Pergamon

Transpn Res.-B, Vol. 31, No. 1, pp. 1–10, 1997

0191-2615/96 $17.00 + 0.00

PII:S0191-2615(96)00020-3

DECOMPOSITION OF THE REACTIVE DYNAMIC ASSIGNMENTS WITH QUEUES FOR A MANY-TO-MANY ORIGIN–DESTINATION PATTERN

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(Received 7 April 1994; in revised form 15 April 1996)

Abstract—This research discusses the formulation and solution algorithm of the reactive dynamic traffic assignment with the link travel time explicitly taking into account the effects of queues under the point queue concept. In the reactive assignment, vehicles are assumed to choose their routes based on present instantaneous travel times. Time dependent many-to-many origin–destination volumes are assumed to be given; that is, the departure times of vehicles from origins are known. We first discuss the formulation of the dynamic assignment so as to satisfy the flow conservation and the First-In-First-Out queue discipline. Then, the reactive assignment is shown to be decomposed with respect to present time, since route choices of vehicles are dependent on the present traffic situation but independent of the past and future traffic condition. An algorithm is finally proposed based upon the decomposition. Copyright © 1996 Elsevier Science Ltd

1. INTRODUCTION

This research analyses the formulations and decomposition of the reactive dynamic assignment with queues on a network. We assume that time dependent many-to-many origin–destination (OD) volumes are given; that is, the departure times of vehicles from origins are known. In the reactive assignment which is sometimes known as the dynamic optimal assignment, vehicles are assumed to choose their routes based on present instantaneous travel times which may be observed by traffic detectors installed on a network and transmitted to travellers through variable message signs, in-vehicle equipment, etc.

Suppose that a network consists of links and nodes and that travel times of all links are observed at present time t. Then, a traveler is assumed to evaluate a route travel time as simply the sum of present link travel times on the route and to choose the shortest route to his destination based on the above present travel time. This possible route choice would become more common as ATIS, such as variable message signs, highway radios, and in-vehicle navigation equipments, frequently supply current traffic information.

In the usual equilibrium assignment travelers are assumed to choose their routes according to the travel times they actually experienced. This is sometimes called predictive assignment in contrast with the reactive assignment for which travelers respond to future traffic conditions as they travel.

Friesz *et al.* (1989), Wie *et al.* (1990), Boyce *et al.* (1993) and Lam and Huang (1995), individually developed mathematically interesting models based upon the control theory, in which link travel time was evaluated from arrival and departure flow rates of the link as well as the number of vehicles existing on the link. However, the First-In-First-Out (FIFO) queue discipline has sometimes been neglected and travellers' perceived route

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costs do not hold a clear relationship with link travel times. Also, their link travel times have not been well explained in conjunction with the arrival and departure flow rates, although the link travel time should be related to these flow rates through the FIFO queue discipline. This point will be further discussed in Section 2.

On the other hand, Kuwahara and Akamatsu (1993, 1994) analyzed the predictive dynamic user equilibrium assignment for a one-to-many (many-to-one) OD pattern taking the effect of queues explicitly into account, given time-dependent OD volume. One of the most important results is the decomposition of the problem with respect to departure time from an origin (arrival time at a destination). This research follows the approach of our previous study and introduces a similar decomposition scheme for the reactive assignment with a many-to-many OD pattern.

2. DYNAMIC NETWORK FLOWS AND LINK TRAVEL TIME WITH QUEUES

2.1. Network and traffic demand

A network consists of links and nodes. Sequential numbers from 1 to N are allocated to N nodes. The number of links is L and a link from node i to j is denoted as link (i, j). A time-dependent many-to-many OD demand generated and absorbed at nodes is assumed to be given, which is denoted as:

$Q_{ij}(t)$ = cumulative OD demand from origin node i to destination node j generated at the origin by time t (given).

Let us also introduce demand functions with respect to the arrival time at a node:

$R_i(t)$ = cumulative trips generated at origin i by time t,
$= \sum_j Q_{ij}(t)$,
$S_j(t)$ = cumulative trips absorbed at destination node j by time t.

Arrival time at node j cannot be known in advance (except when j is an origin node) and hence $S_j(t)$ must be evaluated through the dynamic assignment.

2.2. Cumulative functions and constraints

The cumulative arrival and departure curves are defined as follows:

$$A_{ij}(t) = \text{the cumulative arrivals at link } (i, j) \text{ by time } t, \tag{1}$$

$$D_{ij}(t) = \text{the cumulative departures from link } (i, j) \text{ by time } t. \tag{2}$$

And the derivatives of those with respect to time t are denoted as:

$\lambda_{ij}(t)$ = the arrival rate at link (i, j) at time $t = \mathrm{d}A_{ij}(t)/\mathrm{d}t$,
$\mu_{ij}(t)$ = the departure rate from link (i, j) at time $t = \mathrm{d}D_{ij}(t)/\mathrm{d}t$.

