Default and the Maturity Structure in Sovereign Bonds*

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Abstract

This paper studies the maturity composition of government debt in emerging markets and the dynamics in the term structure of interest rate spreads. We document that in Argentina, Brazil, Mexico and Russia the maturity composition of new debt issuances, the levels of spreads and the spread curve are all highly volatile. When spreads are low, the spread curve is upward sloping and the average maturity is longer term. When spreads are high, the spread curve is inverted and the average maturity is shorter term.

We build a dynamic model of international borrowing and default that can rationalize the dynamics of spreads and in the maturity composition of debt in the data. The spread curve reflects the dynamics of the endogenous probability of default that is persistent yet mean reverting because of the dynamics of debt and output. Long term debt is beneficial because it can hedge against variations in short rates that are negatively related to consumption. Short term debt is also beneficial to deliver the largest immediate consumption. The maturity composition of debt reflects the time variation in the hedging properties of long term debt relative to the cost advantage of short term debt. When calibrated to data from Brazil, the model matches quantitatively the dynamics of the spread curve and the volatility and maturity composition of new debt issuances.

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1 Introduction

Governments in emerging countries issue debt in international financial markets of a volatile maturity structure that covaries with the term structure of interest rate spreads. Debt issuances are mostly short-term when interest rate spreads are high, and the spread on short-term bonds is higher than on long-term bonds. Debt issuances are mostly long-term when interest rate spreads are low, and the long spread is higher than the short spread. This paper documents these patterns in emerging markets and develops a dynamic model of borrowing and default to study the optimal maturity composition of debt when the prices of debt reflect endogenous default probabilities. The dynamics of short and long spreads in the model follow that in the data because default probabilities are persistent and mean reverting due to the persistent dynamics of debt holdings and income. The maturity composition of debt reflects the time variation in the hedging properties of long term debt to fluctuations in short rates relative to its higher cost due to a larger discount for future defaults. It also responds to a time varying supply of credit that endogenously becomes stringent when default probabilities are high, especially for long debt.

We document the dynamics in the maturity composition of international bonds and in the term structure of interest rate spreads for four emerging-market countries: Argentina, Brazil, Mexico, and Russia. We construct a dataset of foreign-currency denominated bonds issued in international financial markets for 1996 to 2004. From bonds' prices, we estimate spread curves: interest rate spreads over comparable US Treasury bonds, as a function of maturity. We find two stylized facts. First, the spread curve is very volatile. When spreads are low the spread curve is upward-sloping. In periods when the 2-year spreads are below their 10 percentile, the average 2-year spread is 1.14% while the average 10-year spread is 3.55% across the 4 countries. When spreads are high the spread curve is downward-sloping. In periods when the 2-year spreads are above their 90 percentile, the average 2-year spread is 15.42% while the average 10-year spread is 11.98% across the 4 countries. Second, the maturity composition is very volatile and covary with the spread curve. Governments issue short-term debt more heavily in times when spreads are high, and issue long-term debt more heavily when the spreads are low. In periods of upward spread curves when 2 years spread are below their 25 percentile, the average duration of new issuances is 7.1 years; whereas in periods of inverted spread curves when 2 year spreads are above their 75 percentile the average duration is 5.7 years.[1]

The paper constructs a dynamic model of borrowing and default to study the maturity

[1] These findings are consistent with those in Broner, Lorenzoni and Schmukler (2007) for eight emerging markets: Argentina, Brazil, Colombia, Mexico, Russia, Turkey, Uruguay, and Venezuela.

composition of sovereign debt in emerging markets and its puzzling co-variation with spread curves. In the model a risk averse borrower who faces income shocks can issue long and short maturity bonds and can default on them at any point in time. The interest rate spreads of long and short bonds reflect the likelihood of default and compensate foreign lenders for the expected loss from default. Default occurs in equilibrium in low income-high debt times because paying un-contingent debt is more costly when consumption is low. Thus the supply of credit is more stringent for all debt instruments in times of low income due to higher default probabilities and persistent shocks.

The model generates the observed dynamics of spread curves because the endogenous probability of default is persistent yet mean reverting as it responds to the dynamics of debt and output. When the short-term spread is low, default is unlikely in the near future. However default might be likely in the far future after a sequence of bad shocks and debt build-up. Thus the spread curve is upward-sloping in low spread times. On the other hand when default becomes likely in the near future, short-term spreads rise sharply to compensate lenders. Long-term spreads rise less, because of the possibility that the borrower's repayment probability increases after a sequence of good shocks and debt reduction. Although cumulative default probabilities on long-term debt are always larger than on short-term debt, the long spread can be lower than the short spread because it reflects an average default probability over time. Thus the spread curve slopes downward in times of high default probabilities because they are expected to revert back to a lower mean.

The model can rationalize the observed covariation of the debt maturity with spread curves as an optimal response to the time variation in the hedging benefits of long-term debt and in the supply of credit for different maturities. Long term debt plays these two roles in the model. First, since short-term interest rates are uncertain and rise during periods of low income, long-term debt can provide insurance against this uncertainty. By issuing long-term debt, the borrower can avoid having to roll over short-term debt at high interest rates in states when consumption is low. Moreover, long term debt insures against future periods of limited credit availability; in particular the borrower can avoid having trade balance surpluses in recessions by borrowing long term. Importantly this benefit is costly in terms of lower prices for long-term loans as default and repayment probabilities are persistent. In fact the larger spread on long term debt during times of heavy long-term issuances can be understood as an insurance premium the borrower is willing to pay.

However when default probabilities are high, larger immediate consumption is most valuable to a credit constraint borrower. Short term debt is the best asset to deliver up-front resources because the borrower can pledge the future income in the no default states towards debt repayment. Long term debt delivers less up-front resources because of the inability of the borrower to savings in the near future towards repayment when the loan is due. We find that when debt contracts lack commitment and are unenforceable short term debt dominates long term debt in terms of delivering higher immediate consumption. Thus short-term debt is the preferred debt class in times of limited credit and high default probabilities.

Second, not only incentives to default are larger in recessions than in booms, but they respond differently to the levels of short-term versus long-term debt. Specifically, in any period, a given level of long-term debt (due in the future) is less likely to trigger a default than the same level of short-term debt (due today). Moreover, the differential in default incentives across maturities changes with expectations for the level output: long-term debt is more likely to be repaid if future output is expected to be high than if it expected to be low. Thus, when shocks are persistent, the terms for long-term debt are more lenient than for short-term debt disproportionately in booms. In response to the cyclical credit conditions across maturities, the economy chooses a larger composition of long term debt in booms. In summary, the observed large issuances of short-term debt during periods of higher short spreads than long spreads can be understood as an equilibrium response of tight credit conditions in the supply credit, especially for long-term debt, and the relatively 'cheaper' short-term debt.

When calibrated to Brazilian data, the model matches quantitatively various facts of the Brazilian economy. First the model matches the volatility of long and short spreads with long bonds spreads being less volatile on average than short spreads. The model generates persistent time varying default probabilities that match the dynamics of the spread curves in the data. The model also matches the highly volatile maturity composition of debt with long term borrowing being actively used in times of good shocks. The model generates short term rates that are volatile and countercyclical as in the data, and thus long term bonds provide insurance against these fluctuations. We find that the insurance benefits of long term debt and the effect of income on credit conditions are quantitatively important for the maturity structure, as they generate a time-varying composition of debt maturities in line with the data.

This paper is related to the literature on the optimal maturity structure of government debt. Angeletos (2001) and Buera and Nicolini (2004) show that, when debt is not state-contingent, a rich maturity structure of government bonds can be used to replicate the allocations obtained with state contingent debt in economies with distortionary taxes as in Lucas and Stokey (1983). In these closed economy models, short and long term interest rate dynamics reflect the variation in the representative agent's marginal rate of substitution, which changes with the state of the economy. Thus, having a rich maturity structure is equivalent to having assets with state contingent payoffs. Lustig, Sleet and Yeltekin (2006) develop a quantitative general equilibrium model with uninsurable nominal frictions, to study the optimal maturity of government debt. They also find that higher interest rates on long-term debt relative to short-term debt reflect an insurance premium paid by the government, for the benefits long-term debt provides in hedging against future shocks.

Our paper shares with these papers the message that managing the maturity composition of debt can provide benefits to the government because of uncertainty over future interest rates. The message is particularly relevant for the case of emerging market economies. As Neumeyer and Perri (2005) have shown, fluctuations in country-specific interest rate spreads play a major role in accounting for business cycle fluctuations in emerging markets, and explain why business cycle magnitudes are so different from those in developed economies. The lesson that our paper provides in this context is that the volatility of the maturity composition of debt in these countries is an optimal response to the significant interest rate fluctuations. However in contrast with these papers, the fluctuations in interest rates reflect time-variation in the countries' own probabilities of default.

The maturity of debt in emerging countries is also of interest because of the general view that countries could alleviate their vulnerability to very costly crises by choosing the appropriate maturity structure. For example, Cole and Kehoe (2000) argue that the 1994 Mexican debt crisis could have been avoided if the maturity of government debt had been longer. Longer maturity debt would allow countries to better manage external shocks and sudden stops. Broner, Lorenzoni, and Schumukler (2005) formalize this idea in a model where the government can avoid a crisis in the short term by issuing long term debt. In their model, with risk averse lenders who face liquidity shocks, long term debt is more expensive, so the maturity composition is the result of a trade-off between safer long-term debt and cheaper short term debt. In line with this paper, we also find that short-term debt is cheaper because the cumulative repayment probability on short-term debt is higher than on long-term debt. Differently than Broner, Lorenzoni and Schmukler, in our model the time-varying availability of short and long-term debt is an equilibrium response to compensate for the economy's default risk, rather than to compensate for foreign lenders' shocks. We find that higher short-term positions in the wake of crises are an optimal response to the tighter availability of long-term debt in times of high default probabilities.

