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Author(s): John R. Hauser and Steven M. Shugan

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DEFENSIVE MARKETING STRATEGIES*

JOHN R. HAUSER[†] AND STEVEN M. SHUGAN[‡]

This paper analyzes how a firm should adjust its marketing expenditures and its price to defend its position in an existing market from attack by a competitive new product. Our focus is to provide usable managerial recommendations on the strategy of response. In particular we show that if products can be represented by their position in a multiattribute space, consumers are heterogeneous and maximize utility, and awareness advertising and distribution can be summarized by response functions, then for the profit maximizing firm:

- it is optimal to decrease awareness advertising,
- it is optimal to decrease the distribution budget unless the new product can be kept out of the market,
- a price increase may be optimal, and
- even under the optimal strategy, profits decrease as a result of the competitive new product.

Furthermore, if the consumer tastes are uniformly distributed across the spectrum

- a price decrease increases defensive profits,
- it is optimal (at the margin) to improve product quality in the direction of the defending product's strength and
- it is optimal (at the margin) to reposition by advertising in the same direction.

In addition we provide practical procedures to estimate (1) the distribution of consumer tastes and (2) the position of the new product in perceptual space from sales data and knowledge of the percent of consumers who are aware of the new product and find it available. Competitive diagnostics, such as the angle of attack, are introduced to help the defending manager.

(Competition; Pricing; Product Entry; Defensive Marketing)

1. Perspective

Many new products are launched each year. While some of these products pioneer new markets, most are new brands launched into a market with an existing competitive structure. For every new brand launched there are often

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[†]Sloan School of Management, Massachusetts Institute of Technology, 50 Memorial Drive, Cambridge, Massachusetts 02139.

[‡]Graduate School of Business, University of Chicago, Chicago, Illinois 60637.

four to five firms (or brands) who must actively defend their share of the market. This paper investigates how the firms marketing the existing brands should react to the launch of a competitive new brand. We will refer to this topic as defensive marketing strategy.

The offensive launch of a new brand has been well studied. See, for example, reviews by Pessemier (1982), Shocker and Srinivasan (1979), Urban and Hauser (1980), and Wind (1982). Good strategies exist for brand positioning, the selection of brand features and price, the design of initial advertising strategies, and the selection of couponing, dealing, and sampling campaigns. But the analysis to support these decisions is often expensive and time consuming. (A typical positioning study takes 6 months and \$50,000 to \$100,000 (Urban and Hauser 1980, Chapter 3).) While such defensive expenditures are justified for some markets and firms, most defensive actions demand a more rapid response with lower expenditures on market research. Indeed, defending firms often routinely must determine if any response is even necessary. Little or no practical analytic models exist to support such defensive reactions.

The competitive equilibrium of markets has also been well studied. See, for example, reviews by Lancaster (1980), Lane (1980), Scherer (1980), Schmalensee (1981, 1982), and Stigler (1964). This literature provides useful insights on how markets reach equilibrium, how market mechanisms lead to entry barriers, and how product differentiation affects market equilibrium. This body of research attempts to describe how markets behave and assess the social welfare implications of such behavior. This economics research does not attempt to prescribe how an existing firm—faced with a competitive new entrant—should readjust its price, advertising expenditures, channel expenditures and product quality to optimally defend its profit.

We are fortunate to have a breadth of related concepts in marketing and economics to draw upon. However, we find that the special nature of defensive strategy requires the development of some additional new theory. Ultimately, researchers will develop a portfolio of methods to address the full complexity of defensive strategies. We choose a more selective focus by addressing an important subclass of problems that are at the heart of many defensive strategies. We limit our research to the general strategic and qualitative issues. Future research can then address other issues.

Problem Definition

Prior Information. We assume that the defending firm knows the positions of existing products (in a multiattribute space) prior to the launch of the competitive new product. We further assume that the defending firm either is (1) willing to commission a positioning study to determine the positioning of the new product or (2) has or can obtain data on sales, awareness, and availability for both the new and existing products. In the second case, we provide a procedure to estimate the new product's position from such data.

Defensive Actions. Our focus is on price, advertising expenditures (broken down by spending on awareness and on repositioning), distribution expenditures, and product improvement expenditures. We do not address detailed

allocation decisions such as the advertising media decisions or timing decisions; we assume that once the level of an advertising or distribution budget is set that standard normative marketing techniques are used to allocate resources within this budget. See Aaker and Myers (1975), Blattberg, Eppen and Lieberman (1981), Stern and El-Ansary (1982), and Kotler (1980). This paper does not explicitly address the counter-launch of a "me-too" or "second-but-better" new product as a defensive strategy. Our analyses enable the firm to evaluate non-counter-launch strategies. Once such strategies are identified they can be compared to the potential profit stream from a counter-launch strategy.

