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Default and Recovery Implicit in the Term Structure of Sovereign *CDS* Spreads

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ABSTRACT

This paper explores the nature of default arrival and recovery implicit in the term structures of sovereign *CDS* spreads. We argue that term structures of spreads reveal not only the arrival rates of credit events ($\lambda^{\mathbb{Q}}$), but also the loss rates given credit events. Applying our framework to Mexico, Turkey, and Korea, we show that a single-factor model with $\lambda^{\mathbb{Q}}$ following a lognormal process captures most of the variation in the term structures of spreads. The risk premiums associated with unpredictable variation in $\lambda^{\mathbb{Q}}$ are found to be economically significant and co-vary importantly with several economic measures of global event risk, financial market volatility, and macroeconomic policy.

THE BURGEONING MARKET FOR SOVEREIGN CREDIT DEFAULT SWAPS (*CDS*) contracts offers a nearly unique window for viewing investors' risk-neutral probabilities of major credit events impinging on sovereign issuers, and their risk-neutral losses of principal in the event of a restructuring or repudiation of external debts. In contrast to many "emerging market" sovereign bonds, sovereign *CDS* contracts are designed without complex guarantees or embedded options. Trading activity in the *CDS* contracts of several sovereign issuers has developed to the point that they are more liquid than many of the underlying bonds. Moreover, in contrast to the corporate *CDS* market, where trading has been concentrated largely in the 5-year maturity contract, *CDS* contracts at several maturity points between 1 and 10 years have been actively traded for several years. As such, a full *term structure* of *CDS* spreads is available for inferring default and recovery information from market data.

This paper explores in depth the time-series properties of the risk-neutral mean arrival rates of credit events ($\lambda^{\mathbb{Q}}$) implicit in the term structure of sovereign *CDS* spreads. Applying our framework to Mexico, Turkey, and Korea, three countries with different geopolitical characteristics and credit ratings, we

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find that single-factor models, in which country-specific $\lambda^{\mathbb{Q}}$ follow lognormal processes,[1] capture most of the variation in the term structures of spreads. The maximum likelihood estimates suggest that, for all three countries, there are systematic, priced risks associated with unpredictable future variation in $\lambda^{\mathbb{Q}}$. Moreover, the time-series of the effects of risk premiums on *CDS* spreads covary strongly across countries. There are several large concurrent "run-ups" in risk premiums during our sample period (March 2001 through August 2006) that have natural interpretations in terms of political, macroeconomic, and financial market developments at the time.

A more formal regression analysis of the correlations between risk premiums and the CBOE U.S. VIX option volatility index (viewed as a measure of event risk), the spread between the 10-year return on U.S. BB-rated industrial corporate bonds and the 6-month U.S. Treasury bill rate (viewed as a measure of both U.S. macroeconomic and global financial market developments), and the volatility in the own-currency options market corroborates our economic interpretations of the temporal changes in risk premiums in the sovereign *CDS* markets. The evidence is consistent with premiums for credit risk in sovereign markets being influenced by spillovers of real economic growth in the United States to economic growth in other regions of the world. Equally notable is that our findings suggest that, during some subperiods, a substantial portion of the co-movement among the term structures of sovereign spreads across countries was induced by changes in investors' appetites for credit exposure at a global level, rather than to reassessments of the fundamental strengths of these specific sovereign economies.

While most of our focus is on the economic underpinnings of the dynamic properties of the arrival rates of credit events, an equally central ingredient to modeling the credit risk of sovereign issuers is the recovery of bond holders in the face of a credit event. Standard practice in modeling corporate *CDS* spreads is to assume a fixed risk-neutral loss rate $L^{\mathbb{Q}}$, largely because the focus has been on the liquid 5-year *CDS* contract.[2] We depart from this literature and exploit the term structure of *CDS* spreads to separately identify both $L^{\mathbb{Q}}$ and the parameters of the process $\lambda^{\mathbb{Q}}$. That we even attempt to separately identify these parameters of the default process may seem surprising in light of the apparent demonstrations in Duffie and Singleton (1999), Houweling and Vorst (2005), and elsewhere of the infeasibility of achieving this objective. We show that, in fact, in market environments where recovery is a fraction of face value, as is the case with *CDS* markets, these parameters can in principle be separately identified through the information contained in the term structure of *CDS* spreads.

[1] In the literature on corporate *CDS* spreads, $\lambda^{\mathbb{Q}}$ was modeled as a square-root process in Longstaff, Mithal, and Neis (2005), while Berndt et al. (2004) argue that corporate *CDS* spreads are better described by a lognormal model. Zhang (2003) had $\lambda^{\mathbb{Q}}$ following a square-root process in his analysis of Argentinean *CDS* contracts.

[2] See, for example, Berndt et al. (2004), Hull, Mirela, and White (2004), and Houweling and Vorst (2005).

The maximum likelihood (*ML*) estimates of the parameters governing $\lambda^{\mathbb{Q}}$ imply that its risk-neutral ($\mathbb{Q}$) distribution shows very little mean reversion and, in fact, in some cases $\lambda^{\mathbb{Q}}$ is $\mathbb{Q}$-explosive. In contrast, the historical data-generating process ($\mathbb{P}$) for $\lambda^{\mathbb{Q}}$ shows substantial mean reversion, consistent with the $\mathbb{P}$-stationarity of *CDS* spreads. This large difference between the properties of $\lambda^{\mathbb{Q}}$ under the $\mathbb{Q}$ and $\mathbb{P}$ measures implies, within the context of our models, that an economically important systematic risk is being priced in the *CDS* market.

Our *ML* estimates are obtained both with fixed $L^{\mathbb{Q}}$ at the market convention 0.75, and by searching over $L^{\mathbb{Q}}$ as a free parameter. In the latter case, the likelihood functions call for much smaller values of $L^{\mathbb{Q}}$ for Mexico and Turkey, more in the region of 0.25, and also slower rates of $\mathbb{P}$-mean reversion of $\lambda^{\mathbb{Q}}$. An extensive Monte Carlo analysis of the small-sample distributions of various moments reveals that many features of the implied distributions of *CDS* spreads for Mexico and Turkey are similar across the cases of $L^{\mathbb{Q}}$ equal to 0.75 or 0.25. For our model formulation and sample *ML* estimates, it is only over long horizons—for most of our countries, longer than our sample periods—that the differences in $\mathbb{P}$-mean reversion in the two cases manifest themselves. This observation, combined with our finding that the unconstrained estimate of $L^{\mathbb{Q}}$ for Korea is similar to the market convention of 0.75, leads us to set $L^{\mathbb{Q}} = 0.75$ for our analysis of risk premiums.

Throughout our analysis we maintain the assumption that a single risk factor underlies the temporal variation in $\lambda^{\mathbb{Q}}$, consistent with most previous studies of *CDS* spreads that have allowed for a stochastic arrival rate of credit events. In the case of our sovereign data, this focus is motivated by the high degree of co-movement among spreads across the maturity spectrum within each country. For our sample period, this co-movement is even greater than that of yields on highly liquid treasury bonds documented, for example, in Litterman and Scheinkman (1991). To better understand the nature of our pricing errors, particularly at shorter maturities, we investigate the potential role for a second risk factor. The behaviors of bid-ask spreads are also examined, with a potential role for liquidity factors in mind.

To our knowledge, the closest precursor to our analysis is the study by Zhang (2003) of *CDS* spreads for Argentina leading up to the default in late 2001. Our sample period begins towards the end of his, is longer in length, and spans a period during which the sovereign *CDS* markets were more developed in breadth and liquidity. The complementary study of Mexican and Brazilian *CDS* spreads in Carr and Wu (2007) explores the correlation structure of spreads on contracts up to 5 years to maturity with implied volatilities on various currency options over the shorter period of January 2002 through March 2005. Relative to both of these studies, we examine a geographically more dispersed set of countries, and we explore in depth the economic underpinnings of the *co-movements* of risk premiums for these countries. Toward this end, we allow for more flexible market prices of risk, and examine a broader array of economic factors underlying market risk premiums.

I. The Structure of the Sovereign *CDS* Market

The structure of the standard *CDS* contract for a sovereign issuer shares many of its features with the corporate counterpart. The default protection buyer pays a semi-annual premium, expressed in basis points per notional amount of the contract, in exchange for a contingent payment in the event one of a pre-specified credit events occurs. Settlement of a *CDS* contract is typically by physical delivery of an admissible bond in return for receipt of the original face value of the bonds,[3] with admissibility determined by the characteristics of the reference obligation in the contract.

Typically, only bonds issued in *external* markets and denominated in one of the "standard specified currencies" are deliverable.[4] In particular, bonds issued in domestic currency, issued domestically, or governed by domestic laws are not deliverable. For some sovereign issuers without extensive issuance of hard-currency denominated Eurobonds, loans may be included in the set of deliverable assets. Among the countries included in our analysis, Turkey and Mexico have sizeable amounts of outstanding loans, and their *CDS* contracts occasionally trade with "Bond or Loan" terms. The contracts we focus on are "Bond only."

The key definition included in the term sheet of a sovereign *CDS* contract is the credit event. Typically, a sovereign *CDS* contract lists as events any of the following that affect the reference obligation: (i) obligation acceleration, (ii) failure to pay, (iii) restructuring, or (iv) repudiation/moratorium. Note that "default" is not included in this list, because there is no operable international bankruptcy court that applies to sovereign issuers.

Central to our analysis of the term structure of sovereign *CDS* spreads is the active trading of contracts across a wide range of maturities. In contrast to the U.S. corporate and bank *CDS* markets, where a large majority of the trading volume is concentrated in 5-year contracts, the 3- and 10-year contracts have each accounted for roughly 20% of the volumes in sovereign markets, and the 1-year contract has accounted for an additional 10% of the trading (see Figure 1).[5] While the total volume of new contracts has been much larger in the corporate than the sovereign market, the volumes for the most actively traded

[3] Physical delivery is the predominant form of settlement in the sovereign *CDS* market, because both the buyers and sellers of protection typically want to avoid the dealer polling process involved in determining the value of the reference bond in what is often a very illiquid post-credit-event market place.

[4] The standard specified currencies are the euro, U.S. dollar, Japanese yen, Canadian dollar, Swiss franc, and the British pound. The option to deliver bonds denominated in these currencies, and of various maturities, into a *CDS* contract introduces a cheapest-to-deliver option for the protection buyer. Our impression, from conversations with traders, is that usually there is a single bond (or small set of bonds) that is cheapest to deliver. So the price of the *CDS* contract tracks this cheapest to deliver bond and the option to deliver other bonds is not especially valuable. In any event, for the purpose of our subsequent analysis, we will ignore this complication in the market.

[5] Figure 1 is a corrected version of the original appearing in Packer and Suthiphongchai (2003).

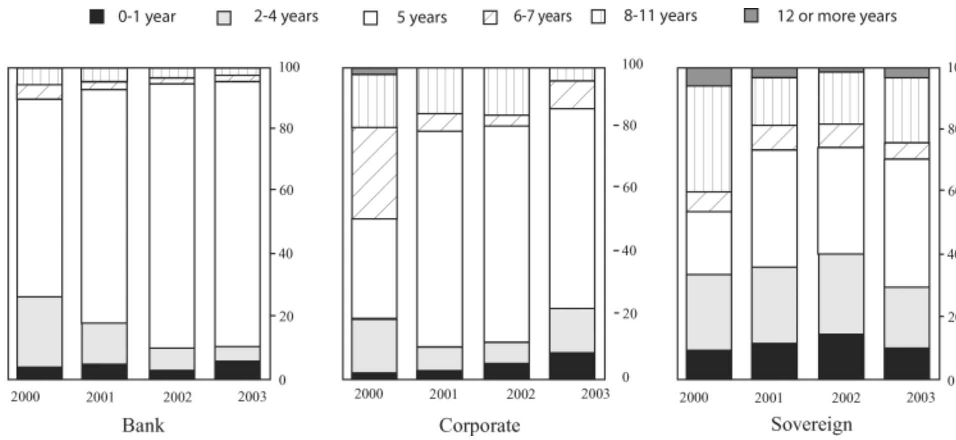

Figure 1. *CDS* volumes by maturity, as a percentage of total volume, based on *BIS* calculations from CreditTrade data. Source: *BIS* Quarterly Review [2003].

sovereign credits are large and growing. We focus our analysis on Mexico, Turkey, and Korea, three of the more actively traded names.[6]

Our sample consists of daily trader quotes of bid and ask spreads for *CDS* contracts with maturities of 1, 2, 3, 5, and 10 years. The sample covers the period March 19, 2001 through August 10, 2006. We focus on the data for three geographically dispersed countries—Mexico, Turkey, and Korea—displayed in figure 2. (Descriptive statistics of these series are displayed on the left-hand side of Table I.) At the beginning of our sample period (March 2001), Mexico had achieved the investment grade rating of Baa3. In February 2002, Mexico was upgraded one notch to Baa2, and it was subsequently upgraded again one notch to Baa1 in January 2005. Turkey maintained the same speculative grade rating, B1, throughout most of our sample period. However, both in April 2001 and July 2002 it was put in the "negative outlook" category. Following the most recent negative outlook, Turkey returned to "stable outlook" in October 2003. Moody's changed its outlook for Turkey to positive in February 2005, and then upgraded Turkish (external) government bonds to Ba3 in December 2005. Korea was upgraded by Moody's from Baa2 to A3 on March 28, 2002 and it maintained this rating throughout our sample period. However, the outlook for Korea was negative towards the end of 2003 (due to concerns about North Korea), it was upgraded to stable in September 2004, and upgraded again to positive in April 2006. Consistent with the relative credit qualities of these countries, the average 5-year *CDS* spreads over our sample period are 62, 166, and 563 basis points, respectively, for Korea, Mexico, and Turkey (see Table I).

[6] Russia as well as several South American credits—Brazil, Colombia, and Venezuela—are also among the more traded sovereign credits. The behavior of the South American *CDS* spreads was largely dominated by the political turmoil in Brazil during the summer/fall of 2002. The co-movements among the *CDS* spreads of these countries is an interesting question for future research.

Table I

Summary Statistics

The sample period is March 2001 until the beginning of August 2006. Med is the sample median; *SD* is the sample standard deviation; a.c. is the first-order autocorrelation statistic.

	CDS Price (bps)						CDS Bid Ask Spread (bps)					
	Mean	Med	*SD*	Min	Max	a.c.	Mean	Med	*SD*	Min	Max	a.c.
	Mexico						Mexico					
1 yr	54.5	33	38.6	14	185	0.993	13.3	10	8.5	5	50	0.940
2 yr	92.4	65	63.7	22	305	0.995	13.1	10	8.9	2	60	0.931
3 yr	123.5	94	78.7	30	370	0.996	13.0	10	8.3	5	50	0.937
5 yr	166.4	147	89.3	46	440	0.997	12.4	10	8.2	4	40	0.951
10 yr	213.0	200	90.2	76	475	0.997	12.6	10	8.5	4	50	0.950
	Turkey						Turkey					
1 yr	378.4	225	355.5	23	1700	0.993	61.1	50	62.3	8	850	0.875
2 yr	458.1	315	357.0	45	1650	0.995	47.5	30	52.1	6	600	0.914
3 yr	505.9	399	347.8	68	1600	0.995	44.3	30	49.6	6	575	0.889
5 yr	563.1	504	327.7	116	1500	0.996	39.5	30	41.1	4	400	0.906
10 yr	607.3	552	304.6	181	1450	0.996	39.4	30	39.0	4	300	0.935
	Korea						Korea					
1 yr	33.7	31	25.0	4	165	0.991	9.2	10	1.0	8	10	0.998
2 yr	41.7	38	27.8	9	176	0.994	9.2	10	1.0	8	10	0.998
3 yr	48.6	45	29.8	13	184	0.995	9.2	10	1.0	6	10	0.995
5 yr	62.0	58	33.2	22	197	0.996	9.2	10	1.0	5	10	0.993
10 yr	81.3	78	38.5	32	212	0.996	9.2	10	1.0	5	10	0.993

In addition to the fact that they cover a broad range of credit quality, two important considerations factor into our choice of these three countries: their regional representativeness in the emerging markets and the relative liquidity and thus better data quality of their *CDS* markets compared to those of many other countries in the same region. The first consideration is important for the economic interpretation of our results. These countries are geographically dispersed—being located in Latin American, Eastern Europe, and Asia—and each, in its own way, has been affected by significant local economic and political events. As such, we are interested in the degree and nature of the co-movements among *CDS* spreads for these countries. The second consideration plays a crucial role in our evaluation of our model's implications for default and recovery implicit in *CDS* spreads, as we will assume that the levels of *CDS* spreads are largely reflective of credit assessments (as opposed to (il)liquidity, for example).

