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Decoherence and entropy production in relativistic nuclear collisions

Rainer J. Fries
Cyclotron Institute and Department of Physics, Texas A&M University, College Station, Texas 77801, USA and RIKEN/BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA

Berndt Müller
Department of Physics, Duke University, Durham, North Carolina 27708, USA

Andreas Schäfer
Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany
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Short thermalization times of less than 1 fm/c for quark and gluon matter have been suggested by recent experiments at the Relativistic Heavy Ion Collider. It has been difficult to justify this rapid thermalization in first-principle calculations based on perturbation theory or the color glass condensate picture. Here, we address the related question of the decoherence of the gluon field, which is a necessary component of thermalization. We present a simplified leading-order computation of the decoherence time of a gluon ensemble subject to an incoming flux of Weizsäcker-Williams gluons. We also discuss the entropy produced during the decoherence process and its relation to the entropy in the final state that has been measured experimentally.

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I. INTRODUCTION

Collisions of nuclei at very high energies have been studied at the Relativistic Heavy Ion Collider (RHIC) in recent years to explore the formation and the properties of the quark gluon plasma (QGP). One striking discovery was the fact that ideal hydrodynamics could describe many salient features of the expansion and cooling of the fireball [1–3]. In particular, the azimuthal asymmetry in collisions with nonzero impact parameter requires an onset of the hydrodynamic expansion at rather early times, between 0.5 and 1 fm/c. This represents a puzzle because the application of ideal hydrodynamics mandates complete thermalization of the system.

Scenarios based on the picture of a dilute system of perturbatively interacting partons and minijets fail to describe early thermalization [4] and the formation of anisotropic transverse flow [5]. Multiparton interactions, involving either ternary collisions [6,7] or collective effects mediated by plasma instabilities [8], have been proposed as solutions to this problem. However, both mechanisms require a large initial entropy production before they can become effective. The color glass condensate model [9,10] seemingly offers a viable explanation of the conundrum. It introduces the saturation scale Q_s that sets the scale for all dynamical processes. For RHIC [11] the magnitude of Q_s^2 is estimated to be approximately 2 GeV^2, and thus thermalization times of order $1/Q_s$ do not seem impossible. However, the production of entropy is a nontrivial problem in any model that is based on the assumption of the dominance of classical fields in the initial state, like the color glass condensate model [12,13].

The situation is further complicated by the fact that it is not clear how close quark-gluon matter must be to complete thermalization for hydrodynamics to be successful. It is also conceivable that a hydrodynamic evolution starting at a later time and supplemented by other mechanisms of transverse dynamics during the long *off-equilibrium* phase can give an equally good or even better description of the data. For example, the anisotropic collective transverse flow may be generated, in part, by interactions of minijets with the bulk medium [14] or by anisotropies in the initial gluon field [15]. Hydrodynamic calculations with viscous corrections describing small deviations from equilibrium start to become available [16–18] and may soon help to test this possibility quantitatively.

One necessary ingredient for thermalization is the decoherence of the initial gluon field. Coherent fields can lead to large anisotropies in pressure and even negative pressure, which are symptoms of a state very far from thermal equilibrium. Thus the decoherence time τ_{dec} should be even smaller than the equilibration time τ_{th}. We argue that the fundamental process at work is somewhat analogous to Coulomb explosion imaging, see Ref. [19], used routinely in molecular physics. If a molecule transverses a very thin metal foil all bonds are broken, the ions decohere and fly apart. From the momentum distribution of the fragments one can then extract information on the original wave function. In heavy-ion collisions each Lorentz contracted ion acts like such a foil for the other.

That the loss of information due to decoherence can generate a rapid increase in entropy in early phases of heavy-ion collisions was realized early, e.g., by Elze [20]. The fact that the hydrodynamic evolution is known to be very close to the ideal one and thus isentropic in heavy-ion collisions, further stresses the need for massive entropy production in very early phases. Indeed, two of us have argued, in Ref. [21], that decoherence can easily generate a large fraction of the total produced entropy. However, the question of the appropriate time scale of entropy production remained open.

A computation of the decoherence time in leading order in perturbation theory was recently presented by two of us in Ref. [22]. Here, we want to strengthen this argument by presenting a calculation of the decoherence time as a function of the gluon two-point function in the nucleus. We then proceed to evaluate our general result within the framework of the McLerran-Venugopalan model [9]. This is made possible by new results for the effects of the running of the coupling constant on the color glass condensate [23], which solve a hitherto unresolved UV problem. Our result agrees with that of Ref. [22] within theoretical uncertainties and suggests that, indeed, $\tau_{\rm dec} \sim Q_s^{-1}$, and that $\tau_{\rm dec}$ is numerically smaller than 1 fm/c at RHIC. In Sec. V we can then revisit some of our previous arguments [21] about entropy production in the framework of the short decoherence times at RHIC.

II. THE DECOHERENCE TIME

We describe the gluons in a nucleus by a density operator D. This nucleus is subjected to the incident gluon field of a second, large, and very fast nucleus scattering off it. We compute the time evolution of the density operator D under the influence of the perturbation presented by the second nucleus. In the following we denote the initial unperturbed gluon field of the first nucleus with A', the final gluon field with A, and the field of the second nucleus with B. The decoherence time of the gluon field A is defined as the inverse decay time of the ratio

$$\frac{\mathrm{Tr}\, D^2(t)}{[\mathrm{Tr}\, D(t)]^2}. \tag{1}$$

Let us introduce some useful notations. We deal with matrix elements $D_{\hat{A},A} = \langle \hat{A}|D|A\rangle$ of the density matrix. We can treat the final gluon field as almost on-shell for long times after the collision. In practice that means that we can decompose it in free modes $|A\rangle = |k, \lambda, a\rangle$ characterized by momentum k, polarization λ, and color a. We can use the same technique for the field B of the fast-moving nucleus 2 that is Weizsäcker-Williams-like. However, the initial-state gluons A' in nucleus 1 are in a bound state and generally off-shell. We do not attempt to describe this field in detail. It turns out that the only two ingredients needed are an ansatz for the matrix elements $D_{\hat{A}',A'}$ of the density matrix of the bound fields and the matrix elements $H_{A',A}$ of the Hamiltonian coupling the fields A, B, and A'.

