# Deeply subwavelength electromagnetic Tamm states in graphene metamaterials

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We study localized modes at a surface of a multilayer structure made of graphene layers separated by dielectric layers. We demonstrate the existence of deeply subwavelength surface modes that can be associated with the electromagnetic Tamm states, with the frequencies in the THz frequency range the negative group velocities. We suggest that the dispersion properties of these Tamm surface modes can be tuned by varying the thickness of a dielectric cap layer.

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## I. INTRODUCTION

An interface separating two different layered media or photonic crystals can support electromagnetic surface waves, and these waves are often called "electromagnetic Tamm states," in analogy to the electronic Tamm states predicted to exist at an interface of crystalline materials [1]. The electromagnetic Tamm states have been studied in different structures, including the waves localized at an interface separating homogeneous and periodic dielectric media [2], an interface between two photonic crystals [3], and an interface between a photonic crystal and a left-handed metamaterial [4].

Conventional electromagnetic Tamm states, which can exist at an interface separating two photonic crystals or a photonic crystal and a homogeneous dielectric, are known to possess a parabolic dispersion. In contrast, the surface modes existing at an interface between two metal-dielectric periodic structures [5] are characterized by a dispersion resembling that of the surface plasmon polaritons. It was shown that these modes can have a negative group velocity as well as a small localization length.

In this paper, we study the properties of the electromagnetic Tamm states supported by a surface of a multilayer graphene structure, or a *terminated graphene metamaterial*. As was shown recently [6–10], the graphene metamaterials exhibit many properties of conventional metal-dielectric multilayer structures, but at lower frequencies. These properties include the existence of the near-field Bloch waves [11] as well as a hyperbolic dispersion. Graphene multilayer structures have two significant differences which can potentially be useful for future optoelectronic devices. First, the period of the graphene multilayer structures can be as small as several nanometers [12], which is about an order of magnitude smaller than that of metal-dielectric structures. Second, due to smaller carrier densities in graphene as compared to metals, the plasma frequency of graphene and all plasmonic features, such as the plasmonic Bloch waves, appear in far-infrared to THz frequency ranges. Since this frequency range is extremely important for biomedical and security applications [13], the field of graphene plasmonics [14,15] is rapidly growing.

## II. MODEL AND IMPEDANCE CONDITIONS FOR SURFACE WAVES

We consider a truncated multilayer graphene structure and study the electromagnetic waves localized near its surface, as shown schematically in Fig. 1. We assume that the first dielectric layer of this multilayer structure has the thickness $d_0$, which in a general case may differ from the period of the structure $d$. As we show below, this allows us to achieve tunability of the dispersion properties of the Tamm modes by varying the parameter $d_0$. While the graphene metamaterials support both transverse magnetic (TM) and transverse electric (TE) polarized surface plasmons, here we consider only the TM polarized modes, because the coupling of the weakly localized TE plasmons does not produce near-field Bloch waves [6].

The dispersion equation for surface waves is derived from the matching surface impedances at the interface of the multilayer structure and the uniform medium. The impedance is defined as the ratio of the tangential components of the electric and magnetic fields, $\tilde{Z} = E_{\parallel}/H_{\parallel}$. As follows from Maxwell's equations, the impedance of the uniform dielectric medium $Z'$ is given by

$$Z' = -i\left(\frac{q'}{k_0\varepsilon'}\right), \tag{1}$$

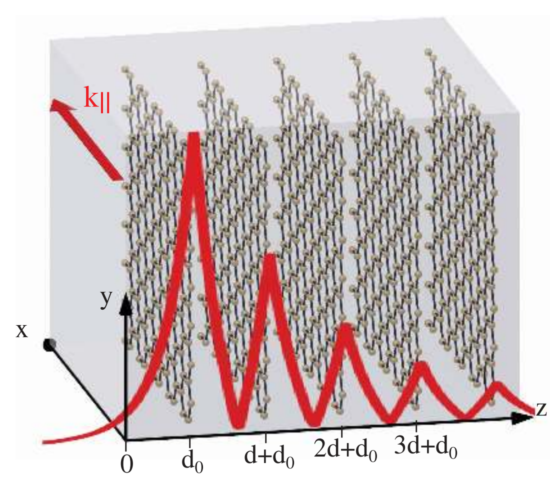


FIG. 1. (Color online) Geometry of the problem: a truncated multilayer structure composed of graphene sheets separated by the dielectric layers with permittivity $\varepsilon$ and thickness $d$. A thin dielectric layer of the thickness $d_0$ is a cap layer. The external dielectric has permittivity $\varepsilon'$. The red curve shows an example of a profile of the surface mode (shown is the tangential electric field component).