As shown in Figure 2, the term-structures of *CDS* spreads exhibit interesting dynamics. One immediately noticeable feature present in all three countries is the high level of co-movement among the 1, 3, 5, and 10 year *CDS* spreads. Indeed, a principal component (*PC*) analysis of the spreads in each country (see Section IV.B) shows that the first *PC* explains over 96% of the variation in *CDS* spreads for all three countries.[7] It is these high levels of explained variation that motivate our focus on one-factor models.

[7] The only exception is the spread on the 1-year contract for Mexico, and 90% of its variation is explained by the first *PC* of Mexican spreads.

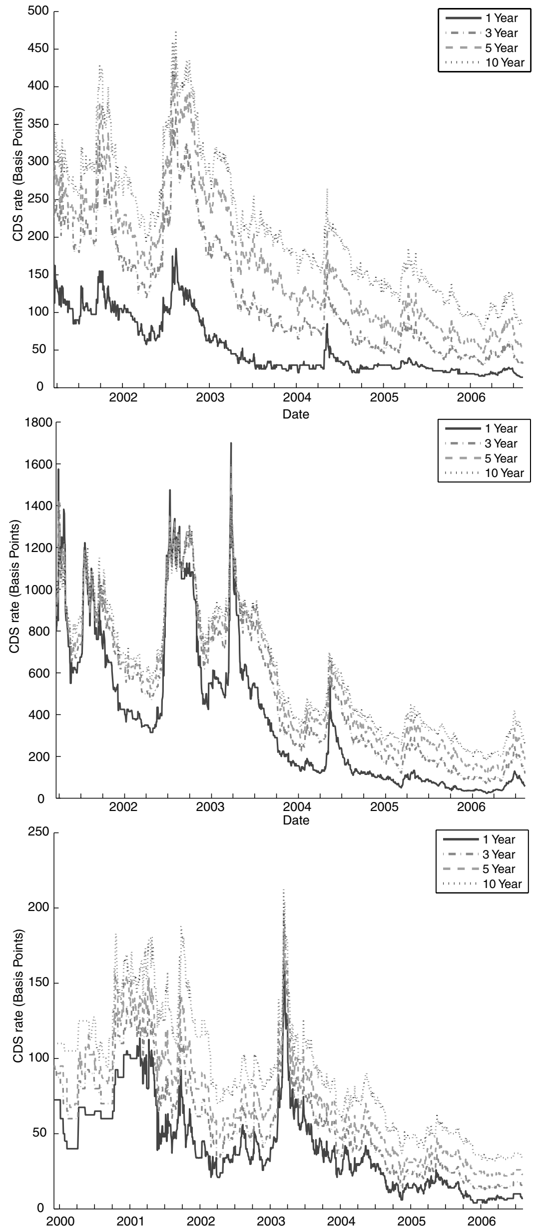

Figure 2. CDS Spreads: Mexico (upper), Turkey (middle), and Korea (lower), mid-market quotes.

Another prominent feature of the *CDS* data is the persistence of upward sloping term structures. This is especially true for the term structures of Mexican and Korean *CDS* spreads: Throughout our sample period, the 1-year *CDS* spreads were always lower than the respective longer maturity *CDS* spreads and, hence, the term structure was never inverted. For example, the difference between the 5-year and 1-year Mexican *CDS* spreads was 112 basis points on average, 31 basis points at minimum, and 275 basis points at maximum. Without resorting to institutional features that might separate the 1-year from the longer maturity *CDS* contracts, this pattern of *CDS* spreads implies an increasing term structure of risk-neutral 1-year forward default probabilities.

The slope of the term structure of *CDS* spreads for Turkey was mostly positive. For example, the difference between the 5- and 1-year *CDS* spreads was on average 185 basis points with a standard deviation of 93 basis points. However, in contrast to the robust pattern of upward sloping spread curves in Mexico and Korea, the term structure of Turkish *CDS* spreads did occasionally invert, especially when credit spreads exploded to high levels due to financial or political crises that were (largely) specific to Turkey. For example, the differences between the 5- and 1-year *CDS* spreads were −250 basis points on March 29, 2001, −150 basis points on July 10, 2002, and −200 basis points on March 24, 2003. The related events were the devaluation of the Turkish lira, political elections in Turkey, and the collapse of talks between Turkey and Cyprus (which had implications for Turkey's bid to join the EU).

Sovereign credit default swaps trade, on average, in larger sizes than in the underlying cash markets: U.S. $5 million, and occasionally much larger, against U.S. $1 million to $2 million. The liquidity of the underlying bond market is relevant, because traders hedge their *CDS* positions with cash market instruments and the less liquid is the cash market, the larger the bid-ask spread must be in the *CDS* market to cover the higher hedging costs. Comparing across sovereign *CDS* markets, a given bid-ask spread will sustain a larger trade in the market for Mexico (up to about $40 million) relative to Turkey (up to about $30 million) (Xu and Wilder (2003)).

For our sample of countries, the bid-ask spreads (in basis points for the 5-year contract) ranged between 4 and 40 for Mexico, 4 and 400 for Turkey, and 2 and 20 for Korea (see Figure 3 and Table I). Korea had the smallest and most stable bid-ask spreads. Notably, when Turkey's spreads widened out due to the "local" events chronicled above, so did the bid-ask spreads. For high-grade countries with large quantities of bonds outstanding like Mexico and Korea, the magnitudes of the bid-ask spreads in the *CDS* markets are comparable to those for their bonds.

Particularly at the short end of the maturity spectrum, there are often limited cash market vehicles available for trading sovereign exposure and this contributes to making the 1-year *CDS* contract an attractive instrument. The bid-ask spreads on the 1-year contract are comparable to those on the longer-dated contracts, though this means that they are larger as a percentage of *CDS* spreads. During turbulent periods, especially in Turkey, when the levels of *CDS* spreads are large, the bid-ask spreads on the 1- are larger then those on the 5-year contracts. We examine the properties of the bid-ask spreads of our data in more depth in Section IV.B in conjunction with our discussion of the challenges of fitting the 1-year (and to a lesser extent the 10-year) spreads within our one-factor term structure model for *CDS* spreads.

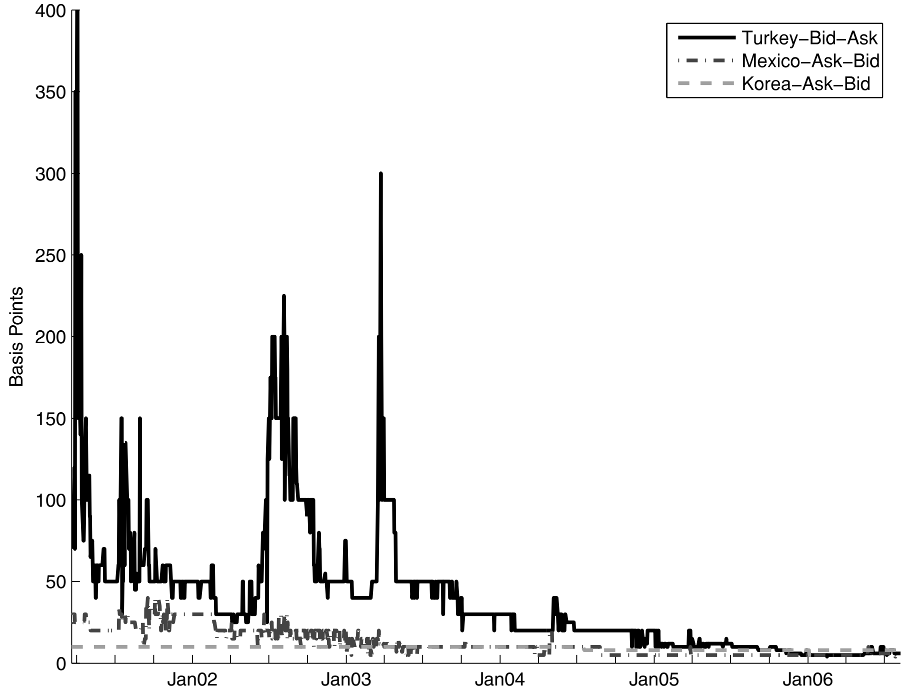

Figure 3. Ask-bid spreads (basis points) for 5-year *CDS* contracts.

II. Pricing Sovereign *CDS* Contracts

The basic pricing relation for sovereign *CDS* contracts is identical to that for corporate *CDS* contracts. Let M denote the maturity (in years) of the contract, $CDS_t(M)$ denote the (annualized) spread at issue, $R^{\mathbb{Q}}$ denote the (constant) risk-neutral fractional recovery of face value on the underlying (cheapest-to-deliver) bond in the event of a credit event, and $\lambda^{\mathbb{Q}}$ denote the risk neutral arrival rate of a credit event. Then, at issue, a *CDS* contract with semi-annual premium payments is priced as (see, e.g., Duffie and Singleton (2003)):

$$\frac{1}{2}\,CDS_t(M)\sum_{j=1}^{2M} E_t^{\mathbb{Q}}\left[e^{-\int_t^{t+.5j}(r_s+\lambda_s^{\mathbb{Q}})\,ds}\right] = (1-R^{\mathbb{Q}})\int_t^{t+M} E_t^{\mathbb{Q}}\left[\lambda_u^{\mathbb{Q}} e^{-\int_t^{u}(r_s+\lambda_s^{\mathbb{Q}})\,ds}\right] du, \tag{1}$$

where r_t is the riskless rate relevant for pricing *CDS* contracts. The left-hand side of (1) is the present value of the buyer's premiums, payable contingent upon a credit event not having occurred. Discounting by $r_t + \lambda_t^{\mathbb{Q}}$ captures the survival-dependent nature of these payments (Lando (1998)). The right-hand side of this pricing relation is the present value of the contingent payment by the protection seller upon a credit event. We normalize the face value of the underlying bond to \$1 and assume a constant expected contingent payment (loss relative to face value) of $L^{\mathbb{Q}} = (1 - R^{\mathbb{Q}})$. In implementing (1), we use a slightly modified version that accounts for the buyer's obligation to pay an accrued premium if a credit event occurs between the premium payment dates.

How should $\lambda^{\mathbb{Q}}$ and $L^{\mathbb{Q}}$ be interpreted, given that default is not a relevant credit event, and *ISDA* terms sheets for plain vanilla sovereign *CDS* contracts reference four types of credit events? To accommodate this richness of the credit process for sovereign issuers, let each of the four relevant credit events have their own associated arrival intensities $\lambda_i^{\mathbb{Q}}$ and loss rates $L_i^{\mathbb{Q}}$. Then, following Duffie, Pedersen, and Singleton (2003) and adopting the usual "doubly stochastic" formulation of arrival of credit events (see, e.g., Lando (1998)), we can interpret the $\lambda_t^{\mathbb{Q}}$ and $L_t^{\mathbb{Q}}$ for pricing sovereign *CDS* contracts as:

$$\lambda_t^{\mathbb{Q}} = \lambda_{acc,t}^{\mathbb{Q}} + \lambda_{fail,t}^{\mathbb{Q}} + \lambda_{rest,t}^{\mathbb{Q}} + \lambda_{repud,t}^{\mathbb{Q}} \tag{2}$$

$$L_t^{\mathbb{Q}} = \frac{\lambda_{acc,t}^{\mathbb{Q}}}{\lambda_t^{\mathbb{Q}}} L_{acc,t}^{\mathbb{Q}} + \frac{\lambda_{fail,t}^{\mathbb{Q}}}{\lambda_t^{\mathbb{Q}}} L_{fail,t}^{\mathbb{Q}} + \frac{\lambda_{rest,t}^{\mathbb{Q}}}{\lambda_t^{\mathbb{Q}}} L_{rest,t}^{\mathbb{Q}} + \frac{\lambda_{repud,t}^{\mathbb{Q}}}{\lambda_t^{\mathbb{Q}}} L_{repud,t}^{\mathbb{Q}} \tag{3}$$

where the subscripts represent acceleration, failure to pay, restructuring, and repudiation. In a doubly stochastic setting, conditional on the paths of the intensities, the probability that any two of the credit events happen at the same time is zero. Thus, $\lambda^{\mathbb{Q}}$ is naturally interpreted as the arrival rate of the first credit event of any type. Upon the occurrence of a credit event of type i, the relevant loss rate is $L_i^{\mathbb{Q}}$ and, given that a credit event has occurred, this loss rate is experienced with probability $\lambda_{it}^{\mathbb{Q}}/\lambda_t^{\mathbb{Q}}$. The corresponding $\lambda_i^{\mathbb{Q}}$ and $L_i^{\mathbb{Q}}$ may, of course, differ across countries.

To set notation, we use the superscript $\mathbb{Q}(\mathbb{P})$ to denote the parameters of the process $\lambda^{\mathbb{Q}}$ under the risk-neutral (historical) distributions, respectively. We highlight a potential ambiguity in our notation here: we are discussing the properties of $\lambda^{\mathbb{Q}}$, as a stochastic process, under two different measures, $\mathbb{Q}$ and $\mathbb{P}$. At this juncture, $\lambda^{\mathbb{P}}$, the arrival rate of default under the historical measure, is playing no role in our analysis. We comment briefly on the relation between $\lambda^{\mathbb{P}}$ and $\lambda^{\mathbb{Q}}$ in subsequent sections.

Under the historical measure $\mathbb{P}$, the risk-neutral mean arrival rate of a credit event is assumed to follow the log-normal process:

$$d \ln \lambda_t^{\mathbb{Q}} = \kappa^{\mathbb{P}}\left(\theta^{\mathbb{P}} - \ln \lambda_t^{\mathbb{Q}}\right) dt + \sigma_{\lambda^{\mathbb{Q}}}\, dB_t^{\mathbb{P}}. \tag{4}$$

The market price of risk η_t underlying the change of measure from $\mathbb{P}$ to $\mathbb{Q}$ for $\lambda^{\mathbb{Q}}$ is assumed to be an affine function of $\ln \lambda_t^{\mathbb{Q}}$:

$$\eta_t = \delta_0 + \delta_1 \ln \lambda_t^{\mathbb{Q}}. \tag{5}$$

This market price of risk allows κ and $\kappa\theta$ to differ across $\mathbb{P}$ and $\mathbb{Q}$, while assuring that $\lambda^{\mathbb{Q}}$ follows a lognormal process under both measures. Specifically, under the risk-neutral measure $\mathbb{Q}$, defined by the market price of risk η_t,

$$d \ln \lambda_t^{\mathbb{Q}} = \kappa^{\mathbb{Q}}\left(\theta^{\mathbb{Q}} - \ln \lambda_t^{\mathbb{Q}}\right) dt + \sigma_{\lambda^{\mathbb{Q}}}\, dB_t^{\mathbb{Q}}, \tag{6}$$

where $\kappa^{\mathbb{Q}} = \kappa^{\mathbb{P}} + \delta_1 \sigma_{\lambda^{\mathbb{Q}}}$ and $\kappa^{\mathbb{Q}}\theta^{\mathbb{Q}} = \kappa^{\mathbb{P}}\theta^{\mathbb{P}} - \delta_0 \sigma_{\lambda^{\mathbb{Q}}}$.

