Deconvolution of multiple images of the same object

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Deconvolution of images of the same object from multiple sensors with different point spread functions as suggested by Berenstein [Proc. IEEE **78,** 723 (1990); *Stochastic and Neural Methods in Signal Processing, Image Processing, and Computer Vision,* S. Chen, ed., Proc. Soc. Photo-Opt. Instrum. Eng. **1569,** 35 (1991)], opens new opportunities in solving the image-deconvolution problem, which has challenged researchers for years. We attack this problem in a more realistic formulation than that used by Berenstein; it explicitly takes into account image sensor noise and the necessity for adaptive restoration with estimation of all required signal and noise parameters directly from the observed noisy signals. We show that arbitrary restoration accuracy can be achieved by the appropriate choice of the number of sensor channels and the signal-to-noise ratio in each channel. The results are then extended to the practically important situation when true images in different sensor channels are not identical.

1. Introduction

Since very early publications in the 1960's, image deconvolution has remained one of the most popular and challenging problems for academic research in image processing. A number of approaches have been investigated since that time (for references, see, for instance Refs. 1–3): inverse and pseudoinverse filtering, least-squares methods, the method of maximum entropy, constrained deconvolution and iterative restoration methods, Bayesian methods based on statistical models of images, etc. All of them show, more or less, applicability for the models involved into substantiation of the methods. However, to our knowledge, no method is regularly used for solving real, practical problems, except possibly the method of maximum entropy, which has been shown to be reasonably effective in stellar astronomy and similar applications in which images represent small objects on more or less uniform background. The reason of the failure for real applicability of the image-restoration methods is connected to the so-called ill posedness of the problem, which means that the number of independent degrees of freedom of the distorted signal is generally less than that of the original one. There are two main sources of the loss of the signal's degrees of freedom (we do not speak now about the sensor's noise, which always accompanies blur introduced by the image sensor). These are boundary effects and zeros in the frequency responses of the imaging systems. Because of the boundary effects, parts of the original signal near the borders of the image frame are lost. Because of zeros in the frequency responses of the imaging systems, some signal spectral components are lost and do not come to the output of the systems. Therefore signal restoration by necessity requires the use of some additional information about the original signal that is not present in the observed distorted signal and should be taken from *a priori* knowledge or from somewhere else. So the practical solution of the deconvolution problem requires a solution to the problem of defining the ways that natural redundancy of the images can be used for compensation of the lack of the degrees of freedom. And this problem is much more involved then the deconvolution problem itself.

Recently a new essential advance in the problem of signal deconvolution was made in the research of Berenstein and his colleagues,[4,5] who addressed the problem from a new position. They considered signal deconvolution from multiple sensors. The main idea of this approach was, of course, the idea of

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Received 16 September 1992; revised manuscript received 29 June 1993.
0003-6935/94/112157-06$06.00/0.

supplying information lost in one sensor from information in another sensor. Berenstein proved a fundamental mathematical result that, while deconvolution of a single convolution equation is generally an ill-posed problem, the inversion of a set of simultaneous convolution equations can be made into well-posed one. Moreover, they provided an exact solution of the following problem: Under what conditions do the compactly supported convolution kernels $\{S_1, S_2, \ldots, S_m\}$ have associated with them compactly supported deconvolution kernels $\{F_1, F_2, \ldots, F_m\}$ such that

$$\sum_{k=1}^{m} S_k \otimes F_k = \delta, \tag{1}$$

where δ is the identity kernel (Dirac delta function) and $\otimes$ denotes convolution?

However, this mathematical result is not sufficient for most image processors and physicists. As a matter of fact, for physicists it has always been clear that exact signal restoration is never possible because of the inevitable measurement errors, or sensor noise, while restoration with some restricted accuracy is always possible. So the more realistic questions are the following: (i) What is potentially the highest restoration accuracy under the given circumstances? (ii) In which way can this ultimate accuracy be achieved? Answering these questions is an objective of this paper.

