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Deductive group symmetry analysis for a free convective boundary-layer flow of electrically conducting non-Newtonian fluids over a vertical porous-elastic surface

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ABSTRACT

General group symmetry analysis so-called deductive group-theoretical method is applied to analyze the free convective boundary-layer problem due to the motion of an elastic surface into an electrically conducting a class of non-Newtonian fluid. The symmetry groups admitted by the corresponding boundary value problem are obtained. Particular attention is paid on the deductive group which provides the similarity solution of the problem. Also, the admissible form of the data, in order to be conformed to the obtained symmetries, is provided. Finally, with the use of the entailed similarity variables the problem is transformed into a boundary value problem of ODEs and is solved numerically for particular non-Newtonian fluid so-called powell-eyring fluid.

Keywords: Deductive symmetry groups, Similarity solutions, non-Newtonian fluid, laminar boundary-layer flow.

MSC 2010 codes: 76A05, 76M55, 54H15.

1 Introduction

The formulation of the group-theoretic method, also called symmetry analysis, is contained in the general theories of continuous transformation groups that were introduced and treated extensively by Lie (1975) [see also Oberlack (1999)]] about 130 years ago. Group analysis is the only rigorous mathematical method to find all symmetries of a given differential equation and no ad-hoc assumptions or a prior knowledge of the equation under investigation is needed. The boundary layer equations are especially interesting from a physical point of view because they have the capacity to admit number of invariant solutions.

In this paper, we apply the so-called deductive group symmetry methods for a particular problem of fluid mechanics. The main advantage of such methods is that they can successfully be applied to non-linear differential equations. The symmetries of a differential equation are those continuous groups of transformations under which the differential equation

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remains invariant, that is, a symmetry group maps any solution to another solution. The interesting point is that, having obtained the symmetries of a specific problem, one can proceed further to find out the group-invariant solutions, which are nothing but the well-known similarity solutions. The similarity solutions are quite popular because they result in the reduction of the independent variables of the problem. In our case, the problem under investigation is two-dimensional. Hence, any similarity solution will transform the system of PDEs into a system of ODEs.

Most of the researchers in the field of fluid mechanics try to obtain the similarity solutions by introducing a general similarity transformation with unknown parameters into the differential equation obtaining in this way an algebraic system. Then, the solution of this system, if exists, determines the values of the unknown parameters. In our opinion, it is better to attack any problem of similarity solutions from the outset, i.e, to find out the full list of the symmetries of the problem and then to study which of them are appropriate to provide group-invariant (or more specifically similarity) solutions.

To obtain symmetry of a differential equation is equivalent to the determination of the transformation group associated with this symmetry. In Olver (1993); Bluman and Kumei (1989); Ibragimov (1985, 1999), one can find the general theory of Lie groups as well as the implied methods for determining transformation group via the infinitesimal generator components. An alternative way being based on exterior calculus for determining the transformation group so-called deductive group can be found in Moran and Gaggioli (1968). It is worth noting that there is an extensive literature where the methods arising from exterior calculus are used to attack symmetry problems of continuum mechanics [Suhubi (1991, 1994); Pakdemirli and Suhubi (1992); Kalpakides (1998, 2001); Koureas *et. al* (2001, 2003)].

We apply this procedure to a boundary layer problem which arises from the motion of an elastic surface into an electrically conducting, incompressible, viscous non-Newtonian fluid. Particular variants of this problem have been studied by numerous of researchers since 1961. We mention here the some work of Sakiadis (1961); Erickson *et. al* (1966); Tsou *et. al* (1967); Gupta and Gupta (1977). It is remarkable that all of them have used the above described heuristic method to obtain the similarity transformations and the associated similarity solutions of the problem. That is, assuming particular boundary conditions and considering a particular form of the magnetic field, they try to fit a similarity solution in these data.

We deal the problem on another base. First, we do not guess any kind of probable symmetry. The question of any possible symmetry for the system of PDEs is examined generally. In the same spirit, we do not make any assumption about the data of the problem. We consider the most general form for the boundary conditions and the magnetic field function involved in the system. Both the specific form of the functions on the boundaries and the form of magnetic field arise as a consequence of the requirement to respect the obtained symmetries. The similarity equations obtained are more general and systematic along with auxiliary conditions. Recently this method has been successfully applied to various non-linear problems [See Malek *et. al* (1999); Darji and Timol (2011, 2012); Adnan *et. al* (2011)]

Next, having established the admissible symmetries of the boundary value problem, we proceed to the determination of the similarity solutions which, in turn, are used to transform the system to a two-point boundary value problem of ODEs. Finally, the reduced problem is solved numerically and its solutions are depicted for different values of the physical parameters.

