

# ‘Deconfined’ quantum critical points

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**The theory of second order phase transitions is one of the foundations of modern statistical mechanics and condensed matter theory. A central concept is the observable ‘order parameter’, whose non-zero average value characterizes one or more phases and usually breaks a symmetry of the Hamiltonian. At large distances and long times, fluctuations of the order parameter(s) are described by a continuum field theory, and these dominate the physics near such phase transitions. In this paper we show that near second order *quantum* phase transitions, subtle quantum interference effects can invalidate this paradigm. We present a theory of quantum critical points in a variety of experimentally relevant two-dimensional antiferromagnets. The critical points separate phases characterized by conventional ‘confining’ order parameters. Nevertheless, the critical theory contains a new emergent gauge field, and ‘de-**

**confined’ degrees of freedom associated with fractionalization of the order parameters. We suggest that this new paradigm for quantum criticality may be the key to resolving a number of experimental puzzles in correlated electron systems.**

## **Introduction and motivation**

Much recent research has focused attention on the behavior of matter near zero temperature ‘quantum’ phase transitions that are seen in several strongly correlated many particle systems (1). Indeed, a currently popular view ascribes many properties of interesting correlated materials to the competition between qualitatively distinct ground states and the associated phase transitions. Examples of such materials include the cuprate high- $T_c$  superconductors, and the rare-earth intermetallic compounds (known as the heavy fermion materials).

The traditional guiding principle behind the modern theory of critical phenomena is the association of the critical singularities with fluctuations of an ‘order parameter’ which encapsulates the difference between the two phases on either side of the critical point (a simple example is the average magnetic moment which distinguishes ferromagnetic iron at room temperature, from its high temperature paramagnetic state). This idea, developed by Ginzburg and Landau (2), has been eminently successful in describing a wide variety of phase transition phenomena. It culminated in the sophisticated renormalization group theory of Wilson (3), which gave a general prescription for understanding the critical singularities. Such an approach has been adapted to examine quantum critical phenomena as well, and provides the generally accepted framework for theoretical descriptions of quantum transitions.

The purpose of this paper is to demonstrate and study specific examples of quantum phase transitions which do not fit into this Ginzburg-Landau-Wilson (GLW) paradigm (4). We will show that in a number of different quantum transitions, the natural field theoretic description

of the critical singularities is not in terms of the order parameter field(s) that describe the bulk phases, but in terms of new degrees of freedom specific to the critical point. In the examples studied in this paper, there is an emergent gauge field which mediates interactions between emergent particles that carry fractions of the quantum numbers of the underlying degrees of freedom. These ‘fractional’ particles are not present (*i.e.* are confined) at low energies on either side of the transition, but appear naturally at the transition point. Laughlin has previously argued for fractionalization at quantum critical points on phenomenological grounds (5).

The specific situations studied in this paper are most conveniently viewed as describing phase transitions in two dimensional quantum magnetism, although other applications are also possible (6). Consider a system of spin  $S = 1/2$  moments  $\vec{S}_r$  on the sites,  $r$ , of a two dimensional square lattice with the Hamiltonian

$$H = J \sum_{\langle rr' \rangle} \vec{S}_r \cdot \vec{S}_{r'} + \dots \quad (1)$$

The ellipses represent other short ranged interactions that may be tuned to drive various zero temperature phase transitions. We assume  $J > 0$ , *i.e.* predominantly antiferromagnetic interactions.

Considerable progress has been made over the last fifteen years in elucidating the nature of the various possible ground states of such a Hamiltonian. The Néel state has long range magnetic order (see Fig. 1) and has been observed in a variety of insulators including the prominent parent compound of the cuprates:  $\text{La}_2\text{CuO}_4$ . Apart from such magnetic states, it is now recognized that models in the class of  $H$  can exhibit a variety of quantum paramagnetic ground states. In such states, quantum fluctuations prevent the spins from developing magnetic long range order. Such paramagnetic states can be broadly subdivided into two groups. First, there are states that can be described as ‘valence bond solids’ (VBS) — a simple example is shown in Fig 2. In such states pairs of nearby spins form a singlet, resulting in an ordered pattern of

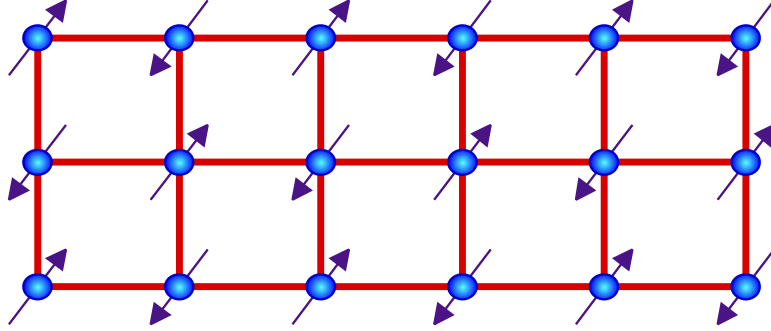


Figure 1: The magnetic Néel ground state of the Hamiltonian (1) on the square lattice. The spins,  $\vec{S}_r$ , fluctuate quantum mechanically in the ground state, but they have a non-zero average magnetic moment which is oriented along the directions shown.

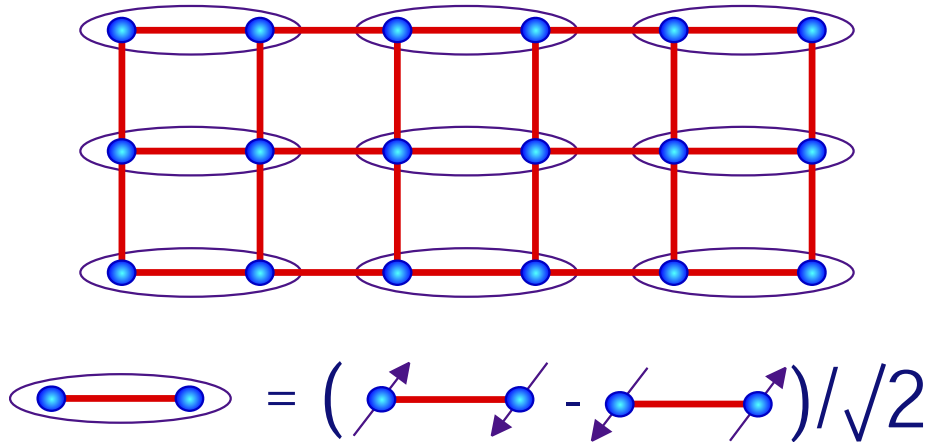


Figure 2: A valence bond solid (VBS) quantum paramagnet. The spins are paired in singlet valence bonds, which resonate among the many different ways the spins can be paired up. The valence bonds ‘crystallize’, so that the pattern of bonds shown has a larger weight in the ground state wavefunction than its symmetry-related partners (obtained by  $90^\circ$  rotations of the above states about a site). This ground state is therefore four-fold degenerate.

