

Defects and Geometry in Condensed Matter Physics

This book describes the key role played by thermally excited defects such as vortices, disclinations, dislocations, vacancies and interstitials in the physics of crystals, superfluids, superconductors, liquid crystals and polymer arrays.

Geometrical aspects of statistical mechanics become particularly important when thermal fluctuations entangle or crumple extended line-like or surface-like objects in three dimensions. In the case of entangled vortices above the first-order flux lattice melting transition in high-temperature superconductors, the lines themselves are defects. A variety of theories combined with renormalization-group ideas are used to describe the delicate interplay among defects, statistical mechanics and geometry characteristic of these problems in condensed matter physics.

This indispensable guide has its origins in Professor Nelson's contributions to summer schools, conference proceedings and workshops over the past twenty years. It provides a coherent and pedagogic graduate-level introduction to the field of defects and geometry.

DAVID NELSON is Mallinckrodt Professor of Physics and Professor of Applied Physics at Harvard University. He received his Ph.D. in 1975 from Cornell University. His research focuses on collective effects in the physics of condensed matter, particularly on the connections between thermal fluctuations, geometry and statistical physics. In collaboration with his Harvard colleague, Bertrand I. Halperin, he has proposed a theory of dislocation- and disclination-mediated melting in two dimensions. Professor Nelson's other interests include the statistical mechanics of metallic glasses, the physics of polymerized membranes, vortex phases in high-temperature superconductors and biophysics.

Professor Nelson is a member of the National Academy of Sciences, a member of the American Academy of Arts and Sciences and a Fellow of the American Physical Society; he has been an A. P. Sloan Fellow, a Guggenheim Fellow and a Junior and Senior Fellow in the Harvard Society of Fellows. He is the recipient of a five-year MacArthur Prize Fellowship, the National Academy of Sciences Prize for Initiatives in Research and the Harvard Ledlie Prize.

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To Patricia, Meredith, Christopher and Peter

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Contents

Preface to the book	xi
Acknowledgements	xiii
1 Fluctuations, renormalization and universality	1
1.1 Fluctuations and universality in condensed matter physics	2
1.2 The universal Prandtl number in two-dimensional hydrodynamics	5
1.3 The universal Poisson ratio in fluctuating polymerized membranes	8
1.4 Defect-mediated phase transitions and hydrodynamic theories	12
1.5 The contents of this book	18
<i>Appendix A Renormalization</i>	21
<i>Appendix B The self-dual point of Ising spins in two dimensions</i>	26
<i>References</i>	28
2 Defect-mediated phase transitions	30
<i>Preface</i>	30
2.1 Introduction	31
2.2 The XY model and superfluidity in two dimensions	41
2.3 Dynamic scaling and third sound in helium films	60
2.4 Statistical mechanics of two-dimensional melting	68
2.5 Melting dynamics	91
2.6 Anisotropic melting	99
2.7 Line singularities in three dimensions	106
<i>References</i>	117
3 Order, frustration and two-dimensional glass	124
<i>Preface</i>	124
3.1 Introduction	126
3.2 Order and frustration in quenched binary arrays	129
3.3 Order and frustration in spaces of incommensurate curvature	136
<i>References</i>	144