Within this setting, closed-form solutions for zero-coupon bond prices and survival probabilities are not known. Accordingly, to price *CDS* contracts we assume that r_t and $\lambda^{\mathbb{Q}}$ are independent, and then construct a discrete approximation to

$$\int_t^{t_M} E_t^{\mathbb{Q}}\left[\lambda_u^{\mathbb{Q}} e^{-\int_t^u (r_s+\lambda_s^{\mathbb{Q}})\,ds}\right] du = \int_t^{t_M} D(t,u) E_t^{\mathbb{Q}}\left[\lambda_u^{\mathbb{Q}} e^{-\int_t^u \lambda_s^{\mathbb{Q}}\,ds}\right] du$$

in terms of the price $D(t, u)$ of a default-free zero-coupon bond (issued at date t and maturing at date u) and the risk-neutral survival probabilities $E_t^{\mathbb{Q}}[e^{-\int_t^u \lambda_s^{\mathbb{Q}}\,ds}]$. The latter are then computed numerically using the Crank–Nicolson implicit finite-difference method to solve the associated Feynman–Kac partial differential equation.

Beyond the specification of the default arrival intensity, a critical input into the pricing of *CDS* contracts is the risk-neutral loss rate due to a credit event, $L^{\mathbb{Q}}$. Convention within both academic analyses and industry practice is to treat this loss rate as a constant parameter of the model. In the context of pricing corporate *CDS* contracts this practice has been questioned in light of the evidence of a pronounced negative correlation between default rates and recovery over the business cycle (see, e.g., Altman et al. (2003) and related publications by the U.S. rating agencies). A business-cycle induced correlation seems less compelling in the case of sovereign risk. Indeed, a theme we consistently heard in conversations with sovereign *CDS* traders is that recovery depends on the size of the country (and the size and distribution of its external debt), but is not obviously cyclical in the same way that corporate recoveries are. In any event, we will follow industry practice and treat $L^{\mathbb{Q}}$ as a constant parameter of our pricing models, appropriately interpreted as the *expected* loss of face value on the underlying reference bond due to a credit event.

Traders are naturally inclined to call upon historical experience in setting loss rates in their pricing models. One source of this information is the agencies that rate sovereign debt issues. For example, Moody's (2003) estimates of the *recoveries* (weighted by issues sizes) on several recent sovereign defaults are: Argentina 28%, Ecuador 45%, Moldova 65%, Pakistan 48%, and Ukraine 69%. As stressed by Moody's, these numbers must be interpreted with some caution, because they are based on the market prices of sovereign bonds shortly after the relevant credit events. Moreover, just as in many discussions of corporate bond and *CDS* pricing, the setting of $L^{\mathbb{Q}}$ based on historical experience requires the assumption that there is no risk premium on recovery, $L^{\mathbb{Q}} = L^{\mathbb{P}}$.

That estimates of recovery may differ, depending on when market prices are sampled and perhaps also across measuring institutions, is confirmed by the recoveries estimated by Credit Suisse First Boston (CSFB), as reported in the *Economist* (2004). The values at default of the bonds involved in Russia's default in May/June 1999 were 23.5% (15.9%) of face value, weighted (unweighted) by issue size. The corresponding numbers for Ecuador's default in October 1999 were 23.4% (30.0%). Interestingly, at the time of restructuring, which in both of these cases was within a year of the default, the restructured values were substantially higher.[8] For Russia they were 36.6% (38%), and for Ecuador they were 36.2% (49.3%). Singh (2003) provides additional examples of the market prices at the time of default being depressed relative to the subsequent amounts actually recovered, and that this phenomenon was more prevalent for sovereign than for corporate credit events. For valuing sovereign *CDS* contracts, it is the loss in value on the underlying bonds around the time of the credit event that matters for determining the payment from the insurer to the insured, regardless of whether or not these values accurately reflect the present values of the subsequently restructured debt.

At a practical level, to match a given day's term structure of new-issue *CDS* spreads, a range of combinations of $L^{\mathbb{Q}}$ and the set of parameters governing the $\mathbb{Q}$-distribution of $\lambda^{\mathbb{Q}}$ will typically give a good fit. Several traders have told us that they set $L^{\mathbb{Q}} = 0.75$ and then either bootstrap $\lambda^{\mathbb{Q}}$ or use a one-factor parametric model for the $\lambda^{\mathbb{Q}}$ process to match a day's cross-section of spreads. This particular standardized choice of $L^{\mathbb{Q}}$ (across maturities and countries) has, as we have just seen, some basis in historical experience. Whether it is in fact consistent with the historical behavior of spreads in the *CDS* contracts for a country is probably not material for the purpose of interpolating new-issue spreads across maturities.

On the other hand, the choice of $L^{\mathbb{Q}}$ is critical for marking to market seasoned *CDS* contracts (e.g., unwinding a seasoned position with a counterparty). In this situation, the price is not given by the market, but rather must be inferred from a model that requires as its inputs $L^{\mathbb{Q}}$ and the parameters of the stochastic $\mathbb{Q}$-process for $\lambda^{\mathbb{Q}}$. Accordingly, one is naturally led to inquire: Can $L^{\mathbb{Q}}$ and the conditional $\mathbb{Q}$ distribution of $\lambda^{\mathbb{Q}}$ be separately identified from a *time-series* of market-provided spreads on newly issued *CDS* contracts?[9] If the answer is yes, then the same pricing model can be used to mark to market the seasoned *CDS* contracts on the same issuer. We turn next to the challenges this separation presents for "reduced-form" *CDS* pricing models.

III. Can We Separately Identify $\lambda^{\mathbb{Q}}$ and $L^{\mathbb{Q}}$?

A common impression among academics and practitioners alike is that fixing $L^{\mathbb{Q}}$ at a specific value is necessary to achieve econometric identification. This is certainly true in an economic environment in which contracts are priced under the *fractional recovery of market* value convention (RMV) introduced by Duffie and Singleton (1999). In such a pricing framework, the product $\lambda^{\mathbb{Q}} \times L^{\mathbb{Q}}$

[8] This is the market value of the new bonds received as a percentage of the original face value of the bonds.

[9] Simply because $L^{\mathbb{Q}} = 0.75$ is market convention is not sufficient, in our minds, for accepting this value as the best description of history. Market makers typically set $L^{\mathbb{Q}}$ in matching the cross-maturity prices of *CDS* contracts on a given day. This does not require (or typically involve) calibrations to history or explicit analyses of the market prices of risk. Therefore, the question of what is the best setting of $L^{\mathbb{Q}}$ for matching the time-series properties of spreads, both in the *CDS* and associated bond markets, is a useful line of inquiry.

determines prices in the sense that the time-t spread on a defaultable bond takes the form

$$CDS_t^{RMV} = g\left(\lambda_t^{\mathbb{Q}} L^{\mathbb{Q}}\right), \tag{7}$$

for some function g. That $\lambda^{\mathbb{Q}}$ and $L^{\mathbb{Q}}$ enter symmetrically implies that they cannot be separately identified using defaultable bond data alone.

In the pricing framework of *fractional recovery of face* value (RFV) (see Duffie (1998) and Duffie and Singleton (1999)), which is the most natural pricing convention for *CDS* contracts, $\lambda^{\mathbb{Q}}$ and $L^{\mathbb{Q}}$ play distinct roles. Specifically, the CDS pricing relation in (1) takes the form

$$CDS_t = L^{\mathbb{Q}} f\left(\lambda_t^{\mathbb{Q}}\right). \tag{8}$$

Comparing equation (7) against (8), we can see that the joint identification problem in the RMV framework is no longer present for *CDS* prices. For example, the explicit linear dependence of CDS_t on $L^{\mathbb{Q}}$ implies that the ratio of two *CDS* spreads on contracts of different maturities does not depend on $L^{\mathbb{Q}}$, but does contain information about $\lambda^{\mathbb{Q}}$.

Now what is conceptually true need not be true in actual implementations of these pricing models, as is illustrated by the very similar prices for par coupon bonds under the pricing conventions RMV and RFV displayed in Duffie and Singleton (1999). To gauge the degree of numerical identification in practice, we perform the following analysis. Suppose that $\lambda^{\mathbb{Q}}$ follows a lognormal process[10], $L^{\mathbb{Q}}$ is constant, and hence $y_t = L^{\mathbb{Q}}\lambda_t^{\mathbb{Q}}$ also follows a lognormal process. More specifically, letting $X_t = \ln(\lambda^{\mathbb{Q}})$ and $Y_t = \ln(y_t)$, we have,

$$\begin{aligned} dX_t &= \kappa_x(\theta_x - X_t)\,dt + \sigma_x dB_t \\ dY_t &= \kappa_y(\theta_y - Y_t)\,dt + \sigma_y dB_t, \end{aligned} \tag{9}$$

where $Y_t = X_t + \ln(L^{\mathbb{Q}}), \kappa_y = \kappa_x, \sigma_y = \sigma_x,$ and $\theta_y = \theta_x + \ln(L^{\mathbb{Q}})$. Using this model we ask what happens to spreads as $L^{\mathbb{Q}}$ is varied holding y fixed. For this exercise, "fixed y" means that the level of $y = L^{\mathbb{Q}}\lambda^{\mathbb{Q}}$ as well as its parameter values θ_y, κ_y, and σ_y are fixed. This, in turn, implies that any variation in $L^{\mathbb{Q}}$ is accompanied by an adjustment of $\lambda^{\mathbb{Q}} = y/L^{\mathbb{Q}}$ and its parameter values.

Figure 4 illustrates the $L^{\mathbb{Q}}$-sensitivity of CDS spreads, under the RFV convention, to variation in $L^{\mathbb{Q}}$ with $y = L^{\mathbb{Q}} \times \lambda^{\mathbb{Q}}$ fixed. The spreads clearly depend on $L^{\mathbb{Q}}$ and their sensitivity to changes in $L^{\mathbb{Q}}$ differs across maturities. This is to be contrasted against the RMV pricing framework in equation (7), under which the sensitivity of a defaultable bond to variation in $L^{\mathbb{Q}}$ is zero with fixed $y = L^{\mathbb{Q}} \times \lambda^{\mathbb{Q}}$. For these calculations we fix the long-run mean of $\ln y$ at $\theta_y = \ln(200\,\text{bps})$ to approximately reproduce the sample average of the 5-year spread for Mexico of around 200 bps;[11] the volatility parameter is set at $\sigma_y = 1$,

[10] The particular dynamics of $\lambda^{\mathbb{Q}}$ are not crucial for the separate identification. For example, the same analysis goes through with the assumption that λ^{Q} follows a square-root process.

[11] To be more precise, the long-run mean of y is $\exp(\theta_y + \sigma_y^2/(\kappa_y \times 4))$.

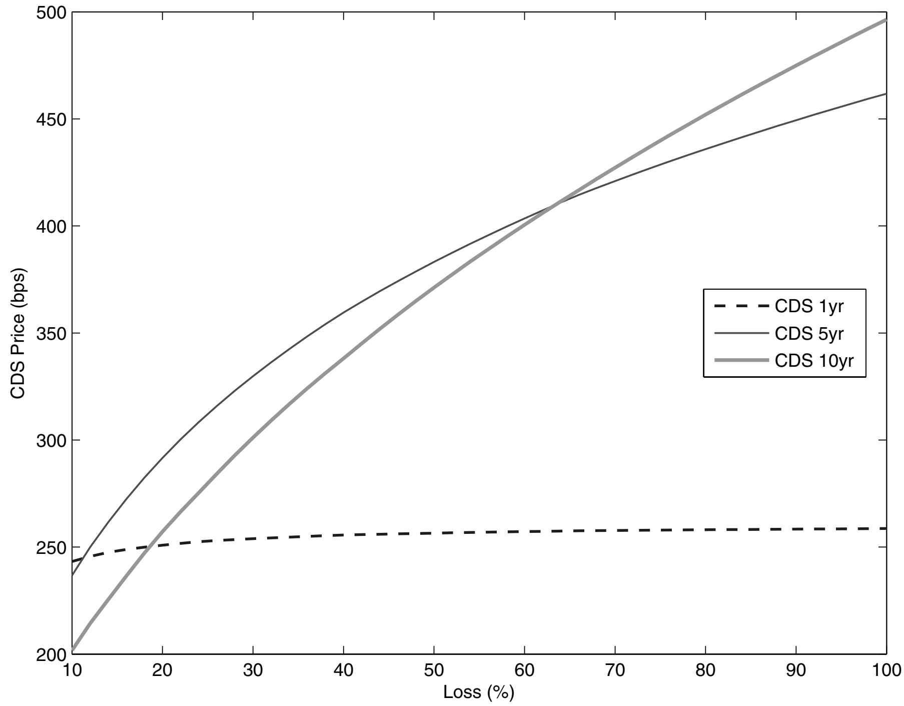

Figure 4. The sensitivity of *CDS* spreads to loss rate $L^{\mathbb{Q}}$ for fixed value of $y = L^{\mathbb{Q}} \times \lambda^{\mathbb{Q}}$. The level of y is fixed at 200 bps and its parameter values are fixed at $\kappa_y = 0.01$, $\sigma_y = 1$ and $\theta_y = \ln(200\,\text{bps})$.

approximately the maximum likelihood estimate for this parameter; and the mean reversion parameter is set at $\kappa_y = 0.01$, between our maximum likelihood estimates for Mexico and Turkey (see Table III).

Of course the degree of econometric identification may be sensitive to the choice of parameter values within the admissible regions of the parameter and state spaces. This is illustrated in Figure 5 by direct calculations of the partial derivatives $\partial CDS/\partial L^{\mathbb{Q}}|_y$. Fixing $L^{\mathbb{Q}} = 75\%$, the top two panels of Figure 5 show that the $\partial CDS/\partial L^{\mathbb{Q}}|_y$ are quite sensitive to changes in volatility (σ_y) and mean-reversion (κ_y). In particular, identification is strong when either volatility is relatively high or when the mean-reversion rate is low. Similarly, the bottom two panels of Figure 5 demonstrate that numerical identification is likely to be achieved over a wide range of values of $y = L^{\mathbb{Q}} \times \lambda_t^{\mathbb{Q}}$ and the loss rate $L^{\mathbb{Q}}$. Moreover, the partial derivatives of the spreads are most sensitive to changes in the parameters for the longer maturity contracts. This is consistent with our prior that access to the term structure of *CDS* spreads enhances the numerical identification of $L^{\mathbb{Q}}$ separately from the parameters governing $\lambda^{\mathbb{Q}}$.

A natural question at this juncture is whether, with sample sizes that are available in the *CDS* markets, one can in fact reliably estimate $L^{\mathbb{Q}}$ in practice. To address this question we conduct a small-scale Monte-Carlo exercise.

