s2}e^{-\frac{\xi_1}{\beta_{s2}}}\left(1-e^{\frac{\xi_1}{\beta_{s2}}-\frac{\xi_1}{\tilde{\beta}_{r2}}}\right)}{\tilde{\beta}_{r2}-\beta_{s2}}+\frac{\tilde{\beta}_{r2}\left(e^{\frac{\xi_1}{\tilde{\beta}_{r2}}-\frac{\xi_2}{\tilde{\beta}_{r2}}-\frac{\xi_1}{\beta_{s2}}}+e^{-\frac{\xi_1}{\tilde{\beta}_{r2}}}-e^{-\frac{\xi_2}{\tilde{\beta}_{r2}}}-e^{-\frac{\xi_1}{\beta_{s2}}}\right)}{\tilde{\beta}_{r2}-\beta_{s2}}, & \tilde{\beta}_{r2}\neq\beta_{s2}\\ 1-e^{-\frac{\xi_1}{\beta_{s2}}}-\frac{\xi_1}{\beta_{s2}}e^{-\frac{\xi_2}{\beta_{s2}}}, & \tilde{\beta}_{r2}=\beta_{s2},\end{cases}$$

$$\widetilde{\mathcal{P}}_3=\begin{cases}e^{-\frac{\xi_1}{\beta_{s2}}}+\frac{\beta_{s2}e^{-\frac{\xi_2}{\beta_{s2}}}}{\tilde{\beta}_{r2}-\beta_{s2}}-\frac{\tilde{\beta}_{r2}e^{\frac{\xi_1}{\tilde{\beta}_{r2}}-\frac{\xi_2}{\tilde{\beta}_{r2}}-\frac{\xi_1}{\beta_{s2}}}}{\tilde{\beta}_{r2}-\beta_{s2}}, & \tilde{\beta}_{r2}\neq\beta_{s2}\\ e^{-\frac{\xi_1}{\beta_{s2}}}-e^{-\frac{\xi_2}{\beta_{s2}}}\left(1+\frac{\xi_2-\xi_1}{\beta_{s2}}\right), & \tilde{\beta}_{r2}=\beta_{s2},\end{cases}$$

$$\widetilde{\mathcal{P}}_4=1-e^{-\frac{\xi_1}{\beta_{s2}}}$$

the received SNRs at the relay, the outage events at $U_1$ and $U_2$ occur regarding the combinations of $\{\varepsilon_i,\hat{\varepsilon}_j,\tilde{\varepsilon}_k\}$. Noting that non-outage events always occur when $\hat{\varepsilon}_1$ happens at $U_1$, the outage probability achieved by the ISDF scheme at $U_1$ can be expressed as

$$\begin{aligned}P_{1,\text{out}}^{\text{isdf}}&=\sum_{i=1}^{3}\sum_{k=1}^{3}\Pr\{\varepsilon_i\}\Pr\left\{\hat{\varepsilon}_2,\tilde{\varepsilon}_k,\gamma_{u_1,\text{isdf}}^{x_1}(\varepsilon_i,\hat{\varepsilon}_j,\tilde{\varepsilon}_k)<\gamma_{\text{th},1}\right\}\\&=\sum_{i=1}^{3}\sum_{k=1}^{3}\Pr\{\varepsilon_i\}\Pr\{\tilde{\varepsilon}_k\}\Pr\left\{\hat{\varepsilon}_2,\gamma_{u_1,\text{isdf}}^{x_1}(\varepsilon_i,\hat{\varepsilon}_j,\tilde{\varepsilon}_k)<\gamma_{\text{th},1}\right\}.\end{aligned}\tag{58}$$

In (58), we have applied the fact that the event $\tilde{\varepsilon}_k$ is independent of $\hat{\varepsilon}_2$. Similarly, the outage probability at $U_2$ can be expressed as

$$\begin{aligned}P_{2,\text{out}}^{\text{isdf}}=&\sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=2}^{3}\Pr\{\varepsilon_i\}\Pr\{\hat{\varepsilon}_j\}\Pr\left\{\tilde{\varepsilon}_k,\gamma_{u_2,\text{isdf}}^{x_1}(\varepsilon_i,\hat{\varepsilon}_j,\tilde{\varepsilon}_k)\right.\\&\left.<\gamma_{\text{th},1}\right\}+\Pr\{\varepsilon_i\}\Pr\{\hat{\varepsilon}_j\}\Pr\left\{\tilde{\varepsilon}_k,\gamma_{u_2,\text{isdf}}^{x_1}(\varepsilon_i,\hat{\varepsilon}_j,\tilde{\varepsilon}_k)\right.\\&\left.\geq\gamma_{\text{th},1},\gamma_{u_2,\text{isdf}}^{x_2}(\varepsilon_i,\hat{\varepsilon}_j,\tilde{\varepsilon}_k)<\gamma_{\text{th},2}\right\}.\end{aligned}\tag{59}$$

In (58) and (59), the probabilities $\Pr\{\hat{\varepsilon}_j\}$ and $\Pr\{\hat{\varepsilon}_k\}$ can be derived as

$$\Pr\{\hat{\varepsilon}_1\}=e^{-\frac{\xi_1}{\beta_{s1}}},\tag{60}$$

$$\Pr\{\hat{\varepsilon}_2\}=1-e^{-\frac{\xi_1}{\beta_{s1}}},\tag{61}$$

$$\Pr\{\tilde{\varepsilon}_1\}=e^{-\frac{\xi_{\max}}{\beta_{s2}}},\tag{62}$$

$$\Pr\{\tilde{\varepsilon}_2\}=\left[e^{-\frac{\xi_1}{\beta_{s2}}}-e^{-\frac{\xi_2}{\beta_{s2}}}\right]^+,\ \text{and}\tag{63}$$

$$\Pr\{\tilde{\varepsilon}_3\}=1-e^{-\frac{\xi_1}{\beta_{s2}}}.\tag{64}$$

**Theorem 3.** *For the ISDF scheme, the achieved outage probabilities at $U_1$ and $U_2$ are respectively given by*

$$\begin{aligned}P_{\text{out},1}^{\text{isdf}}=&\widehat{\mathcal{P}}_1\left(\Pr\{\varepsilon_1\}\Pr\{\tilde{\varepsilon}_1\}+\Pr\{\varepsilon_2\}\right)+\widehat{\mathcal{P}}_2\Pr\{\varepsilon_1\}\\&\times\left(1-\Pr\{\tilde{\varepsilon}_1\}\right)+\Pr\{\varepsilon_3\}\Pr\{\hat{\varepsilon}_2\}\end{aligned}\tag{65}$$