The boundary layer flow of Newtonian fluids past stretching sheeting sheet was first discussed by Crane (1970. Later on same problem was extended by several authors, few of these Soundalgekar and Ramana Murthy (1980); Grubka *et. al* (1985); Dutta *et. al* (1985); Jeng *et. al* (1986); Dutta (1989); Chen and Char (1988) for different physical situations, due to its important applications to polymer industry. These studies restrict their analyses to Newtonian fluids. Flow due to a stretching sheet also occurs in thermal and moisture treatment of materials, particularly in processes involving continuous pulling of a sheet through a reaction zone, as in metallurgy, textile and paper industries, in the manufacture of polymeric sheets, sheet glass and crystalline materials. It is well known that a number of industrial fluids such as molten plastics, polymeric liquids, food stuffs or slurries exhibit non-Newtonian character. Therefore a study of flow and heat transfer in non-Newtonian fluids is of practical importance.

In recent years several industries deal with the non-Newtonian fluids under the influence of magnetic field. In view of this, some researchers [Sarpakaya (1961); Saponkov (1967); Martinson and Pavlov (1971); Samokhen (1987); Andersson *et. al* (1992); Cortell (2005); Liao (2005)] have presented works on MHD flow and heat transfer in an electrically conducting power law fluid over a stretching sheet. However, in the literature rare work has been found regarding other non-Newtonian fluids. This may due to mathematical complication of its strain-stress relationship.

Motivated by this, we produce similarity analysis via deductive group method based on general group transformation is, probably first time, to derive symmetry group and similarity solutions for steady of the laminar free convective boundary layer flow of an electrically conducting all time independent non-Newtonian fluids over a vertical porous and elastic surface. The class of all non-Newtonian fluids is characterized by the property that its stress tensor component τ_{ij} can be related to the strain rate component e_{ij} by the arbitrary continuous functional relation

$$\Omega\left(\tau_{ij}, e_{ij}\right) = 0 \tag{1}$$

2 Mathematical formation

We consider a free convective, laminar boundary-layer flow of an electrically conducting incompressible viscous power-law fluid over a vertical porous and elastic surface. The surface is stretched vertically upward along the positive *x*-axis, with a prescribed velocity

$$u(x, y = 0) = u_0(x) \tag{2}$$

while the origin $(x, y) = (0,0)$ is kept fixed. The *y*-axis is vertical to the surface, as it is depicted in Figure 1. Also, due to the fact that the elastic surface is porous, there is a component of the velocity of the fluid which has vertical direction to the surface given by

$$v(x, y = 0) = v_0(x) \tag{3}$$

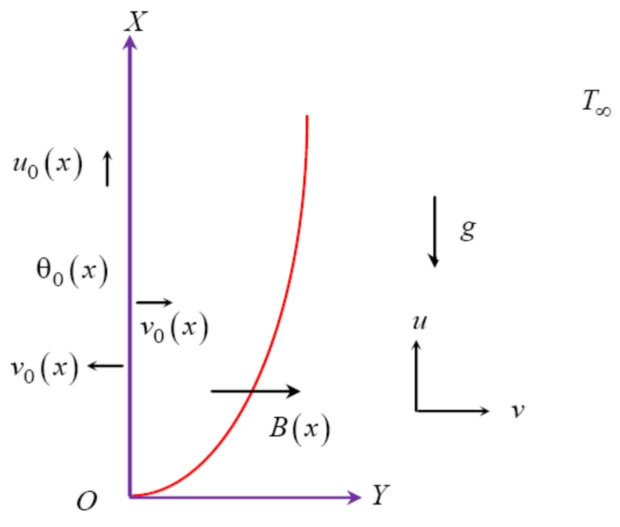

Figure 1: Boundary layer around the stretching surface

The motion of the surface within the fluid creates a boundary layer, which is extended along the *x*-axis. The whole system is under the influence of a magnetic field $B(x)$ which applies to the *y*-direction. We consider that the temperature of the surface changes along the *x*-axis and its distribution is described by a given function $T_0(x)$. The stress-strain relation, under the boundary layer assumption can be found in the form of arbitrary function with only non-vanishing component. Then the relation (2) can be given by τ_{yx}. Then equation (1) can be given by

$$\Omega\left(\tau_{yx}, \frac{\partial u}{\partial y}\right) = 0 \tag{4}$$

Under the assumption that the viscous dissipation term in the energy equation and the induced magnetic field can be neglected, the basic boundary layer equations of the mass, momentum and energy for the steady flow of Boussinesq type are respectively as follows, with the stress-strain relationship given by (4)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\frac{\partial \tau_{yx}}{\partial y} - \frac{\sigma B^2}{\rho}u + g\beta(T - T_\infty) \tag{6}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{7}$$

where σ is the electric conductivity, β is the volumetric coefficient of thermal expansion, ρ is the mass density and α is the thermal diffusivity, which are assumed to be constants.