The arrival rate at link (i, j) at time t, $\lambda_{ij}(t)$, is the unknown variable which must be determined so as to establish the reactive assignment principle defined later. The cumulative arrival, $A_{ij}(t)$, is the integral of $\lambda_{ij}(t)$ over time until time t, and our objective is thus to determine $A_{ij}(t)$ at every link for any t.

2.2.1. Flow conservation at nodes. The first constraint is the flow conservation at a node, which is written with respect to node i:

$$-\sum_k D_{ki}(t) + \sum_j A_{ij}(t) - R_i(t) + S_i(t) = 0, \quad i = 1, 2, \ldots\ldots, N. \tag{3}$$

The first and third terms give the cumulative number of vehicles flowing into node i by time t, while the remaining terms describe the number of vehicles leaving node i by time t.

Since the flow conservation should be satisfied within vehicles travelling to the same destination d, eqn (3) is rewritten only for them:

$$-\sum_k D_{ki}{}^d(t) + \sum_j A_{ij}{}^d(t) - Q_{id}(t) = 0, \quad i = 1, 2, \ldots\ldots, N, \quad i \neq d. \tag{4}$$

where

$A_{ij}^{d}(t)$ = the cumulative arrivals at link (i, j) to destination d by time t,
$D_{ij}^{d}(t)$ = the cumulative departures from link (i, j) to destination d by time t.

Although eqn (4) is written by destinations, the flow conservation can be apparently formulated by origins as well.

2.2.2. First–In–First–Out discipline. Second, under the FIFO discipline, a vehicle must leave link (i, j) in the same order as its order of arrival at the link. Thus, the $A_{ij}(t)$ and $D_{ij}(t)$ must be related to each other through link travel time $T_{ij}(t)$ as shown in Fig. 1.

$$A_{ij}(t) = D_{ij}(t + T_{ij}(t)), \tag{5}$$

where

$T_{ij}(t)$ = travel time on link (i, j) for a vehicle entering the link at time t.

This condition must also be satisfied even by vehicles travelling toward the same destination node d:

$$A_{ij}^{d}(t) = D_{ij}^{d}(t + T_{ij}(t)).$$

This FIFO discipline is also described using arrival and departure rates by taking derivative with respect to time t when these cumulative functions are differentiable:

$$\lambda_{ij}^{d}(t) = \mu_{ij}^{d}(t + T_{ij}(t))\,(1 + \mathrm{d}T_{ij}(t)/\mathrm{d}t), \tag{6}$$

where

$$\lambda_{ij}^{d}(t) = \mathrm{d}A_{ij}^{d}(t)/\mathrm{d}t \text{ and } \mu_{ij}^{d}(t) = \mathrm{d}D_{ij}^{d}(t)/\mathrm{d}t.$$

Therefore, if link (i, j) contains vehicles travelling to destination d as well as a different destination d', the ratio of their arrival and departure rates must be the same under FIFO such that:

$$\lambda_{ij}^{d'}(t)/\lambda_{ij}^{d}(t) = \mu_{ij}^{d'}(t + T_{ij}(t))/\ \mu_{ij}^{d}(t + T_{ij}(t)). \tag{7}$$

From eqn (7), we clearly see the role of the FIFO discipline: the departure rate $\mu_{ij}^{d}(t + T_{ij}(t))$ is controlled not only by its arrival rate $\lambda_{ij}^{d}(t)$ but also by arrival rates to other destinations $\lambda_{ij}^{d'}(t)$'s, $d' \neq d$. Thus, departure rate $\mu_{ij}^{d}(t)$ cannot be simply a function either of its arrival rate $\lambda_{ij}^{d}(t)$ alone or of just the number of vehicles on the link.

2.3. Link travel time

As in Fig. 1, the FIFO discipline clearly defines link travel time $T_{ij}(t)$ such that the horizontal time difference between arrival and departure curves at arrival time t. From eqn (5), $T_{ij}(t)$ is thus written as a function of $A_{ij}(t)$ and $D_{ij}(t)$:

$$T_{ij}(t) = D_{ij}^{-1}(A_{ij}(t)) - t. \tag{8}$$

Since $A_{ij}(t)$ and $D_{ij}(t)$ are integration of $\lambda_{ij}(t)$ and $\mu_{ij}(t)$ over time, link travel time at time t must be described by $\lambda_{ij}(t')$ and $\mu_{ij}(t')$ for $t' \leq t$ under the FIFO discipline. However, as mentioned in Section 1, link travel times defined in the previous studies do not have clear relationships with their arrival and departure rates.