*and*

$$\begin{aligned}P_{\text{out},2}^{\text{isdf}}=&\widetilde{\mathcal{P}}_1\Pr\{\varepsilon_1\}\Pr\{\hat{\varepsilon}_1\}+\widetilde{\mathcal{P}}_2\Pr\{\varepsilon_1\}\\&+\widetilde{\mathcal{P}}_3\Pr\{\varepsilon_1\}\Pr\{\hat{\varepsilon}_2\}+\widetilde{\mathcal{P}}_4\Pr\{\varepsilon_2\}\\&+\Pr\{\varepsilon_2\}\Pr\{\tilde{\varepsilon}_2\}+\Pr\{\varepsilon_3\}(1-\Pr\{\tilde{\varepsilon}_1\}),\end{aligned}\tag{66}$$

*where $\widehat{\mathcal{P}}_i$ ($i=1,2$) and $\widetilde{\mathcal{P}}_j$ ($j=1,2,3,4$) are given in Table VI.*

*Proof:* See Appendix B. ■

The expressions in Theorem 3 show that the outage probability at each user consists of two parts, i.e., one part due to the *Undetermined* SNR events and another part due to the *Outage* SNR events, as marked in Table VII in Appendix B. When $\rho\to\infty$, the *Undetermined* SNR events in Table VII tend to be non-outage. Thus, asymptotic expressions for the outage probabilities are mainly determined by the asymptotic values of the part corresponding to the *Outage* SNR events, which are respectively provided as

$$P_{\text{out},1}^{\text{isdf},\infty} = \frac{\xi_1^2}{\beta_{sr}\beta_{s1}} \tag{67}$$

and

$$P_{\text{out},2}^{\text{isdf},\infty} = \begin{cases} \frac{[\xi_2-\xi_1]^+([\xi_2-\xi_1]^+ + \xi_1)}{\beta_{sr}\beta_{s2}} + \frac{\xi_1(\xi_2-\xi_1)}{\chi\beta_{s2}\beta_{r2}}, & \chi\beta_{r2} \neq \beta_{s2} \\ \frac{[\xi_2-\xi_1]^+([\xi_2-\xi_1]^+ + \xi_1)}{\beta_{sr}\beta_{s2}} + \frac{\xi_1\xi_2}{\beta_{s2}^2}, & \chi\beta_{r2} = \beta_{s2} \end{cases}. \tag{68}$$

Based on (67), it can be seen that $U_1$ achieves a diversity order of two. As such, when $\xi_2 > \xi_1$, it can be seen that $U_2$ also achieves a diversity order of two. Compared to the FR and SDF-CDRT schemes, the ISDF scheme achieves a greater diversity order for both users.

In the ISDF scheme, a relaying transmission does not occur when $\{\hat{\varepsilon}_1, \tilde{\varepsilon}_1\}$ happens. In such a subcase, the corresponding average throughput achieved at $U_1$ is given by $R_1 \Pr\{\hat{\varepsilon}_1\} \Pr\{\tilde{\varepsilon}_1\}$. Moreover, an additional relaying transmission actually occurs only when either $\varepsilon_1$ or $\varepsilon_2$ happens according to Table III, whereas a new transmission block starts when $\varepsilon_3$ happens. In such a subcase, the corresponding outage probability is the first two terms on the righthand side of (65). Therefore, the average throughput achieved by the ISDF scheme for $U_1$ can be expressed as follows:

$$\begin{aligned} R_1^{\text{isdf}} &= R_1 \Pr\{\hat{\varepsilon}_1\} \Pr\{\tilde{\varepsilon}_1\} + \frac{1}{2} R_1 \big(1 - \Pr\{\hat{\varepsilon}_1\} \Pr\{\tilde{\varepsilon}_1\}\big) \\ &\quad \times \Big(1 - \widehat{\mathcal{P}}_1 \big(\Pr\{\varepsilon_1\} \Pr\{\tilde{\varepsilon}_1\} + \Pr\{\varepsilon_2\}\big) \\ &\quad - \widehat{\mathcal{P}}_2 \Pr\{\varepsilon_1\} \big(1 - \Pr\{\tilde{\varepsilon}_1\}\big)\Big). \end{aligned} \tag{69}$$

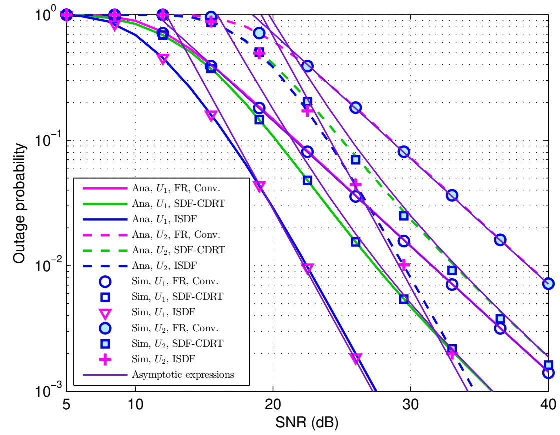
As such, the average throughput achieved by the ISDF scheme for $U_2$ is given by