We are interested in processes in which gluon modes A' of the nucleus at rest (which are centered around rapidity $Y = 0$) are scattered into modes A with large longitudinal momenta ($Y > 1$) so that the overlap with the initial state is very small. In that case the leading contribution in the time evolution comes from second-order perturbation theory

$$D_{\hat{A},A}(t) = \sum_{\hat{A}',A'} \int_0^t d\hat{t}dt' H_{\hat{A},\hat{A}'}(\hat{t}) D_{\hat{A}',A'}(0) H_{A',A}(t'). \tag{2}$$

Note that we have suppressed the field B in the notation of the matrix elements. We treat the field B rather as an external parameter given by the second nucleus. The interpretation of this process is illustrated in Fig. 1. The rapidity distributions of the initial and final gluons are schematically shown in Fig. 2.

We have to specify the relevant part of the Hamiltonian matrix element $H_{A',A}$ that is the three-gluon vertex with fields

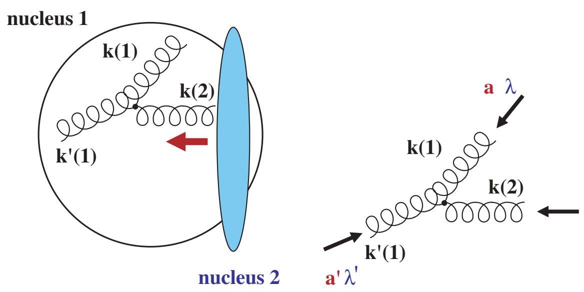

FIG. 1. (Color online) Lowest-order perturbative process that contributes to the time evolution of the matrix element $D_{\hat{A},A}$ for fields $\hat{A}$, A that are separated from the initial fields $\hat{A}'$ and A' by a rapidity gap.

A, A', and B. It is given by

$$H_{A',A} = ig \sum_{a',c,\lambda'} \int d^3x \int \frac{d^4l}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4} \frac{e^{i(k-k'-l)\cdot x}}{\sqrt{2k^+V}}$$
$$\times f^{ca'a} \mathcal{B}^c_\mu(l)[(k+k')^\mu \epsilon^{\nu*}(k,\lambda)\epsilon_\nu(k',\lambda')\mathcal{A}'^{a'}(k',\lambda')$$
$$-(k+l)^\nu \epsilon^{\mu*}(k,\lambda)\epsilon_\nu(k',\lambda')\mathcal{A}'^{a'}(k',\lambda')$$
$$+(l-k')^\nu \epsilon^*_\nu(k,\lambda)\epsilon^\mu(k',\lambda')\mathcal{A}'^{a'}(k',\lambda')]. \tag{3}$$

Here t^a are the adjoint SU(3) generators, f^{abc} are the structure constants of SU(3) and g is the coupling constant. Note that we have used Fourier transformations of the operators of the initial gluon field and the field of the second nucleus

$$A'_\nu(x) = \sum_{a',\lambda'} \int \frac{d^4k'}{(2\pi)^4} [e^{-ik'\cdot x} A'^{a'}_{k',\lambda'} \epsilon_\nu(k',\lambda') t^{a'} + \mathrm{h.c.}] \tag{4}$$

$$B^\mu(x) = \sum_c \int \frac{d^4l}{(2\pi)^4} [e^{-il\cdot x} B^{c\mu}(l) t^c + \mathrm{h.c.}] \tag{5}$$

with operators $A'^{a'}_{k',\lambda'}$ and $B^{c\mu}(l)$. This is similar to the usual expansion of free fields, which we apply in our case to the final

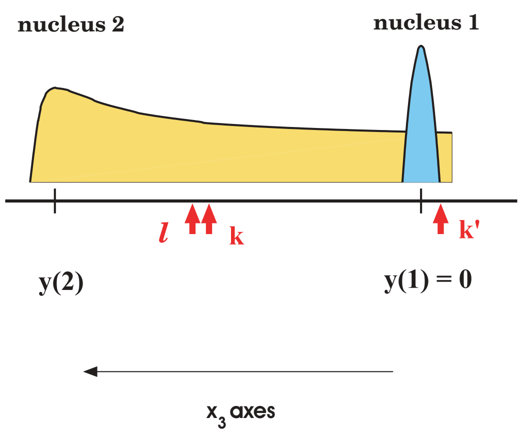

FIG. 2. (Color online) Sketch of the relevant rapidity distributions. The rapidity of the gluon 1 in the nucleus at rest $k'(1)$ is close to zero. The Weizsäcker-Williams gluon with momentum l has a nearly boost-invariant rapidity distribution. The distribution of the final gluon momentum $k(1)$ is, therefore, also nearly boost-invariant.

gluon field A. Equation (3) is then analogous to the familiar case of three interacting fields that are asymptotically free. However, to respect the unknown dynamics of the initial field we do not specify the result of the operators $A'^{a'}_{k',\lambda'}$ and $B^{c\mu}(l)$ acting on the initial states. Rather, we use the matrix elements

$$\mathcal{A}'^{a'}(k',\lambda') = \langle 0|A^{a'}_{k',\lambda'}|A'\rangle, \tag{6}$$

$$\mathcal{B}^{c\mu}(l) = \langle 0|B^{c\mu}(l)|B\rangle. \tag{7}$$

for given initial states $|A'\rangle$ and $|B\rangle$ in Eq. (3).