In order to answer these questions one should first of all select a measure for the restoration accuracy. In this paper we attack the problem by use of the classical least-squares (Wiener) approach, which in image-restoration problems goes back to the study by Helstrom.[6] In the image-processing community it is commonly accepted that the criterion of the least squares is not appropriate for estimation of the image quality. However, we use this criterion because of the following reasons: (i) it is simple to use for analytical treatment; (ii) it gives results that are transparent and well fitted to intuitive expectations, and it contains classical results as a specific case; (iii) it permits straightforward generalization of the results to the practically important case that occurs when original signals for different sensors (sensor channels) are not identical but are only more or less correlated. As far as the authors know, this case has not been investigated before. Moreover, one of the reasons for why the least-squares criterion has appeared not to be appropriate for image-processing problems is the fact that it usually involves averaging of the squared restoration error not only over the sources of measurement errors, or noise, but also over a set of possible images, or an image ensemble, as well. This results in neglect of the individual features of the images. Many restoration algorithms attempt to generate an image as close to an ensemble average image as possible, while just the opposite is required: we need to restore image features distinctive from the average ones. Therefore we eliminate averaging over an image ensemble, and in this way we provide a potential opportunity for the design of adaptive filters that are optimal in the sense of a least-squares criterion for each specific image.[7]

The paper is arranged in the following order. In Section 2 we review briefly the image-deconvolution problem and the least-squares approach to it modified as mentioned above to permit adaptation. In Section 3 we give a least-squares solution of the multiple-image deconvolution problem and illustrate it with the results of computer simulation. In Section 4 we extend the solution to the case of nonidentical images in different sensor channels. In conclusion we give a summary of the results.

2. Image-Deconvolution Problem

Imaging (both good and bad) can be approximated as a linear operation. In particular,

$$i(x, y) = s(x, y) \otimes o(x, y). \tag{1}$$

That is, image i is the object or perfect image o convolved with some spread function s. A more realistic description is (dropping the variables)

$$i = s \otimes o + n, \tag{2}$$

where n is unknown noise, which is always present in any imaging system. The image-restoration problem is a typical inverse problem: given i, and knowing s and something about the expected noise, find a good estimate of o. The obvious first step is the Fourier transformation of both sides of Eq. (2). With obvious notation this becomes

$$I = SO + N. \tag{3}$$

We then select a filter F such that the inverse Fourier transform of FI is a good estimate of o. As are most inverse problems, this one, called image deconvolution, is ill conditioned. This means that it cannot be solved uniquely without additional definition of what we mean by a good estimate.

If no noise is present, a pure inverse filter

$$F_{\text{inv}} = 1/S = S^*/|S|^2 \tag{4}$$

is an obvious solution. But the inverse filter has two insurmountable problems. First, if $|S|^2$ goes to zero at one or more spatial frequencies, F_{inv} is not defined at or near those zeros. Second, F_{inv} takes no account of the expected noise. Precisely at and near zeros, we observe a double disaster with F_{inv}. That is, F_{inv} becomes very large precisely when the signal-to-noise ratio SNR becomes small. Thus it turns out that F_{inv} is a perfect filter for extraction of pure noise from the signal-plus-noise image.

Helstrom[6] was able t show that the optimum (minimum-least-squares restoration error, or Wiener-type) filter is

$$F_W = \frac{1}{S}\frac{\text{SNR}}{1 + \text{SNR}}, \tag{5}$$

where SNR is the ratio of the signal $o = s * i$ power spectrum $|S|^2\langle|O|^2\rangle_0$ to the noise n power spectrum $\langle|N|^2\rangle_n$ on a corresponding frequency:

$$\mathrm{SNR} = |S|^2\langle|O|^2\rangle_0/\langle|N|^2\rangle_n, \tag{6}$$

with $\langle\ \rangle_0$ and $\langle\ \rangle_n$ denoting statistical averaging over initial images and noise ensembles, respectively. The meaning of this formula is transparent: when SNR $\gg 1$,