 

where $q' = (k_\parallel^2 - \varepsilon' k_0^2)^{1/2}$, $k_\parallel$ is the propagation constant, and $k_0 = \omega/c$ is the wave number in a free space. For a periodic structure the surface impedance can be found by employing the transfer-matrix method and applying the Bloch theorem,

$$\begin{pmatrix} E_\parallel(d) \\ H_\parallel(d) \end{pmatrix} = \exp(iKd) \begin{pmatrix} E_\parallel(0) \\ H_\parallel(0) \end{pmatrix} = \hat{T} \begin{pmatrix} E_\parallel(0) \\ H_\parallel(0) \end{pmatrix}, \quad (2)$$

where $K$ is the Bloch wave number. The dispersion relation for the Bloch wave number can be written as

$$\cos Kd = \cosh qd + 2\pi i \left( \frac{q}{k_0 \varepsilon} \frac{\sigma}{c} \right) \sinh qd, \quad (3)$$

and the transfer matrix $\hat{T}$ which relates the tangential components of the electric and magnetic fields at $z = 0, d, 2d \ldots$, with the elements given by the expressions

$$T_{11} = \cosh qd + \frac{2\pi i \sigma}{c} \frac{q}{k_0 \varepsilon} [\sinh qd + \sinh q(d - 2d_0)],$$

$$T_{12} = \frac{-iq}{k_0 \varepsilon} \left( \sinh qd + \frac{2\pi i \sigma}{c} \frac{q}{k_0 \varepsilon} [\cosh qd - \cosh q(d - 2d_0)] \right),$$

$$T_{21} = \frac{i k_0 \varepsilon}{q} \left( \sinh qd + \frac{2\pi i \sigma}{c} \frac{q}{k_0 \varepsilon} [\cosh qd + \cosh q(d - 2d_0)] \right),$$

$$T_{22} = \cosh qd + \frac{2\pi i \sigma}{c} \frac{q}{k_0 \varepsilon} [\sinh qd - \sinh q(d - 2d_0)],$$

where $q = (k_\parallel^2 - \varepsilon k_0^2)^{1/2} \equiv -ik_z$ is the transverse wave number in dielectric layers, and $\sigma$ is the frequency-dependent graphene conductivity which has been derived in a number of papers by using different assumptions [16–18]. In our calculations, neglecting losses and assuming $\hbar\omega < 1.67\mu$ ($\mathrm{Im}\,\sigma > 0$), where $\mu$ is the chemical potential, for doped graphene $k_B T \ll \mu$, we use the result of Ref. [16] simplified to the form

$$\sigma = \frac{ie^2}{\pi\hbar} \left[ \frac{\mu}{\hbar\omega} + \frac{1}{4} \ln \frac{(2\mu - \hbar\omega)}{(2\mu + \hbar\omega)} \right], \quad (4)$$

where $e$ is a charge of electron, $k_B$ is the Boltzmann constant, and $T$ is temperature. As a result, the impedance $Z$ for the graphene metamaterial can be expressed through the matrix elements, and at the surface of the periodic structure (at $z = 0$) it is written as

$$Z = \frac{T_{12}}{e^{iKd} - T_{11}} = \frac{e^{iKd} - T_{22}}{T_{21}}. \quad (5)$$

Once both the impedances of the homogeneous dielectric and periodic structure are found, the dispersion relation $Z = Z'$ can be written implicitly, and then it can be solved numerically in order to find the mode wave number $k_\parallel$.

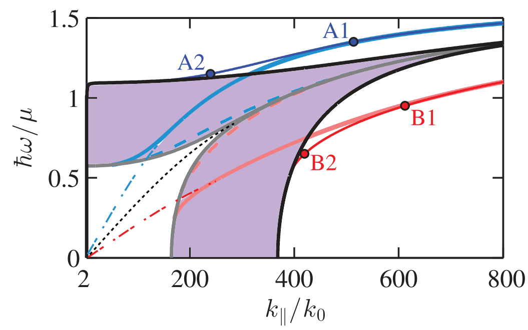


FIG. 2. (Color online) Dispersion of surface waves in the case of positive $\varepsilon'$, and $\varepsilon = 4$. The bright and light purple areas show the bands of the periodic structures with periods $d = 8$ and 40 nm, respectively. The red and blue lines correspond to the dielectic constant of the cap layer $\varepsilon' = 1$ and 10, respectively. The thin black dotted line shows the dispersion of the single graphene sheet with dielectric permittivity $\varepsilon$ on both sides. The dashed-dotted lines show the dispersions of a single graphene sheet given by Eq. (7). The thick solid and dashed lines correspond to $d = 40$ nm, $d_0 = 0$ (thick solid), and $d_0 = 0.2d$ (dashed). The thin solid lines correspond to $d = 8$ nm, $d_0 = 0$. Points A1, A2 and B1, B2 correspond to the parameters for which the mode profiles are presented in Figs. 5(a) and 5(b).