We now consider measurements made at times much larger than the time it takes for the field A' to interact with the Lorentz-contracted fast nucleus, i.e., $\int_0^t dt' \to \int_{-\infty}^{\infty} dt'$. In this case the full integral d^4x over each three gluon vertex can be easily carried out. We define the evolution matrix for the time evolution of the density matrix elements as

$$\begin{aligned} W_{AA',\hat{A}\hat{A}'} &:= \left\langle \int dt H_{A',A} \int dt H^{\dagger}_{\hat{A}',\hat{A}} \right\rangle_2 \\ &= g^2 f^{ca'a} f^{\hat{c}\hat{a}'\hat{a}} \frac{d^4k'}{(2\pi)^4}\frac{d^4\hat{k}'}{(2\pi)^4} \mathcal{A}'^{a'}(k',\lambda')\mathcal{A}'^{\hat{a}'\dagger}(\hat{k}',\hat{\lambda}') \\ &\quad \times \frac{\langle \mathcal{B}^c_\mu(k-k')\mathcal{B}^{\hat{c}\dagger}_{\hat{\mu}}(\hat{k}-\hat{k}')\rangle_2}{2V\sqrt{k^+\hat{k}^+}(VT)^2} \epsilon^*_\sigma(k,\lambda)\epsilon_{\hat{\sigma}}(\hat{k},\hat{\lambda}) \\ &\quad \times \epsilon_\nu(k',\lambda')\epsilon^*_{\hat{\nu}}(\hat{k}',\hat{\lambda}')[(k+k')^\mu g^{\nu\sigma} \\ &\quad - (2k-k')^\nu g^{\sigma\mu} + (k-2k')^\sigma g^{\mu\nu}][(\hat{k}+\hat{k}')^{\hat{\mu}} g^{\hat{\nu}\hat{\sigma}} \\ &\quad - (2\hat{k}-\hat{k}')^{\hat{\nu}} g^{\hat{\sigma}\hat{\mu}} + (\hat{k}-2\hat{k}')^{\hat{\sigma}} g^{\hat{\mu}\hat{\nu}}]. \end{aligned} \tag{8}$$

Because the second nucleus is moving extremely fast and we are not interested the time evolution of its gluon fields B, we have averaged over the fields in the second nucleus $\mathcal{B}^c_\mu(l)\mathcal{B}^{\hat{c}\dagger}_{\hat{\mu}}(\hat{l}) \to \langle\mathcal{B}^c_\mu(l)\mathcal{B}^{\hat{c}\dagger}_{\hat{\mu}}(\hat{l})\rangle_2$, leaving us with an expression that depends only on the initial and final fields in nucleus 1.

Arguing with translational and rotational invariance in the transverse plane [24] and using that it is moving along the light cone, we can decompose the two-point correlation function of the gluon field of nucleus 2 as

$$\begin{aligned} \langle\mathcal{B}^c_\mu(p)\mathcal{B}^{\hat{c}\dagger}_{\hat{\mu}}(q)\rangle_2 &= \langle\mathcal{B}^c_\mu(\mathbf{p}_\perp)\mathcal{B}^{\hat{c}\dagger}_{\hat{\mu}}(\mathbf{q}_\perp)\rangle_2 \frac{\pi^2\delta(p^-)\delta(q^-)}{p^+q^+} \\ &= \delta^{c\hat{c}}\delta_{\mu i}\delta_{\hat{\mu}j}(2\pi)^2\delta^2(\mathbf{p}_\perp-\mathbf{q}_\perp) \\ &\quad \times \frac{\pi^2\delta(p^-)\delta(q^-)}{p^+q^+}\frac{p_i p_j}{p_\perp^2}G(p_\perp), \end{aligned} \tag{9}$$

where $G(p_\perp)$ is the scalar correlation function for the gluon field in the fast-moving nucleus and i, j denote the transverse directions.

Thus far we have not made use of the fact that nucleus 1 is at rest and the results are valid in general as long as the phase space of initial and final gluons is sufficiently different. Now we note that the final gluon momenta have large + components, much larger than the original ones, i.e., $k^+, \hat{k}^+ \gg k'^+, \hat{k}'^+$ with $k^\pm = (k^0 \pm k^3)/\sqrt{2}$. Therefore the dominant terms in Eq. (8) are those with the maximum number of factors k^+ or $\hat{k}^+$. These are

$$(2k-k')^\nu\epsilon_\nu(k',\lambda') \approx 2k^+\epsilon^-(k',\lambda'), \tag{10}$$

$$(2\hat{k}-\hat{k}')^{\hat{\nu}}\epsilon_{\nu*}(\hat{k}',\hat{\lambda}') \approx 2\hat{k}^+\epsilon^-_*(\hat{k}',\hat{\lambda}'). \tag{11}$$