$$\begin{aligned} R_1^{\text{isdf}} &= R_2 \Pr\{\hat{\varepsilon}_1\} \Pr\{\tilde{\varepsilon}_1\} + \frac{1}{2} R_2 \big(1 - \Pr\{\hat{\varepsilon}_1\} \Pr\{\tilde{\varepsilon}_1\}\big) \\ &\quad \times \Big(1 - \widetilde{\mathcal{P}}_1 \Pr\{\varepsilon_1\} \Pr\{\hat{\varepsilon}_1\} - \widetilde{\mathcal{P}}_2 \Pr\{\varepsilon_1\} - \widetilde{\mathcal{P}}_3 \\ &\quad \Pr\{\varepsilon_1\} \Pr\{\hat{\varepsilon}_2\} - \widetilde{\mathcal{P}}_4 \Pr\{\varepsilon_2\} - \Pr\{\varepsilon_2\} \Pr\{\tilde{\varepsilon}_2\}\Big). \end{aligned} \tag{70}$$

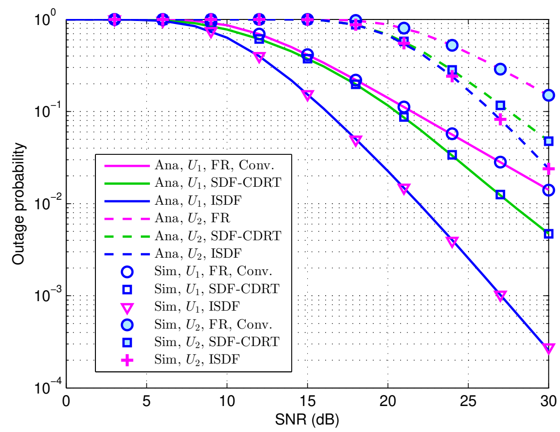
## V. Simulation Results

This section provides Monte Carlo simulation results to verify the correctness of the analytical results and evaluate the outage performance achieved by the proposed schemes. In the figures for the simulation results, the curves corresponding to the conventional FR schemes are denoted by "FR, Conv." and the curves corresponding to the DF relaying without direct links are denoted by "DF w/o DL". For simplicity, we set $\chi = 1$ assuming equal power allocation at the source and the relay.

The outage probability versus the transmit SNR is investigated in Fig. 2, where we consider two scenarios. For scenario 1, we set $\beta_{s1} = \beta_{s2} = 0.8$, and the remained $\beta_{ij} = 1$. For scenario 2, we set all $\beta_{ij} = 1$. The power allocation coefficient is set as $\alpha_1 = 0.8$ for both scenarios. In Fig. 2(a) and Fig. 2(b), the correctness of the analytical outage probability is verified by simulation results. The correctness of the asymptotic expressions for all the considered schemes is also verified in Fig. 2(a). It is shown that the ISDF scheme achieves the smallest outage probabilities for $U_1$ and $U_2$, respectively, in the whole SNR region. The SDF-CDRT scheme also achieves the smaller outage probabilities for $U_1$ and $U_2$ over those of the conventional FR scheme, respectively. In Fig. 2(a), at the $10^{-2}$ outage probability level, the ISDF and SDF-CDRT schemes respectively achieve about 9 dB and 4 dB SNR gains over that of the conventional FR scheme for $U_1$. The similar results can be observed for $U_2$ at the $10^{-2}$ outage probability level. Thus, the superior outage performance of the ISDF scheme is verified by Fig. 2. By applying selective relaying, the SDF-CDRT scheme not only prevents error propagation compared to the conventional FR scheme, but also avoids SIC when the BS-to-relay link is good enough (when $\varepsilon_1$ occurs). Due to error propagation at the relay, the conventional FR scheme achieves the highest outage probability in Fig. 2(a) and Fig. 2(b).



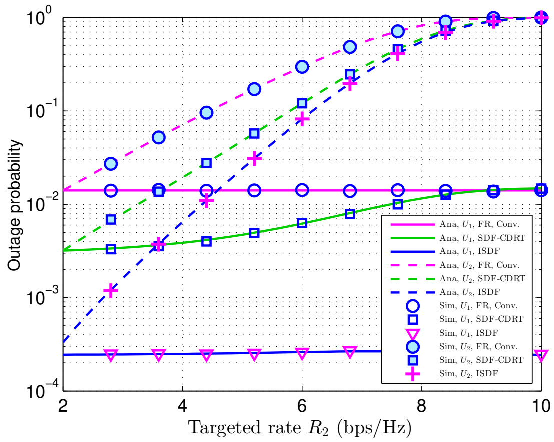

(a) Scenario 1 ($R_1 = 2$ bps/Hz, $R_2 = 4$ bps/Hz).



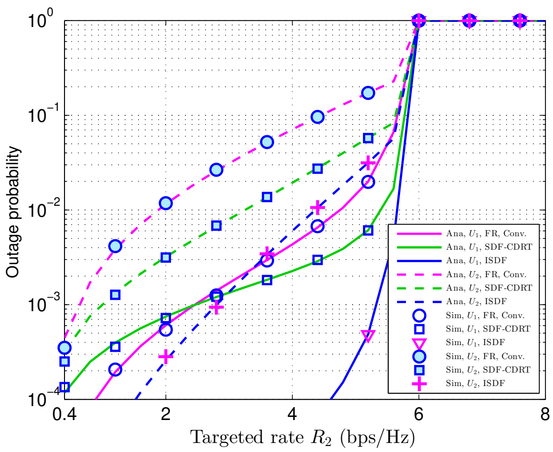

(b) Scenario 2 ($R_1 = 2$ bps/Hz, $R_2 = 5$ bps/Hz).

Fig. 2. Outage probability versus SNR.

