The Annals of Statistics
2003, Vol. 31, No. 4, 1054–1074
© Institute of Mathematical Statistics, 2003

DECOMPOUNDING: AN ESTIMATION PROBLEM FOR POISSON RANDOM SUMS

BY BORIS BUCHMANN AND RUDOLF GRÜBEL

Technische Universität München and Universität Hannover

Given a sample from a compound Poisson distribution, we consider estimation of the corresponding rate parameter and base distribution. This has applications in insurance mathematics and queueing theory. We propose a plug-in type estimator that is based on a suitable inversion of the compounding operation. Asymptotic results for this estimator are obtained via a local analysis of the decompounding functional.

1. Introduction. The statistical problem to be discussed in this pap motivated by applications from insurance mathematics and queueing theory. I standard model of risk theory [see, e.g., Beard, Pentikäinen and Pesonen (1 or Grandell (1991)], claims of random size $X_1, X_2, X_3, \dots$ arrive at random t $T_1, T_1 + T_2, T_1 + T_2 + T_3, \dots$. The random variables $X_1, X_2, X_3, \dots, T_1, T_2, T$ are assumed to be independent, the X_k, $k \in \mathbb{N}$, have distribution P and interarrival times T_k, $k \in \mathbb{N}$, are exponentially distributed with parameter particular, the claim arrival times are given by the points of a Poisson process constant intensity λ. For all $t \geq 0$,

$$S_t = \sum_{k:\, T_1+\cdots+T_k \leq t} X_k \tag{1}$$

is the total claim amount up to and including time t. Similarly, in a queu context as discussed, for example, in Asmussen (1987), if customers arrive service point in bulks of size $X_1, X_2, \dots$ at the time points of a Poisson pro then (1) gives the total number of customers that arrive in the time interval (0

The assumptions imply that the distribution Q of S_1 can be written convolution series,

$$Q = \Psi(\lambda, P) \qquad \text{with } \Psi(\lambda, P) = e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} P^{*k}. \tag{2}$$

Q is the compound Poisson distribution with rate λ and base (or claim siz bulk size) distribution P. (Unfortunately, Poisson distributions with a ran parameter, i.e., mixed Poisson distributions, are often called compound in literature.)

Received July 2001; revised April 2002.