Hence, the leading-order contribution to the evolution operator for the gluon density matrix in our specific kinematic situation is

$$\begin{aligned} W_{AA',\hat{A}\hat{A}'} &= g^2 f^{ca'a} f^{c\hat{a}'\hat{a}} \frac{d^4k'}{(2\pi)^4}\frac{d^4\hat{k}'}{(2\pi)^4}\delta(k^- - k'^-) \\ &\quad \times \delta(\hat{k}^- - \hat{k}'^-)(2\pi)^2\delta^2(\mathbf{k}_\perp - \mathbf{k}'_\perp - \hat{\mathbf{k}}_\perp + \hat{\mathbf{k}}'_\perp) \\ &\quad \times G(|\mathbf{k}_\perp - \mathbf{k}'_\perp|)\frac{\mathcal{P}}{2V\sqrt{k^+\hat{k}^+}}\mathcal{A}'^{a'}(k',\lambda') \\ &\quad \times \mathcal{A}'^{\hat{a}'\dagger}(\hat{k}',\hat{\lambda}')\epsilon^-(k',\lambda')\epsilon^{-*}(\hat{k}',\hat{\lambda}'), \end{aligned} \tag{12}$$

where we have introduced the abbreviation

$$\mathcal{P} = \epsilon^i(k,\lambda)\epsilon^{j*}(\hat{k},\hat{\lambda})\frac{(\mathbf{k}_\perp - \mathbf{k}'_\perp)^i(\mathbf{k}_\perp - \mathbf{k}'_\perp)^j}{(\mathbf{k}_\perp - \mathbf{k}'_\perp)^2}. \tag{13}$$

Note that $\mathcal{P}$ is essentially the product of the projections of the two polarization vectors onto the transverse direction given by the vector $\mathbf{k}_\perp - \mathbf{k}'_\perp = \hat{\mathbf{k}}_\perp - \hat{\mathbf{k}}'_\perp$.

III. THE GLUON DENSITY MATRIX

The evolution of the gluon density matrix of nucleus 1 can now be computed through

$$D_{\hat{A},A}(t) = \sum_{A',\hat{A}'} D_{\hat{A}',A'}(0)W_{AA',\hat{A}\hat{A}'}(t). \tag{14}$$

For our calculation, we do not need the nuclear density matrix itself but just the expectation value of $A'^{a'}_{k',\lambda'}A'^{\hat{a}'\dagger}_{\hat{k}',\hat{\lambda}'}$ in the ground state of nucleus 1. As in Ref. [22] we use an *ansatz* for this expectation value that is diagonal and exhibits a Gaussian momentum distribution with width $1/\zeta$:

$$\begin{aligned} \langle A'^{a'}_{k',\lambda'}A'^{\hat{a}'\dagger}_{\hat{k}',\hat{\lambda}'}\rangle_1 &\equiv \sum_{\hat{A}',A'} D_{\hat{A}',A'}(0)\langle 0|A'^{a'}_{k',\lambda'}|A'\rangle\langle\hat{A}'|A'^{\hat{a}'\dagger}_{\hat{k}',\hat{\lambda}'}|0\rangle \\ &= \sum_{\hat{A}',A'} D_{\hat{A}',A'}(0)\mathcal{A}'^{a'}(k',\lambda')\mathcal{A}'^{\hat{a}'*}(\hat{k}',\hat{\lambda}') \\ &= \delta_{\hat{\lambda}'\lambda'}\delta_{\hat{a}'a'}(2\pi)^4\delta^4(\hat{k}'-k')\mathcal{N}\zeta^2 e^{-\zeta^2(k'^{02}+\mathbf{k}'^2)}. \end{aligned} \tag{15}$$

To determine the normalization constant $\mathcal{N}$ we could calculate the energy density of gluons in the nucleus, $\rho_g \equiv \frac{E_g}{V} = \frac{1}{VT}\int dt\ \mathrm{Tr}\,[HD]$, but we will see later that our final result does not depend on $\mathcal{N}$.

Returning to the expression (14) for the final-state density matrix, we obtain

$$\begin{aligned} D_{\hat{A},A}(t) &= g^2 N_c \int \frac{d^4k'}{(2\pi)^4}\mathcal{P}\sum_{\lambda'}|\epsilon^-(k',\lambda')|^2 \\ &\quad \times (2\pi)^2\delta(k^- - k'^-)\delta(\hat{k}^- - k'^-)\frac{\delta^2(\mathbf{k}_\perp - \hat{\mathbf{k}}_\perp)}{2V\sqrt{k^+\hat{k}^+}} \\ &\quad \times \delta_{\hat{a}a}G(|\mathbf{k}_\perp - \mathbf{k}'_\perp|)\mathcal{N}\zeta^2 e^{-\zeta^2(k'^{02}+\mathbf{k}'^2)}. \end{aligned} \tag{16}$$

The δ function enforces $k'^- = k^-$. Furthermore, we can use $k \approx \hat{k}$ to argue that $\mathcal{P} \approx \frac{1}{2}\delta_{\lambda\hat{\lambda}}$ because the projections in $\mathcal{P}$ are maximal if $\lambda = \hat{\lambda}$ and the average value on integration over the directions of $\mathbf{k}'_\perp$ should be $\langle \cos^2\phi\rangle \approx 1/2$. The same argument also allows us to use the approximation $\sum_{\lambda'} |\epsilon^-(k',\lambda')|^2 \approx 3/4$, allowing for three polarization states of the off-shell gluons in nucleus 1. We can thus write

$$D_{\hat{A},A} = \frac{3g^2 N_c \zeta \mathcal{N}}{32\pi^{3/2}} \delta_{\hat{a}a}\delta_{\hat{\lambda}\lambda} e^{-\zeta^2(k^-)^2}\delta(k^- - \hat{k}^-)\frac{\delta^2(\mathbf{k}_\perp - \hat{\mathbf{k}}_\perp)}{2V\sqrt{k^+\hat{k}^+}} \times \int d^2k'_\perp G(|\mathbf{k}_\perp - \mathbf{k}'_\perp|) e^{-\zeta^2 {k'_\perp}^2}. \quad (17)$$