The theoretical model in this paper builds on the work of Aguiar and Gopinath (2005) and Arellano (2007), who model equilibrium default with incomplete markets, as in the seminal paper on sovereign debt by Eaton and Gersovitz (1981). This paper extends this framework to incorporate debt of multiple maturities. This class of models generates a time-varying probability of default that is linked to the dynamics of debt and income. The dynamics of the spread curve in our model reflect the time-varying default probability, in the same way that Merton (1974) derived for credit spread curves on defaultable corporate bonds. In Merton's model, when default probabilities are low, the credit spread curve is upward-sloping and when default probabilities are high, credit spread curves are downward-sloping or hump-shaped. The spread curve dynamics in this paper follow Merton's results.

The outline of the paper is as follows. Section 2 documents the dynamics of the spread curve and maturity composition for four emerging markets: Argentina, Brazil, Mexico, and Russia. Section 3 presents the theoretical model. Section 4 presents some examples to illustrate the mechanism for the optimal debt portfolio. Section 5 presents all the quantitative results and Section 6 concludes.

2 Emerging Markets Bond Data

We examine data on sovereign, defaultable bonds issued in international financial markets by four emerging-market countries: Argentina, Brazil, Mexico, and Russia. We document the behavior of the term structure of interest rate spreads over default-free bonds, as well as the maturity structure of new bond issues in these countries.

Governments in emerging markets issue bonds at highly volatile interest rates, and the maturity composition of their bond issues changes with the level of interest rate spreads. We find two stylized facts from our dataset. First, spreads increase with maturity during periods when the level of spreads is low, and are flat or decline with maturity during periods when the level of spreads is high. Second, governments issue short-term bonds more heavily in times of high short-term spreads, and issue long-term bonds more heavily when short-term spreads are low.

2.1 Spread curves: definitions and estimation

We define the n-year *yield spread*, or simply *spread*, for an emerging market country as the difference between the yield on a defaultable, zero-coupon bond maturing in n years issued by the country and on a default-free, zero-coupon bond of the same maturity. The spread is the implicit interest rate premium required by investors to be willing to purchase a defaultable bond of a given maturity. The *spread curve* depicts the spread as a function of time to maturity.

Since governments do not issue zero-coupon bonds in a wide range of maturities, we estimate a country's spread curve by using secondary market data on the prices at which coupon-bearing bonds trade.[2] We use a method proposed by Svensson (1994), and used recently by Gurkaynak, Sack, and Wright (2006) for the US, and Broner, Lorenzoni, and Schmukler (2007) for a sample of emerging markets, to fit a spread curve to this data using a simple functional form suggested by Nelson and Siegel (1987).

In this subsection we relate the zero-coupon yield and spread to the price of coupon bonds, then describe the estimation of the spread curve.

We denote the annually compounded yield at date t on a zero-coupon bond issued by country i, maturing in n years as $r_t^i(n)$. The yield is related to the price $q_t^i(n)$ of an n-year zero-coupon bond, with face value 1, through:

$$q_t^i(n) = (1 + r_t^i(n))^{-n} \tag{1}$$

A coupon bond is priced as a collection of zero-coupon bonds, each with maturity given by a coupon payment date, and face value given by the cash flow on that payment date. The price at date t of a bond issued by country i, paying an annual coupon rate c at dates $n_1, n_2, \ldots n_J$ years into the future, is:

$$p_t^i(c, \{n_j\}) = \sum_{j=1}^{J} c(1 + r_t^i(n_j))^{-n_j} + (1 + r_t^i(n_J))^{-n_J} \tag{2}$$

with the face value of the bond paid on the last coupon date.

We define country i's n-year spread as the difference in zero-coupon yields between a bond issued by country i relative to a default free bond. The n-year spread for country i at date t is given by:

$$s_t^i(n) = r_t^i(n) - r_t^*(n)$$

where $r_t^*(n)$ is the yield of a n-year default-free bond.[3]

Conceptually, the zero-coupon yield curve $r_t^i(n)$ is an underlying time-dependent discount rate function that prices any sequence of cash flows promised by country i to investors according to (2). In practice, we estimate the spread curve $s_t^i(n)$ (and thus, given a default-free yield curve $r_t^*(n)$, the yield curve $r_t^i(n)$ as well) using sovereign bond prices from secondary market transactions. In the Appendix we describe how to use these prices, along with US

[2] See the Appendix for information on the bonds used for each country.

[3] Our data include bonds denominated in US Dollars and European currencies, so we take US and Euro-area government bond yields as default-free.

and European government bond yields, to construct an estimate of the spread curve, $s_t^i(n)$ as the following parametric function of maturity n:

$$s_t^i\left(n;\beta_t^i\right)=\beta_{1t}^i+\beta_{2t}^i\left(\frac{1-e^{-\lambda n}}{\lambda n}\right)+\beta_{3t}^i\left(\frac{1-e^{-\lambda n}}{\lambda n}-e^{-\lambda n}\right)$$

where $\beta_t^i=(\beta_{1t}^i,\beta_{2t}^i,\beta_{3t}^i)$ is a time-varying vector of parameters and λ is a constant. As described by Nelson and Siegel (1987) and Diebold and Li (2006), the three components of this curve correspond to a "long-term", or "level" factor (the constant), a "short-term", or "slope" factor (the term multiplying β_2) and a "medium-term", or "curvature" factor (the term multiplying β_3). Linear combinations of these factors can capture a broad range of shapes for the spread curve. We estimate the parameters of each country's spread curve, at each date, by fitting the bond prices predicted by equation (2) to the bond prices in the data. The procedure is described in further detail in the Appendix.

2.2 Spreads in the data

We compute spreads starting in March, 1996, at the earliest, and ending in May, 2004, at the latest, depending on the availability of data for each country. Figure 1 displays the estimated zero-coupon yield spreads for 2-year and 10-year bonds for Argentina, Brazil, Mexico, and Russia.

Spreads are almost always positive, and are very volatile. Long-term spreads are generally higher than short-term spreads; however, when the level of spreads rises, the gap between long- and short-term spread tend to narrow, and sometimes reverses. The series show sharp increases in interest rate spreads associated with Russia's default in 1998, Argentina's default in 2001, and Brazil's financial crisis in 2002.[4] The expectation that the countries would default in these episodes is reflected in the high spreads charged on defaultable bonds.

To emphasize the pattern observed in the time series that short-term spreads tend to rise more than long-term spreads, in Figure 2 we display spread curves averaged across different time periods for each country: the overall average, the average within periods with the 2-year spread below its 10th percentile, and the average within periods with the 2-year spread above its 90th percentile. When spreads are low, the spread curve is upward sloping. When spreads

[4]For Argentina and Russia, we do not report spreads after default on external debt, unless a restructuring agreement was largely completed at a later date. We use dates taken from Sturzenegger and Zettelmeyer (2005). For Argentina, we report spreads until the last week of December, 2001, when the country defaulted. The restructuring agreement for external debt was not offered until 2005. For Russia, we report spreads until the second week of August, 1998, and beginning again after August, 2000, when 75% of external debt had been restructured.

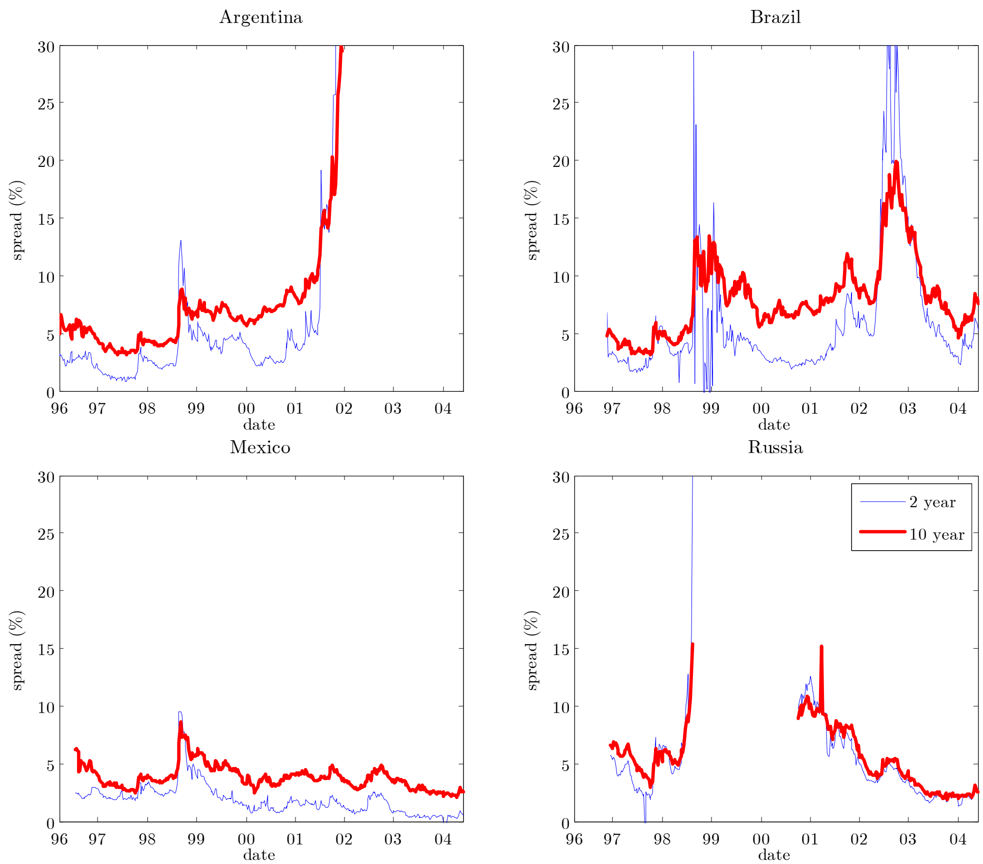

Figure 1: Time series of spreads $s_t^i(n)$ for $n = 2$ and 10 years.

are high, in general, short spreads rise more than long spreads. For Mexico this is reflected in a flatter spread curve. For Argentina, Brazil, and Russia, that display the highest increases in spreads in Figure 1, the spread curve becomes downward sloping. For Mexico the spread curve remains upward sloping, yet much flatter, in the periods with highest spreads. But the level of Mexican spreads in these periods is much lower than in the corresponding periods for Argentina, Brazil, and Russia. The spread curve inverts for the latter three countries when short spreads above 20 percentage points in the cases of Argentina and Brazil, and above 10 percentage points in the case of Russia. In contrast, the short-term spread, even in the upper decile, is below 10 percentage points for Mexico.