Travellers are assumed to perceive $T_{ij}(t)$ as a penalty of travel, although it is possible to introduce perceived costs of travel rather than actual travel time $T_{ij}(t)$ as the conventional traffic assignment. Since the analysis is essentially the same, the link travel time is here considered as the perceived cost to eliminate further complications.

The point queue concept in which a vehicle has no physical length is employed. Consequently, the departure rate from link (i, j) is evaluated as follows independently of traffic condition downstream:

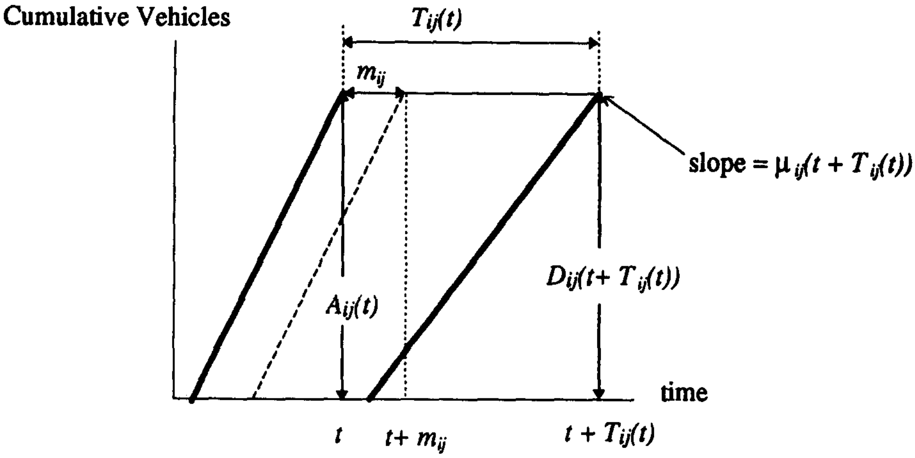

Fig. 1. Cumulative arrival and departures on link (i, j).

$$\mu_{ij}(t + T_{ij}(t)) = \begin{cases} \mu_{ij}^*, & \text{if } T_{ij}(t) > m_{ij} \text{ or } \lambda_{ij}(t) > \mu_{ij}^*, \\ \lambda_{ij}(t), & \text{otherwise,} \end{cases} \tag{9}$$

where

μ_{ij}^* = the maximum departure rate of link (i, j), which is given,

m_{ij} = the minimum link travel time at free flow speed, which is given.

If a vehicle is not delayed, it is assumed to travel on link (i, j) for the minimum travel time m_{ij} which is shown by a broken line in Fig. 1. However, once link travel time $T_{ij}(t)$ becomes larger than m_{ij} at time t or the arrival rate $\lambda_{ij}(t)$ is larger than maximum departure rate μ_{ij}^*, the departure rate $\mu_{ij}(t + T_{ij}(t))$ is assumed to be restricted to μ_{ij}^*. On the other hand, if $T_{ij}(t)$ is equal to m_{ij} and $\lambda_{ij}(t) \leq \mu_{ij}^*$ when no queue exists, $dT_{ij}(t)/dt$ must be zero and consequently from eqn (6), $\mu_{ij}^d(t + T_{ij}(t))$ is equal to $\lambda_{ij}(t)$. Together with eqn (7), when a queue exists on link (i,j) the individual departure rate $\mu_{ij}^d(t + T_{ij}(t))$ becomes:

$$\mu_{ij}^d(t + T_{ij}(t)) = \mu_{ij}^* \cdot \frac{\lambda_{ij}^d(t)}{\sum_{d'} \lambda_{ij}^{d'}(t)}, \tag{10}$$

since $\sum_d \mu_{ij}^d(t + T_{ij}(t))$ is equal to the given μ_{ij}^*.

The above condition implies that if the arrival curve $A_{ij}(t)$ is known up to time t which means $\lambda_{ij}(t)$ is known up to time t as well, $\mu_{ij}(t)$ is determined up to time $t + T_{ij}(t)$ and so is $D_{ij}(t)$. Therefore, from eqn (8), $T_{ij}(t)$ basically becomes a function of only arrival curve $A_{ij}(t')$ for $t' \leq t$ but independent of $A_{ij}(t')$ for $t' > t$. In the deterministic queuing analysis, this result seems apparent under the point queue concept; that is, if arrival curve $A_{ij}(t)$ were known, the departure curve $D_{ij}(t)$ could be drawn as the lower tangent line with the slope of given maximum service rate μ_{ij}^* and travel time $T_{ij}(t)$ could be evaluated from these two cumulative curves.