Also, g is the gravity field assumed to be parallel to the x-axis, $T = T(x, y)$ is the temperature field and T_∞ is the temperature at infinity. According to the above description, the boundary conditions of the problem should be of the form

$$\left.\begin{aligned} &u(x,0) = u_0(x) \\ &v(x,0) = v_0(x), \quad x > 0 \\ &\theta(x,0) = \theta_0(x), \quad x > 0 \\ &u(x,y) \to 0, \quad \theta(x,y) \to 0 \quad \text{as} \quad y \to \infty, \quad x > 0 \end{aligned}\right\} \tag{8}$$

where $\theta = T - T_\infty$. Also, $\theta_0 = T_0 - T_\infty$ is a prescribed function along the boundary surface $y = 0$.

Introducing following non-dimensional quantities:

$$\left.\begin{aligned} &x^* = \frac{Gr}{L}x, \qquad y^* = \left(Re_x \cdot Gr_x\right)^{1/2}\frac{y}{L}, \\ &u^* = \frac{u}{u_0}, \qquad v^* = \left(\frac{Re_x}{Gr_x}\right)^{1/2}\frac{v}{u_0}, \\ &\tau_{yx}^* = \left(\frac{Re_x}{Gr_x}\right)^{-1/2}\frac{\tau_{yx}}{\rho U^2}, \qquad \theta^* = \frac{\theta}{(T_0 - T_\infty)}, \qquad \theta_0^* = \frac{T_0}{(T_0 - T_\infty)}, \\ &Re_x = \frac{u_0 L}{\nu}, \qquad Pr = \frac{\nu}{\alpha}, \qquad Gr_x = \frac{L^3}{\nu^2}g\,\beta\left(T_w - T_\infty\right) \end{aligned}\right\} \tag{9}$$

where L is the reference length, ν is the kinematic viscosity, Re_x is the local Reynolds number, Gr_x is local Grashof number, $\Pr$ is Prandtl number.

Substitute the values in equations (1), (5)-(7) and dropping the asterisks (for simplicity), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\partial \tau_{yx}}{\partial y} - Mu + \lambda\theta \tag{11}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \alpha\frac{\partial^2\theta}{\partial y^2} \tag{12}$$

where $M = \dfrac{\sigma B^2}{\rho}$ is the magnetic field strength and $\lambda = \dfrac{Gr\nu^2}{L^3}$ is the buoyancy parameter.

Now introducing the stream function ψ , which is related to the components of the velocity field such that $u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$, above system of PDEs reduce to:

$$\Omega\left(\tau_{yx}, \frac{\partial^2 \psi}{\partial y^2}\right) = 0 \tag{13}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \frac{\partial \tau_{yx}}{\partial y} - M\frac{\partial \psi}{\partial y} + \lambda\theta \tag{14}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{15}$$

The associated boundary conditions can be written as,

$$\left.\begin{aligned} &\frac{\partial \psi}{\partial y}(x,0) = u_0(x) \\ &\frac{\partial \psi}{\partial x}(x,0) = v_0(x) \\ &\theta(x,0) = \theta_0(x) \\ &\frac{\partial \psi}{\partial y}(x,y) \to 0, \quad \theta(x,y) \to 0 \quad as \quad y \to \infty \end{aligned}\right\} \tag{16}$$

3 Application of deductive group symmetry method

In this section, we will look for any possible symmetry group of the boundary value problem described by PDEs (13) to (15) subject to boundary conditions (16).

The procedure is initiated with the application of the class of a one-parameter continuous deductive group of transformations to the system of PDEs (13) to (16). Under this class, first, we search the subgroup of transformations, through which one will reduce the two independent variables by one and the system of non-linear partial differential equations (13) to (15) will transform to the system of ordinary differential equations.

3.1 Group formulation and invariance analysis

Consider the group C_G, a class of transformation of one-parameter ‘ a ’of the form:

$$C_G: \quad \bar{Q} = \aleph^Q(a)s + \Re^Q(a) \tag{17}$$

Where Q stands for $x, y, \psi, \theta, M, \tau_{yx}$ whereas $\aleph's$ and $\Re's$ are real-valued and are at least differentiable in the real argument a.