We now introduce the convolution of the gluon two-point function with the Gaussian profile

$$F(k_\perp) = \int d^2k'_\perp G(|\mathbf{k}_\perp - \mathbf{k}'_\perp|) e^{-\zeta^2 {k'_\perp}^2}, \quad (18)$$

and thus obtain our final expression for the final-state gluon density matrix (14):

$$D_{\hat{A},A} = \mathcal{N}\frac{3\alpha_s N_c \zeta}{8\sqrt{\pi}V}\delta_{\hat{\lambda}\lambda}\delta_{\hat{a}a}\frac{\delta^3(k-\hat{k})}{2\sqrt{k^+\hat{k}^+}} e^{-\zeta^2(k^-)^2} F(k_\perp), \quad (19)$$

where the three-dimensional δ function refers to the "−" and "⊥" components of the momenta. We note that the density matrix resulting from the interaction with the external gluon field for large times is diagonal in all quantum numbers except for the longitudinal momentum, in accordance with the result obtained in Ref. [22].

Now we proceed to calculate the traces.

$$\mathrm{Tr}\, D = \sum_A D_{A,A} = VT \int \frac{d^4k}{(2\pi)^4} \sum_{a,\lambda} D_{A,A} = T\delta^3(0)\mathcal{N}\frac{3\alpha_s N_c (N_c^2-1)}{32\pi^2}\int \frac{dk^+}{k^+}\int \frac{d^2k_\perp}{(2\pi)^2} F(k_\perp). \quad (20)$$

The integral over k^+ should be regulated by a phase-space projection, because the gluons are (almost) on the mass-shell, but there is no need to specify the details here. We have also assumed that the final-state gluons can carry only transverse polarizations, because they are nearly on mass-shell. The trace of the square is

$$\mathrm{Tr}\, D^2 = \sum_{A,A'} D_{A,A'}D_{A',A} = T^2\delta^3(0)\mathcal{N}^2 \frac{9\alpha_s^2 N_c^2 (N_c^2-1)\zeta}{128\sqrt{2\pi}(2\pi)^6} \times \left(\int \frac{dk^+}{k^+}\right)^2 \int \frac{d^2k_\perp}{(2\pi)^2} F(k_\perp)^2. \quad (21)$$

For the ratio that we are seeking this leads to the expression

$$\frac{\mathrm{Tr}\, D^2}{[\mathrm{Tr}\, D]^2} = \frac{1}{(2\pi)^3\delta^3(0)}\frac{\zeta\sqrt{2\pi}}{2(N_c^2-1)}\frac{I_2}{(I_1)^2}, \quad (22)$$

where

$$I_1 = \int d^2k_\perp g^2 F(k_\perp) \quad (23)$$

$$I_2 = \int d^2k_\perp g^4 F^2(k_\perp). \quad (24)$$

We note that the volume associated with $\delta^3(0) = \delta(0^-)\delta^2(\mathbf{0}_\perp)$ is proportional to the light-cone "time" variable x^+, which is conjugate to k^-. When boosted to midrapidity, x^+ transforms as $x'^+ = x^+/(2\gamma)$, unlike the regular time coordinate x^0, which transforms as $x'^0 = \gamma(x^0 - \beta x^3)$. This fact is in line with our intuitive picture in analogy to Coulomb-explosion: Decoherence occurs due to the fact that Lorentz contracted nucleus 2, which acts as if consisting of incoherent color-charges, passes through nucleus 1.

The result we obtained coincides with that of our previous calculation in predicting a decoherence behavior $\sim 1/x^+$ at leading order [22]. It is given as a function of $F(\mathbf{k}_\perp)$ that can be evaluated using different models for the initial two-gluon correlator G.

IV. DECOHERENCE IN THE McLERRAN-VENUGOPALAN MODEL

In this section we want to compute I_1 and I_2 using the standard two-point gluon function from the McLerran-Venugopalan model. We work with the Fourier transform $G(\mathbf{p}) = \int d^2x e^{-i\mathbf{px}} f(\mathbf{x})$. Here and in the following we suppress the index "⊥" for easier notation. f has first been calculated in the McLerran-Venugopalan model in Ref. [25]. We follow the conventions in Lappi [24] and write

$$f(\mathbf{x}) = \frac{4(N_c^2-1)}{N_c g^2 x^2}\left[1 - e^{-g^4 N_c/(8\pi)\mu^2 x^2 \ln 1/(x\Lambda)}\right], \quad (25)$$

where μ^2 is related to the saturation scale $Q_s \sim g^2\mu$. This result is valid only for $x < 1/\Lambda$, where Λ is a IR cutoff and we set $f = 0$ for $x > 1/\Lambda$. All vectors are two-vectors in the transverse plane.

Note that we have defined factors of the coupling constant g into I_1 and I_2 without canceling them in the ratio. We do so because only the square of the gluon field strength tensor times the running coupling α_s has well-defined properties and we consider $g^2 G(p)$ to be the physical quantity. The correct implementation of the running coupling is a topic of intense investigations. We follow the prescription by Kovchegov and Weigert [23] and substitute

$$g^4 \to g^2(\Lambda^2) g^2(1/x^2). \quad (26)$$

For I_1 we obtain

$$I_1 = (2\pi)^2\frac{\pi}{\zeta^2}\lim_{x\to 0}[g^2 f(\mathbf{x})] = (2\pi)^2 \frac{\pi}{\zeta^2}\lim_{x\to 0}\frac{N_c^2-1}{2\pi}\mu^2 g^2(\Lambda^2) g^2(1/x^2)\ln 1/(x\Lambda) = (2\pi)^4 g^2(\Lambda^2)\frac{(N_c^2-1)\mu^2}{\beta_0\zeta^2} \quad (27)$$

where we used the one-loop running coupling with

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f. \quad (28)$$

Obviously this is a well-defined expression, while I_1 without the running coupling would have led to a logarithmic UV divergence.