## III. DISPERSION AND FIELD PROFILES OF SURFACE MODES

First, we consider the case when a cap dielectric layer is absent, $d_0 = 0$, and the dispersion equation can be written as

$$1 + 4\pi i \frac{\sigma}{c} \frac{q'}{k_0 \varepsilon'} = \frac{q' \varepsilon}{q \varepsilon'} \frac{[e^{iKd} - \cosh(qd)]}{\sinh(qd)}. \quad (6)$$

In the limit $d \to \infty$ or for large wave numbers, Eq. (6) reduces to the dispersion of the $p$-polarized plasmons at the graphene layer sandwiched between two different dielectric media:

$$\frac{k_0 \varepsilon'}{q'} + \frac{k_0 \varepsilon}{q} + i \frac{4\pi\sigma}{c} = 0. \quad (7)$$

In Fig. 2 we show the dispersion curves of the surface waves obtained from Eq. (6) with solid lines for two permittivities of the external dielectric—more and less than the permittivity of the dielectric infilling the separations between graphene layers in the periodic structure. The respective asymptotic solutions of Eq. (7) are plotted with dashed lines. The large-wave-number asymptote of the shaded area (allowed band) corresponds to the plasmon mode localized at the graphene layer surrounded by dielectric layers with permittivity $\varepsilon$. Noticeably, if $\varepsilon' < \varepsilon$, the localized mode's dispersion is blueshifted with respect to the allowed band whereas in the opposite case ($\varepsilon' > \varepsilon$) this branch is redshifted.

With increasing the period of the structure, coupling between individual surface plasmons at the graphene layers decreases, and the allowed band shrinks gradually to a single curve corresponding to the dispersion of an individual surface plasmon polariton. This allows the existence of the surface states in the lower frequency region.

When the thickness of the dielectric cap layer with positive dielectric permittivity is finite, the surface wave is localized at the graphene sheet closest to the interface of the structure. As shown in Fig. 2, increasing $d_0$ from zero gradually makes the dispersion curve of the surface wave approach the allowed band of the periodic structure.

Now we consider the situation of a negative frequency-dependent $\varepsilon'(\omega)$ described by the Drude model $\varepsilon'(\omega) = 1 - \omega_p^2/\omega^2$. Such a plasma substrate with plasma frequency $\omega_p$ in the THz range can be made of a doped semiconductor. In this case, for all values of $d_0$, the large-wave-number asymptote for the localized waves is the dispersion of the surface plasmon at the interface of $\varepsilon$ and $\varepsilon'(\omega)$ which has the asymptotic value of $\omega = \omega_p/\sqrt{\varepsilon+1}$. When $qd \ll 1$ but $q,q' \approx k_\parallel \gg 1$, the mode wave number can be approximated by the following expression,

$$|k_\parallel d| \approx \frac{2(1+\alpha)[(1+\alpha)^{-1/2} - \varepsilon/|\varepsilon'|]}{1 - 2\alpha(1 - d_0/d)}, \qquad (8)$$

where $\alpha = 4\pi\, \mathrm{Im}\,\sigma/ck_0\varepsilon d$. In the particular case $d_0 = d$, the dispersion equation is simplified to the form

$$\left(\frac{q\varepsilon'}{q'\varepsilon}\right)\frac{\sinh(qd)}{e^{iKd} - \cosh(qd)} = 1. \qquad (9)$$

The dispersion of surface waves for two different values of $\omega_p$ is presented in Fig. 3. Again, if $|\varepsilon'(\omega)| < \varepsilon$, the surface mode branch is blueshifted with respect to the allowed band and redshifted in the opposite case, $|\varepsilon'(\omega)| > \varepsilon$.

In Fig. 3 we observe the negative slope for the blueshifted surface wave in a certain frequency range. As we show in Fig. 4, this corresponds to the negative group velocity, i.e., the wave becomes essentially a backward wave. At the same time, for the lower frequencies, where $|\varepsilon'(\omega)| > \varepsilon$, the group velocity is always positive and the surface wave is forward.