‘valence bonds’. Typically, such VBS states have an energy gap to spin-carrying excitations. Furthermore, for spin-1/2 systems on a square lattice, such states also necessarily break lattice translational symmetry. A second class of more exotic paramagnetic states are also possible (7) in principle: in these states, the quantum-mechanical resonance between different valence bond configurations is strong enough to disrupt the VBS, and we obtain a resonating-valence-bond ‘liquid’. The resulting state has been argued to possess excitations with fractional spin and interesting topological structure (8, 9, 10, 11). In this paper we will *not* consider such exotic paramagnetic states. Rather, our focus will be on the nature of the phase transition between the ordered magnet and a VBS. We will also restrict our discussion to the simplest kinds of ordered antiferromagnets - those with collinear order where the order parameter is a single vector (the Néel vector).

Both the magnetic Néel state, and the VBS are states of broken symmetry. The former breaks spin rotation symmetry, and the latter that of lattice translations. The order parameters associated with these two different broken symmetries are very different. A simple Landau-like description of the competition between these two kinds of orders generically predicts either a first-order transition, or an intermediate region of coexistence where both orders are simultaneously present. A direct second order transition between these two broken symmetry phases requires fine-tuning to a ‘multicritical’ point. Our central thesis is that for a variety of physically relevant *quantum* systems, such canonical predictions of Landau’s theory are incorrect. For  $H$ , we will show that a generic second order transition is possible between the very different kinds of broken symmetry in the Néel and VBS phases. Our critical theory for this transition is, however, unusual, and is *not* naturally described in terms of the order parameter fields of either phase. Although we will not explore this case further here, a picture related to the one developed here applies also to transitions between fractionalized spin liquid and VBS states (12), and to transitions between different VBS states (13) in the quantum dimer model (14, 15).

## Field theory and topology of quantum antiferromagnets

In the Néel phase or close to it, the fluctuations of the Néel order parameter are captured correctly by the well-known O(3) nonlinear sigma model field theory (16, 17) with the following action in spacetime (we have promoted the lattice co-ordinate  $r = (x, y)$  to a continuum spatial co-ordinate, and  $\tau$  is imaginary time):

$$\mathcal{S}_n = \frac{1}{2g} \int d\tau \int d^2r \left[ \frac{1}{c^2} \left( \frac{\partial \hat{n}}{\partial \tau} \right)^2 + (\nabla_r \hat{n})^2 \right] + iS \sum_r (-1)^r \mathcal{A}_r. \quad (2)$$

Here  $\hat{n} \propto (-1)^r \vec{S}_r$  is a unit three component vector that represents the Néel order parameter (the factor  $(-1)^r$  is +1 on one checkerboard sublattice, and  $-1$  on the other). The second term is the quantum mechanical Berry phase of all the  $S = 1/2$  spins:  $\mathcal{A}_r$  is the area enclosed by the path mapped by the time evolution of  $\hat{n}_r$  on a unit sphere in spin space. These Berry phases play an unimportant role in the low energy properties of the Néel phase (17), but are crucial in correctly describing the quantum paramagnetic phase (18). We will show here that they also modify the quantum critical point between these phases, so that the exponents are distinct from those of the GLW theory without Berry phases studied earlier (17, 19).

To describe the Berry phases, first note that in two spatial dimensions, smooth configurations of the Néel vector admit topological textures known as skyrmions (see Fig 3). The total *skyrmion number* associated with a configuration defines an integer topological quantum number  $Q$ :

$$Q = \frac{1}{4\pi} \int d^2r \hat{n} \cdot \partial_x \hat{n} \times \partial_y \hat{n}, \quad (3)$$

The sum over  $r$  in (2) vanishes (16, 1) for all spin time histories with *smooth* equal-time configurations, even if they contain skyrmions. For such smooth configurations, the total skyrmion number  $Q$  is independent of time. However, the original microscopic model is defined on a lattice, and processes where  $Q$  changes by some integer amount are allowed. Specifically, such a  $Q$