$$F_w \to F_{\mathrm{inv}} = 1/S, \tag{7}$$

and when SNR $\ll 1$,

$$F_w \to 0, \tag{8}$$

which means that at these frequencies the signal is not restored but suppressed even more, although noise is suppressed as well. Of course, for implementation of the filter [Eq. (5)] one should know *a priori* the statistical power spectra of the noise and image ensembles. The power spectrum of the noise ensemble is not usually a problem. It usually can be derived from the physical description of the source of the noise. In many practical cases a model of white Gaussian noise with known noise spectral density is appropriate. As for the image statistical power spectrum, it is usually assumed that it can be found by averaging of power spectra of individual images taken from a set of images similar to the one expected after restoration.

Numerous experiments with filters of the type described by Eq. (5) have shown that the restoration results are far from acceptable. It suppresses noise efficiently, but at the same time it suppresses the weak signal spectrum components even more, and these components are the most important part of the signal since they are responsible for image contours and other key features that distinguish one image from another. The solution to this problem lies in adaptation to the image under restoration. It means that we should look for the filter that minimizes the squared restoration error for the given observed image. That is, we should eliminate averaging of the error over an image ensemble. It was shown[7] that in this case the optimal restoration filter has the same frequency response [Eq. (5)] except that, instead of the averaged power spectrum of the image ensemble $\langle|O|^2\rangle_0$, one should use for the filter design the power spectrum of the initial image itself, $|O|^2$. Of course, this spectrum is not known since we observe only the signal i distorted by blur and additive noise. Therefore the design of the restoration filter one should first estimate the true signal power spectrum from the observed mixture signal i. If we denote the estimate of the signal power spectrum without noise as $\overline{|S|^2|O|^2}$, we obtain the following formula for the frequency response of the optimal restoration filter:

$$F_W = \frac{1}{S}\frac{\overline{\mathrm{SNR}}}{1 + \overline{\mathrm{SNR}}}, \tag{9}$$

where $\overline{\mathrm{SNR}}$ is the corresponding estimated signal-to-noise ratio:

$$\overline{\mathrm{SNR}} = \overline{|S|^2|O|^2}/\langle|N|^2\rangle_n. \tag{10}$$

There exist different ways for obtaining a good estimation of the spectrum. A detailed analysis of them is out of the scope of the present paper. We confine ourselves to only an illustration of the simplest zero-order estimation:

$$\overline{|S|^2|O|^2} \cong \begin{cases} |I|^2 - \langle|N|^2\rangle_n & \text{if this difference is nonnegative} \\ 0 & \text{otherwise} \end{cases}, \tag{11}$$

for the case of signal-independent white Gaussian noise.

3. Changing the Problem and Its Solution: Deconvolution of Multiple Images of the Same Object

Adaptation to the observed image based on the estimation of the initial image power spectrum can improve restoration quality in comparison with the nonadaptive filtering,[8] but of course it is not able to solve the problem of restoration of the signal frequency components with low SNR. A radical solution of this problem is possible only if additional information about these components is available from a source other than the observed distorted signal, in particular, from another image sensor or multiple sensors. If we have a multiple-sensor system, and if the sensors are not identical, we can choose, for signal restoration on each specific frequency signal, from the sensor with the highest SNR on this frequency. This is basically the main idea of multiple-image deconvolution. We now extend the above least-squares approach to this case. For the sake of simplicity we consider first the case of two sensors generating two images of the same object:

$$i_1 = s_1 * o + n_1,$$
$$i_2 = s_2 * o + n_2. \tag{12}$$

Following to Berenstein,[4,5] we assume that the restored image $\hat{o}$ is obtained as a sum of the restored images from channels 1 and 2. If the frequency responses of the restoration filters in the channels are F_1 and F_2, spectrum $\hat{O}$ of the restored image is then