Direction of First Response. By its very nature, defensive strategy often calls for a rapid response. If firms had full and perfect market research information they might be able to optimize strategy, but often such information is not available. However, we can make statements about the direction of response (e.g., increase or decrease distribution spending) which are surprisingly robust over a wide range of circumstances. As Little (1966) found, such directional responses can be almost as valuable as optimal responses, especially with noisy data. We feel it is important to the development of a defensive marketing theory to know such directional forces and focus our attention on such results.

Equilibrium Issues. Economists such as Lancaster (1971, 1980) and Lane (1980) have identified equilibria to which multiattributed markets evolve. But to do so they have had to simplify greatly the complexity of such markets. For example, Lane (1980, p. 239) assumes that (1) market size is fixed, (2) consumers have uniformly distributed tastes, (3) consumers have perfect information and all products are available to them, (4) all firms face identical cost structures, (5) once firms choose their respective positions they cannot change them, (6) firms choose their prices as if all other firms will not change in response, and (7) various technical conditions. Even so, he must resort to simulation for specific results beyond a proof of the existence of Nash equilibria.

We wish to analyze a more complex problem. We allow the firm to set advertising and distribution as well as pricing, and we allow the firm to reposition. Consumers may vary in awareness of products and products may vary in availability. Consumers' tastes need not be uniform and firms' cost structures may vary.

For such marketing richness we pay a price; we limit our analyses to how the target firm should react to a new product. We assume all products, except the new product and the product of the target firm, remain passive. We feel that this is an important step toward a multiproduct equilibrium. For example, one research direction might be to explore the equilibrium that results if all firms iteratively followed our theories.

There is some evidence that special cases of our analyses, e.g., price response for uniformly distributed tastes, are consistent with special cases of equilibrium analyses found in the economic literature. (Our theorem and Lane's simulation both predict price decreases (Lane 1980, Tables 1 and 2).) Thus, as a working hypothesis, subject to test by future researchers, we posit

the *directional results* of our theorems for a two-product equilibrium will not change for a multiproduct equilibrium. By recognizing this limitation we hope to encourage research into full market equilibria with models that are rich in marketing phenomena.

Deductive Analyses. This paper is deductive from a simple model of consumer response. We attempt to make our assumptions clear. Insofar as our model accurately abstracts reality, the results are true. But we caution the manager to examine his situation and compare it to our model before using our results. We expect future research to examine and overcome the limitations of our analysis both empirically and theoretically.

Overview of the Paper

§2 presents the consumer model and introduces notation and definitions that prove useful in our analyses. §3 derives a set of theorems that indicate normatively how a defending firm should adjust its price, advertising budget, distribution budget, and product positioning in response to a new product. §4 indicates how to estimate the distribution of consumer tastes, §5 indicates how to estimate the new product's position, §6 suggests some competitive diagnostics, §7 discusses the issue of validating deductive models, and §8 discusses limitations of our analyses and suggests future research. All proofs are placed in the appendices. In addition, we provide a glossary of technical definitions.

We begin with the consumer model.

2. Consumer Model

Our managerial interest is at the level of market response. Thus, our primary concern in modeling consumers is to predict how many consumers will purchase our product and our competitors' products, under alternative defensive marketing strategies. However, to promote the evolution of defensive analysis we build up market response as resulting from the response of individual consumers to market forces. Although defensive models may ultimately incorporate complex micro-models such as those reviewed by Bettman (1979), and decision-making cost (e.g., Shugan 1980), we begin with several simplifying assumptions such as utility maximization. In this way, we can deal directly with aggregation across consumers.

A further requirement of a defensive model is that it be sensitive to how a new product differentially impacts each product. Finally, we require a model that incorporates important components of consumer behavior, but is not too complex to be inestimable from available data. The following assumptions state the tradeoffs we have made.