In Fig. 3, we investigate the impact of the target rate on the outage probability, where we set $\rho = 30$ dB and $\alpha_1 = 0.8$. In Fig. 3(a), we fix $R_1 = 2$ bps/Hz and increase $R_2$ from 2 to 10 bps/Hz. As it is shown in Fig. 3(a), the SDF-CDRT and ISDF scheme achieve the smaller outage probabilities than those of the conventional FR scheme for $U_1$ and $U_2$, respectively. For all the considered relaying schemes, the outage probability of $U_2$ increases by increasing $R_2$ and approaches one in the high target rate region. Also, the outage probability of $U_1$ achieved by the SDF-CDRT scheme increases by increasing




(a) Fixed $R_1 = 2$ bps/Hz in scenario 1.




(b) $R_1 = 0.4R_2$ in scenario 2.

Fig. 3. Outage probability versus target rate.

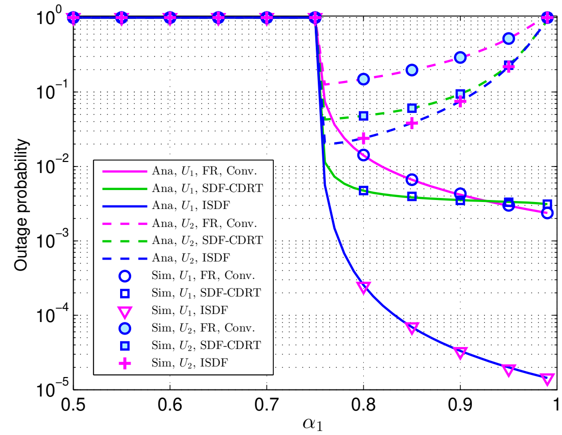
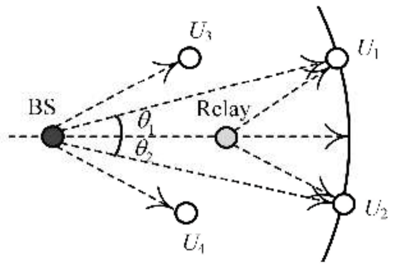



Fig. 4. Outage probability versus power allocation coefficient.

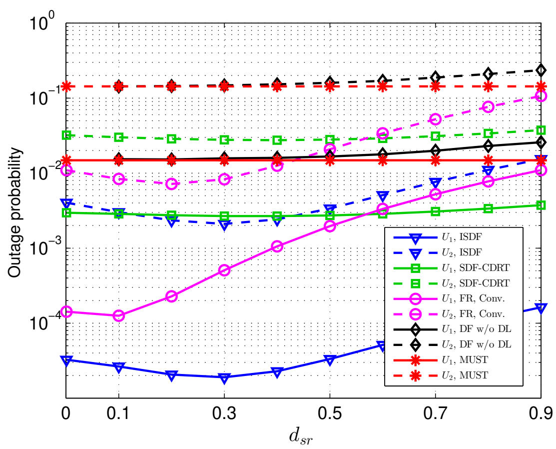
$R_2$. In contrast, the outage probability of $U_1$ achieved by the conventional FR scheme keeps constant irrespective of the variation of $R_2$, while the outage probability of $U_1$ achieved by the ISDF scheme almost keeps constant by increasing $R_2$. In Fig. 3(b), we set $R_1 = 0.4R_2$ and investigate the impact of the target rate on the outage performance for different schemes. Note that when both $R_1$ and $R_2$ increase, the affordable outage performances of the different schemes can be observed. For $U_1$ and $U_2$, Fig. 3(b) shows that the outage probabilities achieved by all the schemes first increase by increasing the target rate and then jump to one as the target rate reaches a large value. It is shown that the ISDF scheme achieves the smallest outage probabilities for $U_1$ and $U_2$, respectively. The SDF-CDRT scheme also achieves the smaller outage probability for $U_1$ than that of the conventional FR scheme. However, for $U_2$, the SDF-CDRT scheme achieves the higher outage probability than that of the conventional FR scheme in the low target region. In the middle and high target rate regions, the SDF-CDRT scheme achieves the smaller outage probability than that of the conventional FR scheme, which is preferred by a higher QoS requirement.

To see the details of the impact of the power allocation on the outage performance, we investigate the outage probability versus $\alpha_1$ in Fig. 4, where we set $\rho = 30$ dB, $R_1 = 2$ bps/Hz, and $R_2 = 5$ bps/Hz in scenario 2. It is shown in Fig. 4 that the ISDF scheme achieves the smallest outage probabilities for $U_1$ and $U_2$ in the whole $\alpha_1$ region. For $U_2$, the SDF-CDRT scheme achieves the smaller outage probability than that of the conventional FR scheme. However, for $U_1$, the conventional FR scheme achieves the smaller outage probability than that of the SDF-CDRT scheme in the high $\alpha_1$ region.

To investigate the impact of the relay's location on the outage performance, we also consider a geometric location model as depicted in Fig. 5(a), in which the BS is located at the center of a unit circle and two far users ($U_1$ and $U_2$) are located on the edge of the circle. For the comparison with the MUST scheme, we assume that two near users ($U_3$ and $U_4$) are located near the BS. In the geometric location model, we set the path loss model by $\beta_{ij} = d_{ij}^{-\tau}$, where the path loss exponent is set as $\tau = 3$ and $d_{ij}$ is the distance between $i \in \{s, r\}$ and $j \in \{r, 1, 2, 3, 4\}$. For the simulation results in Fig. 5(b), we set $\rho = 30$ dB, $R_1 = 2$ bps/Hz, $R_2 = 5$ bps/Hz, $\theta_1 = 15°$, and $\theta_2 = -15°$. In the simulation, the relay moves away from the BS along the vertical direction. For the comparison purpose, the outage probabilities at $U_1$ and $U_2$ achieved by the DF relay without direct links and the direct link transmission using the MUST scheme ($U_1$ and $U_2$ are paired in terms of their target transmission rates) are also presented. It is shown that the ISDF scheme achieves the smallest outage probabilities for $U_1$ and $U_2$, respectively. As the distance $d_{sr}$ increases from 0 to 0.9, the outage probabilities achieved by the ISDF scheme first decreases and then increases. Moreover, as $d_{sr}$ approaches zero, the ISDF scheme becomes the MUST scheme with the retransmission protocol, which is described in Table II. However, this figure shows that the optimal location of the relay is not $d_{sr} = 0$, so that the ISDF scheme can effectively decrease the outage probability even compared with MUST with the retransmission protocol. In the small and middle $d_{sr}$ regions, it is shown that the conventional FR scheme achieves the smaller outage probabilities than those of the SDF-CDRT scheme for $U_1$ and $U_2$, respectively. However, when the relay is located more close to the users, the SDF-CDRT scheme achieves the smaller outage probabilities than those of the conventional FR scheme for $U_1$ and $U_2$, respectively. For the SDF-CDRT scheme, the