$$\hat{O} = F_1I_1 + F_2I_2. \tag{13}$$

Our goal now is to find filters F_1 and F_2 that minimize

$$\|\epsilon\| = \langle\langle\|o - \hat{o}\|\rangle_{n_1}\rangle_{n_2} = \langle\langle\|O - \hat{O}\|\rangle_{n_1}\rangle_{n_2}$$
$$= \langle\langle\|O - F_1I_1 + F_2I_2\|\rangle_{n_1}\rangle_{n_2}, \tag{14}$$

where $\|\ \|$ is the vector norm and $\langle\ \rangle_{n_1}$ and $\langle\ \rangle_{n_2}$ denote statistical averaging over the noise in each channel. Let us assume for simplicity that the noise in two channels is uncorrelated:

$$\langle\langle N_1^* N_2\rangle_{n_1}\rangle_{n_2} = \langle\langle N_1 N_2^*\rangle_{n_1}\rangle_{n_2} = 0. \tag{15}$$

From these noise terms and some bulky but simple algebra we arrive at expressions for the two deconvolution filters:

$$F_{1(2)} = \frac{1}{S_1} \frac{\overline{\mathrm{SNR}_{1(2)}}}{1 + \overline{\mathrm{SNR}_1} + \overline{\mathrm{SNR}_2}}, \tag{16}$$

where

$$\overline{\mathrm{SNR}_{1(2)}} = |S_{1(2)}|^2 |O|^2 / \langle |N_{1(2)}|^2\rangle_{n_{1(2)}} \tag{17}$$

are estimated signal-to-noise ratios in each channel. The meaning of this result is straightforward: each channel contributes, to the restored signal, the signal restored by the corresponding inverse filter, and this contribution is proportional to the signal-to-noise ratio in the channel. This is exactly what one would expect intuitively.

We shall illustrate the above results with a simple numerical experiment. We choose for demonstration of the restoration effect an impulse of Gaussian shape with the spectrum in some frequency scale

$$O(f) = \exp(-f^2/3500),$$

which was blurred by the two imaging systems with frequency responses, respectively,

$$S_1(f) = \frac{\sin(2\pi f/33)}{2\pi f/33},$$

$$S_2(f) = \frac{\sin(2\pi f/47)}{2\pi f/47},$$

and that was observed at the output of these systems in mixture with additive white noise with spectral densities, respectively,

$$\langle |N_1(f)|^2\rangle_{n_1} = 0.001, \langle |N_2(f)|^2\rangle_{n_2} = 0.002.$$

Figure 1 shows the result of restoration of the impulse in the spectral domain by the optimal filter [Eq. (16)] in comparison with the result of independent restoration in each of two channels by an appropriate Wiener filter [Eq. (5)] and subsequent summation of the restored images. The distored signals in each channel are also illustrated in Fig. 1 by their spectra. The effect of optimal restoration is obvious. It is especially high in the vicinity of zeros of the distorting transfer functions, whereas separate Wiener filters fail to restore the signal effectively. The restoration is somewhat defective only when zeros of the distorting transfer functions occur close to each other. This means that it is desirable to choose the two transfer function in such a way that their zeros coincide or are close to each other at the highest possible frequencies. Alternatively we can use three or more transfer functions. Figure 2 shows the frequency responses of the distorting systems, while Fig. 3 presents the frequency responses of the optimal restoration filters in each channel and demonstrates how the filters share the job of signal restoration. These figures seem to be self-explanatory.

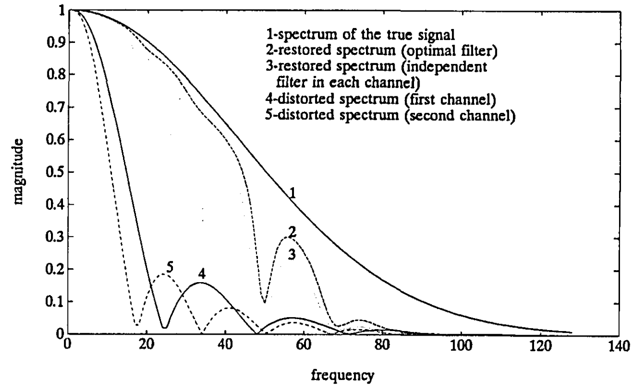

Fig. 1. Comparison of the image restoration from two blurred images by an optimum filter and by two independent Wiener filters in each of two channels.