3. REACTIVE DYNAMIC ASSIGNMENT WITH A MANY-TO-MANY OD PATTERN

3.1. Definition of the reactive assignment principle

Every vehicle is assumed to choose the shortest route to its destination at any time based on the present instantaneous link travel times. Let $\pi_{id}(t)$ be the shortest travel time from node i to destination d prevailing at time t, which means that $\pi_{id}(t)$ is the sum of link travel times along the shortest route p_{id} evaluated at time t:

$$\pi_{id}(t) = \sum_{(i,j) \in p_{id}} T_{ij}(t).$$

Similar to the static assignment, the required condition for the reactive assignment is defined such that

$$\begin{cases} \pi_{id}(t) - \pi_{jd}(t) = T_{ij}(t), & \text{if a vehicle with destination } d \text{ leaving node } i \text{ at time } t \text{ uses link } (i, j), \\ \pi_{id}(t) - \pi_{jd}(t) \leqq T_{ij}(t), & \text{otherwise.} \end{cases} \tag{11}$$

This condition means that if a vehicle with destination d leaving node i at time t uses link (i, j), node j must be on the shortest route to destination d evaluated from the instantaneous link travel times at present time t.

On the other hand, the definition of the predictive assignment is written as follows:

$$\begin{cases} \pi'_{id}(t) - \pi'_{jd}(t + T_{ij}(t)) = T_{ij}(t), & \text{if a vehicle with destination } d \text{ leaving node } i \text{ at time } t \text{ uses link } (i, j), \\ \pi'_{id}(t) - \pi'_{jd}(t + T_{ij}(t)) \leqq T_{ij}(t), & \text{otherwise.} \end{cases} \tag{12}$$

where $\pi'_{id}(t)$ is the shortest travel time from node i to destination d actually experienced by a traveller leaving node i at time t. This description is quite similar to eqn (11) except that the shortest travel time from node j to d is evaluated at time $t + T_{ij}(t)$ when a vehicle actually arrives at node j. Since this difference means that a vehicle must predict future link travel times to find its shortest route from its present node i to the destination d, the predictive assignment is much more difficult compared to the reactive assignment.

3.2. Decomposition with respect to present time

According to the definition of the reactive assignment eqn (11), the route choice of vehicles is clearly dependent only upon the instantaneous link travel times at present time t, but independent of the future link travel times. Therefore, the assignment is decomposed with respect to present time t; that is, we can consider the assignment sequentially from the beginning of the study time period.

If the cumulative arrival curves by destination d have been obtained until time t for link (i, j), the departure curve can be determined until some later time of $t + T_{ij}(t)$ for $\forall(i, j)$ as shown in Fig. 2 which illustrates the cumulative curves on two sequential links (k, i) and (i, j) or (i, l). Then, let us consider how the arrival curves, $A_{ij}^d(t)$s, can be extended from time t, which means that we have to determine arrival rate $\lambda_{ij}^d(t)$ for the individual destination at time t.

At node i, the flow conservation eqn (4) must be satisfied at time t and this conservation can be written in a slightly different form by taking derivatives with respect to time t:

$$-\sum_k \mu_{ki}^d(t) + \sum_j \lambda_{ij}^d(t) - q_{id}(t) = 0, \quad i = 1,2,\ldots\ldots, N, \; i \neq d,$$

where

$$q_{id}(t) = dQ_{id}(t)/dt.$$

The $\mu_{ki}^d(t)$ can be evaluated, since $D_{ki}^d(\cdot)$ has been constructed until time $t + T_{ki}(t) \geq t + m_{ki} > t$ as shown in Fig. 2 and $q_{id}(t)$ has been assumed to be given. Thus, the total arrival rate at node i for destination d is known as:

$$\sum_j \lambda_{ij}^d(t) = \sum_k \mu_{ki}^d(t) + q_{id}(t), \quad i = 1,2,\ldots\ldots, N, \quad i \neq d. \tag{13}$$

However, individual arrival rate $\lambda_{ij}^d(t)$ on link (i,j) has not been determined yet, and it must be evaluated based on the reactive assignment principle. By definition, the above total arrival rate at node i must be loaded on one of the links lying on the shortest route from node i to destination d based on the $T_{ij}(t)$s. In the case of Fig. 2, the instantaneous travel time from node k to j at time t is $T_{ki}(t) + T_{ij}(t)$.