A corollary of the changes in the shape of the spread curve in Figure 2 is that long-term spreads are less volatile than short-term spreads. When spreads rise, long-term spreads tend to rise less than short-term spreads, and when spreads fall, long-term spreads tend to fall less. Table 1 confirms that, for all countries, the volatility of spreads declines with maturity.

The findings represented in Figure 2 are similar to empirical findings on spread curves

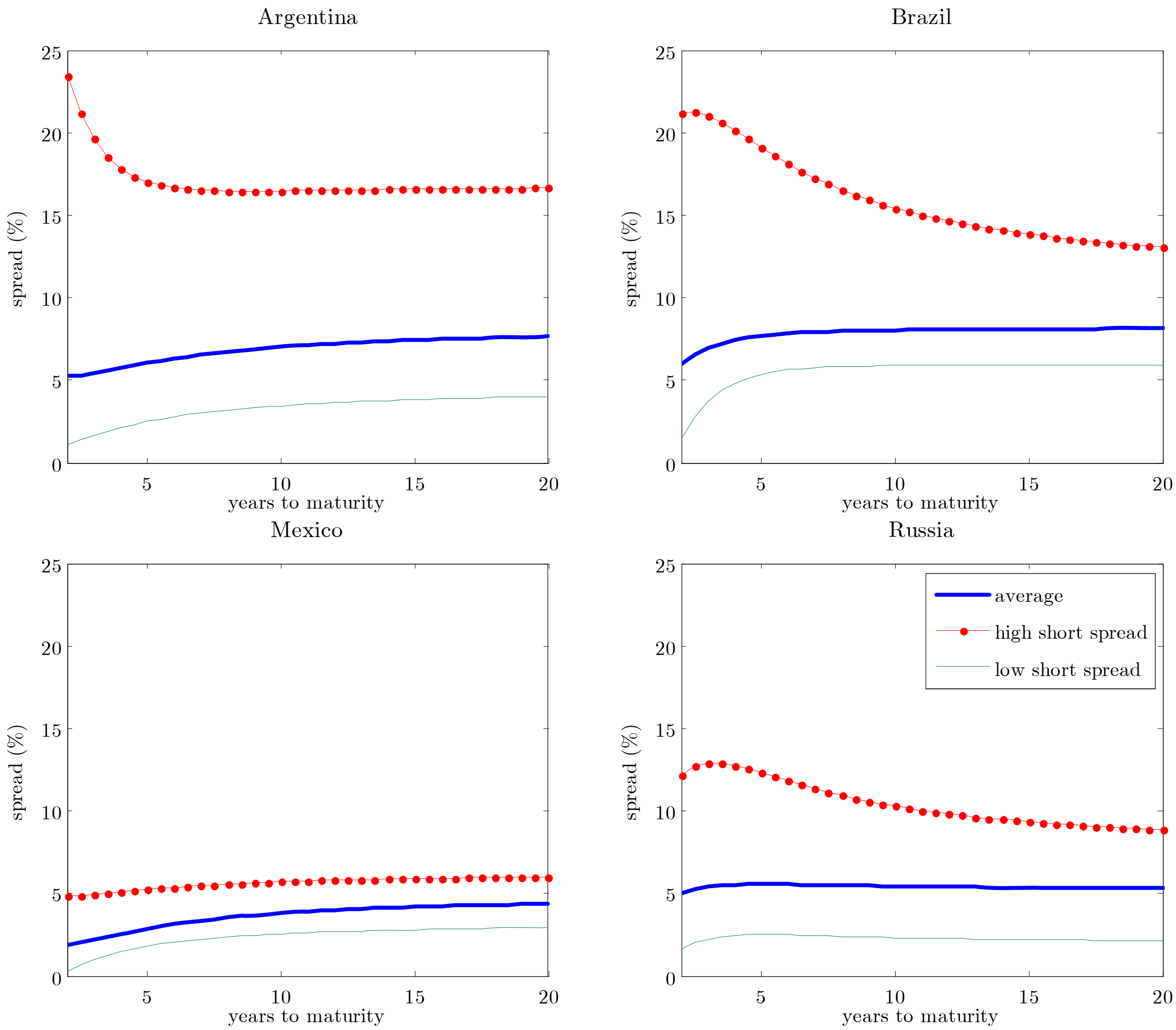

Figure 2: Spread curves: overall average, and average within periods in the highest decile and lowest decile of the short (2-year) spread.

in corporate debt markets. Sarig and Warga (1989), for example, find that highly rated corporate bonds have low levels of spreads, and spread curves that are flat or upward-sloping, while low-grade corporate bonds have high levels of spreads, and average spread curves that are hump-shaped or downward-sloping. Figure 2 suggests that, like corporations borrowing at low interest rate spreads, emerging market governments borrowing during times of low spreads face upward-sloping spread curves. Emerging markets borrowing during times of high spreads face flatter or downward-sloping spread curves, much like corporations that borrow at high spreads.

In Figure 3, we display the time series of discount bond prices for each country relative to

Table 1: Volatility of Spreads

	Standard Deviation			
	Maturity (years)			
	2	5	10	15
Argentina	7.92	4.46	4.15	4.24
Brazil	5.85	4.80	3.39	2.97
Mexico	1.37	1.10	1.05	1.09
Russia	3.45	3.18	2.59	2.44

the price of a US Treasury discount bond. The series are calculated from equation (1) using the estimated US yield curve and each country's estimated spread curve. We plot the ratio $q_t^i(n)/q_t^{\$}(n)$ (with $q_t^{\$}(n)$ defined analogously from $r_t^{\$}(n)$) for $n = 2$ and 10 years.

Figure 3 shows that all four countries generally prices of long-term bonds are lower than for short-term bonds, relative to US bond prices, corresponding to the higher average yield spreads on long-term bonds than short-term bonds in Figure 1. However, during times of high yield spreads, whereas the movements in the spread curve indicate that short-term spreads rise more than long-term spreads, short-term discount bond prices in fact remain above long-term discount bond prices. For example, in Brazil, for the week ending July 5, 2002, yield spreads on 2-year and 10-year bonds were about 24 percent and 16 percent, respectively: short-term spreads were higher. Prices for 2-year and 10-year zero-coupon bonds, however, were 0.65 (which is equal to 1.24^{-2}) and 0.23 (which is equal to 1.16^{-10}), respectively, relative to US bond prices of the same maturities. While annualized interest rates for short-term bonds tend to rise more than for long-term bonds, the cumulative discount, as reflected in prices, charged by investors for holding short-term bonds rises less than for long-term bonds.

2.3 Spreads and the maturity composition of debt

In this section, we examine the maturity composition of new bonds issued by the four emerging market economies during the sample period, and relate the changes in the maturity composition to changes in yield spreads. Since issues of new bonds are relatively rare compared to the weekly price data, we pool the new bond issues into quarterly data, and average the estimated spreads into quarterly time series for comparison.[5]

[5]In addition to external bond debt, emerging countries also have debt obligations with multilateral institutions and foreign banks. However marketable debt constitutes a large fraction of the external debt. The average marketable debt from 1996-2004 is 56% of total external debt in Argentina, 59% in Brazil and 58% in Mexico (Cowan, et al. 2006).

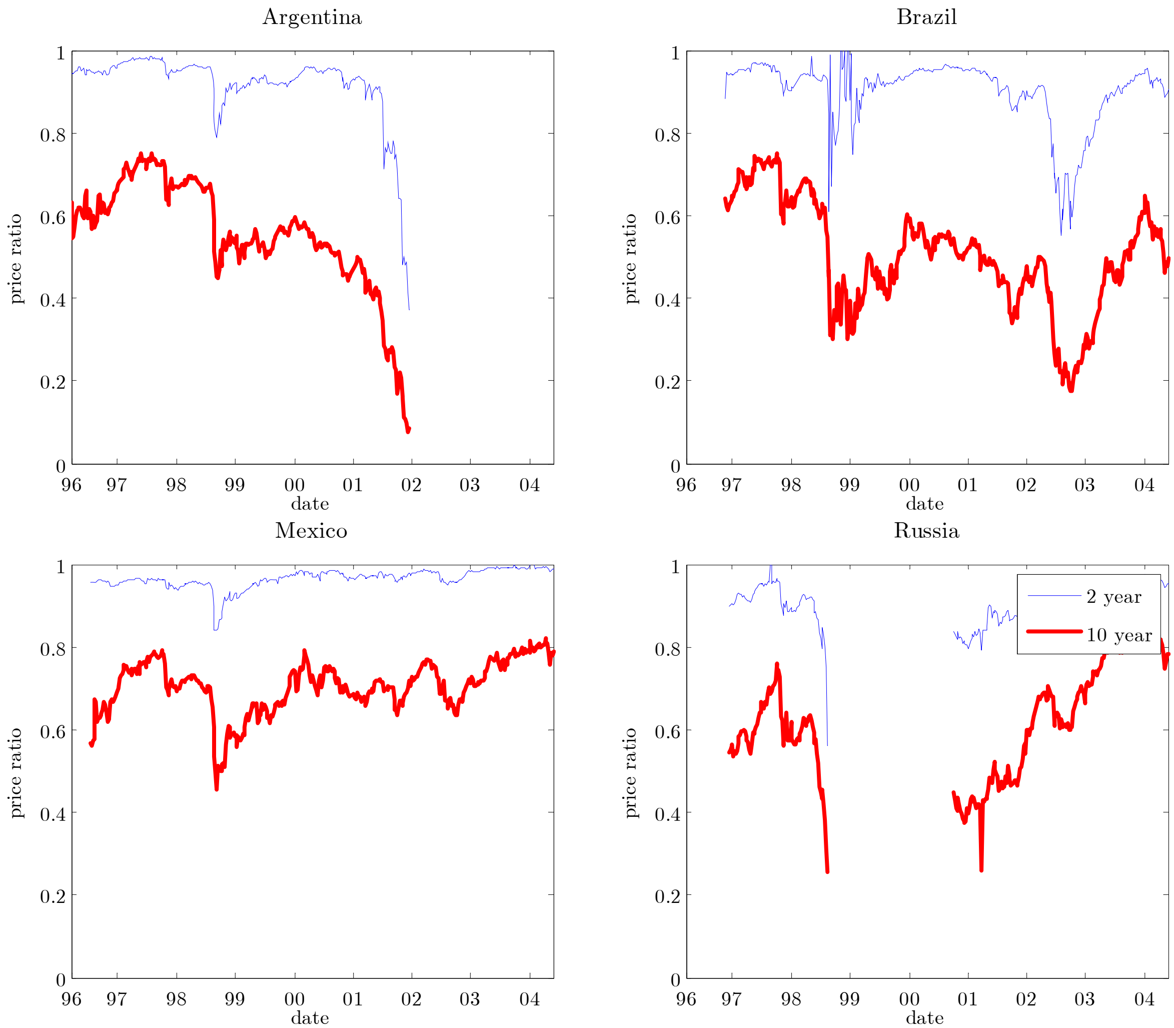

Figure 3: Time series of defaultable bond price relative to default-free US Treasury bond price, $q_t^i(n)/q_t^{\$}(n)$, for $n = 2$ and 10 years.