However, after two Gaussian integrations we see that

$$I_2 = (2\pi)^4 \frac{\pi^2}{\zeta^4} \int \frac{d^2x}{(2\pi)^2} e^{-x^2/(2\zeta^2)} g^4 f^2(x)$$
$$= (2\pi)^3 \frac{\pi^2}{\zeta^4} \frac{16\left(N_c^2-1\right)^2}{N_c^2} \int_0^{\Lambda^{-1}} \frac{dx}{x^3} e^{-x^2/(2\zeta^2)}$$
$$\times \left\{1 - \exp\left[-g^2(\Lambda^2)\mu^2 x^2 \frac{\pi N_c \ln(1/x^2\Lambda^2)}{\beta_0 \ln(1/x^2\Lambda_{\rm QCD}^2)}\right]\right\}^2 . \tag{29}$$

The scale Λ should be chosen such that for typical transverse momenta $1/x$ ideally fulfills [23]

$$\frac{1}{x} \gg \Lambda \gg \Lambda_{\rm QCD}. \tag{30}$$

For x between 0 and $1/\Lambda$ the ratio of the logarithms should hence obey

$$0 \leqslant \ln\left(1/x^2\Lambda^2\right)/\ln\left(1/x^2\Lambda_{\rm QCD}^2\right) \leqslant 1. \tag{31}$$

We conclude that $I_2 < \tilde{I}_2$, where $\tilde{I}_2$ is given by the last expression in Eq. (29) with the ratio of logarithms replaced by 1. This inequality is useful because the remaining integral in $\tilde{I}_2$ can be solved analytically.

After replacing $u = x^2\Lambda^2$ and introducing the short notations $a = (2\zeta^2\Lambda^2)^{-1}$ and $b = \pi N_c \mu^2 g^2(\Lambda^2)/(\Lambda^2\beta_0)$ the integral is

$$\tilde{I}_2 = 2(2\pi)^5 \frac{\Lambda^2}{\zeta^4} \frac{\left(N_c^2-1\right)^2}{N_c^2}$$
$$\times \int_0^1 \frac{du}{u^2} [e^{-au} - 2e^{-(a+b)u} + e^{-(a+2b)u}]. \tag{32}$$

After two partial integrations this can be brought into the form

$$\tilde{I}_2 = 2(2\pi)^5 \frac{\Lambda^2}{\zeta^4} \frac{\left(N_c^2-1\right)^2}{N_c^2} \Bigg\{ -[e^{-au} - 2e^{(a+b)u} + e^{-(a+2b)u}]$$
$$- \int_0^1 du \ln u [a^2 e^{-au} - 2(a+b)^2 e^{(a+b)u}$$
$$+ (a+2b)^2 e^{-(a+2b)u}] \Bigg\}. \tag{33}$$

Now we expand the integration region of the remaining integral to infinity, anticipating that the integrand is rapidly vanishing for $u \to \infty$. Then the integral gives

$$\tilde{I}_2 = 2(2\pi)^5 \frac{\Lambda^2}{\zeta^4} \frac{\left(N_c^2-1\right)^2}{N_c^2} \{-e^{-au} + 2e^{(a+b)u} - e^{-(a+2b)u}$$
$$+ a(\gamma + \ln a) - 2(a+b)[\gamma + \ln(a+b)]$$
$$+ (a+2b)[\gamma + \ln(a+2b)]\}, \tag{34}$$

where γ is Euler's constant.

In the region of applicability for the color glass condensate we expect $b \gg a \sim \mathcal{O}(1)$. Hence the bracket in the last equation is to good approximation equal to $2b \ln 2$. Finally, we obtain

$$\tilde{I}_2 \approx 2 \ln 2 (2\pi)^6 g^2(\Lambda^2) \frac{\mu^2}{\beta_0 \zeta^4} \frac{\left(N_c^2-1\right)^2}{N_c}, \tag{35}$$

giving the following bound for the relevant ratio of integrals:

$$\frac{I_2}{(I_1)^2} < \frac{2\beta_0 \ln 2}{(2\pi)^2 g^2(\Lambda^2) N_c \mu^2}. \tag{36}$$

We now return to Eq. (22). The expression $(2\pi)^3\delta^3(0)$ in the denominator gives the transverse normalization area times the observation time T. To fix the transverse normalization area one can follow two different lines of argument. The incoming gluons from nucleus 1 are effectively localized within the transverse area $\pi\zeta^2$ given by the initial density matrix (15). If the density matrix D is interpreted as that of a completely coherent system Tr $D^2 \approx$ (Tr $D)^2$ then the transverse normalization area has to be chosen as $\pi\zeta^2$. If, however, one prefers to extend the normalization area to the whole area of the nucleus, πR^2, one has to take into account that the starting value of Tr D^2/(Tr $D)^2$ is not close to one but rather of the order ζ^2/R^2 and one should thus ask after which time R^2Tr D^2/ζ^2(Tr $D)^2$ has dropped to $1/e$. (In the latter case D has the form of a block-diagonal matrix with R^2/ζ^2 blocks.) It is reassuring that both lines of argument lead to the same result.

$$\frac{R^2}{\zeta^2} \frac{{\rm Tr}\, D^2}{[{\rm Tr}\, D]^2} < \frac{2\beta_0 \ln 2}{(2\pi)^{5/2} N_c \left(N_c^2-1\right) g^2(\Lambda^2)\mu^2} \frac{1}{\zeta T}. \tag{37}$$