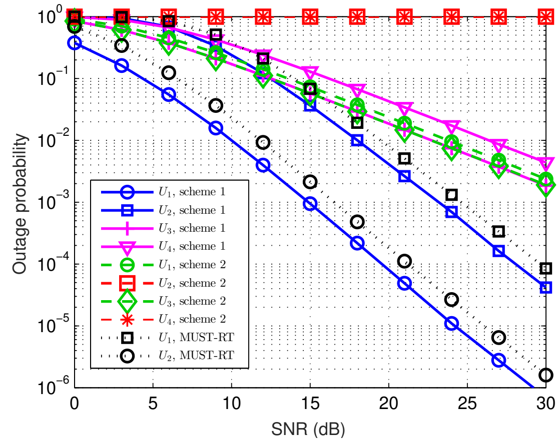



(a) The geometric location model.

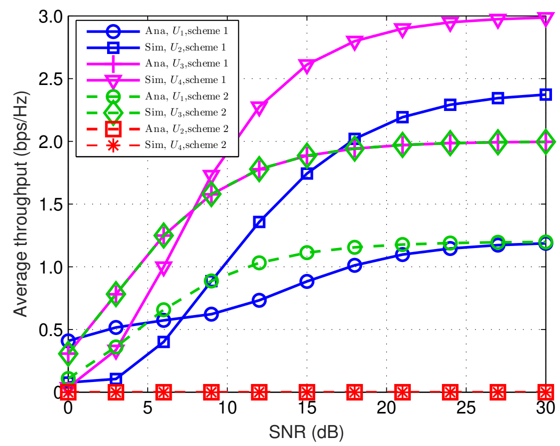



(b) Outage probability versus relay location.

Fig. 5. The impact of the relay location on the outage probability.




(a) Outage probability versus SNR.




(b) Average throughput versus SNR.

Fig. 6. Performance comparison between the ISDF and MUST schemes.

achieved outage probability at $U_1$ increases by increasing $d_{sr}$, whereas the achieved outage probability at $U_2$ first decreases and then increases by increasing $d_{sr}$. For the SDF-CDRT scheme, the achieved outage probabilities also first decrease and then increase by increasing $d_{sr}$. However, the variations are not dramatic compared to those of the ISDF and SDF-CDRT schemes. Moreover, all the considered schemes with direct links achieve the better outage performance over those of the "DF w/o DL" and MUST for the paired $U_1$ and $U_2$.

Considering the MUST scheme that pairs a far user and a near user in terms of their channel qualities, we compare the outage probability achieved by the proposed ISDF and MUST schemes in Fig. 6. The target transmission rates for $U_1$, $U_2$, $U_3$, and $U_4$ are set as 1.2 bps/Hz, 2.4 bps/Hz, 2 bps/Hz, and 3 bps/Hz, respectively. The distances from the BS to the two near users ($U_3$ and $U_4$) are set by $d_{s3} = d_{s4} = 0.5$, as depicted in Fig. 5(a). The relay is located by $d_{sr} = 0.3$ referring to the results in Fig. 5(b). We assume that channel quality based pairing and QoS requirement based pairing are applied in scheme 1 and scheme 2, respectively. That is, in scheme 1, $U_1$ and $U_2$ are paired and the ISDF is deployed with the aid of the relay; $U_3$ and $U_4$ are paired in terms of their target transmission rates. In contrast, MUST is adopted in scheme 2, i.e., the far user $U_1$ is paired with the near user $U_3$, meanwhile $U_2$ is paired with $U_4$. For the two users in each pair, we assume that $\alpha_1 = 0.8$ and $\alpha_2 = 0.2$ are applied for the power allocation.

It is shown in Fig. 6(a) that scheme 1 achieves lower outage probabilities for $U_1$, $U_2$, and $U_4$ than those of scheme 2. Only for $U_3$ do the two schemes achieve the same outage probabilities. When scheme 2 is applied, the outage probabilities of both $U_2$ and $U_4$ are both equal to one, which indicates that MUST cannot support a higher rate transmission for the paired $U_2$ and $U_4$. However, scheme 1 achieves a lower outage probability for $U_2$ than those of $U_3$ and $U_4$ in the middle and high SNR regions, even if the far user $U_2$ has a relatively higher target transmission rate. Note that scheme 1 degenerates to MUST with retransmission (MUST-RT) when $d_{sr} = 0$ regarding Table II. The outage probabilities achieved by MUST-RT for the paired $U_1$ and $U_2$ are also presented in Fig. 6(a). It is shown that MUST-RT achieves larger outage probabilities for the paired $U_1$ and $U_2$ than those of scheme 1. Under the same simulation parameters, the average throughputs achieved by scheme 1 and scheme 2 are plotted in Fig. 6(b). The results in Fig. 6(b) clearly show that scheme 1 achieves non-zero average throughputs for all the users in the whole SNR region, whereas scheme 2 achieves zero throughputs for the paired $U_2$ and $U_4$. For the far user $U_2$ with a relatively higher target transmission rate, scheme 1 achieves a remarkable average throughput in the whole SNR region. The similar results are also observed for $U_4$. Although scheme 2 achieves a larger average throughput for $U_1$ in the middle SNR region than scheme 1, obviously scheme 1 achieves a much larger sum rate than scheme 2. Therefore, the proposed ISDF scheme provides better rate performance than the MUST scheme, especially when the far user has demanding QoS requirements.