To transform the differential equation, transformations of the derivatives of ψ are obtained from C_G via chain-rule operations:

$$\left.\begin{aligned} \bar{Q}_{\bar{i}} &= \left(\frac{\aleph^Q}{\aleph^i}\right) Q_i \\ \bar{Q}_{\bar{i}\,\bar{j}} &= \left(\frac{\aleph^Q}{\aleph^i \aleph^j}\right) Q_{i\,j} \end{aligned}\right\}; \quad Q = \psi, \theta, M, \tau_{yx}; \quad i, j = x, y \tag{18}$$

Now Equation (13)-(15) are said to be invariantly transformed, for some functions $\chi_1(a)$ and $\chi_2(a)$ whenever,

$$\left.\begin{aligned} &\frac{\partial\bar{\psi}}{\partial\bar{y}}\frac{\partial^2\bar{\psi}}{\partial\bar{y}\partial\bar{x}} - \frac{\partial\bar{\psi}}{\partial\bar{x}}\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2} - \frac{\partial}{\partial\bar{y}}\left(\bar{\tau}_{\bar{y}\,\bar{x}}\right) + \bar{M}\frac{\partial\bar{\psi}}{\partial\bar{y}} - \lambda\bar{\theta} \\ &\qquad = \chi_1(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial}{\partial y}\left(\tau_{y\,x}\right) + M\frac{\partial\psi}{\partial y} - \lambda\theta\right] \\ &\frac{\partial\bar{\psi}}{\partial\bar{y}}\frac{\partial\bar{\theta}}{\partial\bar{x}} - \frac{\partial\bar{\psi}}{\partial\bar{x}}\frac{\partial\bar{\theta}}{\partial\bar{y}} - \frac{1}{\Pr}\frac{\partial^2\bar{\theta}}{\partial\bar{y}^2} = \chi_2(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} - \frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}\right] \\ &\Omega\left(\bar{\tau}_{\bar{y}\,\bar{x}}, \frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}\right) = \Omega\left(\tau_{y\,x}, \frac{\partial^2\psi}{\partial y^2}\right) \end{aligned}\right\}$$

Substituting the values from the equation (17) and (18) in above system, yields

$$\begin{aligned} &\frac{(\aleph^\psi)^2}{\aleph^x(\aleph^y)^2}\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right] - \frac{\aleph^{\tau_{yx}}}{\aleph^y}\frac{\partial}{\partial y}\left(\tau_{y\,x}\right) + \left(\aleph^M M + \Re^M\right)\frac{\aleph^\psi}{\aleph^y} - \lambda\left(\aleph^\theta\theta + \Re^\theta\right) \\ &\quad = \chi_1(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial}{\partial y}\left(\tau_{y\,x}\right) + M\frac{\partial\psi}{\partial y} - \lambda\theta\right] \end{aligned} \tag{19}$$

$$\frac{\aleph^\psi\aleph^\theta}{\aleph^x\aleph^y}\left[\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right] - \frac{\aleph^\theta}{(\aleph^y)^2}\left[\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}\right] = \chi_2(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} - \frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}\right] \tag{20}$$

$$\Omega\left(\aleph^{\tau_{yx}}\tau_{y\,x} + \Re^{\tau_{yx}}, \frac{\aleph^\psi}{(\aleph^y)^2}\frac{\partial^2\psi}{\partial y^2}\right) = \Omega\left(\tau_{y\,x}, \frac{\partial^2\psi}{\partial y^2}\right) \tag{21}$$

The invariance of equations (19)-(21) together with boundary conditions, implies that

$$\left.\begin{aligned}
&\Re^{\theta}=\Re^{\tau_{yx}}=\Re^{y}=\Re^{\psi}=\Re^{M}=0,\\
&\frac{\left(\aleph^{\psi}\right)^{2}}{\aleph^{x}\left(\aleph^{y}\right)^{2}}=\frac{\aleph^{\tau_{yx}}}{\aleph^{y}}=\frac{\aleph^{M}\aleph^{\psi}}{\aleph^{y}}=\aleph^{\theta}=\chi_{1}(a)\\
&\frac{\aleph^{\psi}\aleph^{\theta}}{\aleph^{x}\aleph^{y}}=\frac{\aleph^{\theta}}{\left(\aleph^{y}\right)^{2}}=\chi_{2}(a),\\
&\aleph^{\tau_{yx}}=\frac{\aleph^{\psi}}{\left(\aleph^{y}\right)^{2}}=1
\end{aligned}\right\} \tag{22}$$

These yields,

$$\aleph^{x}=\left(\aleph^{y}\right)^{3},\quad \aleph^{\Psi}=\left(\aleph^{y}\right)^{2},\quad \aleph^{\theta}=\frac{1}{\aleph^{y}},\quad \aleph^{\tau_{yx}}=1,\quad \aleph^{M}=\frac{1}{\left(\aleph^{y}\right)^{2}} \tag{23}$$