Since present instantaneous link travel times $T_{ij}(t)$s have been known for all links (i,j)s, the shortest route from node i to destination d can be determined without difficulty by a standard shortest route algorithm. Suppose that link (i,l) lies on the shortest route from node i to destination d: $\pi_{id}(t) - \pi_{ld}(t) = T_{il}(t)$. The total arrival rate of

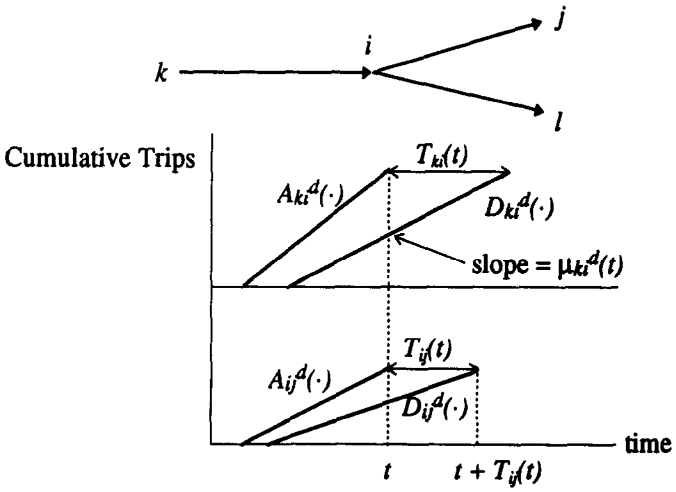

Fig. 2. Cumulative curves on two sequential links.

$\sum_j \lambda_{ij}^d(t)$ must be then loaded on link (i,l):

$$\lambda_{il}^d(t) = \sum_j \lambda_{ij}^d(t),$$
$$\lambda_{ij}^d(t) = 0, \qquad j \neq l. \tag{14}$$

Even if there are two or more equally shortest routes from node i to d, the total rate of:

$$\sum_j \lambda_{ij}^d(t)$$

could be loaded onto one of the shortest routes in order to establish the reactive assignment principle by definition. As the result, $\lambda_{ij}^d(t)$ for $\forall(i,j)$ and $\forall d$ is determined at time t.

Since travellers are assumed to choose the shortest routes simply responding to present traffic situation in this study, it is valid to assign the entire flow rate of:

$$\sum_j \lambda_{ij}^d(t)$$

onto one of the shortest routes as mentioned above. However, this assignment may cause an oscillation of flow rates among the candidates of the shortest routes. To prevent the oscillation, one could split:

$$\sum_j \lambda_{ij}^d(t)$$

into several equally shortest routes so as to equilibrate their travel times. However, an iterative procedure may be needed to determine the split.

3.3. An algorithm to construct the cumulative arrival curves

An algorithm with discrete time intervals of equal length Δt is proposed to evaluate arrival rates at all links for every Δt. As shown in Fig. 3, arrival rate $\lambda_{ij}^d(t)$ is assumed to stay constant during $(t,\ t + \Delta t)$, and departure rate $\mu_{ij}^d(t + T_{ij}(t))$ is also assumed constant during $[t + T_{ij}(t),\ (t + \Delta t) + T_{ij}(t + \Delta t)]$ for $\forall\ (i, j)$. The proposed algorithm is explained step by step as follows:

Step 1: Initialize present time, link travel times, flow rates and the cumulative numbers:

$$
\begin{array}{llll}
t & := 0\,, & & \\
T_{ij}(t) & := m_{ij}\,, & & \forall\ (i, j), \\
\lambda_{ij}^d(t') & := 0, & t' < 0, & \forall\ (i, j) \text{ and } \forall d, \\
\mu_{ij}^d(t') & := 0, & t' < t + T_{ij}(t) = m_{ij}, & \forall\ (i, j) \text{ and } \forall d, \\
A_{ij}^d(t') & := 0, & t' \le 0, & \forall\ (i, j) \text{ and } \forall d, \\
D_{ij}^d(t') & := 0, & t' \le t + T_{ij}(t) = m_{ij}, & \forall\ (i, j) \text{ and } \forall d,
\end{array}
$$

