

Decompositions of binomial ideals

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Abstract We present *Binomials*, a package for the computer algebra system Macaulay 2, which specializes well-known algorithms to binomial ideals. These come up frequently in algebraic statistics and commutative algebra, and it is shown that significant speedup of computations like primary decomposition is possible. While central parts of the implemented algorithms go back to a paper of Eisenbud and Sturmfels, we also discuss a new algorithm for computing the minimal primes of a binomial ideal. All decompositions make significant use of combinatorial structure found in binomial ideals, and to demonstrate the power of this approach we show how *Binomials* was used to compute primary decompositions of commuting birth and death ideals of Evans et al., yielding a counterexample for their conjectures.

Keywords Algebraic statistics · Binomial ideals · Commuting birth and death ideals · Computational commutative algebra · Primary decomposition