We measure the maturity of a bond using two alternative statistics. The first is simply the number of years from the issue date until the maturity date. The second is the bond's *duration*, defined in Macaulay (1938) as a weighted average of the number of years until each of the bond's future payments. A bond issued at date t by country i, paying annual coupon c at dates $n_1, n_2, \ldots n_J$ years into the future, has duration d_t^i defined by:

$$d_t^i\left(c, \{n_j\}\right) = \frac{1}{p_t^i\left(c, \{n_j\}\right)} \left(\sum_{j=1}^{J} n_j c(1 + r_t^i(n_j))^{-n_j} + n_J 100(1 + r_t^i(n_J))^{-n_J} \right)$$

where p_t^i is the bond's price, given in equation (2), and $r_t^i(n)$ is the zero-coupon yield curve.

That is, the time until each future payment is weighted by the discounted value of that payment relative to the price of the bond. Zero-coupon bonds have duration equal to their maturity, but coupon-paying bonds have duration shorter than their maturity.

We calculate the average maturity and average duration of new bonds issued in each week by each country. Table 2 displays each country's overall averages of these weekly maturity and duration series, and averages within periods of high (above median) and low (below median) 2-year spreads.

Table 2: Average Maturity and Duration of New Debt

	Maturity (years)		Duration (years)	
short spread:	$<$ median	$\geq$ median	$<$ median	$\geq$ median
Argentina	9.15	9.05	5.70	5.10
Brazil	14.02	6.60	6.59	4.47
Mexico	13.50	10.30	7.72	6.52
Russia	8.89	10.98	6.11	5.42

The table shows that average duration tends to be much shorter than average maturity. Because the yield on an emerging market bond is high, the principal payment at the maturity date is severely discounted, and much of the bond's value comes from coupon payments made sooner in the future, shortening the duration relative to the maturity. Second, for all countries except Russia, the average maturity of bonds issued during periods of high spreads is shorter than when spreads are low: Mexico, for example, issues bonds that mature three years sooner when spreads are high. For Brazil, the difference is seven and a half years. When looking at duration instead, all countries issue bonds with an average duration of about one year less when spreads are high, compared to when spreads are low.

In Table 3, we emphasize the relationship between the spread *curve* and average duration. The slope of the spread curve, equal to the difference between the 10 year (long term) and 2 year (short term) spread, falls during times when the 2 year spread is high. During these times, however, the countries shift towards short-term debt, even though the spread on long-term debt rise less than for short-term debt. This can be rationalized by considering the ratio of bond prices, as in Figure 3. In the last column of Table 3, we calculate the ratio of the price of 10 year to 2 year bonds, averaged within periods when the 2 year spread is high or low. In times when the short spread is high, the discount price of long-term debt falls by more relative to the price of short-term debt. In this sense, short-term debt becomes more attractive.

Table 3: Slope of Spread Curve and Average Duration of Issuances

	Duration (years)		Spread curve slope (%) $r(10)-r(2)$		Price ratio slope $\frac{q(10)/q^*(10)}{q(2)/q^*(2)}$	
short spread:	$<$ 25th pct	$\geq$ 75th pct	$<$ 25th pct	$\geq$ 75th pct	$<$ 25th pct	$\geq$ 75th pct
Argentina	6.40	5.64	2.47	-1.16	0.70	0.42
Brazil	6.80	4.63	4.01	-1.33	0.58	0.42
Mexico	8.45	6.39	2.30	1.24	0.77	0.68
Russia	6.57	6.19	0.57	-0.67	0.81	0.53

The message of this section is that bond spreads in emerging markets are consistent with underlying default and repayment probabilities that are volatile and persistent, yet mean reverting, over time. Moreover bond issuances are also volatile and the maturity structure is directly related to the country's default probabilities. In what follows, we build a dynamic model where default probabilities and the maturity structure are endogenous to the country's conditions and where default probabilities evolve according to the dynamics of debt holdings and income.

3 The Model

The model is a dynamic model of debt and default as in Arellano (2007) extended to incorporate short and long maturity defaultable debt. In the model a small open economy receives every period a stochastic stream of output y_t of a tradable good. The output shock is assumed to have a compact support and to be a Markov process with a transition function $f(y', y)$. The borrower who is the representative agent of the economy trades with lenders bonds of short and long maturity that pay an un-contingent amount. Financial contracts are unenforceable in that the borrower can default on his debt whenever he wants to. In case of failure to repay in full all its debt obligations, the economy incurs costs that consist on lack of access to international financial markets and direct output costs. In the model two types of bonds are issued by the economy. First, b_{t-1} denotes one-period zero coupon debt outstanding at time t. This bond is a promise to pay one unit of consumption in all states. Second, b_2 denotes the two-period zero coupon debt outstanding at t.

The representative agent has preferences $E\sum_{t=0}^{\infty}\beta^t u(c_t)$. The agent's budget constraint conditional on not defaulting is standard. Its purchases of the single consumption good in the spot markets is constrained by its endowment less payments of the one-period and two-period zero coupon bonds, plus the issues of new zero coupon debt b_t^1 at price q_t^1 and two-period bonds b_t^2 at a price of q_t^2:

$$c_t - q_t^1 b_t^1 - q_t^2 b_t^2 = y_t - b_{t-1}^1 - b_{t-2}^2$$

In particular, in every period the agent chooses its debt holdings from a menu of contracts where prices q_t^1 and q_t^2 for are quoted for each pair (b_t^1, b_t^2).

In case of default, we assume that current debts are erased from the budget constraint of the agent and that he cannot borrow or save such that consumption equals output. In addition, the country incurs output costs:

$$c_t = y_t^{def}$$

where $y_t^{def} = h(y) \leq y$.

3.1 Recursive Problem

For given price schedules for debt, q_t^1 and q_t^2, the recursive problem of the borrower can be represented by the following dynamic programming problem.

The output y_t of the borrower is the exogenous state of the model. Let's define the endogenous states of the economy by the total cash on hand $b_{t-1}^1 + b_{t-2}^2$, which consists of previous period outstanding one-period debt and outstanding long term debt purchased two periods before, and by the outstanding long debt purchased the previous period that is due the following period b_2. The states for the model then include the endogenous and exogenous state $s \equiv (b, b_2, y) = (b_{t-1}^1 + b_{t-2}^2, b_{t-1}^2, y_t)$.

Given that initial states are s, the value of the option to default is given by

$$v^0(b, b_2, y) = \max\left\{v^c(b, b_2, y), v^d(y)\right\}$$

where $v^c(b, b_2, y)$ is the value associated with not defaulting and staying in the contract and $v^d(y)$ is the value associated with default. Given that default costs are incurred whenever the borrower fails to repay in full its obligations, the model will only generate complete default on all outstanding debt short and long term.

When the borrower defaults, the economy is in temporary financial autarky; θ is the probability that it will regain access to international credit markets. If the borrower defaults, output falls and equals consumption. The value of default is given by the following:

$$v^d(y) = u(y^{def}) + \beta \int_{y'} \left[\theta v^o(0,0,y') + (1-\theta)v^d(y')\right] f(y',y)dy' \tag{3}$$

We are taking a simple specification to model both costs of default that seem empirically relevant: exclusion from financial markets and direct costs in output. Moreover we assume that the default value does not depend on the maturity composition of debt prior to default. The idea is that the restructuring procedures include choosing the maturity composition of the new debt obligations.[6]

When the agent chooses to remain in the credit relation, the value conditional on not defaulting is the following:

$$v^c(b, b_2, y) = \max_{\{b', b_2'\}} \left(u(c) + \beta \int_{y'} v^0(b', b_2', y') f(y', y) dy' \right)$$

subject to the law of motion for short term debt:

$$b' = b_2 + \Delta b'$$

and subject to the budget constraint:

$$c - q^1(b', b_2', y)\Delta b' - q^2(b', b_2', y)b_2' = y - b.$$

The borrower decides on optimal contracts b' and b_2' to maximize utility. The borrower understands that each contract $\{b',\ b_2'\}$ comes with specific prices $\{q^1,\ q^2\}$. The decision to remain in the credit contract and not default is a period-by-period decision so that the expected value from next period forward incorporates the fact that the agent could choose to default in the future.

The default policy can be characterized by default sets and repayment sets. Let $A(b, b_2)$ be the set of $y's$ for which repayment is optimal when debt positions for short and long term are (b, b_2), such that*:*

$$A(b, b_2) = \left\{ y \in Y : v^c(b, b_2, y) > v^d(y) \right\},$$

[6] This is consistent with empirical evidence regarding actual restructuring processes, where the maturity composition of the debt defaulted is part of the restructuring agreement (Sturzenegger and Zettelmeyer 2005).

and let $D(B) = \widetilde{A}(B)$ be the set of $y's$ for which default is optimal for debt positions (b, b_2), such that

$$D(b, b_2) = \left\{ y \in Y : v^c(b, b_2, y) \leq v^d(y) \right\}. \tag{4}$$

3.2 Bond Prices

Lenders are risk neutral and have an opportunity cost of funds equal to the risk free rate r. Prices compensate lenders for a loss in the case of default and thus they reflect default probabilities. Price schedules functions of the agent's endogenous states next period which determine the default decision and debt policy, and the current stochastic variables which determine the likelihood of the stochastic shock tomorrow: $\{q^1(b', b_2', y), q^2(b', b_2', y)\}$.