Finally, we get the one-parameter group G, which transforms invariantly the differential equations (13)-(15) and the auxiliary conditions (16), as

$$G:\left\{\begin{aligned}
&G_{H}:\ \begin{cases}\bar{x}=\left(\aleph^{y}\right)^{3}x+\Re^{x}\\ \bar{y}=\aleph^{y}y\end{cases}\\
&\bar{\psi}=\left(\aleph^{y}\right)^{2}\psi\\
&\bar{\theta}=\frac{1}{\aleph^{y}}\theta\\
&\bar{M}=\frac{1}{\left(\aleph^{y}\right)^{2}}M\\
&\bar{\tau}_{\bar{y}\bar{x}}=\tau_{yx}
\end{aligned}\right. \tag{24}$$

3.2 The complete set of absolute invariants

Now we have proceeded in our analysis to obtain a complete set of absolute invariants. If $\eta=\eta(x,y)$ is the absolute invariant of the independent variables then,

$$g_{j}\left(x,y,\psi,M,\theta,\tau_{yx}\right)=\Pi_{j}(\eta),\quad j=1,2,3,4 \tag{25}$$

are absolute invariants of dependent variables.

The application of the basic theorem in group theory, [Moran and Gaggioli (1968); Morgan (1952)], states that:

A function $g\left(x,y,\psi,\theta,M,\tau_{yx}\right)$ is an absolute invariant of a one-parameter group if it satisfies the following first-order linear partial differential equation,

$$\sum_{i=1}^{5}\left(\alpha_i Q_i+\beta_i\right)\frac{\partial g}{\partial Q_i}=0,\quad Q_i=x,y,\psi,\theta,M,\tau_{yx} \tag{26}$$

where

$$\alpha_i=\left.\frac{\partial \aleph^i}{\partial a}\right|_{a=a^0}\quad \text{and}\quad \beta_i=\left.\frac{\partial \Re^i}{\partial a}\right|_{a=a^0}\qquad i=1,\dots 6 \tag{27}$$

and ‘ a^0 ’denotes the value of parameter ‘ a ’ which yields the identity element of the group *G*.

Since $\Re^{\theta}=\Re^{M}=\Re^{y}=\Re^{\psi}=\Re^{\tau_{yx}}=0$ implies that $\beta_2=\beta_3=\beta_4=\beta_5=\beta_6=0$ and from (27) we get $\alpha_1=3\alpha_2=\frac{3}{2}\alpha_3=-\frac{1}{3}\alpha_4=-\frac{3}{2}\alpha_5,\quad \alpha_6=0$.

Hence, equation (26) reduces to

$$(x+\beta)\frac{\partial g}{\partial x}+\left(\frac{y}{3}\right)\frac{\partial g}{\partial y}+\left(\frac{2\psi}{3}\right)\frac{\partial g}{\partial \psi}+\left(-\frac{\theta}{3}\right)\frac{\partial g}{\partial \theta}+\left(-\frac{2M}{3}\right)\frac{\partial g}{\partial M}+(0)\frac{\partial g}{\partial \tau_{yx}}=0,\quad \beta=\frac{\beta_1}{\alpha_1} \tag{28}$$

The absolute invariant of independent variables owing the equation (28) is $\eta=\eta(x,y)$ if it will satisfies the first order linear partial differential equation

$$(x+\beta)\frac{\partial \eta}{\partial x}+\frac{y}{3}\frac{\partial \eta}{\partial y}=0. \tag{29}$$

Applying the variable separable method we get,

$$\eta(x,y)=y(x+\beta)^{-1/3} \tag{30}$$

Further the absolute invariants of dependent variables owing the equation (28) are followed by

$$g_1(x,y,\psi)=\frac{\psi}{(x+\beta)^{2/3}}=\Pi_1(\eta),\qquad g_2(x,y,\theta)=\frac{\theta}{(x+\beta)^{-1/3}}=\Pi_2(\eta)$$

$$g_3(x,y,M)=\frac{M}{(x+\beta)^{-2/3}}=\Pi_3(\eta),\qquad g_4\left(x,y,\tau_{yx}\right)=\tau_{yx}=\Pi_4(\eta)$$

Hence,

$$\begin{aligned}&\psi(x,y)=(x+\beta)^{2/3}\Pi_1(\eta),\quad \theta(x,y)=(x+\beta)^{-1/3}\Pi_2(\eta),\\ &M=(x+\beta)^{-2/3}\Pi_3(\eta)\qquad ,\qquad \tau_{yx}(x,y)=\Pi_4(\eta)\end{aligned} \tag{31}$$