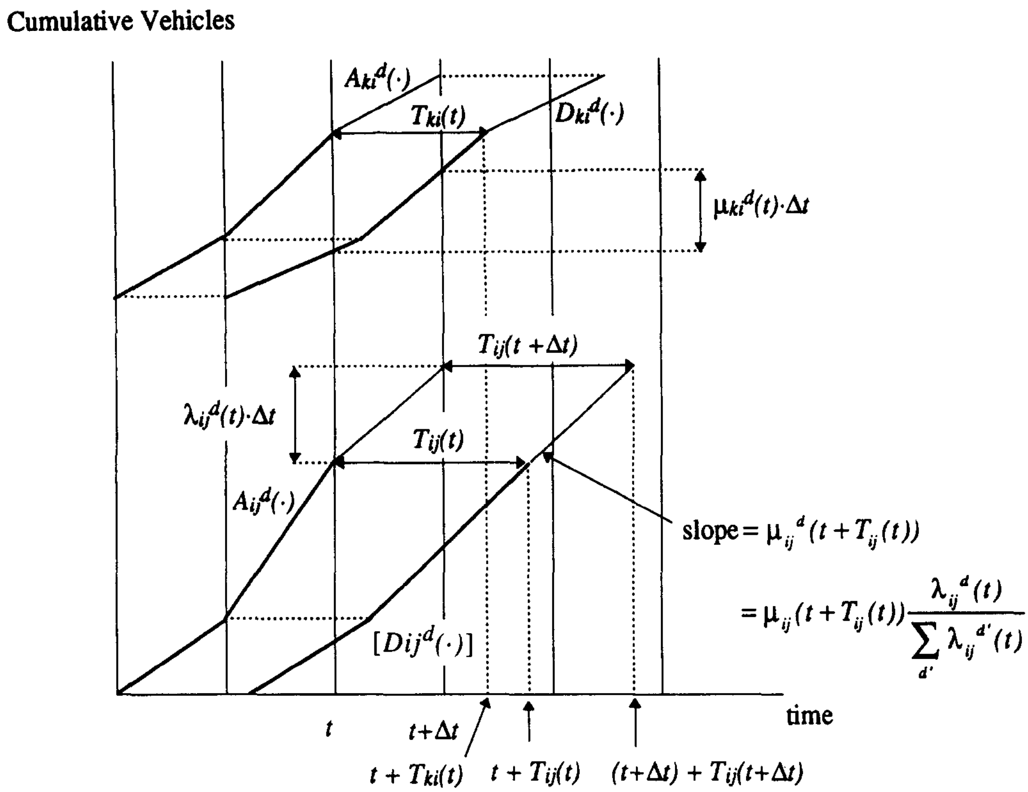

Fig. 3. Construction of cumulative arrival and departure curves on link (i, j).

Step 2: Determine the total arrival demand at node i, $\sum_j \lambda_{ij}^d(t)\cdot\Delta t$, for $(t, t+\Delta t)$ from (13): $\sum_j \lambda_{ij}^d(t)\cdot\Delta t := \sum_k \mu_{ki}^d(t)\cdot\Delta t + q_{id}(t)\cdot\Delta t$, $\forall i$ and $\forall d$, $i \neq d$.

Step 3: Determine the shortest route from node i to destination d, p_{id}, for $\forall i$ and $\forall d$ based on link travel times, $T_{ij}(t)$s.

Step 4: Load the total demand at node i,

$$\sum_j \lambda_{ij}^d(t)\cdot\Delta t,$$

onto a link starting from node i lying on the shortest route p_{id} for $\forall i$ and $\forall d$, and evaluate $\lambda_{ij}^d(t)\cdot\Delta t$ for $\forall(i, j)$ and destination $\forall d$: if link (i, j) lies on route p_{id}, $\lambda_{ij}^d(t)\cdot\Delta t := \sum_j \lambda_{ij}^d(t)\cdot\Delta t$; otherwise $\lambda_{ij}^d(t)\cdot\Delta t := 0$.

Step 5: Evaluate $A_{ij}^d(t+\Delta t)$:

$$A_{ij}^d(t+\Delta t) := A_{ij}^d(t) + \lambda_{ij}^d(t)\cdot\Delta t, \text{ for } \forall(i, j) \text{ and } \forall d.$$

Step 6: Evaluate departure rate $\mu_{ij}^d(t+T_{ij}(t))$ from eqns (9) and (10). By extending $D_{ij}^d(\cdot)$ from $t + T_{ij}(t)$ with the slope of $\mu_{ij}^d(t+T_{ij}(t))$, determine link travel time $T_{ij}(t+\Delta t)$ for $\forall(i, j)$ based on eqn (8).

Step 7: Time $t := t+\Delta t$ and return to step 2 if t is less than the end of the study period; otherwise stop.