## VI. Conclusions

In this paper, we have investigated the FR, SDF-CDRT, and ISDF schemes for the cooperative NOMA system with non-negligible direct links from the BS to two far users. To reduce error propagation resulted from the incorrect detections at the relay, the SDF-CDRT and ISDF schemes take into account the received SNRs at the relay and operate in the DF mode adaptively. When the relay can detect its received signal correctly, the SDF-CDRT can avoid SIC at the receiver side by forming an orthogonal transmission branch with respect to the direct link. For the ISDF scheme, unnecessary relaying can be reduced by exploiting the limited feedback of the received SNR events from the two users. For the considered three cooperative relaying schemes, the outage probabilities have been derived in closed-form for the two users along with the asymptotic expressions for the outage probabilities and diversity orders. Simulation results have verified the superior outage performance achieved by the proposed cooperative relaying schemes over those of the cooperative DF relaying without direct links and MUST without relaying.

## Appendix A: A Derivation for $\mathcal{B}_1$ and $\mathcal{B}_1$

In this part, we evaluate the terms $\mathcal{B}_1$ and $\mathcal{B}_2$. The term $\mathcal{B}_1(\varepsilon_1)$ can be rewritten as

$$\mathcal{B}_1(\varepsilon_1) = \Pr\left\{Y < \frac{2\alpha_1\alpha_2\gamma_{\text{th},1}}{\rho}\right\}, \tag{A.1}$$

where the RV $Y$ is defined as

$$Y \triangleq \frac{2Y_1Y_2}{Y_1+Y_2} \tag{A.2}$$

with $Y_1 \triangleq \alpha_2|h_{s1}|^2$ and $Y_2 \triangleq \alpha_1|h_{r1}|^2$ following i.n.i.d. exponential distributions. Since that $Y$ is the harmonic mean of two i.n.i.d exponential RVs, the cumulative distribution function (CDF) of $Y$ can be expressed as [42]

$$F_Y(y) = 1 - \frac{y}{\sqrt{\alpha_1\alpha_2\beta_{s1}\tilde{\beta}_{r1}}} e^{-\frac{y}{2}\left(\frac{1}{\alpha_2\tilde{\beta}_{r1}}+\frac{1}{\alpha_1\beta_{s1}}\right)} \times K_1\left(\frac{y}{\sqrt{\alpha_1\alpha_2\beta_{s1}\tilde{\beta}_{r1}}}\right), \tag{A.3}$$

where $K_1(\cdot)$ is the first order modified Bessel function of the second kind [43, Eq. (8.432)]. With the obtained (A.3), $\mathcal{B}_1(\varepsilon_1)$ can be evaluated as

$$\mathcal{B}_1(\varepsilon_1) = 1 - \frac{2\gamma_{\text{th},1}}{\rho}\sqrt{\frac{\alpha_1\alpha_2}{\beta_{s1}\tilde{\beta}_{r1}}} e^{-\left(\frac{\alpha_2}{\tilde{\beta}_{r1}}+\frac{\alpha_1}{\beta_{s1}}\right)\frac{\gamma_{\text{th},1}}{\rho}} \times K_1\left(\frac{2\gamma_{\text{th},1}}{\rho}\sqrt{\frac{\alpha_1\alpha_2}{\beta_{s1}\tilde{\beta}_{r1}}}\right). \tag{A.4}$$

Similarly, we can evaluate $\mathcal{B}_2(\varepsilon_1)$ as

$$\mathcal{B}_2(\varepsilon_1) = 1 - \frac{2\gamma_{\text{th},2}}{\rho}\sqrt{\frac{\alpha_1\alpha_2}{\beta_{s2}\tilde{\beta}_{r2}}} e^{-\left(\frac{\alpha_2}{\tilde{\beta}_{r2}}+\frac{\alpha_1}{\beta_{s2}}\right)\frac{\gamma_{\text{th},2}}{\rho}} \times K_1\left(\frac{2\gamma_{\text{th},2}}{\rho}\sqrt{\frac{\alpha_1\alpha_2}{\beta_{s2}\tilde{\beta}_{r2}}}\right). \tag{A.5}$$

For $\mathcal{B}_1(\varepsilon_2)$ and $\mathcal{B}_1(\varepsilon_3)$, it can be easily shown that $\mathcal{B}_1(\varepsilon_2) = \mathcal{B}_1(\varepsilon_3) = 1 - e^{-\frac{\xi_1}{\beta_{s1}}}$. For $\mathcal{B}_2(\varepsilon_2)$ and $\mathcal{B}_2(\varepsilon_3)$, we have $\mathcal{B}_2(\varepsilon_2) = \mathcal{B}_2(\varepsilon_3)$, which can be evaluated by

$$\begin{aligned}\mathcal{B}_2(\varepsilon_i) &= \Pr\{|h_{s2}|^2 < \xi_1\} + \Pr\{|h_{s2}|^2 \geq \xi_1, |h_{s2}|^2 < \xi_2\} \\ &= 1 - e^{-\frac{\xi_1}{\beta_{s2}}} + \left[e^{-\frac{\xi_1}{\beta_{s2}}} - e^{-\frac{\xi_2}{\beta_{s2}}}\right]^+. \end{aligned} \tag{A.6}$$