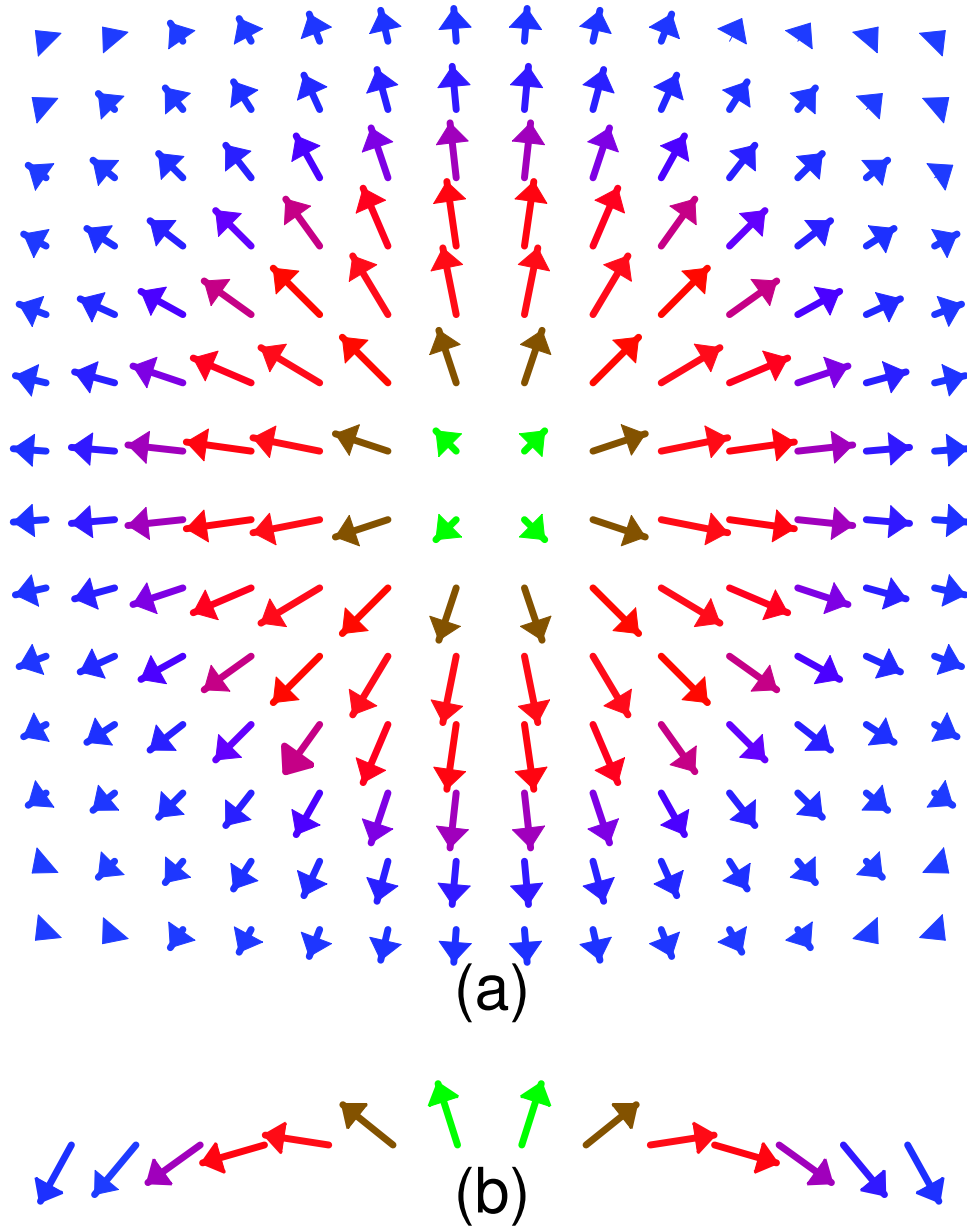


Figure 3: A skyrmion configuration of the field  $\hat{n}(r)$ . In (a) we show the vector  $(n^x, n^y)$  at different points in the  $xy$  plane; note that  $\hat{n} = (-1)^r \vec{S}_r$ , and so the underlying spins have a rapid sublattice oscillation which is not shown. In (b) we show the vector  $(n^x, n^z)$  along a section of (a) on the  $x$  axis. Along any other section of (a), a picture similar to (b) pertains, as the former is invariant under rotations about the  $z$  axis. The skyrmion above has  $\hat{n}(r = 0) = (0, 0, 1)$  and  $\hat{n}(|r| \rightarrow \infty) = (0, 0, -1)$ .

changing event corresponds to a monopole (or ‘hedgehog’) singularity of the Néel field  $\hat{n}(r, \tau)$  in space-time (see Fig 4). Haldane (16) showed that the sum over  $r$  in (2) is non-vanishing in the presence of such monopole events. Precise calculation (16) gives a total Berry phase associated with each such  $Q$  changing process which oscillates rapidly on four sublattices of the dual lattice. This leads to destructive interference which effectively suppresses all monopole events unless they are quadrupled (16, 18) (*i.e* they change  $Q$  by four).

The sigma model field theory augmented by these Berry phase terms is, in principle, powerful enough to correctly describe the quantum paramagnet. Summing over the various monopole tunnelling events shows that in the paramagnetic phase the presence of the Berry phases leads to VBS order (18). Thus  $\mathcal{S}_n$  contains within it the ingredients describing both the ordered phases of  $H$ . However a description of the transition between these phases has so far proved elusive, and will be provided here.

Our analysis of this critical point is aided by writing the Néel field  $\hat{n}$  in the so-called  $\text{CP}^1$  parametrization:

$$\hat{n} = z^\dagger \vec{\sigma} z, \quad (4)$$

with  $\vec{\sigma}$  a vector of Pauli matrices. Here  $z = z(r, \tau) = (z_1, z_2)$  is a two-component complex spinor of unit magnitude which transforms under the spin-1/2 representation of the  $\text{SU}(2)$  group of spin rotations. The  $z_{1,2}$  are the fractionalized ‘‘spinon’’ fields. To understand the monopoles in this representation, let us recall that the  $\text{CP}^1$  representation has a  $\text{U}(1)$  gauge redundancy. Specifically the *local* phase rotation

$$z \rightarrow e^{i\gamma(r,\tau)} z, \quad (5)$$

leaves  $\hat{n}$  invariant, and hence is a gauge degree of freedom. Thus the spinon fields are coupled to a  $\text{U}(1)$  gauge field,  $a_\mu$  (the spacetime index  $\mu = (r, \tau)$ ). As is well-known, the magnetic flux of  $a_\mu$  is the topological charge density of  $\hat{n}$  appearing in the integrand of (3). Specifically,