4 Group invariant solution

Since $M(x)$ is independent of y, $\Pi_3(\eta)$ must be constant say *m*. (Referred as magnetic field parameter)

Thus, finally we get the complete set of absolute invariants for the group *G* that transforms the system of partial differential equations (13)-(15) into ordinary differential equation together with auxiliary conditions (16), as

$$\begin{aligned} &\psi(x,y) = (x+\beta)^{-2/3} F(\eta), \quad &\theta(x,y) = (x+\beta)^{-1/3} G(\eta), \\ &M = m(x+\beta)^{2/3}, &\tau_{yx}(x,y) = H(\eta) \end{aligned} \tag{32}$$

Using the similarity transformation (32) in equation (13)-(15), yields to following non-linear ordinary differential equations

$$\left.\begin{aligned} &\Omega(H, F'') = 0 \\ &(F')^2 - 2FF' - 3H' + mF' - \lambda G = 0 \\ &2FG' + F'G - \frac{1}{\Pr} G'' = 0 \end{aligned}\right\} \tag{33}$$

Further to transform the boundary conditions in to constant form the temperature near surface θ_w must be proportional to $(x+\beta)^{-1/3}$, that is of the form $\theta_0 = c_1(x+\beta)^{-1/3}$, c_1 is non-vanishing arbitrary constant and the prescribe velocities are of the form $u_0(x) = c_2(x+\beta)^{1/3}$, $v_0(x) = c_3(x+\beta)^{-1/3}$. These are the precise restrictions for the existence of similarity solution.

Hence the auxiliary conditions reduce to,

$$\left.\begin{aligned} \eta = 0: &\quad F = c_1, \quad F' = c_2, \quad G = c_3 \\ \eta \to \infty: &\quad G \to 0, \quad F' \to 0 \end{aligned}\right\} \tag{34}$$

Eqs. (33) and (34) describe the new form of our problem. Thus, the initial boundary value problem of PDEs has been transformed into a boundary value problem of ODEs which is generally easier to be solved by some numerical method.

5 Results and discussions

Many Non-Newtonian fluid models based on functional relationship between shear-stress and rate of the strain, are available in real world applications Bird *et. al* (1960). Among these models most research work is so far carried out on power-law fluid model, this is because of its mathematical simplicity. On the other hand rest of fluid models are mathematically more complex and the natures of partial differential equations governing these flows are too non-liner boundary value type and hence their analytical or numerical solution is bit difficult. For the present study the partial differential equation model, although mathematically more complex, is chosen mainly due to two reasons. Firstly, it can be deduced from kinetic theory of liquids rather than the empirical relation as in power-law model. Secondly, it correctly reduces to Newtonian behavior for both low and high shear rate. This reason is somewhat opposite to pseudo plastic system whereas the power-law model has infinite effective viscosity for low shear rate and thus limiting its range of applicability.

Mathematically, the Powell-Eyring model can be written as [Bird et al (1960); Skelland (1967)]

$$\tau_{yx} = \mu \frac{\partial u}{\partial y} + \frac{1}{B} \sinh^{-1}\left(\frac{1}{C} \frac{\partial u}{\partial y} \right) \tag{35}$$

where B and C are rheological parameters.

Introducing the dimensionless quantities into equation (35) and using similarity variables, we get

$$H'(\eta) = f''' + \frac{\varepsilon_1 f'''}{\left\{1 + \varepsilon_2 (f'')^2\right\}}, \quad \text{where} \quad \varepsilon_1 = \frac{1}{\mu BC}, \ \varepsilon_2 = \frac{\rho u_0^3 Gr}{\mu L C^2} \tag{36}$$

where ε_1 and ε_2 are referred as rheological flow parameters.

Substituting the value from (36), the system (33) reduce to,

$$F''' = \frac{\frac{1}{3}\left\{(F')^2 - 2FF'' - c_2^2 + 3mF' - 3\lambda G\right\}\left\{1 + \varepsilon_2^2 (F'')^2\right\}^{1/2}}{\varepsilon_1 + \left\{1 + \varepsilon_2^2 (F'')^2\right\}^{1/2}} \tag{37}$$

$$2FG' + F'G - \frac{1}{\Pr} G'' = 0. \tag{38}$$

Also the dimensionless local skin-friction confident $\left(C_{fx}\right)$ expression is given by

$$\frac{1}{2} C_{fx} \sqrt{\mathrm{Re}_x \cdot Gr_x} \equiv \tau_w \tag{39}$$

where τ_w is local shear stress. That is $\tau_w = \tau_{yx}\big|_{y=0}$.