## Appendix B: A proof of Theorem 3

The end-to-end received SNR events at $U_1$ and $U_2$ can be summarized in Table VI. Based on the results in Table VI, the undetermined SNR events at $U_1$ can be classified into two types, i.e., $\{(33), \hat{\varepsilon}_2\}$ and $\{(36), \hat{\varepsilon}_2\}$. Regarding $\{(33), \hat{\varepsilon}_2\}$ and $\{(36), \hat{\varepsilon}_2\}$, we define

$$\hat{\mathcal{P}}_1 \triangleq \Pr\left\{\tfrac{\rho(\alpha_1|h_{s1}|^2+|\tilde{h}_{r1}|^2)}{\alpha_2\rho|h_{s1}|^2+1} < \gamma_{\text{th},1}, \hat{\varepsilon}_2\right\} \text{ and} \tag{B.1}$$

$$\hat{\mathcal{P}}_2 \triangleq \Pr\left\{\tfrac{\alpha_1\rho(|h_{s1}|^2+|\tilde{h}_{r1}|^2)}{\alpha_2\rho(|h_{s1}|^2+|\tilde{h}_{r1}|^2)+1} < \gamma_{\text{th},1}, \hat{\varepsilon}_2\right\}, \tag{B.2}$$

respectively. Since that all the channel gains follow exponential distributions, $\hat{\mathcal{P}}_1$ and $\hat{\mathcal{P}}_2$ can be respectively evaluated as

$$\begin{aligned}\hat{\mathcal{P}}_1 &= \Pr\left\{|h_{s1}|^2 < \xi_1\big(1 - \tfrac{\rho|\tilde{h}_{r1}|^2}{\gamma_{\text{th},1}}\big), |h_{s1}|^2 < \xi_1\right\} \\ &= \begin{cases} \frac{\beta_{s1}\gamma_{\text{th},1}\left(1-e^{-\frac{\xi_1}{\beta_{s1}}}\right) - \tilde{\beta}_{r1}\rho\xi_1\left(1-e^{-\frac{\gamma_{\text{th},1}}{\tilde{\beta}_{r1}\rho}}\right)}{\beta_{s1}\gamma_{\text{th},1}-\tilde{\beta}_{r1}\rho\xi_1}, & \beta_{s1} \neq \frac{\rho\tilde{\beta}_{r1}\xi_1}{\gamma_{\text{th},1}} \\ \gamma\big(2, \frac{\xi_1}{\beta_{s1}}\big), & \beta_{s1} = \frac{\rho\tilde{\beta}_{r1}\xi_1}{\gamma_{\text{th},1}} \end{cases}\end{aligned} \tag{B.3}$$

and

$$\begin{aligned}\hat{\mathcal{P}}_2 &= \Pr\left\{|h_{s1}|^2 + |\tilde{h}_{r1}|^2 < \xi_1, |h_{s1}|^2 < \xi_1\right\} \\ &= \begin{cases} \frac{\tilde{\beta}_{r1}\left(1-e^{-\frac{\xi_1}{\tilde{\beta}_{r1}}}\right) - \beta_{s1}\left(1-e^{-\frac{\xi_1}{\beta_{s1}}}\right)}{\tilde{\beta}_{r1}-\beta_{s1}}, & \beta_{s1} \neq \tilde{\beta}_{r1} \\ \gamma\big(2, \frac{\xi_1}{\beta_{s1}}\big), & \beta_{s1} = \tilde{\beta}_{r1}. \end{cases}\end{aligned} \tag{B.4}$$

Then, regarding Table VII, the outage probability in (58) can be expressed as

$$\begin{aligned}P_{\text{out},1}^{\text{isdf}} &= \hat{\mathcal{P}}_1\big(\Pr\{\varepsilon_1\}\Pr\{\tilde{\varepsilon}_1\} + \Pr\{\varepsilon_2\}\Pr\{\tilde{\varepsilon}_1\} \\ &\quad + \Pr\{\varepsilon_2\}\Pr\{\tilde{\varepsilon}_2\} + \Pr\{\varepsilon_2\}\Pr\{\tilde{\varepsilon}_3\}\big) \\ &\quad + \hat{\mathcal{P}}_2\big(\Pr\{\varepsilon_1\}\Pr\{\tilde{\varepsilon}_2\} + \Pr\{\varepsilon_1\}\Pr\{\tilde{\varepsilon}_3\}\}\big) \\ &\quad + \Pr\{\varepsilon_3\}\big(\Pr\{\hat{\varepsilon}_2\}\Pr\{\tilde{\varepsilon}_1\} + \Pr\{\hat{\varepsilon}_2\}\Pr\{\tilde{\varepsilon}_2\} \\ &\quad + \Pr\{\hat{\varepsilon}_2\}\Pr\{\tilde{\varepsilon}_3\}\big) \\ &= \hat{\mathcal{P}}_1\big(\Pr\{\varepsilon_1\}\Pr\{\tilde{\varepsilon}_1\} + \Pr\{\varepsilon_2\}\big) \\ &\quad + \hat{\mathcal{P}}_2\Pr\{\varepsilon_1\}\big(1 - \Pr\{\tilde{\varepsilon}_1\}\big) + \Pr\{\varepsilon_3\}\Pr\{\hat{\varepsilon}_2\}, \end{aligned} \tag{B.5}$$

which proves the case for $U_1$.