The price for the one-period economy's loan is then given by:

$$q^1(b', b_2', y) = \frac{1}{1+r} \int_{A(b', b_2')} f(y', y) dy' \tag{5}$$

For every pair (b', b_2') the lender offers a price that compensates for the possible default event where the payoff will be zero.

Given that default occurs for all outstanding debt simultaneously, the price for the two period bond incorporates the default probability for the next period. The equilibrium price for the two-period bond also needs to forecast future choices of debt holdings and the likelihood of default on those levels of debt two periods from now.

Let's first define a transition law such that:

$$Q(b', b_2'; s) = \left\{ \begin{array}{l} 1 \text{ if } b'(b, b_2, y) = b' \text{ and } b_2'(b, b_2, y) = b_2' \\ 0 \text{ elsewhere} \end{array} \right\}$$

The two-period bond price is the present value of one unit of consumption discounted by the possible loss from default in the following two periods.

$$q^2(b', b_2', y) = \left(\frac{1}{1+r} \right)^2 \left[\int_{A(b', b_2')} f(y', y) \left[\int_{A(b'', b_2'') \times B} Q(b'', b_2''; s') f(y'', y') d(b'', b_2'', y'') \right] dy' \right] \tag{6}$$

Note that if default sets are empty in the following two periods, the price of the two-period bonds collapses to the standard default free long discount price $q^2 = \left(\frac{1}{1+r}\right)^2$.

We define the short spread as the difference between the inverse of the one period price relative to the risk free rate: $spr^s = 1/q^1 - (1+r)$ and the long spread as the difference between the discounted long spread relative to the risk free rate: $spr^L = (1/q^2)^{1/2} - (1+r)$.

3.3 Equilibrium

We now define the equilibrium:

Definition. *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumption $c(s)$, short term debt holdings $b'(s)$, long term debt holdings $b_2'(s)$, repayment sets $A(b, b_2)$, and default sets $D(b, b_2)$, and (ii) the price for short term bonds $q^1(b', b_2', y)$ and long term bonds $q^2(b', b_2', y)$ such that:*

1. *Taking as given the bond price functions $q^1(b', b_2', y)$ and $q^2(b', b_2', y)$, the policy functions $b'(s)$, $b_2'(s)$ and $c(s)$, repayment sets $A(b, b_2)$, and default sets $D(b, b_2)$ satisfy the representative domestic agent's optimization problem.*

2. *Bonds prices $q^1(b', b_2', y)$ and $q^2(b', b_2', y)$ reflect the domestic agent default probabilities such that lenders break even in expected value.*

4 Default and Optimal Maturity

In this section we provide two simplified examples to illustrate the mechanisms for the model regarding the optimal debt portfolio. In the first example we show that in a model with incomplete markets and default, the two assets offer different returns to the borrower even if the risk free rate is constant. Long term debt allows the borrower to insure against fluctuations in default risk. Thus long term debt is beneficial for insurance. In the second example we consider the case of linear utility to abstract from any insurance motive. Here we show that in the presence of future default risk, short term debt allows a larger amount of future resources to be transferred to the present. Thus short term debt is the preferred asset when immediate consumption is highly valued.

Consider a three period model with an agent who borrows from a risk neutral lender. The borrower receives stochastic income in periods 1 and 2 and can default at any point in time. In period 0, income equals to zero. If the borrower defaults, he consumes from then on a low income y^{def}. In period 0, the borrower can trade one period and two period bonds and consumption equals the sum of discounted short b_0^1 and long b_0^2 bonds that are bought at discounted prices q_0^1 and q_0^2 :

$$c_0 = q_0^1 b_0^1 + q_0^2 b_0^2.$$

In period 1 conditional on paying off the short term debt due, new short bonds b_1^1 are available at discount price q_1^1. Consumption is equal to income plus net debt:

$$c_1 = y_1 + q_1^1 b_1^1 - b_0^1.$$

In period 2 conditional on not defaulting, the agent pays off long and short term borrowing and consumption equals income minus debt repayment

$$c_2 = y_2 - \left(b_1^1 + b_0^2\right).$$

The risk neutral lenders discount time at rate r and offer debt contracts that compensate them for the risk of default and that gives them zero expected profits.

4.1 Example 1: Long debt provides insurance

For the first example consider the following income process. Income in period 0 is equal to 0, income in period 1 is equal to y and income in period 2 can take 2 values, y^H or y^L with $y^H > y^L$. The probability of y^H is learned in period 1 and can be either g or p with $g < 1$ and $p < 1$. Preferences for the borrower are given by the expected sum of utility over the 3 periods:

$$U = E[u(c_0) + \beta u(c_1) + \beta^2 u(c_2)].$$

To analyze the insurance properties of long term debt, assume that $u(.)$ is strictly increasing and concave and that $\beta = \frac{1}{1+r} = 1$.

The solution of this problem under the assumption that $\frac{y+\frac{(p+g)}{2}y^H}{2+\frac{(p+g)}{2}} > y^{def} > y^L - \frac{2y^H - y}{2+\frac{(g+p)}{2}}$ delivers perfect consumption smoothing for all the no-default states, and default only in period 2 when income is low. Optimal allocations are the following:

$$c_0 = c_1 = c_2(y^H) = \frac{y + \frac{(p+g)}{2}y^H}{2 + \frac{(p+g)}{2}}$$

$$c_2(y^L) = y^{def}$$

In this example the borrower faces risk because of the variation in bond prices due to default risk. In period 1, short bond prices are in equilibrium either p or g. To avoid this risk, the borrower chooses in period 0 to borrow long term at a discount price $\frac{p+g}{2}$ and save enough short term such that he does not need to use any debt in period 1. Thus long term debt allows the borrower to completely insure against fluctuations in short bond prices in period 1. The optimal debt policies are:

$$b_0^1 = -\frac{y(\frac{g+p}{2} - 1) + \frac{(g+p)}{2} y^H}{2 + \frac{(g+p)}{2}}$$

$$b_0^2 = \frac{2y^H - y}{2 + \frac{(g+p)}{2}}$$

$$b_1^1 = 0.$$

Note that in period 0 short debt is cheaper than long debt $1 > \frac{p+g}{2}$, yet the borrower prefers long term debt. The lower discount price on long debt is the insurance premium the borrower is willing to pay for insurance against default risk in period 1.

If long term debt provides insurance, is short debt ever preferable? In the next example we illustrate that short debt can dominate long term debt in terms of providing up-front resources.

4.2 Example 2: Short debt provides larger up-front resources

For example 2 consider the following income process. Income in period 0 is equal to 0, income in period 1 is equal to y and income in period 2 can take 2 values, y^H or y^L with $y^H > y^L = 0$. Also assume that the probability of y^H is equal to g with $g < 1$ and that $y^{def} = 0$. To abstract from any insurance properties of debt, we assume that preferences are linear in consumption and given by:

$$U = E[c_0 + \beta c_1 + \beta^2 c_2].$$

We assume that the borrower likes to front-load consumption: $\beta < \frac{1}{1+r} = 1$. We also impose non-negativity in consumption $c_t \geq 0$ for $t = 0, 1,$ and 2.

It is easily verifiable that the solution of the borrower's problem under the assumption that $gy^H > y^L$ and $\beta < (gy^H - y^L)/(y^H - y^L)$ is the following. In period 2 the borrower defaults when income equals y^L. In period 0 the borrower consumes the expected value of income for the no-default states $y + gy^H$. The borrower then consumes zero thereafter. Allocations are :

$$c_0 = y + gy^H$$

$$c_1 = c_2(y^H) = 0$$

$$c_2(y^L) = y^{def} = 0$$

It is optimal for the borrower front-load all his income for consumption in period 0 because the borrower does not have preferences for smoothing. These allocations can be achieved by multiple portfolios of short and long term debt in period 0, but the portfolio requires the use of short term debt in period 0. For example one possible portfolio to achieve the above allocations is to borrow only short term debt in period 0, and then roll over in period 1 the unpaid debt to pay it off in the high state in period 2. Borrowing short term in period 0 allows the borrower to consume up-front his lifetime income

$$b_0^1 = y + gy^H \text{ with } q_0^1 = 1$$

$$b_0^2 = 0$$

$$b_1^1 = y^H \text{ with } q_1^1 = g$$

However if the borrower were to use only long term debt in period zero, he could not consume up-front his lifetime income. In particular if short term debt were forced to be zero in period 0, allocations would be the following:

$$c_0 = gy^H$$

$$c_1 = y$$

$$c_2(y^H) = 0, \; c_2(y^L) = y^{def} = 0$$

By using only long term debt in period 0, the borrower is unable to transfer any of the period 1 income to period 0 because this would require the agent to save his period 1 income to repay in period 2. However the borrower cannot commit to save his income because in period 1 it is optimal for any level of b_0^2 to borrow and transfer income from period 2 to period 1. A long term debt contract that allows the borrower to consume $q_0^2 b_0^2 = (y + gy^H)$ cannot be offered because the probability of default on this contract in period 2 is equal to 1 given that $y^H < (y + gy^H)/g$. Predicting the borrower policies the equilibrium price on such loan would be equal to 0. The largest long term contract that the borrower can credibly repay in the second period in the high state is one where $b_0^2 = y^H$.

Long term borrowing is costly because it shrinks the borrower's budget set. Restricting short term borrowing and allowing only long term borrowing denies the possibility of transferring resources from period 1 to period 0 when borrowers cannot commit to future saving and can default on their debt. This example illustrates that in the presence of lack of commitment and default risk, short term debt is a superior instrument to provide up-front resources.

4.3 Summary

In a standard incomplete markets model with fluctuating output and without default, a borrower will find the portfolio of long and short debt indeterminate if the risk free rate is constant across time because the two assets have payoffs that make them equivalent. However, in our model with default risk and a risk averse borrower, long term debt and short term provide distinct benefits. The first example illustrated how long term is beneficial because it hedges against variations in short rates and provides insurance for default risk. However long term debt is costly when borrowers can default because it reduces the budget set of the borrower relative to using short term debt due to the inability for commitment of future policies. Thus in this model with default even with a constant risk free rate, the borrower has incentives to hold a precise portfolio of both assets.