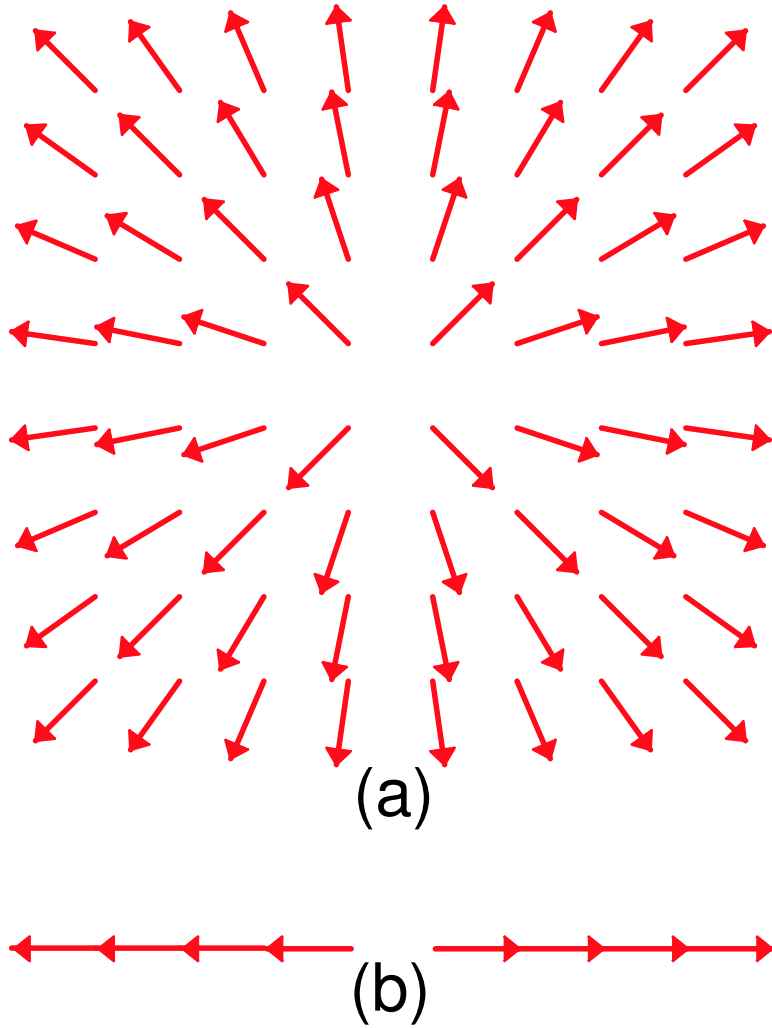


Figure 4: A monopole event, taken to occur at the origin of spacetime. An equal-time slice of spacetime at the tunnelling time is represented following the conventions of Fig 3. So (a) contains the vector  $(n^x, n^y)$ ; the spin configuration is radially symmetric, and consequently a similar picture is obtained along any other plane passing through the origin. Similarly, (b) is the representation of  $(n^x, n^z)$  along the  $x$  axis, and a similar picture is obtained along any line in spacetime passing through the origin. The monopole above has  $\hat{n}(r) = r/|r|$ .

configurations where the  $a_\mu$  flux is  $2\pi$  correspond to a full skyrmion (in the ordered Néel phase). Thus the monopole events described above are space-time ‘magnetic’ monopoles (instantons) of  $a_\mu$  at which  $2\pi$  gauge flux can either disappear or be created. That such instanton events are allowed, means that the  $a_\mu$  gauge field is to be regarded as ‘compact’.

We now state our key result for the critical theory between the Néel and VBS phases. As we will demonstrate below, the Berry phase-induced quadrupling of monopole events renders monopoles *irrelevant* at the quantum critical point. So in the critical regime (but *not* away from it in the paramagnetic phase), we may neglect the compactness of  $a_\mu$ , and write down the simplest continuum theory of spinons interacting with a non-compact U(1) gauge field with action  $\mathcal{S}_z = \int d^2r d\tau \mathcal{L}_z$ , and

$$\mathcal{L}_z = \sum_{a=1}^N |(\partial_\mu - ia_\mu) z_a|^2 + s|z|^2 + u(|z|^2)^2 + \kappa (\epsilon_{\mu\nu\kappa} \partial_\nu a_\kappa)^2, \quad (6)$$

where  $N = 2$  is the number of  $z$  components (later we will consider the case of general  $N$ ), we have softened the length constraint on the spinons with  $|z|^2 \equiv \sum_{a=1}^N |z_a|^2$  allowed to fluctuate, and the value of  $s$  is to be tuned so that  $\mathcal{L}_z$  is at its scale-invariant critical point. Note that the irrelevance of monopole tunneling events at the critical fixed point implies that the total gauge flux  $\int d^2r (\partial_x a_y - \partial_y a_x)$ , or equivalently the skyrmion number  $Q$ , is asymptotically conserved. This emergent global topological conservation law provides precise meaning to the notion of deconfinement. It is important to note that the critical theory described by  $\mathcal{L}_z$  (21) is distinct from the GLW critical theory of the O(3) non-linear sigma model obtained from (2) by dropping the Berry phases and tuning  $g$  to a critical value (22). In particular, the latter model has a non-zero rate of monopole tunneling events at the transition, so that the global skyrmion number  $Q$  is not conserved.

A justification for the origin of  $\mathcal{L}_z$  is provided in the remainder of this paper. We will begin in the following section by considering antiferromagnets with an ‘easy plane’ anisotropy, so

that the spins prefer to lie in the  $xy$  plane. Subsequent sections generalize the arguments to fully isotropic antiferromagnets.

## Duality transformations with easy plane anisotropy

For the easy-plane case, duality maps and an explicit derivation of a dual form of  $\mathcal{L}_z$  are already available in the literature (6, 24). Moreover, an easy plane  $S = 1/2$  model with two- and four-particle ring exchanges has recently been studied numerically (25), and a direct transition between Néel and VBS phases was found. Here we will obtain the theory for this transition (6, 24) using simple physical arguments which enable generalization to the isotropic case.

The easy plane anisotropy reduces the continuous SU(2) spin rotational invariance to the U(1) subgroup of rotations about the  $z$ -axis of spin. An additional important discrete symmetry is time reversal, under which

$$\vec{S}_r \rightarrow -\vec{S}_r. \quad (7)$$

This may be combined with a rotation in the  $xy$  plane which restores the sign of  $S_{x,y}$  to simply change the sign of  $S^z$  alone, comprising a  $Z_2$  symmetry. With these symmetries, (2) allows an additional term  $u_{ep} \int d\tau d^2r (n^z)^2$ , with  $u_{ep} > 0$ .

Let us first think classically about this easy plane model. The classical (Néel) ground state simply consists of letting  $\hat{n}$  be independent of position, and lying entirely in the spin  $xy$  plane. Topological defects in this ground state will play an important role. With the easy plane anisotropy, these are vortices in the complex field  $n^+ = n_x + in_y$ . More precisely, on going around a large loop containing a vortex the phase of  $n^+$  winds around by  $2\pi m$  with  $m$  an integer.