Assumptions

We assume: (1) Existing brands can be represented by their position in a multiattribute space such as the one shown in Figure 1. The position of the brand represents the amount of attributes the consumer can obtain by spending one dollar on that brand. (2) Each consumer chooses the brand that

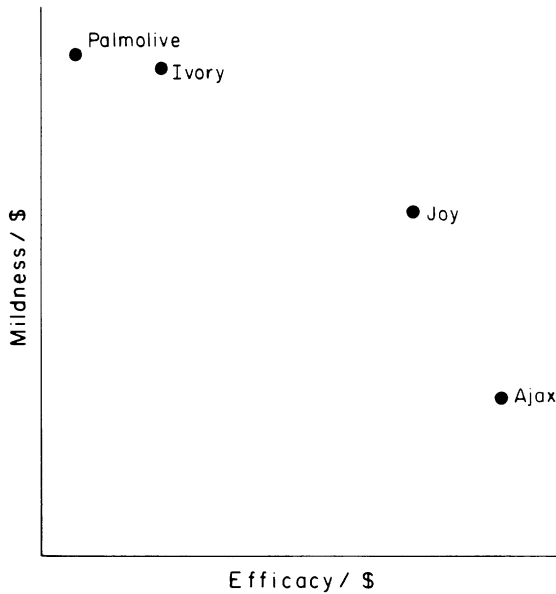


FIGURE 1. Hypothetical Perceptual Map of Four Liquid Dish Washing Detergents.

maximizes his utility, and (3) The utility of the product category is a concave function of a summary measure linear in the product attributes (or some linear transformation of the product attributes). This assumption allows us to represent the brand choice decision with a linear utility function; however, we do not require the actual utility function to be linear. (4) The effect of “awareness” and “availability” can be modeled by advertising and distribution response functions as defined below. This fourth assumption allows us to model consumer response when consumers do not have full information about all products, when they are interested in only a subset of the products, or when products are not available to all consumers. This assumption is particularly important when markets are in turmoil as they are after a new product launch.

Assumption 1, representation by a product position, is commonly accepted in marketing. See Green and Wind (1975), Johnson (1971), Kotler (1980), Pessemier (1982), and Urban (1974). Scaling by price comes from a budget constraint applied to the bundle of consumer purchases and from separability conditions that allow us to model behavior in one market (say liquid dish washing detergents) as independent of another market (say deodorants). (That is, the brand of deodorant used does not influence the brand of liquid dishwashing detergent selected.) See Blackorby, Primont, and Russell (1975) and Strotz (1957). This implicit assumption is discussed in Hauser and Simmie (1981), Hauser and Urban (1982), Keon (1980), Lancaster (1971, Chapter 8), Ratchford (1975, 1979), and Srinivasan (1982).

Dividing by price implicitly assumes that the dimensions of the multiattributed space are ratio scaled. This is clearly true for most physical attributes,

say miles per gallon, but may require special measurement techniques for more qualitative attributes such as 'efficacy.' We require only that such scales exist, not that they are easy to measure. Practical measures remain an important empirical question; however some progress has been made. For example, Hauser and Simmie (1981) present methods to obtain appropriate ratio scales for perceptual dimensions and Hauser and Shugan (1980) derive statistical tests for the necessary ratio-scaling properties.

Assumption 2, utility maximization, is reasonable for a market level model. At the individual level stochasticity (Bass 1974, Massy, Montgomery and Morrison 1970), situational variation, and measurement error make it nearly impossible to predict behavior with certainty. At the market level, utility maximization by a group of heterogeneous consumers appears to describe and predict sufficiently well to identify dominant market forces. For example, see Givon and Horsky (1978), Green and Rao (1972), Green and Srinivasan (1978), Jain *et al.* (1979), Pekelman and Sen (1979), Shocker and Srinivasan (1979), Wind and Spitz (1976), and Wittink and Montgomery (1979). Utility maximization is particularly reasonable when we define utility on a perceptual space because perceptions are influenced by other product characteristics and psychosocial cues such as advertising image, and hence already incorporate some of the effects due to information processing. Finally, any individual level stochasticity that results from randomness in consumers' tastes can be readily incorporated in our model by including such randomness in the market distribution of consumers' tastes. Such stochasticity when independent across individuals in no way affects our results.

Assumption 3, linearity, is a simplification to obtain analytic results. With linearity we sacrifice generality but obtain a manageable model of market behavior. In many cases the linearity assumption can be viewed as an approximation to the tangent of each consumer's utility function in the neighborhood of his chosen product. Furthermore, Green and Devita (1975) show that linear preference functions are good approximations, and Einhorn (1970) and Einhorn, D. Kleinmuntz and S. Kleinmuntz (1979) present evidence based on human information processing that justifies a linear approximation. In our theorems we treat the utility function as linear in the product attributes; however it is a simple matter to modify these theorems for any utility function that is linear in the 'taste' parameters that vary across the population. For example, we can handle the standard ideal point models since utility is linear in the squared distance, along each attribute, from the ideal point. However, for the pricing theorem we must be careful in modeling the price scaling involved in any transformation. In general, our analysis can incorporate much of the same class of nonlinear utility functions discussed in McFadden (1973).