In terms of defined rheological flow parameters (39) yields,

$$\frac{1}{2}C_{fx}\sqrt{\mathrm{Re}_x \cdot Gr_x} = \varepsilon_2 F''(0) + \frac{\varepsilon_1}{\sqrt{\varepsilon_2}}\sinh^{-1}\left\{\sqrt{\varepsilon_1}F''(0)\right\} \tag{40}$$

In order to face numerically problem (33)–(34), we have used a numerical solver of MATLAB package which solves any two-point boundary value problem for ODEs by collocation. To enhance the effect of magnetic field, without loss of generality, each parameter assumed appropriately in boundary conditions (34). The numerical solutions are produced graphically in Figures (2)-(4).

The effect of magnetic field *M*, on the functions $F'(\eta)$ related to velocity along *x* direction, $F''(\eta)$ related to relate to local skin-friction and $G(\eta)$ related to temperature is analyzed graphically in the Figures (2)-(4).

Figure (2) shows that boundary layer decrease as the magnetic field increase. Figure (3) depicts behavior of $F''(\eta)$ throughout the domain. In particular it is interesting to observe that, as *M* increases $F''(0)$ decrease and hence the local shear-stress (see Table 1), which decreases local skin-friction C_f .

Influence of magnetic field on thermal boundary layers displayed by Figure 4. It shows that increase in magnetic field will precisely increase thermal boundary layer within the boundary layer domain.

Table 1: Local shear stress

M	0.01	0.1	0.3	0.8	1
$F''(0)$	0.3374	0.2725	0.1385	-0.1477	-0.2473
τ_w	0.6687	0.5417	0.2766	-0.2949	-0.4921

6 Conclusion

First time the general group of transformations using the deductive group symmetry method a typical case of lie group method for a particular boundary layer problem including a class of Non-Newtonian fluids. The governing system of PDEs transformed into the system of ODEs subject to the similarity requirement, by employing the derived transformations. Numerical solutions for special Non-Newtonian fluid so-called prendtl-etring fluid, are produced by MATLAB computational algorithm. An interesting effect of magnetic field is observed. All the numerical solutions are generated for dimensionless quantity and hence it is executed for all types of under considered fluids. An interesting effect of magnetic field is observed.