What is the nature of the core of these vortices? In the core, the XY order will be suppressed and the  $\hat{n}$  vector will point along the  $\pm\hat{z}$  direction. Physically, this corresponds to a non-zero staggered magnetization of the  $z$ -component of the spin in the core region. Thus, at the classical

level there are two kinds of vortices – often called ‘merons’ in this context – depending on the direction of the  $\hat{n}$  vector at the core (see Fig 5). Either kind of vortex *breaks* the Ising-like  $n^z \rightarrow -n^z$  symmetry at the core. For future convenience, let us denote by  $\psi_1$  ( $\psi_2$ ) the quantum field that destroys a vortex whose core points in the up (down) direction.

Clearly, this breaking of the Ising symmetry is an artifact of the classical limit - once quantum effects are included, the two broken symmetry cores will be able to tunnel into each other and there will be no true broken Ising symmetry in the core. This tunnelling is often called an ‘instanton’ process that connects two classically degenerate states.

Surprisingly, such an instanton event is physically the easy plane avatar of the space-time monopole described above for the fully isotropic model. This may be seen pictorially. Each classical vortex of Fig 5 really represents half of the skyrmion configuration of Fig 3. Now imagine a  $\psi_2$  meron at time  $\tau \rightarrow -\infty$  with a spatial configuration as in Fig 5a,c, and the  $\psi_1$  meron as in Fig 5a,b at time  $\tau \rightarrow \infty$ . These two configurations cannot be smoothly connected, and there must be a singularity in the  $\hat{n}$  configuration, which we place at the origin of spacetime. A little imagination now shows that the resulting configuration of  $\hat{n}$  can be smoothly distorted into the radially symmetric monopole event of Fig 4 (indeed, the union of Figs 5b,c placed as shown is easily seen to be similar to Fig 4a). Thus, the tunnelling process between the two merons is equivalent to creating a full skyrmion. This is precisely the monopole event of Fig 4. Hence, as pictorially reinforced in Figs. 3,5, a skyrmion may be regarded as a composite of an “up” meron and a “down” antimeron, and the skyrmion number is hence the difference in the numbers of up and down merons.

The picture so far has not accounted for the Berry phases terms. The interference effect discussed above for isotropic antiferromagnets applies here too, leading to an effective cancellation of instanton tunnelling events between single  $\psi_1$  and  $\psi_2$  merons. The only effective tunnelling are those in which *four*  $\psi_1$  merons come together and collectively flip their core spins

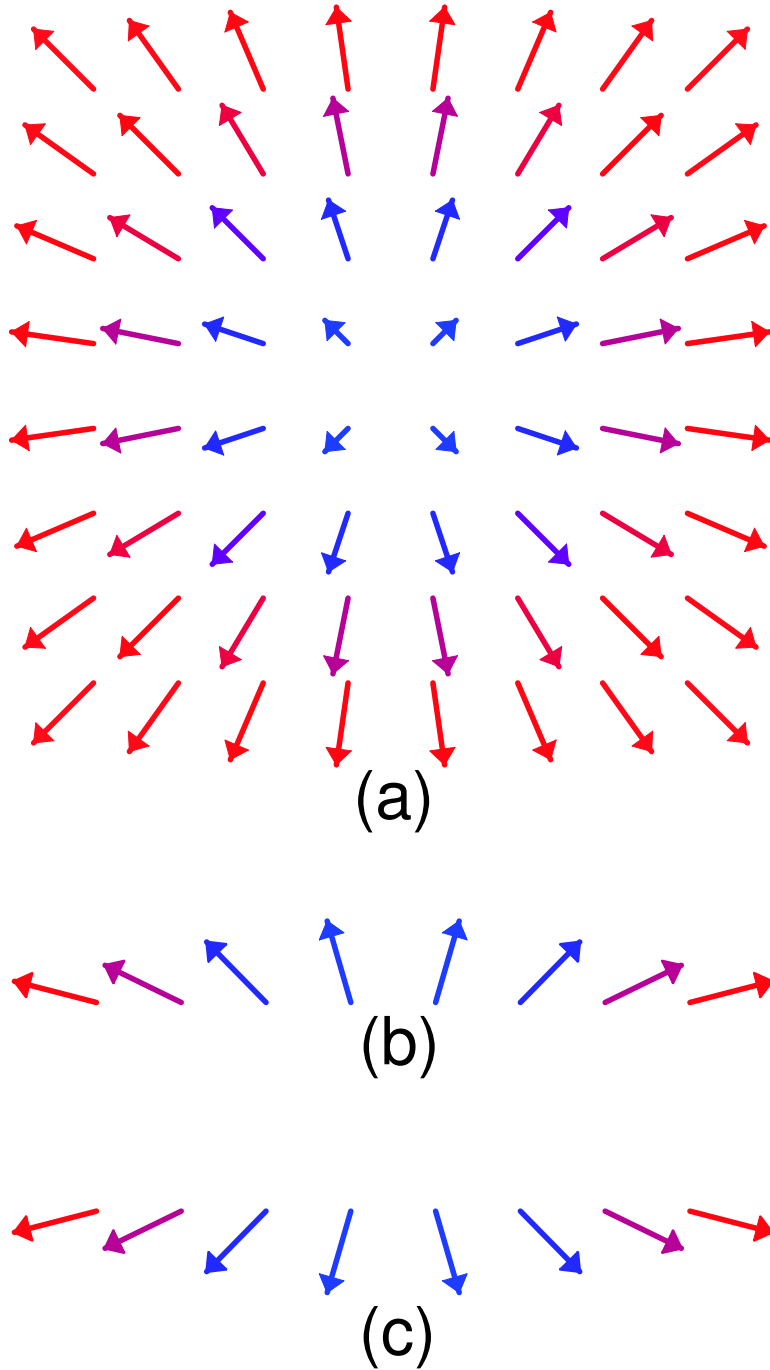


Figure 5: The ‘meron’ vortices in the easy plane case. There are two such vortices,  $\psi_{1,2}$ , and  $\psi_1$  is represented in (a) and (b), while  $\psi_2$  is represented by (a) and (c), following the conventions of Fig 3. The  $\psi_1$  meron above has  $\hat{n}(r = 0) = (0, 0, 1)$  and  $\hat{n}(|r| \rightarrow \infty) = (x, y, 0)/|r|$ ; the  $\psi_2$  meron has  $\hat{n}(r = 0) = (0, 0, -1)$  and the same limit as  $|r| \rightarrow \infty$ . Each meron above is ‘half’ the skyrmion in Fig 3: this is evident from a comparison of (b) and (c) above with Fig 3b. Similarly, one can observe that a composite of  $\psi_1$  and  $\psi_2^*$  makes one skyrmion.

to produce four  $\psi_2$  merons, or vice versa.