Assumption 4, response function analysis, is a standard approximation in marketing that is based on much empirical experience (e.g., see Blattberg and Golanty 1978, Hauser and Urban 1982, Kotler 1980, Little 1979, Stern and El-Ansary 1982, Urban 1974). A response function relates firm expenditures to outcomes. For example, if k_d dollars are spent optimally on distribution (transportation, inventory, channel incentives, sales force, etc.), then we as-

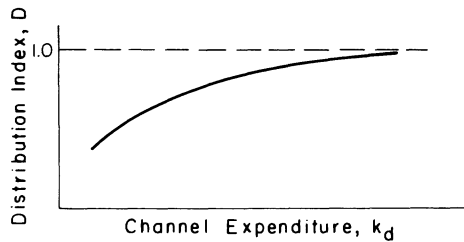


FIGURE 2. An Example Distribution Response Function Exhibiting Decreasing Marginal Returns over the Relevant Range.

sume that sales are proportional to $D(k_d)$ where $D(\cdot)$ is the response function. Such functions always exist. Our assumption concerns their properties, i.e., that beyond a certain point the functions are marginally decreasing ($\partial^2 D / \partial k_d^2 < 0$) as in Figure 2 and proportionality holds over the range of analysis.

Assumption 4 is not as restrictive as it might appear. Our theorems require only that the firm is operating on the concave portion of the response function prior to the entry of the new product. This is reasonable, since, even if the response function were S-shaped, most optimization analyses would suggest operating on either the concave portion of the response function or at zero expenditure. Assumption 4 also implies multiplicative interactions among marketing expenditures but rules out more complex interactions. Based on the evidence summarized by Little (1979) we believe Assumption 4 is a valid beginning.

For ease of exposition we call the response analysis effect of advertising expenditures, "awareness," and the response analysis effect of distribution expenditures, "availability." We caution the reader that advertising and distribution affect, but are not limited to, consumer awareness of a product and the availability of the product in the store. Response functions, and our theorems, refer to the more general effects. The labels are for convenience of exposition. Finally, in addition to response analysis, we consider advertising expenditures directed at specific attributes.

Sensitivity to Assumptions

The set of four assumptions about consumer response is sufficient to prove a set of normative theorems which indicate qualitatively how a defending firm should adjust its marketing strategy. However, as summarized in Table 1, each assumption only affects some of the theorems. The others are independent of that particular assumption.

We have selected theorems which we feel are reasonably robust with respect to our consumer model. We have already indicated that the results hold for stochasticity of tastes and some nonlinearities in utility. It is possible that the theorems may hold for other relaxations of the assumptions. We leave such relaxations for future research.

We make a final assumption for ease of exposition in the pricing and repositioning theorems. We limit our analyses to two product attributes. Two attributes make possible pictorial representation and provide valuable intu-

TABLE I
Assumptions of Theorems

Theorems	Assumptions			Response Functions
	Price Scaling	Utility Maximization	Linear Tastes	
1. Profit				
2. Price (Uniform tastes)	x	x	x	
3. Price (Dominance)	x	x	x	
4. Price (Monotonic tastes)	x	x	x	
5. Price (Segmented tastes)	x	x	x	
6. Price decoupling				x
7. Distribution				x
8. Pre-emption				x
9. Repositioning (product improvement)		x	x	
10. Advertising				x
11. Repositioning (advertising)		x	x	
12. Price (Irregular markets)	x	x	x	
13. Price (General)	x	x	x	
14. Evoked Sets				

"x" indicates our proof of the theorem uses of this assumption.

ition into the marketing forces which drive the theorems. As Schmalensee (1982) indicates, there is some evidence that results derived for one or two price-scaled attributes hold for three attributes. However, he also suggests that little is known about extensions to an arbitrary number of attributes.

Notation

By assumption, products are represented by their attributes. Let x_{1j} be the amount of attribute 1 obtainable from one unit of brand j . Define x_{2j} to be the amount of attribute 2 obtainable from one unit of brand j . (Note: $\infty > x_{ij} \geq 0$ for $i = 1, 2$ and x_{ij} is ratio-scaled.) Let p_j be the price per unit of brand j . Let \tilde{u}_j be the utility that a randomly selected consumer places on purchasing brand j . Note that \tilde{u}_j is a random variable due to consumer heterogeneity. If every

consumer is aware of each brand and finds it available, all brands will be in everyone’s choice set, otherwise choice sets will vary. Let A be the set of all brands, let A_l , a subset of A , be the set of brands called choice set l , and let S_l be the probability that a randomly chosen consumer will select from choice set l for $l = 1, 2, \dots, L$. See Silk and Urban (1978) for empirical evidence on the variation of choice sets.