We see therefore that two images with different but known point spread functions can yield a much better combined estimate of the true image than either alone can. Naturally the more image channels we have, the better. Extending these results to three or more images is straightforward. Let us observe a set of images

$$\{I_k = S_k O + N_k;\quad k = 1, 2, \ldots, K\}$$

in K channels, subject each image to filtering F_k, and generate the restored image as

$$\hat{O} = \sum_{k=1}^{K} F_k I_k. \tag{18}$$

From the criterion of minimization of averaged squared restoration error [Eq. (14)] one can obtain the following system of equations:

$$\left\{F_k S_k + \overline{\mathrm{SNR}_k} \sum_{l=1}^{K} F_1 S_1 = \overline{\mathrm{SNR}_k}\right\}, \tag{19}$$

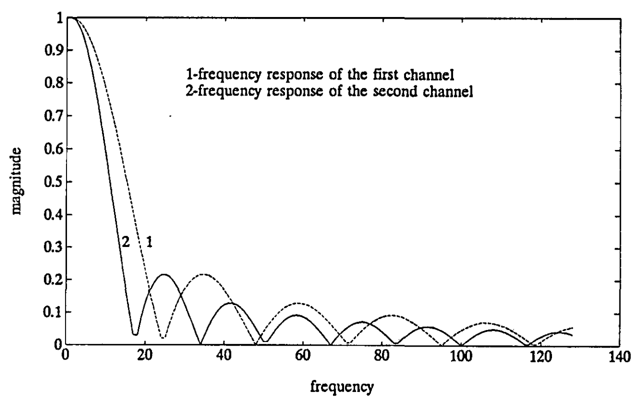

Fig. 2. Frequency responses of the two distorting channels.

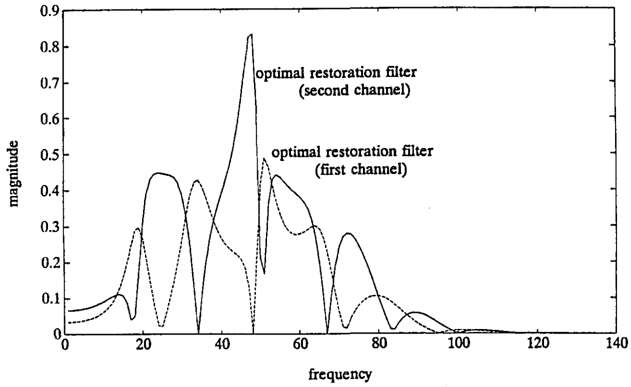

Fig. 3. Frequency responses of the two restoration filters.

where

$$\overline{\mathrm{SNR}_k} = |S_k|^2 |O|^2 / \langle N_k^2 \rangle_{n_k} \tag{20}$$

is the signal-to-noise ratio in the kth channel estimated from the corresponding observed image. The obvious solutions of this system are

$$\left\{ F_k = \frac{1}{S_k} \frac{\overline{\mathrm{SNR}_k}}{1 + \sum_{k=1}^{K} \overline{\mathrm{SNR}_k}}; \quad k = 1, 2, \ldots, K \right\}. \tag{21}$$

It is instructive to substitute Eq. (22) into Eq. (18) and to see how the restored object looks in comparison with the true object:

$$\hat{O} = \frac{\sum_{k=1}^{K} \overline{\mathrm{SNR}_k}}{1 + \sum_{k=1}^{K} \overline{\mathrm{SNR}_k}} O + \frac{\sum_{k=1}^{K} N_k \overline{\mathrm{SNR}_k} / S_k}{1 + \sum_{k=1}^{K} \overline{\mathrm{SNR}_k}}. \tag{22}$$

The first term in this expression describes the restored image, and the second one describes the restoration noise. The ratio of the image energy to the noise energy on each frequency in the restored image can be easily found from

$$\frac{\text{image energy}}{\text{restoration noise energy}} = \sum_{k=1}^{K} \overline{\mathrm{SNR}_k}. \tag{23}$$

Equations (22) and (23) show that any desirable restoration quality is possible given an appropriate choice of different image sensors and their number.