The hedging properties of long term bonds present in our model share similarities with results in Angeletos (2002) and Buera and Nicolini (2004). These papers show that a sufficiently rich maturity structure of bonds can achieve the complete markets allocation in a model in which bonds are traded between risk-averse households and a benevolent government. The difference in our model is that short rates vary because of endogenous default probabilities, rather than because of variation in the marginal rate of substitution of borrowers.

However in our model, short term debt provides benefits in terms of delivering larger up-front consumption, due to the inability of the borrower to commit to saving and not defaulting in the future. This result is related to the literature in corporate finance as in Jensen (1986) and Stultz (1991) where the capital structure of firms is determined by agency costs arising from a conflict of interest between managers and shareholders. In these papers lenders finance firms' projects with short term debt because it reduces the amount of "free" cash available to managers to engage in personal pursuits such as managers' perks or unprofitable overinvestment. In our model short term debt allows the greatest possibilities for immediate consumption because it reduces the amount of resources available to the borrower in the near future, eliminating the need for savings (which might not be optimal for the borrower at that future date). The total amount of resources that can be raised at any point in time is bounded by the present discounted value of income in the states of no default. If borrowers cannot commit to saving in the near future, these resources cannot be pledged for long term debt repayment, i.e. probabilities of default on these larger long term loans would be zero. Thus long term debt contracts take into account the future actions of the borrower and in equilibrium long term debt prices adjust to reflect higher probabilities of default.

Insurance and readily up-front resources shape the optimal maturity structure of debt for a borrowing government. The quantitative relevance of each force depends on the specifics of preferences and the income process. Thus in the next section we quantify these two sources by calibrating our general model to an actual emerging market.

5 Quantitative Analysis

5.1 Calibration

The model is solved numerically to evaluate its quantitative predictions regarding the term structure of sovereign bonds in emerging markets and optimal maturity composition. We calibrate our model to the Brazilian economy.

We use quarterly series for GDP to calibrate the stochastic structure for output. The series are for 1990-2004 deflated by CPI and taken from IBGE (Instituto Brasileiro de Geografia e Estatistica). The stochastic process for output is assumed to be a log-normal AR(1) process $\log(y_t) = \rho \log(y_{t-1}) + \varepsilon$ with $E[\varepsilon^2] = \eta_y^2$. Shocks are discretized into a 11 state Markov chain by using a quadrature based procedure (Tauchen and Hussey 1991). The spread series for the long and short bond are the 10 year and 2 year spreads from the bond data discussed in section 2.[7] The series for short debt issuances is the ratio of all new bond issuances of 5 years or less relative to all new issuances in every quarter.

The utility function of the borrower used in the numerical simulations is $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$. The risk aversion coefficient is set to 2 which is a common value used in real business cycle studies. The interest rate is set to 1% quarterly which equals the average quarterly yield of a 2 year U.S. bond from 1996 to 2004.

Following Arellano (2007) we assume output after the default before re-entering financial markets is assumed to remain low and below some threshold. Output after default evolves in the following form:

$$h(y) = \left\{ \begin{array}{ll} y & if\ y \leq (1-\lambda)\overline{y} \\ (1-\lambda)\overline{y} & if\ y > (1-\lambda)\overline{y} \end{array} \right\}$$

The output cost after default λ, the time preference parameter β, and the probability of reentering financial markets after default θ are calibrated jointly to match three moments in Brazil: the historical default probability in Brazil of 4%[8], the mean debt service to GDP

[7] The statistics are not exactly equal to those of Table 1 because these are quarterly series to make them consistent with the business cycle statistics.

[8] Brazil has defaulted 4 times during the 1900s in their international debt according to Beim and Calomiris; in 1902, 1914 , 1931 and 1983.

ratio in Brazil from 1990 to 2004 of 8.3% and the mean short spread in Brazil when short spreads are above their 50 percentile of 9.5%. Table 4 summarizes the parameter values.

Table 4: Parameters

	Parameter value	Target
Discount factor lender	$\left(\frac{1}{1+r}\right) = 0.99$	U.S. quarterly interest rate 1%
Risk aversion	$\sigma = 2$	Standard value
Stochastic structure	$\rho = 0.9$, $\eta = 0.0235$	Brazil output
Probability of re-entry	$\theta = 0.125$	$\overline{spr}^s$ >50 pct=9.5
Output after default	$\lambda = 0.03$	mean(b/y)=8%
Discount factor borrower	$\beta = 0.938$	4% default probability

5.2 Results

We simulate the model, and in the following subsections, we report statistics on the dynamic behavior of spreads and the maturity composition of debt from the limiting distribution of asset holdings. The model contains a dynamic portfolio problem where the borrower chooses holdings of two assets: 1-period and 2-period bonds. Below, we show how movements in the probability of default generate time-varying differences in the prices and risk structures of these two assets, which rationalize the movements in spread curves and maturity composition observed in the data.

5.2.1 Spreads and Prices

We start with the model's predictions for the spread curve: the difference between equilibrium bond yields and the default-free interest rate r, for the two maturities.

Spreads reflect the probability of default, which varies with the state of the economy. The borrower defaults when output is very low and debt is very high. Thus, the price schedules for short and long debt are decreasing in the amount of debt that will be due in the next period. Due to the dynamic behavior of output and debt, the probabilities of default in one period and in two periods differ. Therefore, the spread curve is not flat and is not time invariant –spreads on long and short bonds are generally different and the relationship between them changes over time. Table 5 presents the model's short and long spreads, in periods in which the short spread (i.e., the probability of default in the next period) is high or low.

Table 5: Spread Curves

	MODEL		DATA	
short spread	Short spread	Long spread	Short spread	Long spread
< 25	0.01	2.14	2.26	5.43
< 50	1.20	2.35	2.84	5.72
≥ 50	9.13	7.62	9.50	9.67
≥ 75	11.03	8.58	13.90	11.76
Overall Mean	5.07	4.92	6.16	7.69

When default is unlikely, both spreads are low, and the spread curve is upward sloping: when the short spread is below its 25th percentile, for example, the average short spread is 0.01%, and the average long spread is 2.14%. In contrast, when the probability of default is higher, both spreads rise, and the spread curve becomes downward sloping: when the short spread is above 75th percentile, the average short spread is 11.03%, and the average long spread is 8.58%. Compared to the data for Brazil, the model captures the difference in the slope of the spread curve associated with periods of high and low short spreads observed. The model's overall average short and long spreads, however, are both pinned down by the average probability of default, which is 4.3%, so the average spread curve is flat.

In the model, when the probability of default in the next period is zero, the spread is zero: $1/q^1(b', b_2', y) = 1 + r$, from equation (5). However, the probability of defaulting two periods into the future could be positive. In this case the long spread is positive. From equation (6),

$$1/q^2(b', b_2', y) = (1+r)^2 \Bigg/ \int f(y', y) \int_{A(b'', b_2'') \times B} Q(b'', b_2'', s') f(y'', y') d(b'', b_2'', y'') dy' \ .$$

Therefore, in the periods with the lowest short spread, the long spread is higher than the short spread: $[1/q^2(b', b_2', y)]^{1/2} > 1/q^1(b', b_2', y)$.

When the probability of default in the next period is high, the model can generate a downward-sloping spread curve if the probability of defaulting in two periods falls, conditional on not defaulting in the next period. For example, if $\int_{A(b', b_2')} f(y', y) dy' \leq 1$, but conditional on repaying at state y', the probability of default is zero for all states y'' in two periods, then

$$1/q^1(b', b_2', y) = (1+r) \Bigg/ \int_{A(b', b_2')} f(y', y) dy'$$

but

$$1/q^2(b', b_2', y) = (1+r)^2 \Bigg/ \int_{A(b', b_2')} f(y', y) dy'$$

so that the short spread is higher than the long spread: $1/q^1(b', b_2', y) > [1/q^2(b', b_2', y)]^{1/2}$.

Note, however that as in the data, the price of a long bond relative to the price of default-free debt is always less than for the short bond, $q^2(b', b_2', y)(1 + r)^2 < q^1(b', b_2', y)(1 + r)$. This is because when default occurs, both short and long bonds are defaulted upon. The probability of repayment on the short debt must always be at least as high as the probability of repayment on the long debt. In this sense short debt is always a cheaper asset for the economy. However, interest rate spreads on long bonds are, in a sense, an average of current and future short spreads, so that when the probability of default is currently high, the long spread reflects a lower expected default probability next period.

The preceding discussions indicate that the important feature of the model for generating the dynamics of the spread curve is that the probability of default is mean-reverting: a period with high probability of default is followed by a period with lower probability of default, and vice versa. The effects of mean-reverting default probabilities on the spread curve are the same as those highlighted by Merton (1974) in the case of credit spreads for corporate debt. Endogenous mean-reverting default probabilities in our model are a result of the dynamics of the output process and debt accumulation. When output is high, it is also expected to be high in the near future, so the probability of default in the next period is low. The economy borrows a large amount at low interest rate spreads, so that in states where the economy is hit by a bad shock, default becomes more likely farther in the future. In contrast, when the likelihood of default is imminent, the economy avoids default in the next period only in states with high output. Conditional on not defaulting, then, output is expected to remain high, and the probability of default farther in the future falls. The persistence and mean reversion of default and repayment probabilities driven by the dynamics of debt and income therefore rationalize the dynamic behavior of the spread curve observed in the data.

5.2.2 Maturity composition

We now present the model's predictions for the maturity composition of debt. It is important to note that the optimal composition of debt is analyzed in a framework that generates the empirically observed dynamics of debt prices.

Two forces in the model shape the dynamic behavior of the maturity composition. First, as discussed in Section 4, long-term bonds insure against short rate fluctuations but they provide less up-front resources; we find that the insurance motive is more valuable in times of high output and low short-term spreads, while the liquidity advantage for short debt is more valuable in times of low output and high short-term spreads. Second, we find that the incentive to default in a given state is stronger for high levels of short-term debt relative to long-term debt; we show that this is asymmetry between short-term and long-term debt is magnified when output is high. These two forces lead the borrower to use long term debt more heavily in times when output is high and short-term spreads are low.