Note that, in step 2, to evaluate

$$\sum_j \lambda_{ij}^d(t)\cdot\Delta t$$

during $(t, t+\Delta t)$, we need to know $\mu_{ki}^d(t)\cdot\Delta t$ which means $D_{ki}^d(\cdot)$ must be known at least until time $t+\Delta t$. On the other hand, $D_{ki}^d(\cdot)$ can be drawn until time $t + T_{ki}(t)$ as mentioned earlier. Thus, time interval Δt must be decided so that $\Delta t < T_{ki}(t)$ for any (k, i) and t, which means that Δt should be smaller than or equal to:

$$\min_{(i,j)} m_{ij}.$$

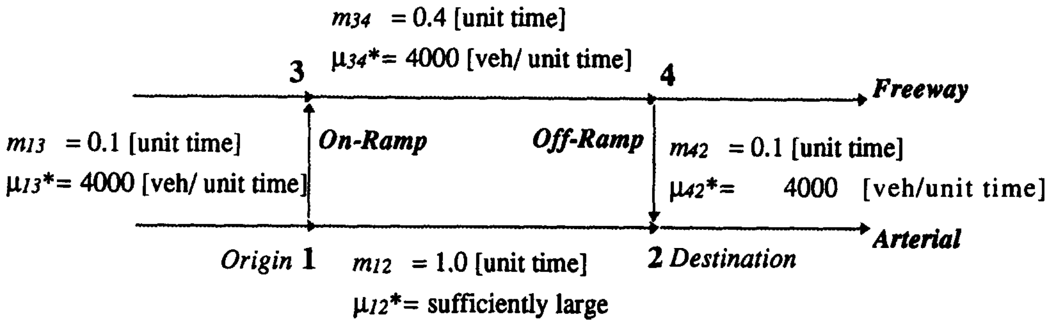

Fig. 4. A simple network.

Also, to simplify the algorithm, the computation starts from the zero-flow condition by setting $A_{ij}^d(t)$ and $D_{ij}^d(t)$ equal to zero in step 1. However, one can start with some positive cumulative numbers with the corresponding link travel times as initial values.

3.4. An example

A simple network with a freeway and an arterial running parallel is shown in Fig. 4, in which free flow link travel times and maximum link service rates are also shown. Although only one arterial is illustrated in Fig. 4, it is considered to the aggregation of several arterials running parallel with the freeway, and its maximum capacity is thus assumed to be sufficiently large to handle the demand.

Let us consider a case with only one OD pair from node 1 to 2 so that the estimated cumulative figures can be easily compared with the theoretical figures under continuous time, which can be easily drawn in this example. For a typical rush hours, the time-dependent OD demand rate is assumed as follows:

$$q_{12}(t) = 2000 \text{ [vehicles/ unit time], for } 0 \le t < 1 \text{ and } 2 \le t,$$
$$q_{12}(t) = 8000 \text{ [vehicles/ unit time], for } 1 \le t < 2.$$

Since link travel times on the freeway are smaller, everyone would use the freeway at the beginning. At time 1.0 [unit time], a queue forms on link $(1,3)$ because the demand increases from 2000 to 8000 [vehicles/ unit time]. The entire OD demand still uses the freeway (links $(1,3)$, $(3,4)$ and $(4,2)$) until time 1.4 [unit time] even after the queue forms. However, at time 1.4 [unit time], travel time via the freeway reaches 1.0 [unit time] due to the queue, and the OD demand should be split into two routes so as to equalize the travel times: $A_{12}(t) = A_{13}(t) = 4000$ [veh/unit time] until time 2.0 [unit time]. (Note that a suffix of destination d is eliminated because of one destination in this example.) Then, at time 2.0 [unit time], the OD demand returns to 2000 [veh/unit time] which is less than the maximum service rate of link $(1,3)$ and therefore everyone again uses the freeway thereafter. In this example, the delay on link $(1,3)$ controls the assignment, since the maximum service rates of downstream links $(3,4)$ and $(4,2)$ are not less than that of link $(1,3)$.

Figure 5 shows the estimated cumulative curves with three different values of Δt, in which we clearly see that as Δt gets smaller, the cumulative curves become close to the theoretical figures. However, general trends of the cumulative curves seem to be reproduced even with $\Delta t = 0.1$ [unit time].

4. SUMMARY AND FUTURE EXTENSIONS

This research deals with the reactive dynamic assignment on an over saturated network with queues, given a time dependent many-to-many OD demand. In the reactive assignment, vehicles are assumed to choose routes based on the current instantaneous link travel times. First, we formulate the flow conservation and the FIFO queue disci-