Defining the decoherence time $\tau_{\rm dec}$ as the time where this ratio has dropped to a value $1/e$ and fixing the physical saturation scale as $Q_s = g^2(\mu^2)\mu$, we obtain the upper bound

$$\tau_{\rm dec} < \left[\frac{8e \ln 2}{\sqrt{2\pi} N_c (N_c^2-1)}\right] \left[\frac{g^2(\mu^2)}{g^2(\Lambda^2)}\right]$$
$$\times \left[\frac{\beta_0 g^2(\mu^2)}{16\pi^2}\right] \left(\frac{1}{\zeta Q_s}\right) \frac{1}{Q_s}$$
$$\approx 0.25 \left(\frac{g_\mu^2}{g_\Lambda^2}\right) \left(\frac{\beta_0 g_\mu^2}{16\pi^2}\right) \left(\frac{1}{\zeta Q_s}\right) \frac{1}{Q_s}. \tag{38}$$

All factors in parentheses being of order unity, we thus conclude that $\tau_{\rm dec} \sim Q_s^{-1}$ in agreement with the result obtained in Ref. [22].

V. DECOHERENCE ENTROPY

We now turn to the question how much entropy can be produced by the rapid decoherence of the initially coherent nuclear gluon field. To illustrate the mechanism, we first discuss a simple model for which the relevant calculations can be performed exactly [21] but that is sufficiently general to permit a semiquantitative estimate of the entropy produced by decoherence in a heavy-ion reaction.

The quantum mechanical analog of a classical field is a coherent state [26]

$$|\Psi[J]\rangle = \prod_{\mathbf{k},\lambda} \exp(i\alpha_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda}^\dagger - i\alpha_{\mathbf{k}\lambda}^* a_{\mathbf{k}\lambda})|0\rangle, \tag{39}$$

where the amplitude $\alpha_{\mathbf{k}\lambda}$ is determined by the classical current $\mathbf{J}$ creating the field

$$\alpha_{\mathbf{k}\lambda} = (\hbar\omega_{\mathbf{k}} V)^{-1/2}\epsilon_{\mathbf{k}\lambda}\cdot\mathbf{J}(\mathbf{k},\omega_{\mathbf{k}}). \quad (40)$$

Let us begin by considering a single mode $\mathbf{k}\lambda$. The coherent state can be written as a superposition of particle number eigenstates:

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle. \quad (41)$$

Being a pure quantum state, $|\alpha\rangle$ is described by a density matrix

$$\rho_{mn} = \langle m|\alpha\rangle\langle\alpha|n\rangle, \quad (42)$$

which satisfies the relation $\rho^2 = \rho$ and has no entropy: $S = -\mathrm{Tr}\,\rho\ln\rho = 0$.

Complete decoherence of this quantum state corresponds to the total decay of all off-diagonal matrix elements of the density matrix, yielding the diagonal density matrix

$$\rho_{mn}^{\mathrm{dec}} = |\langle n|\alpha\rangle|^2\delta_{mn} = e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}\delta_{mn}. \quad (43)$$

The particle number in this mixed state follows the Poisson distribution, and the average number of particles is $\bar{n} = |\alpha|^2$. The entropy content of the mixed state is given by

$$S_{\mathrm{dec}}^{(\mathrm{cs})} = \sum_{n=0}^{\infty} e^{-\bar{n}}\frac{\bar{n}^n}{n!}\ln\left(e^{-\bar{n}}\frac{\bar{n}^n}{n!}\right)$$
$$= e^{-\bar{n}}\sum_{n=0}^{\infty}\frac{\bar{n}^n}{n!}(n\ln\bar{n} - \bar{n} - \ln n!), \quad (44)$$

where the superscript "cs" indicates that the result holds for a coherent state. With the help of Stirling's formula and the integral representation of the logarithm,

$$\ln n = \int_0^{\infty}\frac{ds}{s}(e^{-s} - e^{-ns}), \quad (45)$$

the sum in Eq. (44) can be performed yielding an analytical result that is valid asymptotically for $\bar{n} \gg 1$ (actually, the approximation is excellent already for $\bar{n} \approx 1$):

$$S_{\mathrm{dec}}^{(\mathrm{cs})} = \frac{1}{2}\left(\ln(2\pi\bar{n}) + 1 - \frac{1}{6\bar{n}} + \cdots\right). \quad (46)$$

It is not surprising that the entropy is proportional to $\ln\sqrt{\bar{n}}$, because we have deleted all information about the relative signs of the amplitudes $\langle\alpha|n\rangle$ by eliminating the off-diagonal elements of the density matrix. The number of significantly contributing elements is given by the width, $\Delta n = \sqrt{\bar{n}}$, of the Poisson distribution. That the decoherence entropy is controlled by Δn, rather than by $\bar{n}$, can be seen by considering more general pure quantum states, for which the average occupation number $\bar{n}$ and the occupation number uncertainty Δn are not related. For a pure state with $\bar{n} \gg \Delta n \gg 1$ and in the Gaussian approximation, it is straightforward to show that the decoherence entropy is given by

$$S_{\mathrm{dec}} = \tfrac{1}{2}(\ln(2\pi(\Delta n)^2) + 1 + \cdots), \quad (47)$$

confirming our assertion. For a classical coherent state (41), the expression (47) coincides with (46).