7 Figures

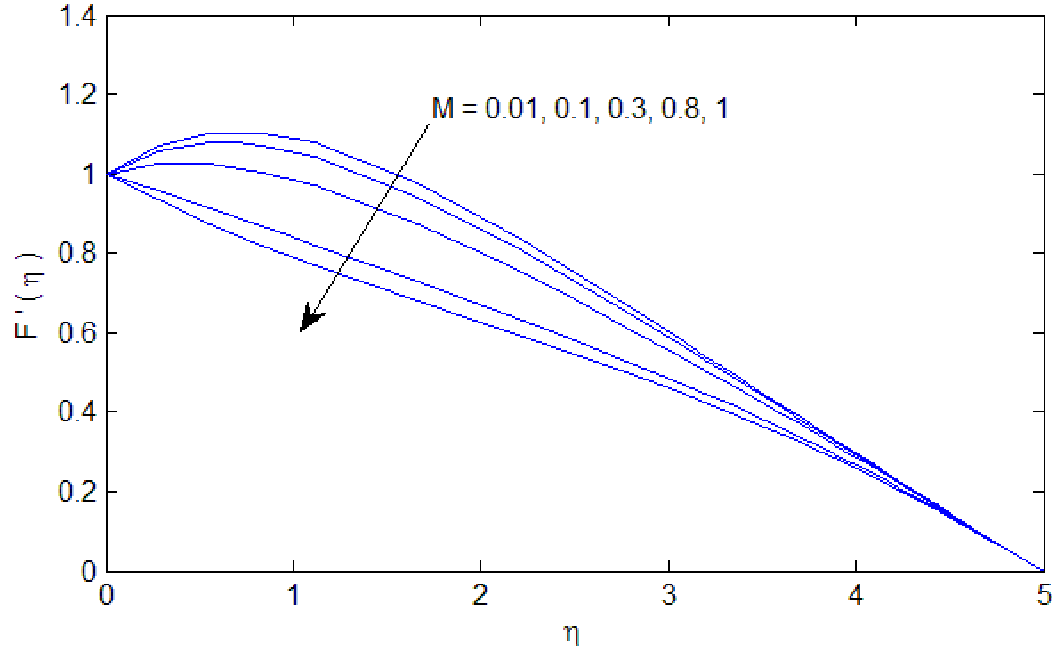

Figure 2: Influence of magnetic field on horizontal velocity

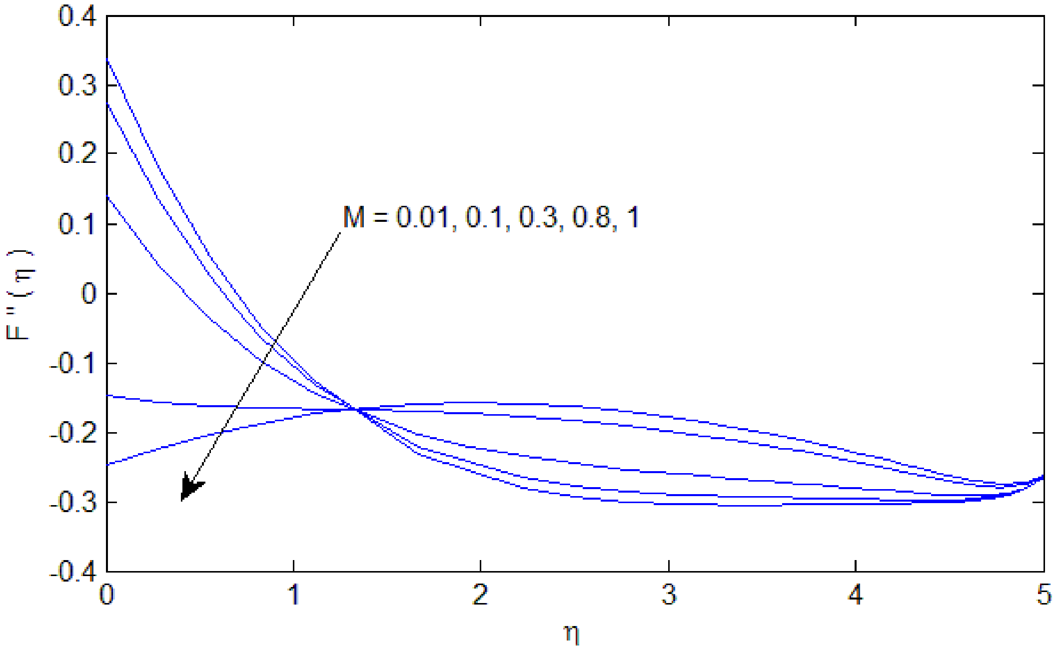

Figure 3: Influence of magnetic field on shear stress within boundary layer domain

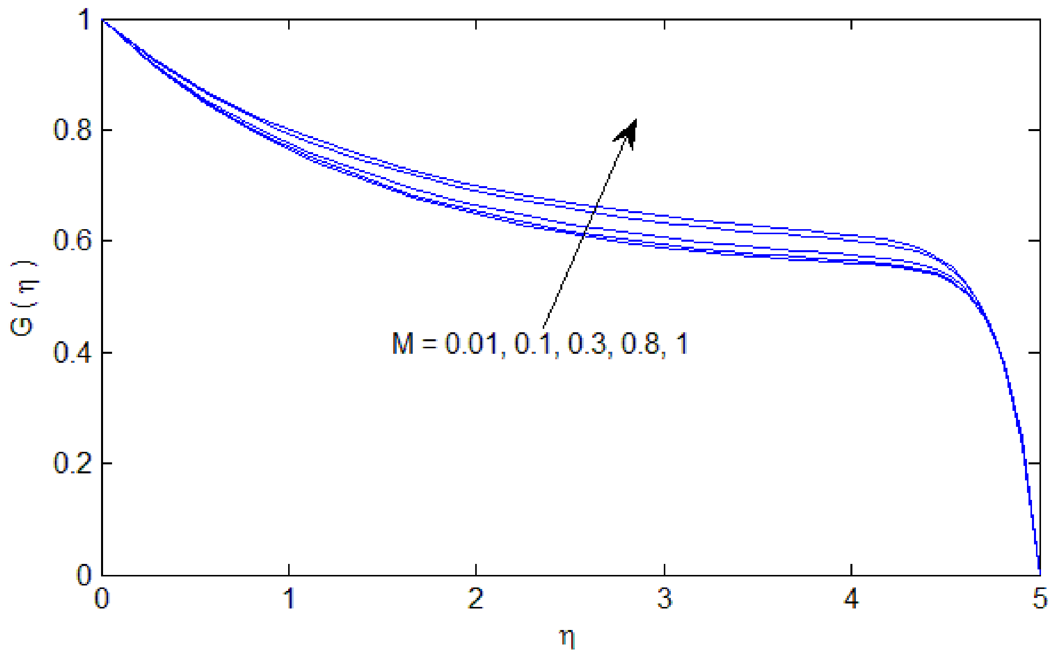

Figure 4: Thermal boundary layer domain under the effect of magnetic field

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