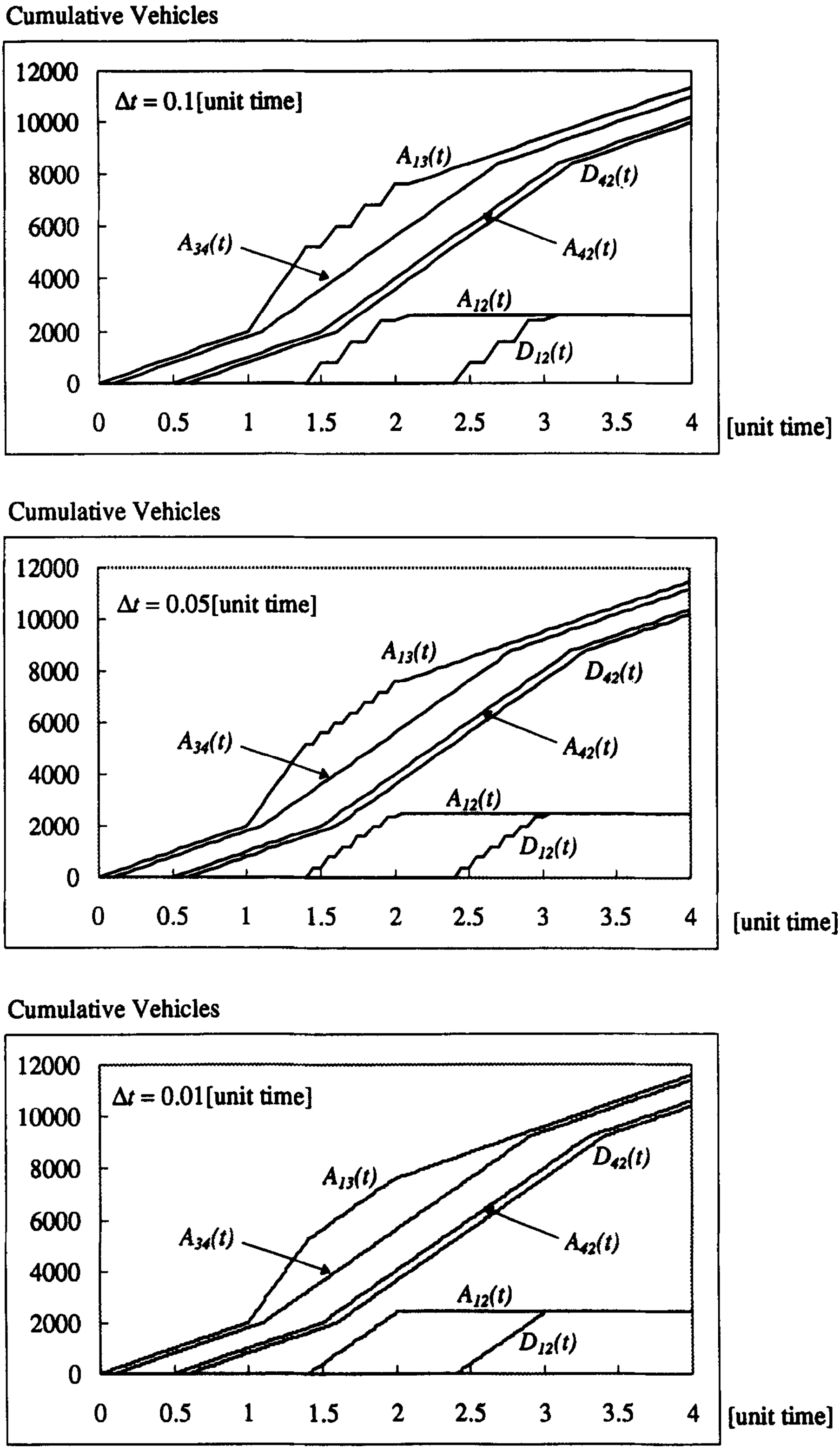

Fig. 5. Estimated cumulative arrivals and departures with various Δt.

pline, which must be satisfied not only in the reactive assignment but generally in any dynamic assignment. We then argue that the link travel time should be defined in relation to the arrival and departure flow rates of the link. Second, the reactive assignment principle is formulated and the assignment is shown to be decomposed with respect to present time. Third, a procedure to draw the cumulative arrival curves at all links is proposed and applied to a simple network.

The analysis presented here is fairly straightforward because the assignment is independent of future traffic conditions. For immediate extension of this analysis, we may introduce the perceived cost function as the criteria of route choice into the same decomposition framework, although here we employ travel time itself to avoid the complication. Also, the stochastic route choice model can be easily included instead of the shortest route choice, as long as travellers are assumed to choose routes based upon present traffic condition. To consider queue back up phenomena based on physical queues rather than point queues would be also covered by the reactive assignment framework, since the queue back up phenomena (kinematic waves) are in principle dependent only upon past and present traffic history. For future research, inclusion of the departure time choice in addition to the route choice seems to be some interest.

Acknowledgement—The authors much appreciate valuable comments and suggestions from anonymous referees.

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