Mathematical Derivation

We first compute the probability, m_j , that a randomly chosen consumer purchases brand j . In marketing terms, m_j is the market share of product j if all consumers purchase the same number of products per time period.¹ Applying assumption 2, utility maximization, we obtain equation (1) by definition.

$$m_j = \text{Prob}[\tilde{u}_j > \tilde{u}_i \text{ for all } i \neq j] \quad \text{where Prob}[\cdot] \text{ is a probability function.} \tag{1}$$

In addition, define $m_{j|l} = \text{Prob}[\tilde{u}_j > \tilde{u}_i \text{ for all } i \neq j \text{ in } A_l]$. In marketing terminology, $m_{j|l}$ is the market share of product j among those customers who evoke A_l . Now, equation (1) and assumption 3, linearity, imply

$$m_{j|l} = \text{Prob}[(\tilde{w}_1 x_{1j} + \tilde{w}_2 x_{2j})/p_j > (\tilde{w}_1 x_{1i} + \tilde{w}_2 x_{2i})/p_i \text{ for all } i \in A_l], \tag{2}$$

where $i \in A_l$ denotes all products contained in choice set l . Here \tilde{w}_1 and \tilde{w}_2 are relative “weights” a randomly selected consumer places on attributes 1 and 2, respectively. Suitable algebraic manipulation of equation (2) yields

$$m_{j|l} = \text{Prob}[(x_{1j}/p_j - x_{1i}/p_i) > (\tilde{w}_2/\tilde{w}_1)(x_{2i}/p_i - x_{2j}/p_j) \text{ for all } i \in A_l]. \tag{3}$$

Moreover, equation (3) is equivalent to

$$m_{j|l} = \text{Prob}[\tilde{w}_2/\tilde{w}_1 < r_{ij} \text{ for all } i \in A'_l \text{ and } \tilde{w}_2/\tilde{w}_1 > r_{ij} \text{ for all } i \in A''_l], \tag{4}$$

where

$$r_{ij} = (x_{1j}/p_j - x_{1i}/p_i)/(x_{2i}/p_i - x_{2j}/p_j),$$

$$A'_l = \{i \mid i \in A_l \text{ and } x_{2i}/p_i > x_{2j}/p_j\},$$

$$A''_l = \{i \mid i \in A_l \text{ and } x_{2i}/p_i < x_{2j}/p_j\}.$$

¹We show later that m_j is still the market share if consumers vary in the quantity that they purchase. This will become clear when we introduce consumer heterogeneity with a taste distribution, $f(\alpha)$, that can incorporate multiple purchases by counting consumers in proportion to their purchase frequency. Our empirical procedure automatically adjusts for this phenomenon. See also footnote 3.

Note that equation (4) illustrates that \tilde{w}_2/\tilde{w}_1 is a sufficient parameter for computing choice probabilities and that $r_{ij} = r_{ji}$. Now consider Figure 3a. Pictorially, 'Joy' will be chosen if the angle of the indifference curve defined by (w_2, w_1) lies between the angle of the line connecting 'Ivory' and 'Joy' and the line connecting 'Joy' and 'Ajax!' Note that consumers will only choose those brands, called *efficient brands*, that are not dominated on both dimensions by another product in the evoked set.² Hence, we consider only efficient brands in evoked sets. (See the glossary for a verbal summary of our definitions.)

We simplify equation (4) and make it easier to visualize by introducing new notation. Let $\tilde{\alpha} = \tan^{-1}(\tilde{w}_2/\tilde{w}_1)$ and let $\alpha_{ij} = \tan^{-1}r_{ij}$. The angle, $\tilde{\alpha}$, represents a measure of consumer preference that varies between 0° and 90° . As indicated in Figure 3b, $\tilde{\alpha}$ is the angle the utility indifference curve makes with the vertical axis. Figures 3a, b, and c graphically illustrate the mathematical formulation. As can be seen from Figure 3a, three brands of liquid dishwashing detergent are efficiently positioned. The angles α_{23} and α_{12} are $\tan^{-1}r_{23}$ and $\tan^{-1}r_{12}$ respectively. Moreover, these angles represent the relative positions of Ivory, Joy and Ajax. Figure 3b shows how consumer preferences can be represented. Consumers with w_1 and w_2 as shown in the figure (w_1 and w_2 are different for different consumers) would prefer Ivory because Ivory touches those consumers' highest indifference curve. Note that whenever $\tan^{-1}(w_2/w_1)$ is greater than α_{23} then Ivory is preferred. Figure 3c illustrates the regions where each product is preferred. If $\tan^{-1}(w_2/w_1)$ exactly equalled α_{23} , then those consumers would be indifferent between Joy and Ivory. For continuous distributions, these ties occur only on a set of measure zero.