In the Wiener formulation the defect of restoration and remaining noise are considered with equal weights when the root-mean-squared error of restoration is calculated. Generally in image restoration it is better to evaluate the error of restoration with different weights for the defect of restoration and for the remaining noise and even to take these weights frequency dependent. The corresponding generalization is straightforward, but it goes out of the scope of this paper.

4. Deconvolution of Multiple but Nonidentical Images of the Same Object

The practical significance of the above result is the possibility of producing an image with any desirable image quality with real, imperfect image formation devices. However, this requires special design of multiple imaging devices for one object instead of a single device. There are, however, many practical situations when multiple imaging of the same object is performed for other reasons with no regard for the restoration problem. The most evident example is color images formed from three components: red, blue, and green. Among others examples, one can mention multispectral imaging for remote sensing and multimodality medical imaging. Of course, images in individual channels are not identical in these cases, but they are still the images of the same object, and therefore they are more or less correlated. This correlation creates potential for mutual correction of the images in the channels.

Let us assume that we observe a set of images $\{i_k; k = 1, 2, \ldots, K\}$ such that

$$\{I_k = S_k O_k + N_k\}, \tag{24}$$

with the same denotations as above except now $\{O_k\}$ are the spectra of true images in the corresponding channels. We can take advantage of the cross-channel image correlation if we restore the images in each channel by appropriate filtering and subsequent summation of the filtered images of all the channels:

$$\hat{O}_k = \sum_{l=1}^{K} F_{k,l} I_l. \tag{25}$$

For the same criterion of least squares in each channel we then obtain, assuming that noise in the channels is uncorrelated, that the optimal interchannel filters $\{F_{k,l}\}$ are defined by the following relationship:

$$F_{k,l} = \frac{1}{S_1} \frac{O_k O_1^*}{|O_1|^2} \frac{\overline{\mathrm{SNR}_1}}{1 + \sum_{m=1}^{K} \overline{\mathrm{SNR}_m}}, \tag{26}$$

with $*$ denoting complex conjugation. This formula is a natural generalization of Eq. (22) and requires for its implementation knowledge of the cross correlations $\{O_k O_1^* / |O_1|^2\}$ between true image spectra in individual channels in each frequency. Within a statistical approach these cross correlations should be measured *a priori* over an appropriate set of images. There exists, however, a possibility of adaptive estimation of them from the observed signals; this possibility was discussed above for the adaptive Wiener filter.

5. Conclusion

Within the least-squares approach we have shown that deconvolution of images subjected to linear blur is possible with arbitrary accuracy from multiple-sensor data provided that an appropriate choice of number of sensors is made. The deconvolution is performed by a set of deconvolution filters acting in each sensor's channel whose outputs are summed up to form a restored image. Each filter produces an inverse filtered image in the corresponding channel, weighted in the spectral domain proportional to the channel's signal-to-noise ratio on the corresponding frequency. The formulas are given for potential restoration accuracy. A similar result is obtained for restoration of multiple-sensor images when true images in the individual channels are nonidentical. The frequency-domain weights of the inversely filtered images in the channels are additionally proportional in this case to cross-correlation coefficients between true images in the corresponding channels. All results are obtained under the assumption that all the parameters needed for the restoration filter design should be estimated from the observed distorted images, and some methods for such estimation are briefly outlined.

L. Yaroslavsky gratefully acknowledges the partial support of the Deutsche Forschunggemainschaft through contract ER16/94-1. H. J. Caulfield was partially supported by the U.S. Air Force Office of Scientific Research under contract F49620-93-1-0123.

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