It should be noted that in real structures, where the losses in the graphene layers and semiconductor substrate play a significant role, the stronger localized modes corresponding to larger wave numbers decay faster due to the larger localization at the interfaces. Thus, for structures of smaller periods, there will be an interplay between larger localization and at the same time larger damping.

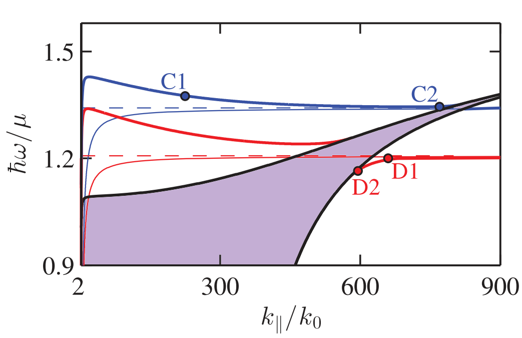


FIG. 3. (Color online) Dispersion of the surface waves in the case of negative frequency-dependent $\varepsilon'(\omega)$. The parameters of the periodic structure are $\varepsilon = 4$ and $d = 8$ nm. The shaded area is the allowed band. The red and blue thick lines depict the dispersion of surface waves for $\hbar\omega_p = 2.7\mu$ and $\hbar\omega_p = 3\mu$, respectively. The asymptotes $\omega = \omega_p/\sqrt{\varepsilon+1}$ are shown by dashed lines. The thin solid lines are dispersion curves of conventional plasmons supported by the interface of $\varepsilon'(\omega)$ and $\varepsilon$. Points C1, C2 and D1, D2 were taken for the profile calculation in Figs. 5(c) and 5(d).

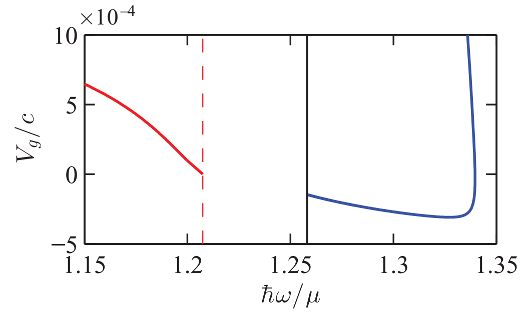


FIG. 4. (Color online) Spectrum of the group velocity for the surface wave when the plasma frequency of the adjoined semiconductor is $\hbar\omega_p = 2.7\mu$. The red dashed and black solid lines mark the asymptotic frequency $\omega = \omega_p/\sqrt{\varepsilon+1}$ and the boundary of the allowed band, respectively.

Concerning the transverse field profile, two different types of surface states can be naturally distinguished—lying in proximity to the allowed band and far from it. Far from the allowed band (points A1, B1 in Fig. 2 and C1, D1 in Fig. 3), the character of the propagating waves is very similar to plasmonic [see Figs. 5(a) and 5(c)], which means they have a strong localization—the field almost completely vanishes in two periods of the structure. In essence, only the boundary influences the propagating surface wave and the extension of the periodic multilayer graphene structure has little impact. By contrast, closer to the allowed bands (points A2, B2 in Fig. 2 and C2, D2 in Fig. 3), the coupling is higher and the field penetrates deeper into the periodic multilayer graphene structure [see Figs. 5(b) and 5(d)].

It is also worth mentioning the effect of losses in the graphene sheet on the dispersion properties of the surface

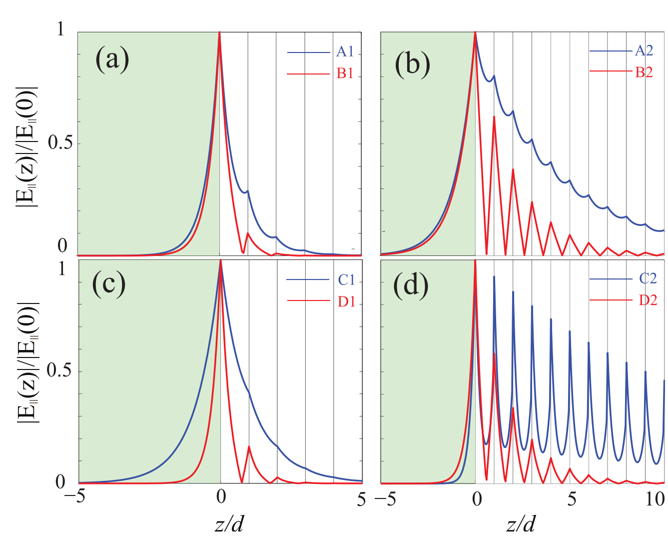


FIG. 5. (Color online) Field profiles for the surface waves corresponding to the points marked in Figs. 2 and 3. Shown is the normalized tangential component of the electric field. (a), (c) Plasmoniclike surface waves strongly localized in the vicinity of the boundary. (b), (d) Surface waves decaying slowly inside the graphene metamaterial. Branching from the bottom of the allowed band, modes B1, B2, D1, D2 are staggered.