Moreover, we introduce the convention of index numbering brands such that $x_{2j}/p_j > x_{2i}/p_i$ if $j > i$. This will assume that index numbers (and α_{ij} 's) increase counter-clockwise for efficient brands in A_l for any l . For boundary conditions let $\alpha_{01} = 0^\circ$ and $\alpha_{j\infty} = 90^\circ$.

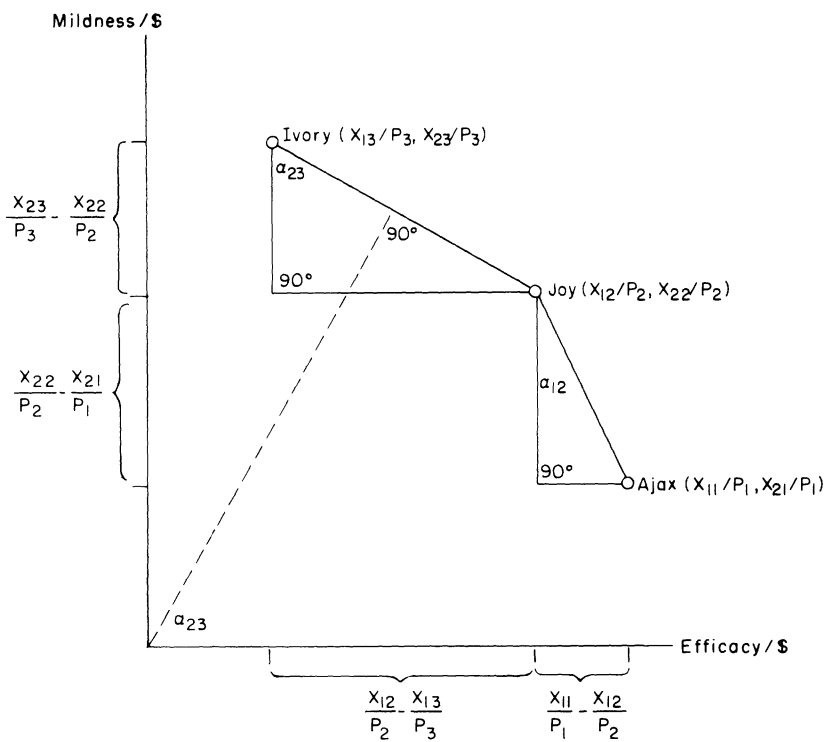
Finally, we introduce the definition of *adjacency*. In words, a lower (upper) adjacent brand is simply the next efficient brand in A_l as we proceed clockwise (counter-clockwise) around the boundary. Mathematically, a brand, $j -$, is lower adjacent to brand j if (1) $j - < j$, (2) $j - \geq k$ for all $k < j$ where both k and j are contained in A_l and (3) both brands are efficient. Define the upper adjacent brand, $j +$, similarly. Note that $j -$ and $j +$ depend on the choice set A_l . With these definitions $m_{j|l}$ is simply computed by

$$m_{j|l} = \text{Prob}[\alpha_{j-} < \tilde{\alpha} \leq \alpha_{j+} \text{ for } \alpha_{j-}, \alpha_{j+} \text{ defined by } A_l]. \quad (5)$$

We now introduce consumer heterogeneity in the form of a variation in tastes across the consumer population. Since each consumer's utility function is defined by the angle, $\tilde{\alpha}$, we introduce a distribution,³ $f(\alpha)$, on the angles.

²Our definition of efficiency assumes utility is monotonic in x_{1j} and x_{2j} . Ideal points can be handled by redefining the scale to be monotonic in distance from the ideal point.

³To assure that $m_{j|l}$ is the market share, we assume that consumers are counted in proportion to purchase frequency. This will be true automatically when $f(\alpha)$ is calibrated on sales data.



where $\alpha_{23} = \tan^{-1} (X_{12}/P_2 - X_{13}/P_3) / (X_{23}/P_3 - X_{22}/P_2)$

$\alpha_{12} = \tan^{-1} (X_{11}/P_1 - X_{12}/P_2) / (X_{22}/P_2 - X_{21}/P_1)$

FIGURE 3a. Geometric Illustration of the Relationship in Equation (3) for a Three Brand Market.