4	The structure and statistical mechanics of three-dimensional glass	146
	<i>Preface</i>	146
	4.1 A physical picture	147
	4.2 The model free energy	154
	<i>References</i>	163
5	The statistical mechanics of crumpled membranes	165
	<i>Preface</i>	165
	5.1 Flat surfaces	166
	5.2 Crumpled membranes	170
	5.3 Normal–normal correlations in liquid membranes	179
	5.4 Tethered surfaces with bending energy	181
	5.5 Defects and hexatic order in membranes	186
	<i>References</i>	191
6	Defects in superfluids, superconductors and membranes	194
	<i>Preface</i>	194
	6.1 Introduction	195
	6.2 Two-dimensional superfluids and superconductors	198
	6.3 Defects in membranes and monolayers	217
	<i>Appendix A Superfluid density and momentum correlations</i>	238
	<i>References</i>	241
7	Vortex-line fluctuations in superconductors from elementary quantum mechanics	245
	<i>Preface</i>	245
	7.1 Introduction	246
	7.2 Correlated pinning and quantum bound states	249
	7.3 Flux melting and the quantum harmonic oscillator	258
	7.4 Vortex entanglement in the liquid phase	261
	<i>Appendix A The transfer-matrix representation of the partition function</i>	266
	<i>Appendix B Vortex probability distributions</i>	267
	<i>References</i>	269
8	Correlations and transport in vortex liquids	271
	<i>Preface</i>	271
	8.1 Introduction	272
	8.2 Statistical mechanics of flexible lines	285

8.3 Correlations in flux liquids with weak disorder	301
8.4 Dynamics near the irreversibility line	311
<i>References</i>	316
9 Statistical mechanics of directed polymers	322
<i>Preface</i>	322
9.1 Introduction	323
9.2 One polymer in a nematic solvent	327
9.3 A model of polymer nematics	332
9.4 Mapping onto boson quantum mechanics	335
9.5 Correlations in polymer nematics with soft broken symmetry	341
9.6 The hydrodynamic treatment of line liquids	344
9.7 Defects in hexagonal columnar crystals	347
<i>References</i>	361
<i>Index</i>	365

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Preface to the book

Considerable relief accompanies completion of a project like this, especially one pursued (intermittently) for a period of eighteen years! There are many people to thank, not the least of which are the numerous graduate students, postdocs and other colleagues mentioned in Chapter 1 and in the acknowledgements of the remaining chapters. However, I owe a special debt to Michael Fisher, Leo Kadanoff and Bert Halperin, who provided inspiring examples of how to do theoretical physics early in my career. I am also grateful to Harvard University and the National Science Foundation of the USA, for creating the environment and freedom to do curiosity-driven research for the past twenty-five years.

I was fortunate to find a sympathetic publisher in Cambridge University Press. Simon Capelin and Rufus Neal provided expert editorial guidance and extraordinary patience in face of distractions caused by, among other things, my three years as Chair of the Harvard Physics Department. The original documents on which the last eight chapters are based illustrate the recent history of scientific publishing. At least one chapter evolved from an old-fashioned typewritten manuscript. Others originally appeared via a photographic offset printing process. The later chapters were created using LaTeX and at least one is available (in an early form) on the World Wide Web (<http://arXiv.org/abs/cond-mat/9502114>). I appreciate the willingness of the original publishers to allow me to adapt my contributions to various proceedings, summer schools and workshops. I owe a special debt to Sally Thomas, Steven Holt, Jo Clegg and Jayne Aldhouse for the expert way in which they created a seamless high-quality book from these disparate media. Saul Teukolsky, Paul Horowitz and Renate D'Archangelo kindly provided advice and assistance with the index.

Farid Abraham generously assisted with the preparation of the illustration on the front cover. For more beautiful images created by Farid and his collaborators, see his gallery (<http://www.almaden.ibm.com/vis/membrane/gallery.html>). The pictures on the back cover are double-sided decorations of flux lines in high-temperature superconductors, due to Zhen Yao, Charles Lieber and their associates. I am grateful to Zhen and Charlie for permission to use their remarkable images, which

xii Preface to the book

led to the first experimental measurement of a bosonic “phonon–roton” spectrum for entangled vortex lines. For a more detailed discussion, see the article with George Crabtree in the April 1997 issue of *Physics Today* and references therein. While going over the final page proofs, I was struck again by the many striking and experimentally observable manifestations of geometry, defects and statistical mechanics in condensed-matter physics. I hope others will be able to capture some of this excitement while reading this book.

David R. Nelson
Rhineland, Wisconsin
September, 2001

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