According to Table VII, the undetermined SNR events at $U_2$ can be classified into four types, i.e., $\{(29), \tilde{\varepsilon}_2\}$,

TABLE VII
RECEIVED SNR EVENTS IN THE ISDF SCHEME

| | $\{\hat{\varepsilon}_1, \tilde{\varepsilon}_1\}$ | | $\{\hat{\varepsilon}_1, \tilde{\varepsilon}_2\}$ | | $\{\hat{\varepsilon}_1, \tilde{\varepsilon}_3\}$ | |
|---|---|---|---|---|---|---|
| | $U_1$ | $U_2$ | $U_1$ | $U_2$ | $U_1$ | $U_2$ |
| $\varepsilon_1$ | $\gamma_{1,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} \geq \gamma_{\text{th},2}$ | $\gamma_{1,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} = (29)$ | $\gamma_{1,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} = (31)$<br>$\gamma_{2,\text{isdf}}^{x_2} = (32)$ |
| $\varepsilon_2$ | $\gamma_{1,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_1 2} \geq \gamma_{\text{th},2}$ | $\gamma_{1,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} < \gamma_{\text{th},2}$ | $\gamma_{1,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} = (38)$<br>$\gamma_{2,\text{isdf}}^{x_2} = (39)$ |
| $\varepsilon_3$ | $\gamma_{1,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} \geq \gamma_{\text{th},2}$ | $\gamma_{1,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} < \gamma_{\text{th},2}$ | $\gamma_{1,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} < \gamma_{\text{th},1}$ |
| | $\{\hat{\varepsilon}_2, \tilde{\varepsilon}_1\}$ | | $\{\hat{\varepsilon}_2, \tilde{\varepsilon}_2\}$ | | $\{\hat{\varepsilon}_2, \tilde{\varepsilon}_3\}$ | |
| | $U_1$ | $U_2$ | $U_1$ | $U_2$ | $U_1$ | $U_2$ |
| $\varepsilon_1$ | $\gamma_{1,\text{isdf}}^{x_1} = (33)$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} \geq \gamma_{\text{th},2}$ | $\gamma_{1,\text{isdf}}^{x_1} = (36)$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} = (32)$ | $\gamma_{1,\text{isdf}}^{x_1} = (36)$ | $\gamma_{2,\text{isdf}}^{x_1} = (31)$<br>$\gamma_{2,\text{isdf}}^{x_1 2} = (32)$ |
| $\varepsilon_2$ | $\gamma_{1,\text{isdf}}^{x_1} = (33)$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} \geq \gamma_{\text{th},2}$ | $\gamma_{1,\text{isdf}}^{x_1} = (33)$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} < \gamma_{\text{th},2}$ | $\gamma_{1,\text{isdf}}^{x_1} = (33)$ | $\gamma_{2,\text{isdf}}^{x_1} = (38)$<br>$\gamma_{2,\text{isdf}}^{x_2} = (39)$ |
| $\varepsilon_3$ | $\gamma_{1,\text{isdf}}^{x_1} < \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} \geq \gamma_{\text{th},2}$ | $\gamma_{1,\text{isdf}}^{x_1} < \gamma_{\text{th},1}$ | $\gamma_{2,\text{isdf}}^{x_1} \geq \gamma_{\text{th},1}$<br>$\gamma_{2,\text{isdf}}^{x_2} < \gamma_{\text{th},2}$ | $\gamma_{1,\text{isdf}}^{x_1} < \gamma_{\text{th},1}$ | $\gamma_{1,\text{isdf}}^{x_1} < \gamma_{\text{th},1}$ |

$U_1$: Non-outage Outage Undetermined

$U_2$: Non-outage Outage Undetermined

$\{(31), (32), \tilde{\varepsilon}_3\}$, $\{(32), \tilde{\varepsilon}_2\}$, and $\{(38), (39), \tilde{\varepsilon}_3\}$. With respect to these four types of the undetermined SNR events, we define

$$\widetilde{\mathcal{P}}_1 \triangleq \Pr\{\rho(\alpha_2|h_{s2}|^2 + |\tilde{h}_{r2}|^2) < \gamma_{\text{th},2}, \tilde{\varepsilon}_2\}, \quad \text{(B.6)}$$

$$\begin{aligned} \widetilde{\mathcal{P}}_2 \triangleq \; & \Pr\left\{\frac{\alpha_1\rho(|h_{s2}|^2+|\tilde{h}_{r2}|^2)}{\alpha_2\rho(|h_{s2}|^2+|\tilde{h}_{r2}|^2)+1} < \gamma_{\text{th},1}, \tilde{\varepsilon}_3\right\} \\ & + \Pr\left\{\frac{\alpha_1\rho(|h_{s2}|^2+|\tilde{h}_{r2}|^2)}{\alpha_2\rho(|h_{s2}|^2+|\tilde{h}_{r2}|^2)+1} \geq \gamma_{\text{th},1},\right. \\ & \left.\alpha_2\rho(|h_{s2}|^2 + |\tilde{h}_{r2}|^2) < \gamma_{\text{th},2}, \tilde{\varepsilon}_3\right\}, \end{aligned} \quad \text{(B.7)}$$

$$\widetilde{\mathcal{P}}_3 \triangleq \Pr\{\alpha_2\rho(|h_{s2}|^2 + |\tilde{h}_{r2}|^2) < \gamma_{\text{th},2}, \tilde{\varepsilon}_2\}, \quad \text{and} \quad \text{(B.8)}$$

$$\begin{aligned} \widetilde{\mathcal{P}}_4 \; \triangleq \; & \Pr\left\{\frac{\rho(\alpha_1|h_{s2}|^2+|\tilde{h}_{r2}|^2)}{\alpha_2\rho|h_{s2}|^2+1} < \gamma_{\text{th},1}, \tilde{\varepsilon}_3\right\} \\ & + \Pr\left\{\frac{\rho(\alpha_1|h_{s2}|^2+|\tilde{h}_{r2}|^2)}{\alpha_2\rho|h_{s2}|^2+1} \geq \gamma_{\text{th},1},\right. \\ & \left.\alpha_2\rho|h_{s2}|^2 < \gamma_{\text{th},2}, \tilde{\varepsilon}_3\right\}, \end{aligned} \quad \text{(B.9)}$$

respectively. Taking into that all the channel gains follow exponential distributions, $\widetilde{\mathcal{P}}_1$, $\widetilde{\mathcal{P}}_2$, $\widetilde{\mathcal{P}}_3$, and $\widetilde{\mathcal{P}}_4$ can be respectively evaluated as those in Table VI. As such, the outage probability at $U_2$ can be expressed as (66).

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