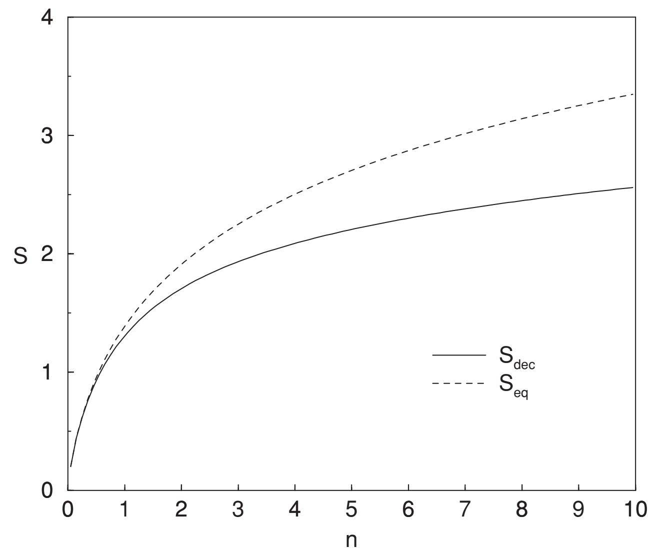

FIG. 3. Decoherence entropy S_{dec} for a coherent state of a single field mode and equilibrium entropy S_{eq} for the same average total energy as a function of the average occupation number $\bar{n}$.

We also note that the entropy for a single quantum oscillator in equilibrium at temperature T is given by

$$S_{\mathrm{eq}} = \ln(\bar{n}+1) + \bar{n}\ln\left(1 + \frac{1}{\bar{n}}\right), \quad (48)$$

where $\bar{n} = (e^{\omega/T} - 1)^{-1}$ is the average occupation number. Asymptotically, for large $\bar{n}$, one obtains $S_{\mathrm{eq}} \approx 2S_{\mathrm{dec}}^{(\mathrm{cs})}$, i.e., the thermal entropy becomes twice as large as the decoherence entropy. However, for small to moderate occupation numbers the ratio $S_{\mathrm{dec}}^{(\mathrm{cs})}/S_{\mathrm{eq}}$ is close to unity. Figure 3 shows the decoherence and equilibrium entropies as a function of the average occupation number $\bar{n}$. For not too large values of $\bar{n}$, the decoherence process generates a large fraction of the equilibrium entropy, and any subsequent equilibration process adds only a small amount of entropy to it.

What does this imply for quantum field theory, where the field is a system of infinitely many coupled oscillators? Assume that, after decoherence, the system can be described as a collection of N particles, given by some distribution function over single-particle states, which were generated by the decoherence of N_{cs} coherent quantum states. Examples of such states include the internal wave functions of nucleons forming a large nucleus and a quark with its comoving gluon cloud. Each coherent state contributes on average $\bar{n} = N/N_{\mathrm{cs}}$ partons. Then, after full equilibration, the thermal entropy is of the order of $S_{\mathrm{th}} \sim N_{\mathrm{cs}}\bar{n} = N$, whereas for the decoherence entropy we get $S_{\mathrm{dec}} \sim N_{\mathrm{cs}}\frac{1}{2}\ln(2\pi\bar{n})$. The ratio of the two entropies is

$$\frac{S_{\mathrm{dec}}}{S_{\mathrm{th}}} \sim \frac{\ln(2\pi\bar{n})}{2\bar{n}}, \quad (49)$$

i.e., for large-amplitude quantum states, which turn into many particles per coherent mode, the decoherence contribution to the thermal entropy is small. However, if the individual occupation numbers are of order 1, the contribution is sizable. This case applies to our problem of interest, the collision of two nuclei at high energy, as we will discuss in the following.

For the coherent color fields in colliding nuclei, the average number of decohering gluons per transverse area has been given by [27]

$$\frac{dN}{d^2b\,dy} \approx \frac{C_F \ln 2Q_s^2}{\pi^2 \alpha_s}, \quad (50)$$

where $C_F = 4/3$. The characteristic transverse area, over which the color fields in nucleus 2 are coherent, is π/Q_s^2, and one can argue that the longitudinal coherence length is of the order of $\Delta y \approx 1/\alpha_s$ [28]. We thus obtain for the average number of decohering partons per coherence domain

$$\bar{n} = \frac{dN}{d^2b\,dy}\frac{\pi}{Q_s^2}\Delta y \approx \frac{C_F \ln 2}{\pi \alpha_s^2} \approx 3. \quad (51)$$

For this value, our arguments presented above indicate that the entropy produced in the decoherence process is about half of the equilibrium entropy. Applying Eq. (46) and using that the initial number of coherent domains per transverse area is $(Q_s R)^2$, we find that the total entropy per unit rapidity produced by decoherence in a Au + Au collision at RHIC is

$$\frac{dS_{\rm dec}}{dy} \approx \frac{Q_s^2 R^2}{2\Delta y}\,[\ln(2\pi\bar{n}) + 1] \approx \frac{Q_s^2 R^2 \alpha_s}{2}\left(\ln\frac{2C_F \ln 2}{\alpha_s^2} + 1\right) \approx 1500, \quad (52)$$

where we used the values $Q_s^2 \approx 2$ GeV^2, $R = 7$ fm, and $\alpha_s \approx 0.3$ [27]. This value accounts for about one-third of the entropy measured in the final hadron distribution [29,30].

VI. CONCLUSIONS

We advocate the idea that a large fraction of the total entropy produced in high-energy heavy-ion collisions is generated by decoherence of the many-body quark-gluon wave functions of the colliding nuclei in the very first phase of the collision.

We presented an improved determination of the decoherence time $\tau_{\rm dec}$ as a function of the initial gluon correlation function. Within the color glass condensate formalism this leads to a decoherence time $\tau_{\rm dec} \leqslant 1$ fm/c that agrees with the result of an earlier calculation. We also estimate the entropy produced through decoherence of the initial gluon field and find that it could contribute about one-third of the total entropy observed at RHIC.

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