A different perspective on the  $\psi_{1,2}$  meron vortices is provided by the  $\text{CP}^1$  representation. Ordering in the  $xy$  plane of spin-space requires condensing the spinons,

$$|\langle z_1 \rangle| = |\langle z_2 \rangle| \neq 0, \quad (8)$$

so that  $n^+ = z_1^* z_2$  is ordered *and* there is no average value of  $n^z = |z_1|^2 - |z_2|^2$ . Now, clearly, a full  $2\pi$  vortex in  $n^+$  can be achieved by either having a  $2\pi$  vortex in  $z_1$  and not in  $z_2$ , or a  $2\pi$  antivortex in  $z_2$  and no vorticity in  $z_1$ . In the first choice, the amplitude of the  $z_1$  condensate will be suppressed at the core, but  $\langle z_2 \rangle$  will be unaffected. Consequently  $n^z = |z_1|^2 - |z_2|^2$  will be non-zero and negative in the core, as in the  $\psi_2$  meron. The other choice also leads to non-zero  $n^z$  which will now be positive, as in the  $\psi_1$  meron. Clearly, we may identify the  $\psi_2$  ( $\psi_1$ ) meron vortices with  $2\pi$  vortices (anti-vortices) in the spinon fields  $z_1$  ( $z_2$ ). Note that in terms of the spinons, paramagnetic phases correspond to situations in which neither spinon field is condensed.

The above considerations, and the general principles of boson duality in three spacetime dimensions (23) determine the form of the dual action,  $\mathcal{S}_{\text{dual}} = \int d\tau d^2r \mathcal{L}_{\text{dual}}$  for  $\psi_{1,2}$  (6, 24):

$$\begin{aligned} \mathcal{L}_{\text{dual}} = & \sum_{a=1,2} |(\partial_\mu - iA_\mu) \psi_a|^2 + r_d |\psi|^2 + u_d (|\psi|^2)^2 \\ & + v_d |\psi_1|^2 |\psi_2|^2 + \kappa_d (\epsilon_{\mu\nu\kappa} \partial_\nu A_\kappa)^2 - \lambda [(\psi_1^* \psi_2)^4 + (\psi_2^* \psi_1)^4], \end{aligned} \quad (9)$$

where  $|\psi|^2 \equiv |\psi_1|^2 + |\psi_2|^2$ .

The correctness of this form may be argued as follows: First, from the usual boson-vortex duality transformation (23), the dual  $\psi_{1,2}$  vortex fields must be minimally coupled to a dual *non-compact*  $\text{U}(1)$  gauge field  $A_\mu$ . Note that this dual gauge invariance is *not* related to (5), but is a consequence of the conservation of the total  $S^z$ : the ‘magnetic’ flux  $\epsilon_{\mu\nu\kappa} \partial_\nu A_\kappa$  is the conserved  $S^z$  current (23). Second, under the  $Z_2$   $T$ -reversal symmetry the two vortices get interchanged,

*i.e.*  $\psi_1 \leftrightarrow \psi_2$ . The dual action must therefore be invariant under interchange of the 1 and 2 labels. Finally, if monopole events were to be disallowed by hand, the total skyrmion number – *i.e.* the difference in number of up and down meron vortices – would be conserved. This would imply a global U(1) symmetry (not to be confused with the U(1) spin symmetry about the  $z$  axis) under which

$$\psi_1 \rightarrow \psi_1 \exp(i\alpha) ; \quad \psi_2 \rightarrow \psi_2 \exp(-i\alpha), \quad (10)$$

where  $\alpha$  is a constant. However, monopole events destroy the conservation of skyrmion number, and hence this dual global U(1) symmetry. But, as the monopoles are effectively quadrupled by cancellations from Berry phases, *skyrmion number is still conserved modulo 4*. Thus the symmetry (10) must be broken down to the discrete cyclic group of four elements,  $Z_4$ .

The dual Lagrangian in (9) is the simplest one that is consistent with all these requirements. In particular, we note that in the absence of the  $\lambda$  term the dual global U(1) transformation in Eqn. 10 leaves the Lagrangian invariant. The  $\lambda$  term breaks this down to  $Z_4$  as required. Thus we may identify this term physically with the (quadrupled) monopole tunnelling events.

In this dual vortex theory, the XY ordered phase is simply characterized as a dual ‘paramagnet’ where  $\langle \psi_{1,2} \rangle = 0$  and fluctuations of  $\psi_{1,2}$  are gapped. On the other hand, spin paramagnetic phases such as the VBS states correspond to condensates of the fields  $\psi_{1,2}$  which break the dual gauge symmetry. In particular if both  $\psi_1$  and  $\psi_2$  condense with equal amplitude  $|\langle \psi_1 \rangle| = |\langle \psi_2 \rangle| \neq 0$ , then we obtain a paramagnetic phase where the global Ising symmetry is preserved. Note the remarkable complementarity between the description of the phases in this dual theory with that in terms of the spinon fields of the  $CP^1$  representation: the descriptions map onto one another upon interchanging both  $z_{1,2} \leftrightarrow \psi_{1,2}$  and the role of the XY ordered and paramagnetic phases. As discussed below, this is a symptom of an exact duality between the two descriptions that obtains close to the transition.