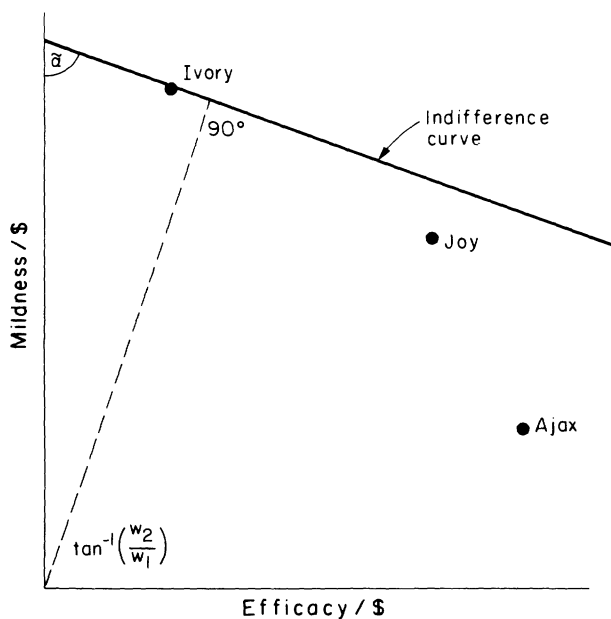


FIGURE 3b. Geometric Illustration of the Relationship in Equation (3) for a Three Brand Market.

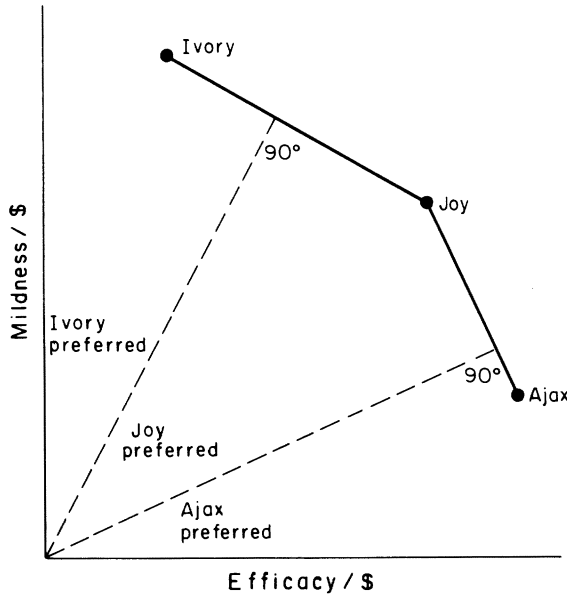


FIGURE 3c. Geometric Illustration of the Relationship in Equation (3) for a Three Brand Market.

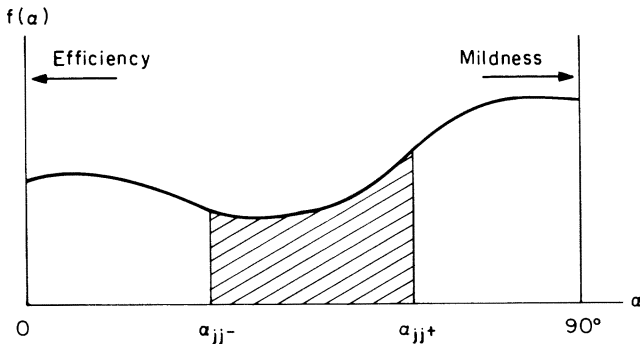


FIGURE 4. Hypothetical Distribution of Tastes with Respect to Efficacy and Mildness. The Shaded Region Represents Those Consumers Who Will Choose 'Joy.'

($f(\alpha)$ may depend on l .) One hypothetical distribution is shown in Figure 4. As shown, small angles, $\alpha \rightarrow 0$, imply that consumers are more concerned with attribute 1, e.g., efficacy, than with attribute 2, e.g., mildness; large angles, $\alpha \rightarrow 90^\circ$, imply a greater emphasis on attribute 2. This method of introducing heterogeneity is similar to models proposed by Blin and Dodson (1980), Pekelman and Sen (1975), and Srinivasan (1975). Equation (5) now becomes

$$m_{j|l} = \int_{\alpha_{jj-}}^{\alpha_{jj+}} f(\alpha) d\alpha. \tag{6}$$

Thus, $m_{j|l}$ is the shaded area in Figure 4. Finally, if the choice sets vary, the

total market share, m_j , of brand j is given by

$$m_j = \sum_{l=1}^L m_{j|l} S_l \quad \text{where } l = 1, 2, \dots, L. \tag{7}$$

An interesting property of equation (7) is that a brand can be inefficient on A , but have nonzero total market share if it is efficient on some subset with nonzero selection probability, S_l . This property, which is consistent with empirically observed consumer behavior, distinguishes our marketing model from the models assumed by Lancaster (1980) and Lane (1980).