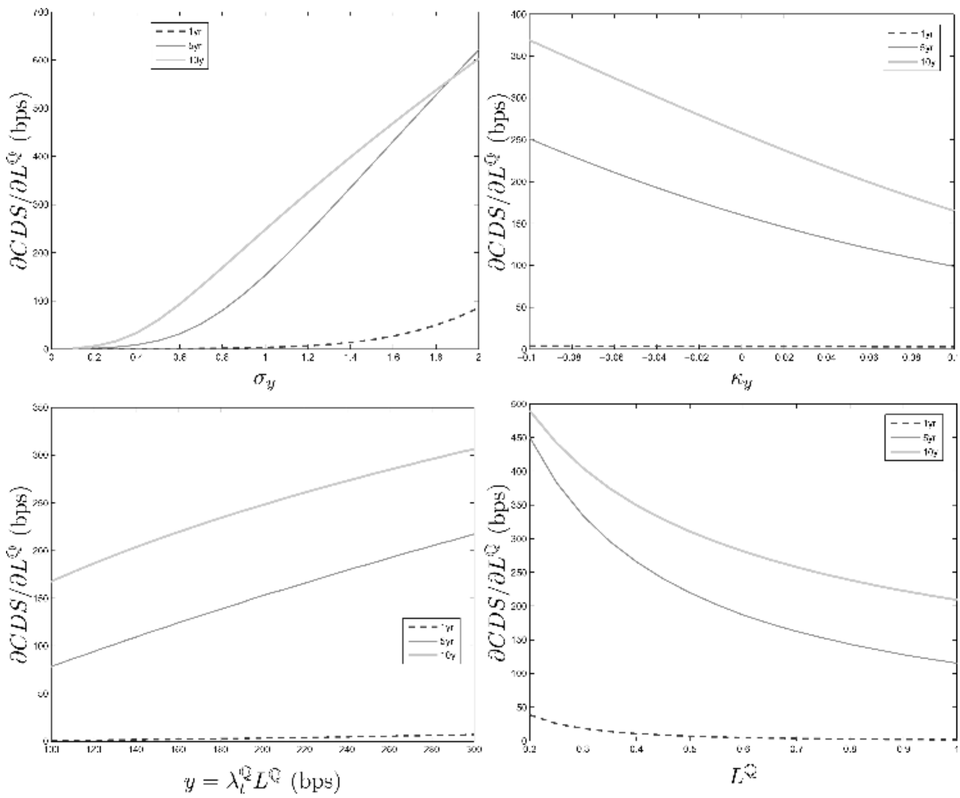

Figure 5. The partial derivative of CDS spread with respect to loss rate $L^{\mathbb{Q}}$ with fixed *y*. The level and parameter values of $\lambda^{\mathbb{Q}}$ are adjusted so that the process $y = L^{\mathbb{Q}} \times \lambda^{\mathbb{Q}}$ is kept fixed (both level and parameter values). In all figures, the base case parameters are: $\theta_y = \ln(200\,\text{bps})$, $\kappa_y = 0.01$, $\sigma_y = 1$, and $L^{\mathbb{Q}} = 0.75$.

Specifically, we simulate affine model-implied 1-, 3-, 5-, and 10-year *CDS* spreads, and add normally distributed pricing errors to the 1-, 3- and 10-year *CDS* spreads.[12] The resulting (noisy) simulated *CDS* data is then used to construct *ML* estimates of the underlying parameters. We repeat this 100 times, and the means and standard deviations of the *ML* estimates are displayed in Table II. To gauge the effect of $\kappa^{\mathbb{Q}} < 0$, we consider two cases: one with explosive $\mathbb{Q}$-intensity ($\kappa^{\mathbb{Q}} < 0$), and the other with stationary $\mathbb{Q}$-intensity

[12] For reasons of tractability, we turn to an affine specification of $\lambda^{\mathbb{Q}}$. The components of the *CDS* prices can be computed analytically in this model and this substantially reduces the computational burden of our Monte Carlo analysis. To incorporate the variation in bid-ask spreads into the conditional volatilities of the pricing errors we start with the sample averages of $(Ask_t - Bid_t)/CDS_t$, say *PBA*, for the 1-, 3- and 10-year contracts. The pricing errors are then assumed to be normally distributed with zero mean and standard deviation $PBA * CDS(t) * \sigma_\epsilon$, where $\sigma_\epsilon = 0.5$ for all three maturities. So, under this scheme, there is no time-series variation in percentage bid-ask spreads, but there is time-series variation in bid-ask spreads driven by the variation in *CDS* prices.

Table II

Simulation Results for the Affine Model

Simulations are performed under the "true" parameter values with the same sample size as that of our *CDS* data. The mean and standard deviation of the estimates are calculated with 100 simulation runs.

	$\theta^{\mathbb{P}}$	$\kappa^{\mathbb{P}}$	$\sigma_{\lambda^{\mathbb{Q}}}$	$\kappa^{\mathbb{Q}}$	σ_{ϵ}	$L^{\mathbb{Q}}$	$\theta^{\mathbb{Q}}\kappa^{\mathbb{Q}}$
Explosive case							
True param	219 bp	2.7880	0.1691	−0.3361	0.5000	0.7500	12 bp
Mean(estm)	224 bp	3.1417	0.1704	−0.3458	0.5043	0.7265	12 bp
SD(estm)	41 bp	0.8002	0.0007	0.0017	0.0069	0.0278	1bp
Stationary case							
True param	219 bp	2.7880	0.1691	0.1000	0.5000	0.7500	611 bp
Mean(estm)	232 bp	3.2271	0.1711	0.0848	0.5046	0.7148	633 bp
SD(estm)	55 bp	0.9935	0.0044	0.0073	0.0074	0.0135	7 bp

($\kappa^{\mathbb{Q}} > 0$). To reduce the computational burden of estimation, we use a common coefficient $\sigma_{\epsilon}(M)$ for the volatilities of the 1-, 3- and 10-year *CDS* pricing errors.

The standard deviations of the simulated estimates are of the same orders of magnitude as the standard errors reported from the *ML* results for the affine model using the actual data, and the means of the simulated estimates are close in magnitude to the true parameter values. Moreover, for econometric identification, whether or not the default intensity is $\mathbb{Q}$-explosive appears to be inconsequential. The degree of persistence in $\kappa^{\mathbb{Q}}$ matters, of course, as was documented in Figure 5, but so long as $\lambda^{\mathbb{Q}}$ is reasonably persistent the likelihood function appears to exhibit sufficient curvature for reliable estimation of $L^{\mathbb{Q}}$.

IV. Maximum Likelihood Estimates

The parameters are estimated by the method of maximum likelihood, with the conditional distribution of the spreads derived from the known conditional distribution of the state, which is lognormal.[13] The 5-year *CDS* contract was assumed to be priced perfectly, so that the pricing function could

[13] More formally, within the framework outlined in the remainder of this paragraph, we make the following auxiliary assumptions in deriving our likelihood function. Letting BA_t denote the four-vector of bid-ask spreads at date t for maturities $M = 1, 3, 7$, and 10, we assume that $BA_t = g(\lambda_t^{\mathbb{Q}}) + \nu_t$, with ν_t statistically independent of the process $\{\lambda_t^{\mathbb{Q}}\}$. This allows for the joint determination of $\lambda^{\mathbb{Q}}$ and BA_t, possibly through a nonlinear mechanism. Further, letting ϵ_t denote the four-vector of pricing errors for the contracts priced with error and $\mathcal{I}_t$ denote the econometrician's information at date t, we assume that

$$f^{\mathbb{P}}(\lambda^{\mathbb{Q}}, \nu_t, \epsilon_t \mid \mathcal{I}_{t-1}) = f^{\mathbb{P}}(\lambda_t^{\mathbb{Q}} \mid \mathcal{I}_{t-1}) \times f^{\mathbb{P}}(\epsilon_t \mid \lambda_t^{\mathbb{Q}}, \nu_t, \mathcal{I}_{t-1}) \times f^{\mathbb{P}}(\nu_t \mid \lambda_t^{\mathbb{Q}}, \mathcal{I}_{t-1})$$
$$= f^{\mathbb{P}}(\lambda_t^{\mathbb{Q}} \mid \lambda_{t-1}^{\mathbb{Q}}) \times f^{\mathbb{P}}(\epsilon_t \mid BA_t, \mathcal{I}_{t-1}) \times f^{\mathbb{P}}(\nu_t \mid \mathcal{I}_{t-1}).$$

The form of the first component of $f^{\mathbb{P}}(\lambda^{\mathbb{Q}}, \nu_t, \epsilon_t \mid \mathcal{I}_{t-1})$ follows from the Markov assumption on $\lambda^{\mathbb{Q}}$; the second amounts to assuming that the dependence of the conditional distribution of be inverted for $\lambda^{\mathbb{Q}}$.[14] The 1-, 2-, 3-, and 10-year contracts are assumed to be priced with normally distributed errors with mean zero and standard deviations $\sigma_\epsilon(M)|Bid_t(M) - Ask_t(M)|$, where the $\sigma_\epsilon(M)$ are constants depending on the maturity of the contract, M. Time-varying variances that depend on the bid-ask spread allow for the possibility that the fits of our one-factor models deteriorate during periods of market turmoil when bid-ask spreads widen substantially. Conveniently, $\sigma_\epsilon(M)$ measures the degree of mispricing by the model relative to bid-ask spreads.

The risk-free interest rate (term structure) is assumed to be constant. We experiment with using a two-factor affine model (an $A_1(2)$ model in the nomenclature of Dai and Singleton (2000)) for r_t, but we obtain virtually identical results to those with a constant riskfree rate.[15] A simple arbitrage argument (see, e.g., Duffie and Singleton (2003)) shows that *CDS* spreads are approximately equal to the spreads on comparable maturity, par floating rate bonds from the same issuer as the reference bonds underlying the *CDS* contract. The prices of these bonds are not highly sensitive to the level of interest rates and this underlies the insensitivity of our findings to the introduction of a stochastic riskfree rate.

A. ML Estimates of One-Factor Models

The *ML* estimates of the parameters (expressed on an annual time scale) and their associated standard errors are presented in Table III. Across all three countries, and regardless of whether $L^{\mathbb{Q}}$ is a fixed or free parameter, there is a striking contrast between the parameters governing the $\mathbb{Q}$- and $\mathbb{P}$-dynamics of $\lambda^{\mathbb{Q}}$. Indeed, in the cases of Mexico (constrained or unconstrained) and Turkey (unconstrained), the point estimates for $\kappa^{\mathbb{Q}}$ are negative, implying that the default intensity $\lambda^{\mathbb{Q}}$ is explosive under $\mathbb{Q}$; whereas $\kappa^{\mathbb{P}} > 0$ so $\lambda^{\mathbb{Q}}$ is $\mathbb{P}$-stationary for all three countries. These large differences between the $\mathbb{Q}$ and $\mathbb{P}$ distributions are indicative of substantial market risk premiums related to uncertainty about future arrival rates of credit events.

From these parameter estimates, we can back out the coefficients for the market prices of risk, δ_0 and δ_1, as defined in equation (5). The values for (Mexico, Turkey, Korea) are $\delta_0 = (-7.36, -2.29, -6.16)$ and $\delta_1 = (-1.35, -0.48, -0.98)$

ϵ_t on $\lambda_t^{\mathbb{Q}}$ and ν_t can be summarized by BA_t, which itself is fully determined by $\lambda_t^{\mathbb{Q}}$ and ν_t; and the third follows from the independence assumption underlying our assumed decomposition of BA_t. Finally, the assumptions that $f^{\mathbb{P}}(\nu_t \mid \mathcal{I}_{t-1})$ does not depend on the parameters governing $f^{\mathbb{P}}(\lambda_t^{\mathbb{Q}} \mid \lambda_{t-1}^{\mathbb{Q}})$ and $f^{\mathbb{P}}(\epsilon_t \mid BA_t, \mathcal{I}_{t-1})$, and that the M^{th} element of $f^{\mathbb{P}}(\epsilon_t \mid BA_t, \mathcal{I}_{t-1})$ is the density of a $N(0, \sigma_\epsilon^2(M)(Bid_t(M) - Ask_t(M))^2)$ imply our likelihood function.

[14] The 5-year contract was chosen because of its relative liquidity. The liquidities of the 5-year contracts are enhanced, for all three countries examined, by their inclusion in the Dow Jones CDX.EM traded index of emerging market *CDS* spreads.

[15] For checking the sensitivity of our results to the presence of stochastic interest rates we once again shifted to an affine model for reasons of computational tractability. Within the affine setting we can allow for stochastic interest rates that are correlated with $\lambda^{\mathbb{Q}}$ and still obtain closed-form solutions for survival probabilities and zero-coupon bond prices.

Table III

Maximum Likelihood Estimates

Daily data from March 19, 2001 through August 8, 2006. The sample size is 1,357 for Mexico, 1,377 for Turkey, and 1,308 for Korea. llk is the sample average of log-likelihood.

	$L^{\mathbb{Q}}$ Fixed at 0.75			$L^{\mathbb{Q}}$ Unconstrained		
	Mexico	Turkey	Korea	Mexico	Turkey	Korea
$\kappa^{\mathbb{Q}}$	−0.0638	0.0239	0.0651	−0.119	−0.0351	0.0673
	(0.0015)	(0.0013)	(0.0015)	(0.003)	(0.0012)	(0.0039)
$\theta^{\mathbb{Q}}\kappa^{\mathbb{Q}}$	0.268	−0.015	−0.384	0.661	0.480	−0.414
	(0.007)	(0.004)	(0.007)	(0.014)	(0.006)	(0.043)
$\sigma_{\lambda^{\mathbb{Q}}}$	1.086	1.144	0.921	0.773	0.811	0.934
	(0.004)	(0.004)	(0.007)	(0.015)	(0.006)	(0.018)
$\kappa^{\mathbb{P}}$	1.40	0.57	0.97	0.78	0.28	0.99
	(1.15)	(0.56)	(0.66)	(0.67)	(0.31)	(0.68)
$\theta^{\mathbb{P}}$	−5.51	−4.61	−6.25	−4.45	−4.23	−6.35
	(0.59)	(1.54)	(0.69)	(0.69)	(2.44)	(0.71)
$\sigma_\epsilon(1)$	1.436	1.056	0.619	1.472	1.069	0.618
	(0.032)	(0.021)	(0.028)	(0.035)	(0.021)	(0.028)
$\sigma_\epsilon(2)$	1.084	0.858	0.442	1.057	0.839	0.442
	(0.018)	(0.026)	(0.026)	(0.018)	(0.026)	(0.026)
$\sigma_\epsilon(3)$	0.933	0.595	0.296	0.935	0.586	0.296
	(0.031)	(0.018)	(0.009)	(0.032)	(0.017)	(0.009)
$\sigma_\epsilon(10)$	0.838	1.350	0.869	0.855	0.885	0.867
	(0.022)	(0.040)	(0.028)	(0.023)	(0.018)	(0.029)
$L^{\mathbb{Q}}$	= 0.75	= 0.75	= 0.75	0.231	0.236	0.833
	N/A	N/A	N/A	(0.010)	(0.004)	(0.129)
Mean llk	32.030	27.213	36.626	32.126	27.700	36.626

in the constrained models with $L^{\mathbb{Q}} = 0.75$, and $\delta_0 = (-5.35, -2.03, -6.27)$ and $\delta_1 = (-1.16, -0.38, -0.98)$ in the unconstrained models. Recalling that $\kappa^{\mathbb{Q}} = \kappa^{\mathbb{P}} + \delta_1\sigma_{\lambda^{\mathbb{Q}}}$ and $\kappa^{\mathbb{Q}}\theta^{\mathbb{Q}} = \kappa^{\mathbb{P}}\theta^{\mathbb{P}} - \delta_0\sigma_{\lambda^{\mathbb{Q}}}$, the negative signs of δ_0 and δ_1 imply that the credit environment is much worse under $\mathbb{Q}$ than under $\mathbb{P}$. More precisely, $\kappa^{\mathbb{Q}}\theta^{\mathbb{Q}} > \kappa^{\mathbb{P}}\theta^{\mathbb{P}}$ so, even at low arrival rates of credit events, $\lambda^{\mathbb{Q}}$ will tend to be larger under $\mathbb{Q}$ than under $\mathbb{P}$. Moreover, for a given level of $\lambda^{\mathbb{Q}}$, there is more persistence under $\mathbb{Q}$ than under $\mathbb{P}$ (bad times last longer under $\mathbb{Q}$). It is this pessimism about the credit environment that allows risk-neutral pricing to recover market prices in the presence of investors who are averse to default risk.