The combination  $\psi_1^* \psi_2 \equiv |\psi_1 \psi_2| e^{i(\theta_1 - \theta_2)}$  actually serves as an order parameter for the trans-

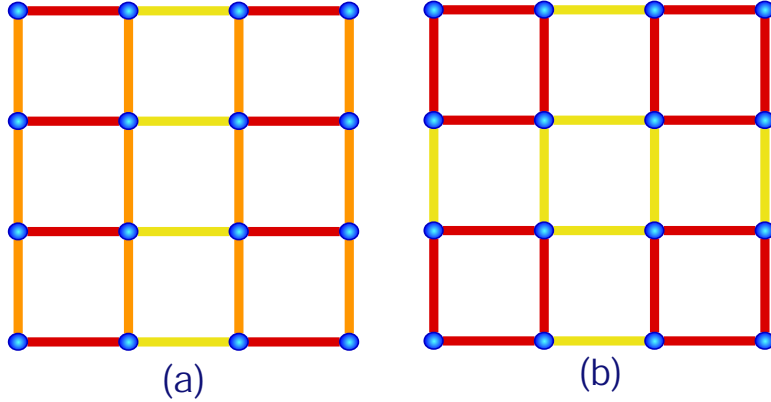


Figure 6: Pattern of symmetry breaking in the two possible VBS states predicted by (9). The last term in (9) leads to a potential,  $-\lambda \cos(4(\theta_1 - \theta_2))$ , and the sign of  $\lambda$  chooses between the two states above. The distinct lines represent distinct values of  $\langle \vec{S}_r \cdot \vec{S}_{r'} \rangle$  on each link. Note that the state in (a) is identical to that in Fig 2.

lation symmetry broken VBS ground state. This may be seen from the analysis of Refs. (6, 24). Alternatively, we may use the identification, due to Read and Sachdev (18), of the skyrmion creation operator with the order parameter for translation symmetry breaking. Such a condensate of  $\psi_{1,2}$  breaks the global  $Z_4$  symmetry of the action in Eqn. 9. The preferred direction of the angle  $\theta_1 - \theta_2$  depends on the sign of  $\lambda$ . The two sets of preferred directions correspond to columnar and plaquette patterns of translational symmetry breaking (see Fig 6).

Let us now consider the transition between the XY ordered and VBS phases described by  $\mathcal{L}_{\text{dual}}$  at  $\lambda = 0$ , *i.e.* in the absence of instanton events. This model has been studied (21, 22), and has a remarkable self-dual (22) critical point. Here, our arguments demonstrate the self duality quite simply: recall that we had argued earlier that in the absence of instantons, the continuum limit of the direct SU(2)-invariant theory led to  $\mathcal{L}_z$  in (6). The easy plane anisotropy will allow an additional  $v|z_1|^2|z_2|^2$  term in  $\mathcal{L}_z$ , and then  $\mathcal{L}_z$  has exactly the same form as the dual theory  $\mathcal{L}_{\text{dual}}$  under  $z_{1,2} \rightarrow \psi_{1,2}$  and  $a_\mu \rightarrow A_\mu$ ! Note that in both the direct and dual representations, the degrees of freedom in the Lagrangian are not those associated with the ‘physical’ boson operator (either  $n^+$  or the skyrmion creation operator). Rather the theory is expressed most simply in



terms of ‘fractionalized’ fields – namely the spinons or the meron vortices. In particular, the physical  $n^+$  field is a composite of two spinon fields, and the skyrmion field is likewise a composite of the two meron fields.

Let us now include monopole events. This is where the existence of a dual representation pays off, as the non-trivial, non-local physics of instantons in the direct theory are represented by a local perturbation in the dual theory: simply set  $\lambda \neq 0$  in  $\mathcal{L}_{\text{dual}}$ . The relevance/irrelevance of monopoles at the self-dual  $\lambda = 0$  fixed point is determined by the scaling dimension,  $\Delta$ , of the  $(\psi_1^* \psi_2)^4$  operator, *i.e.* the fourth power of the creation operator of the physical boson. Provided  $\Delta > d + 1 = 3$ , monopoles will be irrelevant. The  $\lambda = 0$  critical fixed point describes an XY ordering transition where the physical boson field is a composite of the fundamental fields of the theory. We therefore expect that correlators of the physical boson (and its various powers) will decay with a scaling dimension that is *larger* than the corresponding one for the ordinary XY transition in  $D = 2 + 1$  dimensions. Now for the usual XY fixed point four-fold symmetry breaking perturbations are known to be *irrelevant* (26). This then implies that a small  $\lambda$  will be irrelevant by power counting at the  $\lambda = 0$  fixed point of the present model as well. Note the crucial role played by the Berry phase term for the monopoles in reaching this conclusion – quadrupling the monopoles and thereby increasing their scaling dimension renders them irrelevant.

Although the  $\lambda$  term is irrelevant at the critical fixed point, it is clearly very important in deciding the fate of either phase. In particular, in the paramagnetic phase it picks out the particular pattern of translation symmetry breaking (columnar versus plaquette) and forces linear confinement of spinons. In critical phenomena parlance, it may be described as a *dangerously irrelevant* perturbation (19).

Thus, in the easy plane case there is the possibility of a direct second order transition between the Néel and VBS phases. Remarkably the critical theory is ‘deconfined’ in the sense that

the spinons emerge as natural degrees of freedom right at the critical point. We note that the spinons are *confined* in both phases and do not appear in the excitation spectrum. The length scale at which this confinement occurs however *diverges* on approaching the critical point.

At a more sophisticated level, the critical fixed point is characterized by the emergence of an extra global U(1) symmetry (10) that is not present in the microscopic Hamiltonian. This is associated with conservation of skyrmion number and follows from the irrelevance of monopole tunnelling events only at the critical point.

## Isotropic magnets

We now provide evidence supporting the possibility that a direct second transition separates Néel and VBS states of isotropic (*i.e.* SU(2) invariant) spin-1/2 magnets with ‘deconfinement’ obtaining at the critical point. For this purpose, it is convenient to work with the direct CP<sup>1</sup> representation in terms of spinon fields. Following Ref. (27), consider a generalization to CP<sup>*N*-1</sup> models of an *N*-component complex field  $z_a$  that is coupled to a compact U(1) gauge field with the same Haldane Berry phases as in the  $N = 2$  case of interest. We argue that both at  $N = 1$  and at  $N$  large, the model displays a second-order transition between a ‘Higgs’ phase (this phase is the analog of the antiferromagnetic Néel phase for  $N = 2$ ) and a paramagnetic VBS phase with confined spinons which breaks lattice translation symmetry; moreover, instanton events are irrelevant at this critical point.