Table 6 compares the average maturity when spreads are high relative to when spreads are low in the model with the average duration of bonds issued in the Brazilian data.

Table 6: Average Maturity

	MODEL	DATA
$<$ 50 pct / overall	1.11	1.09
$<$ 50 pct / $\geq$ 50 pct	1.27	1.35

In the model, average maturity is 11% higher when spreads are low than average, and 27% higher when spreads are low than when spreads are high. In Brazil, the average duration of bonds issued when spreads are low is 9% higher than average, and 35% higher than when spreads are low. The dynamics in the maturity composition of debt holdings in the model matches the patterns observed in the data.

Table 7 provides more details about the maturity composition. The first two columns of Table 7 show the model's debt portfolio, conditional on different levels of the short spread. When the short spread is below its median, the borrower issues on average 78% of its debt in long-term bonds, and 22% in short-term bonds. When the short-term spread is above its median, the average maturity composition shifts, to only 60% long-term bonds, and 40% short-term bonds. Overall, the borrower issues about 60% of its debt long-term and 40% short-term.

Table 7: Model Maturity Composition

short spread	b^1/b	b^2/b	$b/mean(y)$	$q^1(1+r)$	$q^2(1+r)^2$	$cov(m', q^{1\prime})$
$<$ 50 pct	0.22	0.78	8.7	0.997	0.989	-0.07
$\geq$ 50 pct	0.60	0.40	7.6	0.980	0.966	-0.03
Overall Mean	0.40	0.60	8.2	0.989	0.978	-0.06

The next three columns provide one part of the rationalization for these patterns in maturity composition. Long debt is beneficial as it provides insurance to interest rate fluctuations because the covariance between the borrower's intertemporal marginal rate of substitution,

$m' = \beta u'(c')/u'(c)$, and the short-term bond price next period, $q^{1\prime}$ is negative. However long term debt delivers less up-front resources and is more costly in terms of lower repayment probabilities $q^2(1+r)^2 < q^1(1+r)$.

In times when the short spread is low, the covariance between the borrower's intertemporal marginal rate of substitution and the short-term bond price next period is equal to -0.07. In states with high marginal utility of consumption, the short bond price is expected to be low. Issuing long debt today allows the borrower to avoid having to issue short-term debt tomorrow in these states. Therefore, the larger discount of long bonds relative to short bonds (about 1% versus 0%, as reflected in $q^2(1+r)^2$ and $q^1(1+r)$), can be interpreted as an insurance premium. The borrower is willing to issue a lot of long term debt at lower prices because it insures against movements in the short-term price next period, $q^{1\prime}$.

Times of high short spreads are times when credit limits are tight and more binding as income and wealth are low. Debt equals 8.7% of output when the short spread is below its median while it is 7.6% of output when short spreads are above their median. The liquidity advantage of short term debt is the dominating factor in the portfolio decision as the borrower can transfer more of the future resources to the present with short debt. Thus, during times of tight credit conditions the borrower exploits the cost advantage of short term debt. Moreover, the insurance benefit of long term debt is smaller as the covariance between the marginal rate of substitution and the short bond price tomorrow reduces in magnitude, to only -0.03.

In addition to the borrower's motive for using costly long-term bonds for insurance, the equilibrium pricing schedules vary with output in a way that favors long-term debt when output is high. To illustrate this, we examine the effect of the maturity composition on the decision to default. Default is chosen for the various $\{b, b^2, y\}$ combinations such that $v^c(b, b^2, y) < v^d(y)$. Figure 4 displays the regions of (b, b^2)-space for which default and repayment are chosen, for three different levels of output: the mean level, $\overline{y}$, a high level, $y = 1.02\overline{y}$, and a low level, $y = 0.98\overline{y}$.

For higher output levels, the boundary between the regions shifts outwards: higher levels of debt can be sustained without default. Therefore, when shocks are persistent, a higher level of output means that price schedules are lenient, and the economy can borrow more. In addition, the boundary in the figure becomes flatter for higher levels of output: more long debt can be sustained, relative to short debt, for higher levels of output. Lenders are always willing to hold more long term debt, because the borrower is less likely to default on a given level of long-term debt than on the same level of short-term debt. When output is high, it becomes increasingly easier to roll over a large level of long debt because it is likely that in

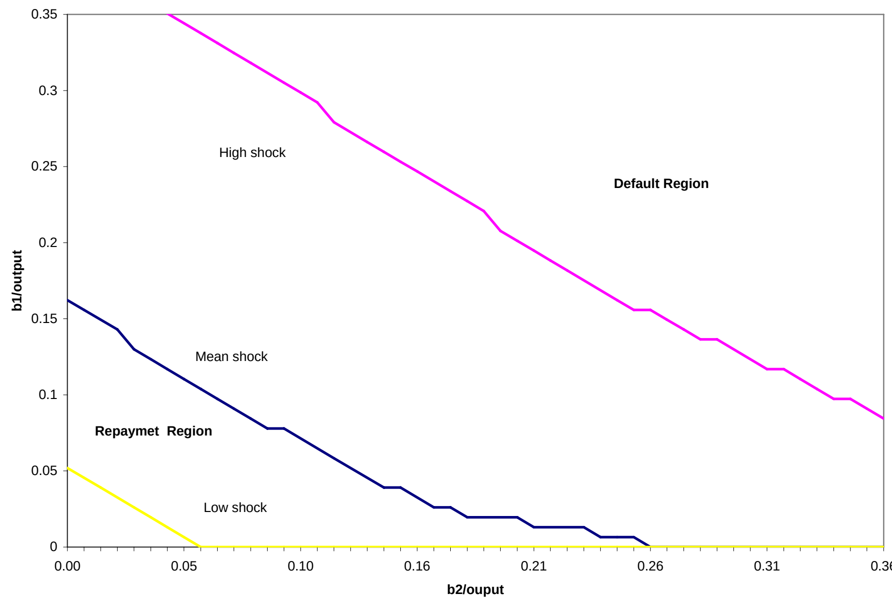

Figure 4: Default boundary for three levels of income

the future the economy will continue to boom, and thus be able to sustain a large level of debt. Therefore, although lenders in our model are risk neutral and their payoff is the same in expected value both in high and low output states, the economy's time varying default probability creates a time varying supply of long term credit. And in response to the very lenient credit conditions for long-term debt in booms, the economy finds it optimal to hold a large portfolio of long-term debt obligations.

Figure 5 further illustrates the insurance properties of long term debt. In our model, the economy faces schedules of debt and prices (i.e. not face a perfectly elastic supply of credit), thus the covariation calculations in table 7 underestimate the strength of the insurance benefit of long debt. The figure plots the trade balance, $y - c$, assuming that the borrower does not default, as a function of output (mean output equals 1). The two lines in each plot correspond to the case in which the debt with which the economy enters the period is all long-term or all short-term. The three plots correspond to different levels of total debt.

The trade balance decreases with output, because of the fact that price schedules for debt are more lenient in high output states, and hence the economy runs a trade deficit when output is high. When debt is all short term, if the economy is hit by low output, the debt becomes very expensive to roll over. For example, if the short debt due is 6% of mean output, and the economy receives output 10% below the mean, then the economy has a trade surplus of almost 3%, because of high cost of borrowing. However if the same level of total debt

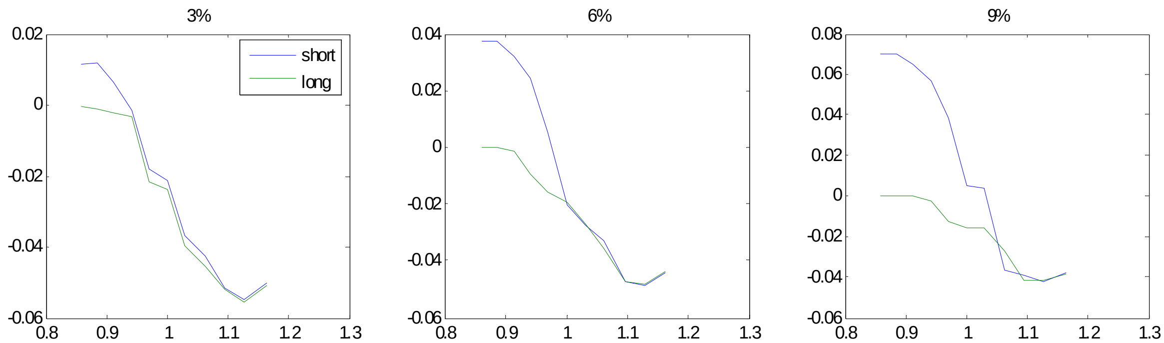

Figure 5: Trade Balance and Income when Debt is only Short or Long Term

is held in long term loans, the economy avoids the high cost of borrowing, and thus avoids a trade surplus when output is low. Therefore, long term debt provides insurance against experiencing trade surpluses in recessions. In addition, the gain from having issued long term debt before a low output shock increases with the level of total debt due, as the trade surplus required to roll over debt increases with the level of short-term debt due.

In summary through the lens of our model, the observed maturity structure in emerging markets can be rationalized by two factors: a time-varying role of long term debt for insuring against fluctuations in short term interest rates relative to a 'cost' advantage of short term debt, and a time-varying supply of long-term credit that dries up when default probabilities are high.

5.2.3 Model dynamics

In this section we report the model's business cycle statistics, and compare them to those of Brazil. We find that the model matches the business cycle statistics in Brazil well. Therefore, our theory of the optimal maturity structure of foreign debt in emerging markets is contained in a model that is consistent with the data.

Table 8 reports standard deviations and correlations of selected series in the data, and analogous series from the limiting distribution of debt holdings in the model. The model series are treated in a similar fashion as the data.

The model matches the higher volatility of short spreads relative to long spreads: short spreads are twice as volatile as long spreads in the model because they are more sensitive to

Table 8: Business Cycles

	MODEL Std. Dev.	MODEL Corr (x, spr^s)	DATA Std. Dev.	DATA Corr (x, spr^s)
spr^s	10.08		5.28	
spr^L	4.81	0.93	3.11	0.92
y	5.26	-0.11	5.38	-0.21
c	6.20	-0.15	5.17	-0.40
tb/y	2.38	0.16	3.15	0.48
b^1/b	68.86	0.12	42.1	0.10

the time variation of default probabilities. However, the model overestimates the volatility of the short term spread. The model reproduces the negative correlation of spreads with output and consumption, because default is more likely in recessions. The economy faces higher interest rates in times of low income because default probabilities are higher in recessions. The negative comovement of the short spread with consumption is a significant source of risk, and as discussed above, long term debt helps to alleviate this risk.