Turning to the magnitudes of the pricing errors for the *CDS* contracts with maturities of 1, 2, 3, and 10 years, the estimates of $\sigma_\epsilon(M)$ in Table III measure the standard deviations of the pricing errors in units of the bid-ask spreads. Typically, $\sigma_\epsilon(M)$ is less than about one, the most notable exceptions being $\sigma_\epsilon(1)$ for Mexico (with or without $L^{\mathbb{Q}}$ constrained) and $\sigma_\epsilon(10)$ for Turkey with $L^{\mathbb{Q}} = 0.75$. Korea shows the best fit in that the $\sigma_\epsilon(M)$ are relatively small, as are the bid-ask spreads on these contracts (see Figure 3). For a given country, the $\sigma_\epsilon(M)$ tend to be smaller for the intermediate maturities, and the bid-ask spreads fall (on average, as seen from Table I) with increasing maturity, so our models tend to fit somewhat better for $M = 2$ and 3 than for $M = 1$ or 10.

The time-series of *CDS* pricing errors, measured by the market minus the model-implied spreads and evaluated at the parameters with $L^{\mathbb{Q}} = 0.75$, are plotted in Figure 6. The high degree of co-movement in the *CDS* spreads across maturities and countries is much less evident in the corresponding pricing errors. In the cases of Mexico and Turkey, the pricing errors on the 1- and 10-year contracts are negatively correlated, suggesting that there is some tension in fitting both of these spreads simultaneously. For Korea, on the other hand, our one-factor model appears to price the short-dated contracts equally well in that $\text{Corr}(\epsilon(1), \epsilon(3)) = 0.89$. The pricing errors on long-dated Korean contracts move in a largely uncorrelated way with those at the short end. A more indepth analysis of these pricing errors and the potential role of a second factor is explored in Section IV.B. At this juncture we simply highlight the small magnitudes of the standard deviations of these errors, typically less than one bid-ask spread.

There are several notable differences between the maximum likelihood estimates of the models with and without $L^{\mathbb{Q}}$ fixed. Perhaps most striking is the fact that the unconstrained estimates of $L^{\mathbb{Q}}$ for Mexico and Turkey are approximately 0.23, much smaller than the market convention of 0.75. Standard likelihood ratio statistics reject the constraint $L^{\mathbb{Q}} = 0.75$ at conventional significance levels. On the other hand, for Korea the estimate is quite close to the market convention. Accompanying the relatively small values of $L^{\mathbb{Q}}$ for Mexico and Turkey are relatively larger values of $\kappa^{\mathbb{Q}}\theta^{\mathbb{Q}}$ and smaller values of both $\kappa^{\mathbb{Q}}$ and $\kappa^{\mathbb{P}}$ (compared to their counterparts in the models with $L^{\mathbb{Q}} = 0.75$). The larger values of $\kappa^{\mathbb{Q}}\theta^{\mathbb{Q}}$ are intuitive: To match spreads with a lower loss rate, the "intercept" of the $\lambda^{\mathbb{Q}}$ process under the $\mathbb{Q}$ distribution must be larger.[16]

The relatively larger value of the log-likelihood function in the unconstrained model is attributable to the component associated with the dynamic properties of $\lambda^{\mathbb{Q}}$ under $\mathbb{P}$, and not to the component associated with the pricing errors. Accordingly, to gain further insight into the relative goodness-of-fits of the constrained and unconstrained models, we examine the model-implied small-sample distributions of various moments of the *CDS* spreads and their first differences (time changes). Ten-thousand time series, each of length 1,500 (the approximate length of our samples), are simulated and the means and standard deviations of the small-sample distributions of various moments are computed. Among the moments examined are the mean, standard deviation, skewness, and kurtosis, and the autocorrelations of the levels of *CDS* spreads and the slope of the *CDS* curve.

Table IV displays the means and standard deviations of the small-sample distributions of mean, skewness, and kurtosis for Mexico and Turkey, along with their sample counterparts. For the first through fourth central moments, the differences between the means of the small-sample distributions across the corresponding models with and without $L^{\mathbb{Q}}$ constrained are small, certainly

[16] Conditional on $\lambda_t^{\mathbb{Q}}$, $\lambda_{t+1}^{\mathbb{Q}}$ will tend to be larger in the model with the lower estimate of $L^{\mathbb{Q}}$. Since $\kappa^{\mathbb{Q}} < 0$ in the unconstrained models for Mexico and Turkey, $\lambda^{\mathbb{Q}}$ does not have a finite $\mathbb{Q}$-mean.

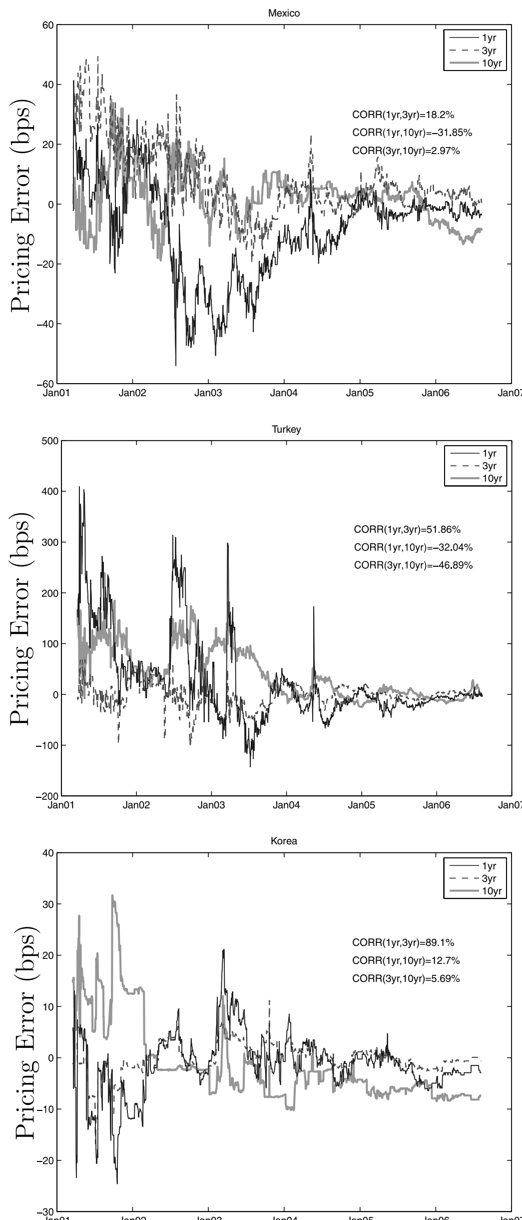

Figure 6. The *CDS* pricing errors, market *CDS* price minus the model implied, for maturities of 1, 3, and 5 years. These errors are evaluated using the constrained maximum likelihood estimates with $L^{\mathbb{Q}} = 0.75$.

relative to the standard deviations of these distributions. Moreover, the means of the small sample distributions of the first, second (not shown), and third moments are quite close to their historical counterparts, particularly in the case of Mexico. There is a tendency for the sample kurtoses to be below their

Table IV

The Small-Sample Moments of *CDS* Spreads

The small-sample means and standard deviations (in brackets) of the 1-, 5-, and 10-year *CDS* spreads, along with their sample counterparts, are reported in basis points. MC^C refers to Monte Carlo results for the model with $L^{\mathbb{Q}} = 0.75$, and MC^U refers to the Monte Carlo results for the models with unconstrained $L^{\mathbb{Q}}$. *ACF*1 and *ACF*2 refer to the first- and second-order autocorrelations, respectively, and *slope* is the 10-minus 1-year spread.

	Mexico			Turkey		
Moment	Sample	MC^C	MC^U	Sample	MC^C	MC^U
E[1 yr]	55	59 [18]	57 [21]	355	306 [183]	294 [168]
E[5 yr]	166	155 [40]	151 [47]	563	504 [191]	495 [193]
E[10 yr]	213	200 [39]	195 [47]	607	520 [152]	531 [175]
Skew[1 yr]	0.95	1.28 [.56]	1.16 [.60]	1.09	1.50 [.69]	1.31 [.73]
Skew[5 yr]	0.74	0.94 [.49]	0.84 [.54]	0.51	0.97 [.57]	0.88 [.61]
Skew[10 yr]	0.62	0.71 [.45]	0.67 [.50]	0.48	0.89 [.54]	0.92 [.60]
Kurt[1 yr]	2.64	4.86 [2.3]	4.34 [2.2]	3.24	5.53 [3.3]	4.75 [3.0]
Kurt[5 yr]	2.65	3.75 [1.6]	3.44 [1.6]	2.10	3.75 [1.8]	3.49 [1.7]
Kurt[10 yr]	2.56	3.26 [1.2]	3.11 [1.2]	2.02	3.58 [1.6]	3.60 [1.8]
*ACF*1(5 yr)	0.996	0.989 [.005]	0.992 [.004]	0.995	0.992 [.004]	0.994 [.003]
*ACF*2(5 yr)	0.991	0.978 [.009]	0.984 [.007]	0.991	0.985 [.007]	0.988 [.006]
*ACF*1(*slope*)	0.993	0.990 [.004]	0.993 [.003]	0.963	0.985 [.008]	0.991 [.006]
*ACF*2(*slope*)	0.988	0.981 [.008]	0.986 [.007]	0.940	0.970 [.016]	0.983 [.012]

model-implied small-sample counterparts, but the former are within one standard deviation of the latter.

At first glance, we expected larger differences in the implied autocorrelations of *CDS* spreads across the constrained (C) and unconstrained (U) models, because $\kappa^{\mathbb{P}C} > \kappa^{\mathbb{P}U}$ (see Table III). However our models are parameterized on an annual time scale so, over moderate horizons, the differences in model-implied (first- and second-order) autocorrelations of *CDS* spreads are small. The model-implied autocorrelations for the slope for Turkey are a bit larger than their sample counterparts, but otherwise the model and sample autocorrelations are very similar (Table IV). Of course the higher degree of $\mathbb{P}$ persistence with $L^{\mathbb{Q}}$ treated as a free parameter will manifest itself over sufficiently long horizons. However, the effects of $\kappa^{\mathbb{P}C} > \kappa^{\mathbb{P}U}$ on our analysis of risk premiums in Section V are negligible at the 1-year horizon. At the 5-year horizon, the differences are again negligible for Mexico, though they are material for Turkey.

In the light of these findings, how should we set $L^{\mathbb{Q}}$? Consistent with our theoretical and small-sample analyses in Section III, the choice of $L^{\mathbb{Q}}$ does matter. Yet the primary differences across values of $L^{\mathbb{Q}}$ as dispersed as 0.23 and 0.75 (at least as revealed by the moments we examine) are in the $\mathbb{P}$-persistence properties of $\lambda^{\mathbb{Q}}$, and these differences revealed themselves only over quite long horizons. Additionally, there is the possibility that specification error is compromising our models' abilities to fit the highly persistent and volatile nature of spreads for Mexico and Turkey. Korean spreads are equally persistent, but they are smaller and less volatile, and it seems plausible that our lognormal model

Table V

OLS Regressions of *CDS* Spreads on Principal Components

For each country, *PC1* and *PC2* are the first and second principal components of the respective country's term structure of *CDS* spreads. $\hat{\beta}$ is the estimated loading and R^2 is the coefficient of determination for the regression.

	Mexico				Turkey				Korea			
	PC1		*PC2*		*PC1*		*PC2*		*PC1*		*PC2*	
Mat.	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
1 yr	0.22	89.8%	0.59	8.4%	0.46	97.2%	0.78	2.8%	0.35	95.4%	0.56	4.3%
2 yr	0.38	97.5%	0.49	2.1%	0.47	99.8%	0.13	0.1%	0.40	98.2%	0.39	1.7%
3 yr	0.47	99.3%	0.25	0.4%	0.46	99.8%	−0.16	0.1%	0.43	99.5%	0.19	0.4%
5 yr	0.54	99.4%	−0.31	0.4%	0.43	99.1%	−0.40	0.8%	0.48	99.8%	−0.10	0.1%
10 yr	0.54	98.6%	−0.50	1.1%	0.40	98.6%	−0.44	1.2%	0.55	97.1%	−0.70	2.8%

is a somewhat better approximation for these spreads. Given that our results for Korea are supportive of market convention and that most of our subsequent analysis is (qualitatively) robust to the choice of $L^{\mathbb{Q}}$, we henceforth focus on the case of $L^{\mathbb{Q}} = 0.75$.

B. Is One Factor Enough?

Up to this point we have chosen to focus on a single-factor model for $\lambda^{\mathbb{Q}}$, largely because, for a given sovereign, the first *PC* of the *CDS* spreads explains a very large percentage of the variation for all maturities. However, the preceding discussion of pricing errors in one-factor models leads us naturally to inquire about the dimensions along which an additional factor might improve the fit of our model, if at all.

Table V displays the factor loadings and the percentage variation explained from projections of the *CDS* spreads onto the first two *PC*s of the data.[17] As noted at the outset of our analysis, *PC1* explains a large percentage of the variation in spreads for all countries and all maturities. Indeed, for maturities of 3 years and longer, *PC1* accounts for at least 97% of the variation in all of the spreads. Moreover, parallel to the findings for the term-structures of the U.S. Treasury or swap markets (Litterman and Scheinkman (1991)), the first *PC* emerges as a "level" factor, as reflected in the roughly constant factor loadings across maturities (for a given sovereign). As expected, our one-factor model with default intensity $\lambda^{\mathbb{Q}}$ picks up this level factor: Regressing the time series of model-implied $\lambda^{\mathbb{Q}}$ onto *PC1* yields an R^2 of 99.0% for Mexico, 98.6% for Turkey, and 98.7% for Korea.

As an additional, more demanding check on the fit of our models, we display in Table VI the correlations between the *CDS* spreads and the slopes of the

[17] This *PC* analysis was conducted using the covariance matrix of the levels of spreads.

Table VI

The Small-Sample Moments of the *CDS* Slope

The *CDS* slope measures the difference between the 10-year and the 1-year *CDS* spreads, in basis points. Both the sample moments and the small-sample moments (MC) are reported. For the latter case, 10,000 time series, each of length 1,500, were simulated and the sample moments for each series were computed. The top panel reports moments relating to the level of the slope, and the bottom panel reports moments relating to the change in the slope. Standard deviations of the small-sample distributions are given in brackets.

	Korea		Mexico		Turkey	
$S = 10$ yr $-$ 1 yr	Sample	MC	Sample	MC	Sample	MC
$E[S]$	34	46 [5]	158	141 [21]	229	214 [45]
Corr(S, 1 yr)	0.60	0.87 [.12]	0.77	0.96 [.02]	−0.60	−0.33 [.57]
Corr(S, 2 yr)	0.67	0.88 [.11]	0.87	0.96 [.02]	−0.48	−0.30 [.57]
Corr(S, 3 yr)	0.72	0.88 [.11]	0.90	0.97 [.02]	−0.43	−0.27 [.58]
Corr(S, 5 yr)	0.77	0.90 [.11]	0.95	0.98 [.01]	−0.37	−0.23 [.60]
Corr(S, 10 yr)	0.85	0.93 [.10]	0.96	0.99 [.01]	−0.35	−0.21 [.61]
	Korea		**Mexico**		**Turkey**	
	Sample	MC	Sample	MC	Sample	MC
Corr(ΔS,Δ 1 yr)	−0.36	0.58 [.26]	−0.04	0.88 [.08]	−0.77	−0.63 [.35]
Corr(ΔS,Δ 2 yr)	−0.09	0.60 [.26]	0.40	0.89 [.07]	−0.58	−0.58 [.36]
Corr(ΔS,Δ 3 yr)	−0.005	0.62 [.25]	0.52	0.90 [.06]	−0.50	−0.53 [.38]
Corr(ΔS,Δ 5 yr)	0.11	0.67 [.24]	0.67	0.94 [.05]	−0.39	−0.47 [.41]
Corr(ΔS,Δ 10 yr)	0.33	0.74 [.22]	0.80	0.97 [.04]	−0.16	−0.44 [.43]

CDS curves, using levels and first differences, for the historical sample and as implied by our models.[18] Though the patterns in these correlations are quite different across countries (most notably the different signs for Turkey versus Korea and Mexico), our one-factor models match the correlations of levels of *CDS* spreads and slopes quite closely. The models do less well at matching the correlations among the first differences of these variables, though this is to be expected as first differences are essentially daily innovations in these variables. Even for changes, the match is quite good for Turkey at all maturities and for Mexico and Korea at the longer maturities.