Consider first  $N = 1$  where  $z \equiv e^{i\phi}$  is simply a complex number of unit magnitude. This  $N = 1$  model displays a transition between a Higgs and a VBS phase (27). The latter has a four-fold degenerate ground state due to lattice symmetry breaking. Simple symmetry arguments suggest a transition modelled by a  $Z_4$  clock model. Since the four fold anisotropy is irrelevant at the  $D = 3$  XY fixed point (26), this is in the 3D XY universality class. Ref. (27) also provided numerical evidence supporting this expectation. All of this is also readily established by the

analog of the duality discussed above for  $N = 1$ . The dual global XY fields simply represent  $2\pi$  vortices in  $z$ , and the four-fold anisotropy is the quadrupled instanton event. Once again the quadrupling is due to the Berry phase term. Thus the irrelevance of the four-fold anisotropy may be interpreted as the irrelevance of instanton events. The resulting  $3D$  XY universality is simply the dual of the condensation transition of the  $z$  boson coupled to a non-compact  $U(1)$  gauge field (23).

Now let us consider  $N$  large and begin with the model with *all* monopoles excluded. This is the non-compact  $CP^{N-1}$  model and has an ordering transition associated with the condensation of  $z$ . It is clear that the crucial question is whether the four-monopole event is relevant/irrelevant at the fixed point of this non-compact model. The scaling dimension of the  $q$ -monopole operator in this model was computed by Murthy and Sachdev (28) and their results give a scaling dimension  $\propto N$ . For large  $N$  this is much larger than  $D = 3$ , and hence the  $q = 4$  monopoles are strongly irrelevant for large- $N$  (29).

Thus the  $N = 1$ ,  $N = \infty$ , and easy plane  $N = 2$  models all support the same picture. A direct second order transition between the spinon-condensed and VBS phases is possible with a ‘deconfined’ critical point. Right at this point, monopole tunnelling events become irrelevant, spinon degrees of freedom emerge as the natural fields of the critical theory, and there is an extra global conservation law of skyrmion number which is absent in the microscopic Hamiltonian. This provides strong evidence that the same scenario obtains for the  $SU(2)$  symmetric model (*i.e* at  $N = 2$ ). However, the self-duality found in the easy plane  $N = 2$  model is special and *not likely* to generalize to the isotropic case. In the direct representation, the critical theory of the isotropic case is the critical point of  $\mathcal{L}_z$  in (6). Remarkably, as claimed earlier, the complications of compactness and Berry phases (both required by the microscopics) have cancelled one another, leading ultimately to a much simpler critical theory ! Equivalently, the critical theory is that of the  $D = 3$  classical  $O(3)$  model with monopoles suppressed. Direct numerical

simulations of such an  $O(3)$  model, and of the non-compact  $CP^1$  model, have been performed in Ref. (22), and the results are consistent with a common critical theory. Earlier work (30,31) had examined related  $O(3)$  models, and the results for critical exponents (31) are also consistent with Ref. (22).

## Physical properties near the ‘deconfined’ critical point

We now briefly mention the consequences of our theory for the physical properties near the quantum phase transition between the Néel and VBS phases. The presence of a dangerously irrelevant coupling implies that there are two distinct length scales which diverge as we approach the critical point from the VBS side: the spin correlation length  $\xi$ , and a *longer* length scale  $\xi_{\text{VBS}} \sim \xi^{(\Delta-1)/2}$  (where  $\Delta > 3$  is the scaling dimension of 4-monopole operator) which determines the thickness of a domain wall between two VBS states. Remarkably, on length scales  $\xi < L < \xi_{\text{VBS}}$ , the low energy excitations of the VBS state are more similar to Goldstone modes (associated with spontaneous breaking of a continuous symmetry), than domain walls, despite the discrete broken symmetry of the phase. Standard scaling arguments can be used to predict a variety of physical properties, among which are: (i) the emergence of spinons at the critical point leads to a large anomalous dimension of the Néel order, estimated to be  $\eta \approx 0.6$  (31,22); (ii) the self-duality of the critical point for the easy plane case implies the remarkable result that the columnar dimer, plaquette, and staggered XY magnetization all decay with the same power law at the critical point, and the  $\beta$  exponents for the VBS and XY orders are the same.

One intriguing aspect of our theory is the physics of the vortices of the XY ordered phase (in the easy plane case) close to the transition. As discussed extensively above, there are two kinds of classical meron vortices which tunnel into each other in the quantum theory. However, the irrelevance of these instanton events near the transition implies that the tunnelling rate of the Ising order in the core of merons will diverge on approaching the transition. This Ising order

may be detectable in numerical studies (25) - this is despite the transition actually being to a VBS phase with no such staggered order !

## Discussion

Our results offer a new perspective on the phases of Mott insulators in two dimensions: liquid resonating-valence-bond-like states, with gapless spinon excitations, can appear at isolated critical points between phases characterized by conventional confining orders. It appears probable that similar considerations apply to quantum critical points in doped Mott insulators, between phases with a variety of spin- and charge-density-wave orders and  $d$ -wave superconductivity. If so, the electronic properties in the quantum critical region of such critical points will be strongly non-Fermi-liquid like, raising the prospect of understanding the phenomenology of the cuprate superconductors.

On the theoretical side, our results also illuminate studies of frustrated quantum antiferromagnets in two dimensions. A theory of the apparent critical point between the Néel and VBS phases observed by Sandvik *et al.* (25) is now available, and precise tests of the values of critical exponents should now be possible. A variety of other SU(2)-invariant antiferromagnets have been studied (32), and many of them exhibit VBS phases. It would be interesting to explore the characteristics of the quantum critical points adjacent to these phases, and test our prediction of gapless, liquid, resonating-valence-bond-like behavior.

Our results also caricature interesting phenomena (33, 34) found in the vicinity of the onset of magnetism in the heavy fermion metals. Remarkably the Kondo coherence that characterizes the non-magnetic heavy Fermi liquid seems to disappear at the same point that magnetic long range order sets in. Furthermore strong deviations from Fermi liquid theory are seen in the vicinity of the quantum critical point. All of this is in contrast to naive expectations based on the Landau paradigm for critical phenomena. However this kind of exotic quantum criticality

between two conventional phases is precisely the physics discussed in the present paper.

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