Tamm states. There are several mechanisms for damping of THz plasmons in graphene [19], such as electron scattering, interband absorption, and optical phonon coupling. While the latter two mechanisms can be suppressed by choosing frequencies below the graphene chemical potential and optical phonon frequencies (about 0.2 eV), the finite electron free propagation time will cause the damping of the surface states and limit the minimal available localization lengths. While for the case of a single graphene sheet surface plasmon a figure of merit which is the ratio of the real and imaginary part of the in-plane wave vector is a function of $\mathrm{Re}\,\sigma/\mathrm{Im}\,\sigma$ and frequency, in our case it will also depend on the period of the structure and the thickness of the cap layer. The calculation of the figure of merit for the graphene Tamm surface states is the subject of a future work.

## IV. CONCLUSIONS

We have studied the electromagnetic surface waves existing at the surface of the graphene multilayer structure, the graphene metamaterial. We have shown that the dispersion properties of these modes can be finely tuned with the thickness of the cap dielectric layer, and these types of modes can acquire a negative group velocity.

## ACKNOWLEDGMENT

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[1] I. Tamm, Zh. Eksp. Teor. Fiz. **3**, 34 (1933).
[2] P. Yeh, A. Yariv, and A. Y. Cho, Appl. Phys. Lett. **32**, 104 (1978).
[3] A. P. Vinogradov, A. V. Dorofeenko, A. M. Merzlikin, and A. A. Lisyansky, Phys.-Usp. **53**, 243 (2010).
[4] A. Namdar, I. V. Shadrivov, and Yu. S. Kivshar, Appl. Phys. Lett. **89**, 114104 (2006).
[5] I. Iorsh, A. Orlov, P. A. Belov, and Yu. S. Kivshar, Appl. Phys. Lett. **99**, 151914 (2011).
[6] I. V. Iorsh, I. S. Mukhin, I. V. Shadrivov, P. A. Belov, and Yu. S. Kivshar, Phys. Rev. B **87**, 075416 (2013).
[7] M. A. K. Othman, C. Guclu, and F. Capolino, Opt. Express **21**, 7614 (2013).
[8] C. S. R. Kaipa, A. B. Yakovlev, G. W. Hanson, Y. R. Padooru, F. Medina, and F. Mesa, Phys. Rev. B **85**, 245407 (2012).
[9] Y. Fan, Z. Wei, H. Li, H. Chen, and C. M. Soukoulis, Phys. Rev. B **88**, 241403 (2013).
[10] K. V. Sreekanth, S. Zeng, J. Shang, K. Yong, and T. Yu, Sci. Rep. **2**, 737 (2012).
[11] A. P. Vinogradov and A. V. Dorofeenko, J. Commun. Technol. Electron. **50**, 1153 (2005).
[12] Y. J. Li, Q. Sun, L. Chen, P. Zhou, P. Wang, S. Ding, and D. W. Zhang, AIP Adv. **2**, 012191 (2012).
[13] J. M. Chamberlain, Philos. Trans. R. Soc., A **362**, 199 (2004).
[14] F. H. L. Koppens, D. E. Chang, and F. J.-G. de Abajo, Nano Lett. **11**, 3370 (2011).
[15] A. N. Grigorenko, M. Polini, and K. S. Novoselov, Nat. Photonics **6**, 749 (2012), and references therein.
[16] L. A. Falkovsky and A. A. Varlamov, Eur. J. Phys. B **56**, 281 (2007).
[17] F. T. Vasko and V. Ryzhii, Phys. Rev. B **76**, 233404 (2007).
[18] T. Stauber, N. M. R. Peres, and A. K. Geim, Phys. Rev. B **78**, 085432 (2008).
[19] H. Yan, T. Low, W. Zhu, Y. Wu, M. Freitag, X. Li, F. Guinea, P. Avouris, and F. Xia, Nat. Photonics **7**, 394 (2013).