Among the various maturities, the one-factor model misprices the 1-year contract most severely. As we have just seen, our models are also challenged by the low degree of correlation between innovations in the 1-year *CDS* spreads and the slopes of the *CDS* curves. Taken together, these observations suggest that there are components of the short ends of the *CDS* curves that are not well captured by our one-factor models. Further support for this assessment comes from regressing, for each country, the 1-year pricing error on the second *PC* of the *CDS* spreads, which gives R^2s of 67.6% for Mexico, 45.9% for Turkey, and 65.1% for Korea. The corresponding R^2s for the pricing errors on longer maturity contracts decline substantially with maturity in the cases of Mexico and

[18] The first row of Table VI confirms that our models do a reasonable job of matching the average slopes of the *CDS* curves for our sample period.

Table VII
OLS Regressions of *CDS* Bid/Ask Spreads on the First Two Principal Components of Bid/Ask Spreads

	Mexico				Turkey			
	PC1		*PC2*		*PC1*		*PC2*	
Mat.	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
1 yr	0.44	89.7%	−0.79	8.2%	0.57	93.7%	0.65	4.9%
2 yr	0.47	93.2%	−0.18	0.4%	0.48	95.2%	0.17	0.5%
3 yr	0.44	93.8%	0.18	0.4%	0.45	94.6%	−0.25	1.1%
5 yr	0.44	95.6%	0.37	1.9%	0.37	92.8%	−0.38	4.0%
10 yr	0.45	93.9%	0.42	2.3%	0.33	81.6%	−0.59	10.4%

Turkey, suggesting that what *PC2* is picking up is primarily a short-maturity phenomenon in these markets.

Based on conversations with traders, it seems that the most likely explanation for this "anomalous" behavior of the 1-year contract is due to a liquidity or supply/demand premium. We are told that large institutional money management firms often use the short-dated *CDS* contract as a primary trading vehicle for expressing views on sovereign bonds. The sizable trades involved in these transactions introduce an idiosyncratic "liquidity" factor into the behavior of the 1-year contract. Consistent with this view, the bid-ask spreads as a percentage of the underlying *CDS* spreads are notably larger for the 1-year contract.

Of interest then is whether or not there is a component of the bid-ask spreads that is orthogonal to the first *PC* of spreads, that is, whether there are large idiosyncratic components of the bid-ask spreads for specific maturities.[19] This question is answered in Table VII where we report the results from regressing the bid-ask spreads of the individual *CDS* contracts onto the first two principal components of the bid-ask spreads for Mexico and Turkey. There is a small role for a second factor in the bid-ask spreads, concentrated almost entirely at the 1- and 10-year maturity points. These patterns suggest that there might indeed be something special about the 1- and possibly 10-year contracts from a liquidity perspective. The roles of such illiquidity or trading pressures on *CDS* spreads are issues that we hope to explore in future research.

V. On Priced Risks in Sovereign *CDS* Markets

The large differences between the parameters governing $\lambda^{\mathbb{Q}}$ under the risk-neutral and the actual measures suggest that there is a systematic risk related to changes in future arrival rates of sovereign credit events that is priced in

[19] The bid-ask spreads are highly correlated with the corresponding levels of spreads. In particular, the correlations between *PC1* of the *CDS* spreads (contract prices) and *PC1* of the bid-ask spreads are 80.7% for Mexico and 86.3% for Turkey.

the *CDS* market. To examine the economic underpinnings of the priced risks in the sovereign *CDS* markets, we take the *ML* estimates obtained in Section IV and construct two measures of fitted *CDS* spreads. The first is the actual fitted spread $CDS_t(M)$ from (1). The second is

$$CDS_t^{\mathbb{P}}(M) = \frac{2(1 - R^{\mathbb{Q}}) \int_t^{t+M} E_t^{\mathbb{P}} \left[\lambda_u^{\mathbb{Q}} e^{-\int_t^u (r_s + \lambda_s^{\mathbb{Q}})\, ds} \right] du}{\sum_{j=1}^{2M} E_t^{\mathbb{P}} \left[e^{-\int_t^{t+.5j} (r_s + \lambda_s^{\mathbb{Q}})\, ds} \right]}, \tag{10}$$

obtained from (1) by replacing all of the expectations $E^{\mathbb{Q}}$ with expectations under the physical measure $\mathbb{P}$, $E^{\mathbb{P}}$. If market participants are neutral towards the risk of variation over time in $\lambda^{\mathbb{Q}}$, then $CDS_t^{\mathbb{P}}(M)$ should replicate the corresponding market price $CDS_t(M)$. Put differently, a mark-up in the *CDS* spread relative to the pseudo-spread implies that the buyer of the *CDS* contract is willing to pay a premium for holding the *CDS* contract, while the seller demands a premium. This is similar to what is found in equity options markets where the time-variation of volatility is a priced risk. To quantify the role of risk premiums regarding variation in $\lambda^{\mathbb{Q}}$, in percentage terms, we report[20]

$$CRP_t(M) \equiv \left(CDS_t(M) - CDS_t^{\mathbb{P}}(M)\right) / CDS_t^{\mathbb{P}}(M). \tag{11}$$

The percentage contribution of the risk premiums to spreads at the 1-year maturity ($CRP_t(1)$) are displayed in Figure 7. The correlations between the *CRP*s are 93.6% for (Mexico, Turkey), 89.6% for (Mexico, Korea), and 88.0% for (Turkey, Korea). This high degree of co-movement in the *CRP*s is striking given the very different credit qualities and geo-political features of the three countries examined. Risk premiums induced more volatility in the spreads during the early part of our sample, with the gap between CDS_t and $CDS_t^{\mathbb{P}}$ (on a percentage basis) being most volatile for Mexico. During the later period of our sample, when spreads in the credit markets were tight and when talks of "reaching for yield" were prevalent, the *CRP*s (as seen through our lognormal model) turned negative. Figure 8 shows that $CRP_t(M)$ tends to increase with maturity.[21] Evidently, not only does risk increase with horizon, but its effect on premiums increases on a *percentage* basis as the maturity of the contract increases. Additionally, unlike in the case of the 1-year contract, the *CRP*s do not become negative at the long end of the maturity spectrum.

To assist in interpreting the various "peaks" in the contributions of risk premiums to spreads during our sample period, in Figure 8 we mark the dates of several key economic events around the times of these peaks. The early part of our sample was dominated by economic and political events in South America. Argentina faced an economic crisis in the spring of 2001 and President de

[20] We stress that neither CDS_t nor $CDS_t^{\mathbb{P}}$ involve the physical intensity $\lambda^{\mathbb{P}}$. As emphasized by Jarrow, Lando, and Yu (2005) and Yu (2002), this information cannot be extracted from bond or *CDS* spread data alone.

[21] This measure of the effects of premiums on spreads is larger still when $M = 10$.

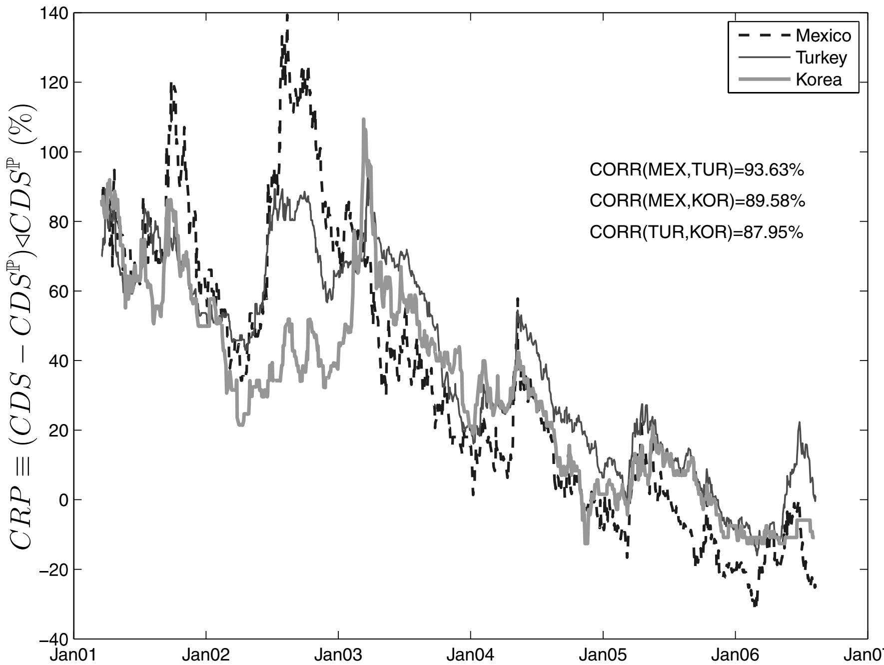

Figure 7. The percentage difference between the 1-year *CDS* price and the 1-year pseudo-*CDS* for Mexico, Turkey and Korea.

la Rua removed his Minister of Economics and introduced a fiscal austerity program. This was followed in the summer of 2001 by a "zero-deficit" plan in an attempt to avoid major bank runs and reverse the depletion of foreign reserves (Zhang (2003)). A year later, in the summer of 2002, the prospect of the left-wing candidate Lula Da Silva winning the presidential elections in Brazil roiled sovereign debt markets. He subsequently won the election in October of that year. Perhaps not surprisingly, all of these political developments in South America had much larger effects on the risk premiums for Mexico than on those for Turkey or Korea.

The simultaneous and large jumps both in *CDS* spreads and the *CRP*s during May 2004 have their roots in investors' portfolio reallocations due to macroeconomic developments in the U.S. During the second quarter of 2004 there was a substantial increase in nonfarm payrolls in the United States. This, combined with comments by representatives of the Federal Reserve, led market participants to expect a tightening of monetary policy. A reason that these concerns had large and widespread effects on spreads is that both financial institutions and hedge funds had substantial positions in "carry trades." They were borrowing short-term in dollars and investing in long-term bonds, often high-yield and emerging market bonds issued in various currencies. The unexpected strength

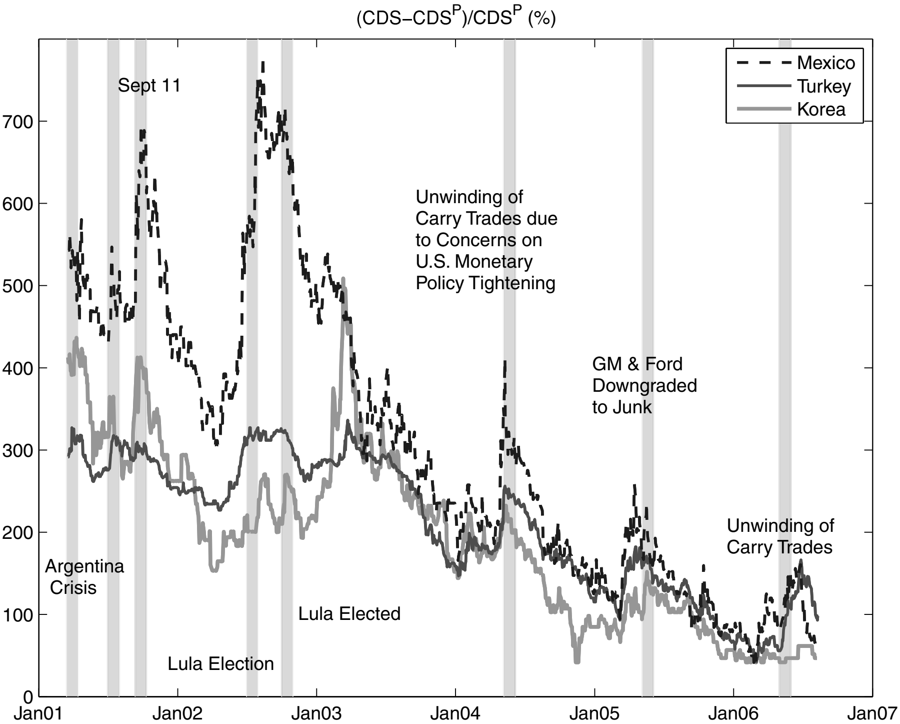

Figure 8. $CRP(5) \equiv (CDS - CDS^{\mathbb{P}})/CDS^{\mathbb{P}}$ **for Mexico, Turkey, and Korea, computed using the 5-year *CDS* contract.**

in the U.S. economy led to an unwinding of some of these trades and, consequently, an across-the-board adjustment in spreads on corporate and sovereign credits.[22] This episode illustrates the importance of changes in investors' appetite for exposure to credit, as a global risk class, for co-movements in yields. The induced changes in yields on the sovereign credits examined here (apparently) had nothing directly to do with the inherent credit qualities of the issuers.

In March of 2005 there were similarly sized run-ups in $CRP_t(5)$ associated with the deteriorating credit quality of General Motors and Ford in the U.S. In the middle of March Fitch downgraded GM, S&P changed its rating outlook to negative, and Moody's placed GM on review for a downgrade. These changes were followed with similarly negative outlooks on Ford in early April 2005. Concurrently, there was a substantial widening of spreads not only on the individual-name *CDS* contracts for these issuers, but also on high-yield

[22] These concerns were widely noted in the media at the time. "In a single day, May 7, yields on Brazilian bonds jumped 1.52 percentage points as the unexpectedly strong jobs report in the U.S. increased the likelihood of higher short-term rates. (Henry (2004))" See also the discussion in Cogan (2005).

corporate indices (e.g., Packer and Wooldridge (2005)). Figure 8 shows that the retrenchment in high-yield positions extended to emerging markets as well.

Finally, *CRP*(5) shows a sizable increase during the late spring of 2006. Once again the evidence supports an increased aversion to exposure to emerging market credit risk rather than reassessments of the fundamental economic strengths of individual countries. There was a broad sell-off in emerging market equities and a concurrent correction in foreign currency markets as hedge funds and other leveraged investors unwound carry trades in the emerging market currencies (e.g., IMF (2006)). During this episode Turkey in particular experienced large balance of payments pressures on its currency, as well as domestic political uncertainties related to its EU accession.

An interesting feature of the time-series of *CRP*(5)s in Figure 8 is that adjustments to Mexico's risk premiums had the largest percentage effects on spreads throughout most of our sample period. During the first half of our sample this is no doubt attributable to the political and economic upheavals in Latin America. The gaps between the countries' *CRP*(5) are smaller during the second half of our sample, and the events in early 2006 had the largest effect on Turkey. As noted, this was most likely a manifestation of domestic policy and political issues in Turkey at the time.

Another striking country-specific episode in Figure 8 is the brief, but large, run-up in *CRP*(5) for Korea in the early part of 2003. This was a period of rising delinquencies on credit card debts following a very rapid expansion in consumer borrowing. Concurrently, the financial stability of several credit card companies and investment trusts was called into question (Kang (2004)). In addition, the conglomerate SK Global reported material accounting irregularities in March, 2003 and this contributed to existing concerns about the stability of the Korean financial system (Cooper and Madigan (2003)).

Comparison of Figures 3 and 8 suggests that episodes in which the risk premiums associated with variation in $\lambda^{\mathbb{Q}}$ were large (as measured by *CRP*) were also episodes in which the bid-ask spreads on the *CDS* contracts were large.[23] This is true of Mexico to some degree and, on an absolute basis, it is particularly true of Turkey over the early part of our sample. However, other than for a brief period in early 2002 for Mexico, the changes in bid-ask spreads for Mexico and Korea were much smaller and their ratios $(ask - bid)/bid$ remained below 10%. Thus, although the gradual increase in the liquidity of the sovereign *CDS* markets during our sample period no doubt contributed to the downward trend in spreads, changes in liquidity do not appear to have been a major source of variation in the $CRP_t(M)$.