The model matches the positive correlation between spreads and the trade balance. When default probabilities are high, the economy has generally low income and large debt. As discussed above, the price of debt is low in these times, so spreads are high and the economy experiences trade balance surpluses. The economy borrows more in booms, both short and long term, because of the state-contingent price schedules; the correlation between the trade balance and output equals -0.22. However, the economy mostly borrows long term debt in booms to insure against even larger trade balance surpluses in recessions. The correlation of output and the share of long debt relative to total debt is 0.18. The state contingent price schedules also imply that consumption is more volatile than output, which is a common feature of most emerging markets. The Brazilian government has issued a time-varying maturity composition of debt. The model matches the data in generating a very volatile share of short-term debt.

6 Conclusion

This paper has constructed a dynamic model of borrowing and default to study the maturity structure of sovereign bonds and the spread curve. In the data, debt spreads are volatile, and the spread curve follows a pattern: when spreads on short term debt are low, long term spreads are higher than short-term spreads, and when short-term spreads rise, long-term spreads rise less. In our model, spreads on long-term bonds are higher during times of low interest rate spreads because the risk of default occurring is far into the future. When the risk of immediate default becomes high, the short-term spread rises, but if the economy avoids default, then it becomes much more likely to repay its debt, and the long-term spread reflects a relatively lower average risk of default. Issuing long-term bonds insures against future increases in short-term spreads in the presence of default risk. Issuing short-term bonds allows the borrower to consume upfront the greatest amount of resources, and this is beneficial when additional immediate consumption is highly valued. The model generates the pattern of issuances observed in the data. Long term debt is issued mostly in times of low spreads when the insurance motive is the strongest while short-term bonds are used more heavily in times when their spreads are high because here is when credit conditions are the tightest.

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Appendix

Data Description

All the sovereign bond data are from Bloomberg. For the four countries we examine, we use all bonds with prices quoted at some point between March, 1996 and May, 2004, with the following exceptions. We exclude all bonds with floating-rate coupon payments, and at every date, we exclude bonds that are less than three months to maturity, following Gurkaynak, Sack and Wright (2007). For each country, we estimate spreads starting from the first week for which at least four bond prices are available every week through the end of the sample. We use data from 110 bonds for Argentina, 71 for Brazil, and 25 for Russia. To estimate default-free yield curves, we use data on US and European government bond yields. The US data are from the Federal Reserve Board, and the European data are from the European Central Bank.[9] For constructing the quarterly maturity and duration statistics, we also include bonds issued during the sample period that did not have prices quoted, and use the estimated spread curve to construct their prices according to equation (2).

For constructing quantity-weighted statistics of bond maturity, we use Bloomberg's reported amount issued for each bond, and convert quantities to US dollars using quarterly exchange rates from the IMF's *International Financial Statistics*.

Spread Curve Estimation

We repeat the basic equations that relate the prices of coupon bonds to the underlying yield and spread curves:

$$p_t^i(c, \{n_j\}) = \sum_{j=1}^{J} c(1 + r_t^i(n_j))^{-n_j} + (1 + r_t^i(n_J))^{-n_J} \tag{7}$$

and

$$s_t^i(n) = r_t^i(n) - r_t^*(n)$$

where $r_t^*(n)$ is a default-free yield curve.

We introduce another measure of a bond's price, the *yield to maturity*, that is useful in estimating spreads. For a bond with coupon c and payments in $n_1, n_2, \ldots n_J$ years, the yield

[9] The US data are the "Treasury constant maturities" yields, available at: http://www.federalreserve.gov/releases/h15/data.htm.
The European data are "Euro area benchmark government bond yields", which is an average of European national government bond yields available at: http://sdw.ecb.europa.eu.

to maturity is the rate $y(c,\{n_j\})$ that solves:

$$p_t^i(c,\{n_j\}) = \sum_{j=1}^{J} c(1+y)^{-n_j} + (1+y)^{-n_J} \tag{8}$$

with $p_t^i(c,\{n_j\})$ given by (7). That is, the yield to maturity is the constant rate of interest at which the bond's price equals the discounted value of its payments.

To estimate spread curves using data on coupon bond prices, we use a functional form suggested by Nelson and Siegel (1987), to fit a curve through zero-coupon yields for each country and for default-free bonds. We define

$$s_t^i(n;\beta_t^i) = \beta_{1t}^i + \beta_{2t}^i\left(\frac{1-e^{-\lambda n}}{\lambda n}\right) + \beta_{3t}^i\left(\frac{1-e^{-\lambda n}}{\lambda n} - e^{-\lambda n}\right) \tag{9}$$

for each country i, where $\beta_t^i = (\beta_{1t}^i, \beta_{2t}^i, \beta_{3t}^i)$ and λ are parameters. For default-free bonds, we define

$$r_t^{\$}(n;\beta_t) = \beta_{1t}^{\$} + \beta_{2t}^{\$}\left(\frac{1-e^{-\lambda n}}{\lambda n}\right) + \beta_{3t}^{\$}\left(\frac{1-e^{-\lambda n}}{\lambda n} - e^{-\lambda n}\right) \tag{10}$$

and

$$r_t^{€}(n;\beta_t) = \beta_{1t}^{€} + \beta_{2t}^{€}\left(\frac{1-e^{-\lambda n}}{\lambda n}\right) + \beta_{3t}^{€}\left(\frac{1-e^{-\lambda n}}{\lambda n} - e^{-\lambda n}\right) \tag{11}$$

for US (\$) and Euro (€) bonds.

We first estimate the parameters $\beta_t^{\$}$ and $\beta_t^{€}$ by OLS, using US and Euro bond yields. Throughout, we follow Diebold and Li (2006) by setting the parameter $\lambda = 0.714$, so that the term multiplying β_3 in all countries' spread curves is maximized when $n = 2\frac{1}{2}$ years.

Then, given a set of parameters β_t^i, we use equation (7) to price each of country i's bonds at date t using the riskfree yield given by (10) or (11) and the spread given by (9):

$$p_t^i(c,\{n_j\};\beta_t^i) = \sum_{j=1}^{J} c(1+s_t^i(n_j;\beta_t^i)+r_t^*(n_j))^{-n_j} + (1+s_t^i(n_J;\beta_t^i)+r_t^*(n_J))^{-n_J}$$

where r_t^* refers to $r_t^{\$}$ if the bond is denominated in US dollars, or $r_t^* = r_t^{€}$ if the bond is denominated in a European currency. We use equation (8) to compute a yield-to-maturity for each bond, given the parameters β_t^i, solving the following for $y(c,\{n_j\};\beta_t^i)$:

$$p_t^i(c,\{n_j\};\beta_t^i) = \sum_{j=1}^{J} c(1+y(c,\{n_j\};\beta_t^i))^{-n_j} + (1+y(c,\{n_j\};\beta_t^i))^{-n_J}$$

We choose the parameters β_t^i to minimize the sum of squared deviations of the predicted yields-to-maturity, $y(c, \{n_j\}; \beta_t^i)$ from their actual values. That is, our estimated parameters solve

$$\min_{\beta_t^i} \sum (y(c, \{n_j\}; \beta_t^i) - y(c, \{n_j\}))^2$$

where the summation is taken over all bonds issued by country i with prices available at date t. As discussed in Svensson (1994), minimizing yield to maturity errors rather than price errors gives a better fit for short-term yields to maturity, because short-term bond prices are less sensitive to their yields to maturity than long-term bond prices.

The following features present in the data require modification of the basic bond pricing equation (7):

1. Between coupon periods, the quoted price of a bond does not include accrued interest, so we subtract from the bond price the portion of the next coupon's value that is attributed to accrued interest.

2. For bonds with principal payments guaranteed by US Treasury securities, we discount the payment of principal by the risk-free yield only, without the country spread.

3. For bonds with coupon payments that increase or decrease over time with certainty ("step-up" and "step-down" bonds, respectively), we modify the sequence of payments in equation (2) accordingly.

Further Statistics on Conditional Average Spreads

Table 9: Average Spreads (percent)

	Maturity (years)	Overall	When 2-year spread is above/below nth percentile					
			$<$ 10th	$<$ 25th	$<$ 50th	$\geq$ 50th	$\geq$ 75th	$\geq$ 90th
Argentina	2	5.23	1.11	1.63	2.16	8.30	12.64	23.41
	5	6.03	2.50	3.08	3.76	8.30	11.06	17.02
	7	6.53	3.01	3.62	4.39	8.67	11.20	16.51
	10	7.02	3.45	4.10	4.95	9.08	11.49	16.48
	15	7.43	3.82	4.49	5.41	9.44	11.78	16.58
Brazil	2	6.01	1.47	2.12	2.75	9.25	13.55	21.19
	5	7.69	5.36	5.03	5.11	10.27	13.55	19.11
	7	7.92	5.76	5.67	5.67	10.18	12.92	17.27
	10	8.04	5.89	6.12	6.08	9.99	12.23	15.43
	15	8.10	5.92	6.47	6.40	9.80	11.61	13.85
Mexico	2	1.87	0.31	0.57	0.95	2.78	3.51	4.85
	5	2.87	1.81	2.01	2.27	3.47	4.01	5.24
	7	3.36	2.23	2.47	2.81	3.91	4.39	5.49
	10	3.81	2.55	2.86	3.30	4.33	4.76	5.72
	15	4.19	2.81	3.18	3.70	4.68	5.07	5.91
Russia	2	5.04	1.66	2.00	2.69	7.37	9.56	12.22
	5	5.57	2.51	2.66	3.30	7.82	10.13	12.34
	7	5.53	2.46	2.64	3.40	7.65	9.56	11.37
	10	5.45	2.33	2.56	3.46	7.43	8.90	10.30
	15	5.37	2.20	2.48	3.50	7.22	8.29	9.35