The strengths of the economies in all three of the countries examined depend, to varying degrees and through various economic channels, on the strength of

[23] Concurrent movements in liquidity and credit quality are often observed in credit markets. As shown by Duffie and Singleton (1999), the pricing formulas we use can be adapted to accommodate liquidity risk by adjusting the discount rate from $r_t + \lambda_t^{\mathbb{Q}}$ to $r_t + \lambda_t^{\mathbb{Q}} + \ell_t$, where ℓ_t is a measure of liquidity costs. Longstaff et al. (2005) use this extended framework in their analysis of corporate bond and *CDS* contracts. They assume that $\ell_t = 0$ in their pricing of corporate *CDS* contracts or, equivalently, that *CDS* spreads are driven nearly entirely by variation in $\lambda^{\mathbb{Q}}$.

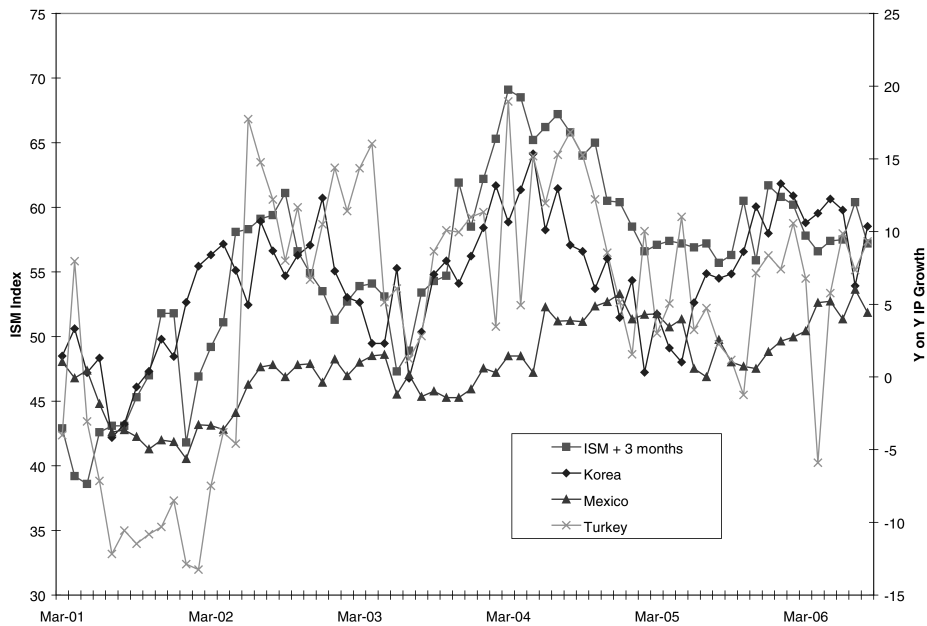

Figure 9. The year-on-year growth rates of industrial production in Korea, Turkey, and Mexico (right-hand scale), and the ISM Index of U.S. Manufacturing led 3 months (left-hand scale).

the U.S. economy. This is apparent from Figure 9, which displays the year-on-year growth rates of industrial production (right scale) and the Institute for Supply Management (ISM) index of U.S. manufacturing shifted one quarter ahead (left scale).[24] The sample correlations of *ISM* and the growth rates (one quarter hence) for Korea, Turkey, and Mexico are 0.66, 0.65, and 0.58, respectively. For Korea, the most persistent gap between these measures of economic growth occurred during 2004 when Korea experienced a marked slowdown in private consumption expenditures in part as a consequence of the consumer debt overhang from 2003 noted above. Turkey shows much more country-specific variation in growth, though one can visually see the secular co-movement with the U.S. economy.

With these observations about the economic events associated with peaks in the *CRP*s in mind, we turn next to a more in-depth exploration of the relationships between $CRP_t(M)$ and various measures of global risk and financial market developments. Figure 10 displays the (standardized) CBOE VIX volatility index and the spread between the U.S. Industrial 10-year BB Yield and the

[24] The data on industrial production were obtained from the International Monetary Fund. The ISM index of manufacturing, based on a monthly survey of purchasing and supply executives throughout the U.S., is constructed by weighting seasonally adjusted new orders, production, employment, supplier deliveries and inventories.

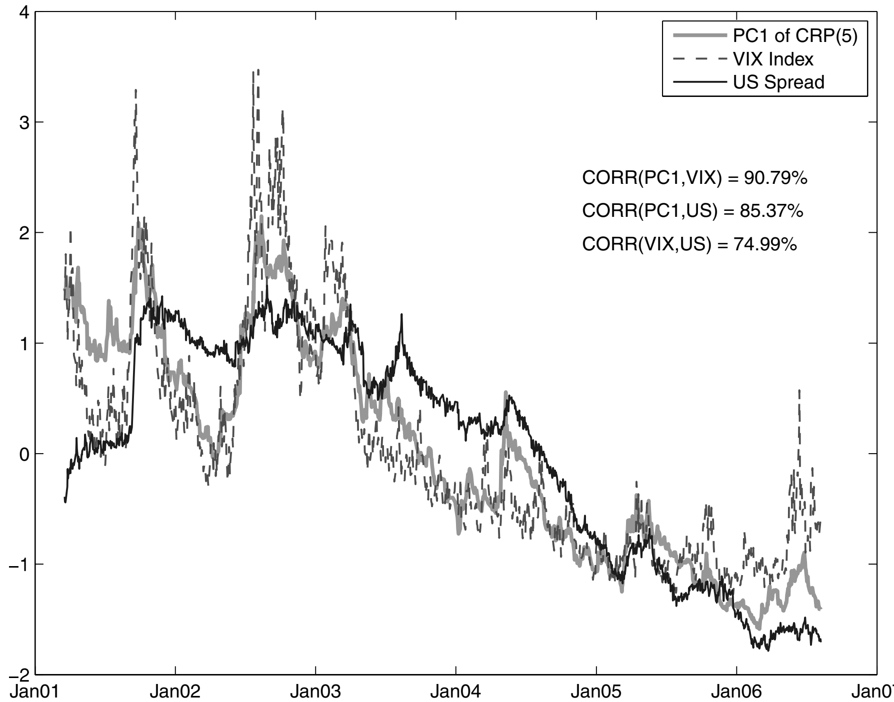

Figure 10. The CBOE VIX index, the spread between the U.S. BB corporate yield the 6-month Treasury bill rate (U.S.-Spread), and the first principal component of $CRP(5) = (CDS(5) - CDS^{\mathbb{P}}(5))/CDS^{\mathbb{P}}(5)$, for Mexico, Turkey, and Korea. All variables are demeaned and scaled by their respective sample standard deviations.

6-month Treasury bill yield (U.S.-Spread)[25] plotted against the first principal component (CRP-PC1) of the $CRP_t(5)$ for Mexico, Turkey, and Korea. We view VIX, a widely watched measure of event risk in credit markets, as a central ingredient in investors' appetite for exposure to the high-yield bond credit class. The view that a significant component of the recent declines in VIX is due to changing investors' appetite for risk is widely expressed in the financial press (see, e.g., "Drop in volatility measure 'reflects investors growing appetite for risk,' Dennis (2006))." More formally, for the overlapping portions of our samples, the risk premium component of VIX computed by Todorov (2006) (see his Figure 5) appear to track our standardized VIX series in Figure 10 quite closely.

We view the positive correlation between *CRP* and U.S.-Spread as having at least two economic sources. First, we have seen from our discussion of Figure 8 that the unwinding of carry trades had large effects on sovereign *CDS* spreads, especially during 2004 and 2006. The "long long-dated corporate, short

[25] The yield data were downloaded from Bloomberg Financial Services.

short-dated Treasury" exposure captured by U.S.-Spread should reflect changes in risk or liquidity premiums associated with the desirability of carry-trade positions. While we do not have direct measures of the interest rate or default risk premiums underlying movements in U.S.-Spread, we highlight two complementary studies that suggest that changes in these premia were important during our sample period. Berndt et al. (2004) show that, within their lognormal framework, a majority of the variation in corporate *CDS* spreads between the summer of 2002 and the end of 2004 was due to variation in default risk premiums and not expected loss rates on these bonds. For the U.S. Treasury markets, Cochrane and Piazzesi (2006) find that the seemingly anomalous behavior of yields on long-term U.S. Treasury bonds in recent years is largely explained by declining risk premiums and not by changes in expectations about future yields. U.S.-Spread reflects, in addition, the slope of the U.S. Treasury yield curve, which is widely watched as an indicator of the stance of U.S. monetary policy and, thus, of the condition of the U.S. economy.

The co-movement of CRP-PC1 with VIX in Figure 10 is notable; their sample correlation is over 90%. A strong correlation between VIX volatility and U.S. corporate credit spreads has been extensively documented (see, e.g., Collin-Dufresne, Goldstein, and Spencer Martin (2001) and Schaefer and Strebulaev (2004)). That VIX, a domestic equity volatility index, is also highly correlated with spreads on sovereign entities as widely dispersed as Mexico and Turkey supports the view that VIX is a key factor in investors' appetite for global "event risk" in credit markets. Turning to U.S.-Spread, the association with CRP-PC1 is relatively weak over the first half of our sample. However, particularly during the run-up in *CRP*(5)s in the springs of 2004 and 2006, U.S.-Spread and CRP-PC1 track each other closely. This appears to be a graphical depiction of the effects on risk premiums of the widespread unwinding of carry trades.

More formally, we next examine the relative contributions to the variation in the individual country *CRP*(5)s of the risk factors VIX, U.S.-Spread, and the own-country implied currency option volatility (CVOL). The risk factor CVOL is included to assist in capturing the effects of capital flows induced both by external macroeconomic developments and their effects on the flows of goods and capital, and the effects of local political and economic events on the credit qualities of sovereign issuers. Table VIII displays the regression estimates with Newey–West *t*-statistics reported in squared brackets, for our entire sample period and the second half of our sample.[26] Focusing first on the univariate regressions over the entire sample period, VIX has the most explanatory power for *CRP*(5) for Mexico, U.S.-Spread for Turkey, and CVOL has slightly more explanatory power than U.S.-Spread for Korea. In the multivariate regressions for Mexico and Turkey both VIX and U.S.-Spread have significant explanatory

[26] Reliable data for Turkey on the implied volatilities of currency options was available only for the second half of our sample and this explained the partial results for Turkey in the upper half of Table VIII. Fifty lags were used in computing the Newey–West standard errors.

Table VIII

OLS Regressions of the 5-year Risk Premium,

Measured by $(CDS - CDS^{\mathbb{P}})/CDS^{\mathbb{P}}$, on the CBOE VIX Index, the U.S. Industrial 10-year BB Yield Minus the 6-month Treasury, and the 6-month Currency Implied Volatility, CVOL, for the Respective Countries

Mexico (2001/4/24–2006/8/10)				Turkey (2001/3/19–2006/8/9)				Korea (2001/3/19–2006/8/8)			
VIX	U.S. Spread	CVOL	R-sqr (%)	VIX	U.S. Spread	CVOL	R-sqr (%)	VIX	U.S. Spread	CVOL	R-sqr (%)
22.84			86.04	9.20			68.66	10.50			52.65
[19.06]				[9.54]				[4.73]			
	78.75		75.13		40.92		76.91		46.17		57.08
	[8.80]				[15.27]				[9.76]		
		80.73	61.27							55.29	59.78
		[7.44]								[9.87]	
15.69	34.50		92.04	4.38	27.22		83.84	5.25	29.52		62.84
[10.27]	[8.14]			[4.44]	[6.48]			[1.75]	[3.52]		
16.63	36.68	−7.18	92.18					2.32	24.02	35.13	79.32
[11.81]	[10.19]	[−1.54]						[1.16]	[4.90]	[5.37]	
Mexico (2003/12/12–2006/8/10)				**Turkey (2003/12/12–2006/8/9)**				**Korea (2003/12/12–2006/8/8)**			
VIX	U.S. Spread	CVOL	R-sqr (%)	VIX	U.S. Spread	CVOL	R-sqr (%)	VIX	U.S. Spread	CVOL	R-sqr (%)
17.95			37.50	14.44			39.73	12.27			29.66
[3.89]				[3.64]				[2.73]			
	46.30		76.49		36.67		76.57		37.15		81.10
	[8.80]				[7.51]				[12.37]		
		30.89	24.90			11.46	68.06			31.86	43.68
		[2.92]				[5.73]				[3.49]	
5.04	41.14		78.50	5.60	31.39		80.96	1.22	35.90		81.30
[2.59]	[8.34]			[3.35]	[7.39]			[0.74]	[12.69]		
5.88	42.07	−3.81	78.71	0.45	24.73	6.34	90.59	1.23	34.82	1.73	81.36
[3.17]	[7.93]	[−0.57]		[0.39]	[9.14]	[10.77]		[0.74]	[7.02]	[0.30]	

power. In the case of Korea, U.S.-Spread and CVOL contribute explanatory power, while VIX is statistically insignificant.

While the reduced-form nature of our regressions introduces some ambiguity into the interpretation of these regressions, the evidence is consistent with the view that much of the effect of risk premiums on *CDS* spreads for Mexico was associated with investors' appetite for exposure to event risk. At least over the early part of our sample, there is notable co-movement between VIX, the *CRP*s, and major political/economic events throughout Latin America. The incremental explanatory power from U.S.-Spread for Mexico is, at least partially, owing to investors' reallocations of capital through the unwinding of carry trades. That these "price pressure" effects of capital flows mattered for Mexico is further supported by the results for the second half of our sample during which U.S.-Spread had by far the most explanatory power for *CRP*(5) (see the lower half of Table VIII). The correlation between U.S.-Spread and *CRP*(5) may also reflect real economic risk associated with the close trading relationship between Mexico and the U.S. Notably, after accounting for VIX and U.S.-Spread, the coefficient on CVOL is statistically insignificant, both for the entire and the second half of our sample period. This suggests that the currency option volatilities in Carr and Wu's (2006) analysis of Mexican *CDS* spreads may have served as stand-ins for the more fundamental macroeconomic and event risks embodied in VIX and U.S.-Spread.

Turning to the results for Turkey, over the entire sample period for which we only have data on VIX and U.S.-Spread, these two risk factors had comparable explanatory power, suggesting that many of the considerations discussed for Mexico were relevant for Turkey as well. Over the second half of our sample, for which we have data on all three risk factors, once again U.S.-Spread is a key explanatory variable for variation in *CRP*(5). However, unlike for Mexico, CVOL has significant explanatory power and the coefficient on VIX is statistically insignificant. The economic underpinnings of the substantial incremental explanatory power of CVOL for Turkey were the large current account deficits and substantial portfolio inflows during the latter half of our sample. Turkey experienced a consumption-led expansion that was partly financed by these large portfolio inflows. Many foreign investors hedged the currency risk of their local bond positions so it seems natural that the placement and subsequent unwinding of carry trades induced a significant correlation between CVOL and *CRP*(5).

The results for Korea show that all three risk factors had significant explanatory power for *CRP*(5) for the entire sample period. Interestingly, the contribution of VIX is muted, certainly relative to the case of Mexico, and, like Turkey, the coefficient on VIX is statistically insignificant for the second half of our sample. We have been told by some investment bankers that investors in Korean bonds have a more "local" focus and that, consequently, spreads are not as highly correlated with VIX as for some other countries. Comparing our results for all three countries, it may simply be that, at least recently, VIX has served more as a measure of risk in the U.S. (or perhaps regionally in the Americas). The explanatory power of U.S.-Spread and CVOL for Korea are surely in part a reflection of the dependence of Asian economies, through exports and imports, on the strength of the U.S. economy (see Figure 9).

VI. On the Sensitivity of Results to Modeling Choices

All of our model-based findings on the structure of risk premiums in sovereign markets are premised on our having adopted a plausible model for $\lambda^{\mathbb{Q}}$, and on our results being largely robust to alternative specifications of $f^{\mathbb{P}}(\epsilon \mid BA_t, \mathcal{I}_{t-1})$, the conditional density of the pricing errors (see footnote 13).

For the purpose of valuing *CDS* contracts, the literature has typically assumed either that $\lambda^{\mathbb{Q}}$ follows a square-root diffusion (e.g., the Zhang (2003) analysis of Argentinean *CDS* spreads and the Longstaff et al. (2005) analysis of U.S. corporate *CDS* spreads) or that $\ln(\lambda^{\mathbb{Q}})$ follows an Ornstein-Uhlenbeck (Gaussian) process (e.g., the Berndt et al. (2004) analysis of U.S. corporate *CDS* spreads). We explored in-depth the relative goodness of fits of these two models, as well as a model in which $\lambda^{\mathbb{Q}}$ follows a "three-halves" diffusion (see Ahn and Gao (1999) for a discussion of this model in a term structure setting), and concluded that, for the sovereign *CDS* spreads and sample period examined, the lognormal model fit the best.

More concretely, as is illustrated in Figure 11 for the case of Mexico with $L^{\mathbb{Q}} = 0.75$, the magnitudes of the pricing errors tend to be ordered, with those of the lognormal model lying between those of the three-halves and affine (square-root) models. The differences between the pricing errors are relatively large during relatively volatile periods as might be expected given their different specifications of the conditional variances of $\lambda^{\mathbb{Q}}$.[27] For example, during 2002 when risk premiums in Mexican markets were historically large (Figure 8), the errors for the lognormal model were closest to zero, with those of the affine (three-halves) model being notably positive (negative). That is, the affine (three-halves) model gives *CDS* spreads that are too low (high). The ordering of mispricing is reversed at the 10-year maturity, with the affine model fitting somewhat better than the lognormal model during 2002. All three models fit comparably well during the low-volatility period in 2003. Comparisons like these, along with the additional information provided in an earlier draft of this paper (that is available from the authors' web sites) led us to favor, on balance, the lognormal model for this study. Berndt (2006) formally assesses the goodness-of-fit of similar models for corporate default using the specification test proposed by Hong and Li (2005) and also concludes that the lognormal model provides a better fit than affine models.

As is evident in Figures 6 and 11, the pricing errors from all three models exhibit positive autocorrelation. The likelihood functions underlying the parameter estimates used to compute these pricing errors presumed that they are serially independent (see footnote 13). Given the relatively small variances of the pricing errors compared to the variances of the fitted *CDS*

[27] The instantaneous standard deviations of $\lambda^{\mathbb{Q}}$ depend on the square-root, level, and three-halves power in the affine, lognormal, and three-halves models, respectively.

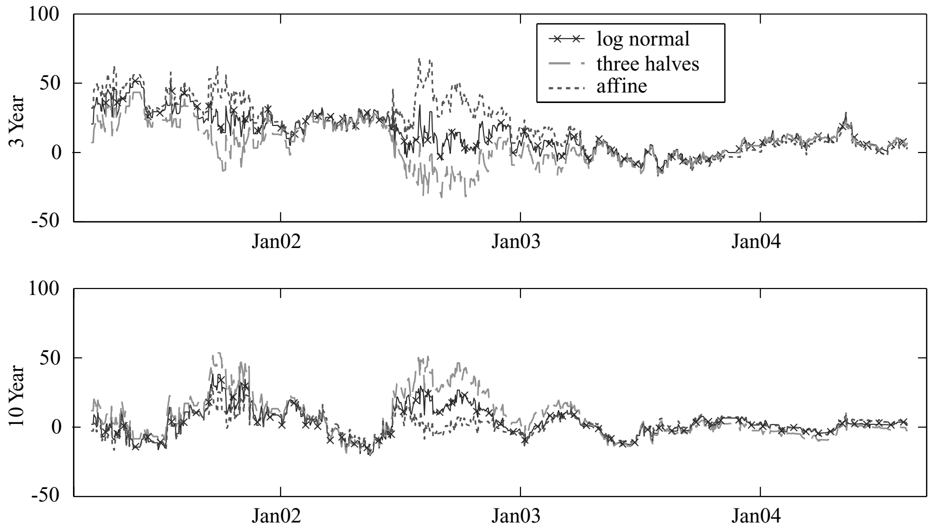

Figure 11. Pricing errors from the affine (square-root), lognormal, and three-halves models of $\lambda^{\mathbb{Q}}$ for Mexico, based on *ML* estimates with $L^{\mathbb{Q}} = 0.75$.

spreads, our prior was that our *ML* estimates of the parameters governing $\lambda^{\mathbb{Q}}$ would be largely robust to alternative formulations of the conditional mean of the errors ϵ, so long as $E[\epsilon_t] = 0$.[28] To verify that our findings are robust, we explored two alternative formulations of $E[\epsilon_t(M) \mid BA_t(M), \mathcal{I}_{t-1}]$ using the Mexican *CDS* data: first-order serial correlation, $0.95\epsilon_{t-1}(M)$, and linear dependence on $BA_t(M) - E[BA_t(M)]$. The latter model accommodates serial correlation indirectly through the inherent persistence of bid-ask spreads. In both cases the *ML* estimates obtained were quantitatively similar to those reported above.

Another notable feature of the pricing errors is that they tend to differ from zero much more in the first than in the second half of our sample. This could simply be a reflection of the fact that the first half of the sample was more turbulent owing to political and economic events around the world. However, we are also mindful of the significant strengthening in the economies of many "emerging" countries, to the point that, in recent years, official reserve positions of these countries are strong and the emerging markets investment class is increasingly being viewed as investment grade. These developments were accompanied by increased trading activity in many sovereign *CDS* contracts and declines in the bid-ask spreads. The median bid-ask spreads on the 5-year contract for the entire sample (Table I) were (10, 30, 10) for (Mexico, Turkey, Korea), and the corresponding medians for the second half of the sample (December 2003 through July 2006) were (5, 12, 8). These observations led us to inquire

[28] Forcing the unconditional mean of ϵ_t to be zero is important for disciplining the model to fit the historical *CDS* spreads on average.

Table IX

Maximum Likelihood Estimates for the Second Half of the Sample

Daily data from November 26, 2003 through August 8, 2006. The sample size is 679 for Mexico and 699 for Turkey. llk is the sample average of log-likelihood.

	$L^{\mathbb{Q}}$ Fixed at 0.75		$L^{\mathbb{Q}}$ Unconstrained	
	Mexico	Turkey	Mexico	Turkey
$\kappa^{\mathbb{Q}}$	−0.035	−2.83e-006	−0.0339	−0.0365
	(0.0043)	(0.0021)	(0.0091)	(0.0044)
$\theta^{\mathbb{Q}}\kappa^{\mathbb{Q}}$	0.132	0.0722	0.12	0.391
	(0.02)	(0.0088)	(0.082)	(0.032)
$\sigma_{\lambda^{\mathbb{Q}}}$	1.10	1.18	1.11	0.972
	(0.0087)	(0.0059)	(0.046)	(0.027)
$\kappa^{\mathbb{P}}$	5.13	2.21	5.19	1.60
	(2.5)	(1.8)	(2.6)	(1.3)
$\theta^{\mathbb{P}}$	−5.87	−4.94	−5.91	−4.25
	(0.21)	(0.51)	(0.34)	(0.55)
$\sigma_{\epsilon}(1)$	0.954	0.802	0.954	0.835
	(0.038)	(0.029)	(0.038)	(0.033)
$\sigma_{\epsilon}(3)$	0.778	0.862	0.778	0.86
	(0.027)	(0.048)	(0.027)	(0.047)
$\sigma_{\epsilon}(7)$	0.722	0.663	0.722	0.674
	(0.038)	(0.037)	(0.039)	(0.038)
$\sigma_{\epsilon}(10)$	1.09	0.995	1.08	0.95
	(0.065)	(0.039)	(0.064)	(0.034)
$L^{\mathbb{Q}}$	0.75	0.75	0.781	0.367
	N/A	N/A	(0.18)	(0.029)
Mean llk	31.497	27.358	31.497	27.419

whether the parameter estimates—and, in particular, the characterizations of the credit-event arrival and recovery processes—are different when the model is fit to the second half of our sample.[29]

Focusing first on Mexico, the point estimates of $\sigma_{\lambda^{\mathbb{Q}}}$ (Table IX) are comparable to those in Table III based on the entire sample period. However, both $\kappa^{\mathbb{Q}}$ and $\kappa^{\mathbb{Q}}\theta^{\mathbb{Q}}$ are closer to zero for the second half of the sample, implying that $\lambda^{\mathbb{Q}}$ is less explosive and, risk-neutrally, the mean arrival rate of a credit event is smaller at small values of $\lambda^{\mathbb{Q}}$. Notably, this (risk-neutrally) improved credit environment during the second half of our sample is being traded off against a much larger value of $L^{\mathbb{Q}}$: The unconstrained estimate of $L^{\mathbb{Q}}$ is 0.78, which is very close to the value set by traders in marking their sovereign *CDS* books (though with a standard error of 0.18).[30] Although the relative contributions of

[29] We focus on Mexico and Turkey, because these are the cases where the unconstrained *ML* estimates of $L^{\mathbb{Q}}$ were low relative to the values set by most traders. As with most split-sample analyses, examination of the second half of a sample sheds light on parameter drift due to changes in economic regimes, but it comes with the potential cost of using a sample that is not representative of the data generating process.

[30] To assure ourselves that this result was not due to our choice of starting values we considered five random seeds for $L^{\mathbb{Q}}$ drawn from the Uniform [0.1,0.9] distribution and optimized the likelihood

event arrival and recovery have changed, the risk premiums associated with unpredictable variation in $\lambda^{\mathbb{Q}}$ remain large. In fact, with unconstrained $L^{\mathbb{Q}}$, the model-implied estimate of δ_1, the slope coefficient on $\lambda^{\mathbb{Q}}$ in our specification of market prices of risk, is -1.16 over the entire sample and -4.71 over the second half of the sample.[31]

Our findings for Turkey are closer to those obtained over the entire sample period. There is a moderate decline in $\kappa^{\mathbb{Q}}\theta^{\mathbb{Q}}$, with $\kappa^{\mathbb{Q}}$ remaining largely unchanged. Accompanying the lower value of $\kappa^{\mathbb{Q}}\theta^{\mathbb{Q}}$ is a larger value of $L^{\mathbb{Q}}$, an increase from 0.24 to 0.37. The overall fit, as measured by the smaller estimated $\sigma_{\epsilon}(M)$ (and smaller bid-ask spreads) for several maturities, also improves over the second half of the sample. This is perhaps to be expected given that this subsample was a relatively less turbulent time.

Within our limited sample of three countries, it is intriguing that the estimates of $L^{\mathbb{Q}}$ are larger for the more highly rated countries. That is, for the higher rated countries, our likelihood function calls for relatively favorable risk-neutral processes for the arrival rates of credit events, balanced against larger values of $L^{\mathbb{Q}}$. Analogous to the relatively larger jump-at-default risk premiums for more highly rated corporate bonds (see, e.g., Huang and Huang (2003) and Berndt et al. (2004)), it is plausible that (risk-neutral) loss rates are indeed much larger for highly rated countries like Korea or Mexico. The economic circumstances that would bring either of these countries to restructure its external debt would surely have hugely adverse consequences globally, relative to events that would bring, say, Turkey to restructure. As the market for recovery swaps on sovereign debts develops, it will be interesting to compare our findings to the market's ordering of $L^{\mathbb{Q}}$, as reflected in the values of these contracts.

VII. Concluding Remarks

We have documented systematic, priced risks associated with unpredictable future variation in the credit-event arrival intensity $\lambda^{\mathbb{Q}}$ for three countries: Mexico, Turkey, and Korea. The effects of these risk premiums on *CDS* spreads co-vary strongly across countries, and large moves in these premiums have natural interpretations in terms of political, macroeconomic, and financial market developments during our sample period. Most notably, our results suggest that, during some subperiods, a substantial portion of the co-movement among the term structures of sovereign spreads across countries was induced by changes in investors' appetites for credit exposure at a global level, rather than by reassessments of the fundamental strengths of these specific sovereign economies. That is, our findings support the view that there is a global high-yield credit class and that spreads in all markets are affected simultaneously as both the financing costs of "risk arbitrage" positions change and investors' attitudes towards bearing the risks of these positions change over time. Spillovers of real economic

function for all five seeds. The seed giving the largest value of the likelihood function was then pursued to a higher degree of accuracy. For the estimates reported in Table IX, the starting values for $L^{\mathbb{Q}}$ ranged between 0.78 and 0.47.

[31] Recall that our model is parameterized at annual intervals, so the implications of these estimates of δ_1 for moderate time horizons are not hugely different.

growth in the U.S. to economic growth in other regions of the world also contribute to the co-movements among the risk premiums in the sovereign markets examined.

Country-specific and regional economic risks were also present and reflected in our models' estimates. This was particularly the case with Turkey, though such specific risks were also present for Mexico during the early part of our sample, and for Korea during 2003 to 2004. Even in the presence of these specific risks, our one-factor lognormal models do quite a good job of capturing the variation over time in the entire term structure of *CDS* spreads (maturities ranging from 1 to 10 years). Further examination of the pricing errors revealed that the 1-year contracts were the least well priced by our one-factor models. Both anecdotal evidence from conversations with *CDS* traders and the co-movements of these errors with bid-ask spreads suggest that liquidity or the effects of supply/demand pressures on prices might underlie this localized mispricing.

Finally we argued, using both analytic calculations of the scores of the likelihood functions and Monte Carlo analysis of small-sample distributions, that the parameters governing the conditional distribution of the arrival rate of credit events ($\lambda^{\mathbb{Q}}$) and the loss given an event ($L^{\mathbb{Q}}$) are separately identifiable using time-series data on the term structure of *CDS* spreads. In practice, the model-implied distributions of $\lambda^{\mathbb{Q}}$ were indeed different as $L^{\mathbb{Q}}$ was varied over the admissible parameter space, confirming that times-series information on the term structure of *CDS* spreads is informative about loss rates. However, the unconstrained maximum likelihood estimates of $L^{\mathbb{Q}}$ for our full sample period were in the region of the market convention, $L^{\mathbb{Q}} = 0.75$, only in the case of Korea.

This left open the question of whether or not there are features of the distribution of *CDS* spreads for countries like Mexico or Turkey that our lognormal models are not capturing and that, once captured, would give rise to estimates for $L^{\mathbb{Q}}$ closer to market convention. We took a small step toward exploring this possibility by re-estimating our models for these countries over the second half of our sample period, a less turbulent period with smaller bid-ask spreads. The estimate of $L^{\mathbb{Q}}$ for Mexico over this subsample was very close to the value of 0.75 set by traders, while for Turkey it was larger than in the full sample, but still less than 0.50.

Looking ahead, further insights into the default and recovery processes for sovereign issuers may also be revealed by a joint analysis of *CDS* spreads and other credit-sensitive derivative products. The expanding offerings of options on *CDS* contracts and various basket or index products offers hope in this direction.[32] Even when additional sources of market information about sovereign credit are readily available, our analysis suggests that it will be useful

[32] In this spirit, though in the context of credit risk for corporate issuers, Das and Hanouna (2006) and Le (2006) achieve the separate identification of the parameters governing $L^{\mathbb{Q}}$ and $\lambda^{\mathbb{Q}}$ by combining models for pricing *CDS* contracts with models for pricing the issuer's (firm's) equity, under the assumption that equity is a "zero-recovery" instrument. Both of these studies rely on a parametric functional dependence of $\lambda^{\mathbb{Q}}$ on observable state variables, however.

to incorporate the rich information embodied in the term structure of *CDS* spreads.

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