As an illustration for

$$A = \{ \text{Ajax } (j = 1), \text{ Ivory } (j = 3), \text{ Palmolive } (j = 4) \}$$

suppose that the positions of the three brands are given by $(x_{1j}/p_j) = (5, 1), (2, 4.6),$ and $(1, 5)$ for $j = 1, 3, 4,$ respectively. We first compute r_{ij} , finding $r_{13} = 3/3.6 = 0.83$ and $r_{34} = 1/(0.4) = 2.5$. Finding the angle whose tangent is r_{ij} gives us $\alpha_{01} = 0^\circ, \alpha_{13} = 40^\circ, \alpha_{34} = 68^\circ,$ and $\alpha_{4\infty} = 90^\circ$. For this example 'Ajax' is lower adjacent to 'Ivory' and 'Palmolive' is upper adjacent to 'Ivory.' See Figure 5.

If tastes are uniformly distributed over $\hat{\alpha}$, then $f(\alpha) = (1/90^\circ)$ and $m_{j|l} = (\alpha_{jj+} - \alpha_{jj-})/90^\circ$. For our example, if everyone evokes choice set A , market shares are 0.44 for 'Ajax,' 0.32 for 'Ivory,' and 0.24 for 'Palmolive.'

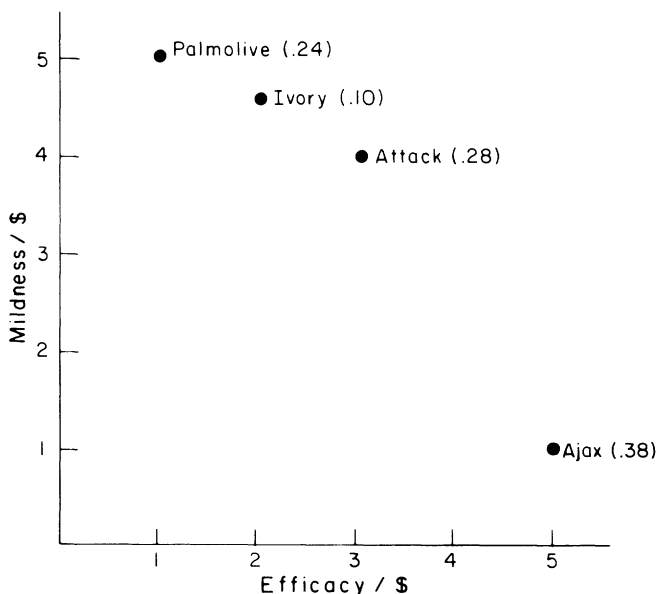


FIGURE 5. Hypothetical Market after the Entry of the New Product 'Attack' (Numbers in Parentheses Indicate New Market Shares).

Suppose a new brand, 'Attack,' is introduced at (3, 4) and suppose it is in everyone's choice set. Since 'Attack' is positioned between 'Ajax' and 'Ivory' we number it $j = 2$. See Figure 5. We compute $r_{12} = 0.67$, $r_{23} = 1.67$, $\alpha_{12} = 34^\circ$, and $\alpha_{23} = 59^\circ$. The new market shares are 0.38 for 'Ajax,' 0.28 for 'Attack,' 0.10 for 'Ivory' and 0.24 for 'Palmolive.' Thus 'Attack' draws its share dominantly from 'Ivory,' somewhat from 'Ajax,' and not at all from 'Palmolive.' 'Ivory' will definitely need a good defensive strategy!

Clearly the threat posed by the new brand depends upon how it is positioned with respect to the defending firm's brand. We formalize this notion in §6. §4 derives estimates for $f(\alpha)$ and §5 derives estimates of the new product's position.

We turn now to the analysis of general strategies for defensive pricing, advertising, distribution incentives, and product improvement which depend only on general properties of $f(\alpha)$, the new product's position, and response functions. The theorems in the following section are based on the consumer model derived above. For analytic simplicity, unless otherwise noted, we base our derivations on the full product set, A . Extension to variation in evoked sets is discussed in §5.

3. Strategic Implications

We sequentially examine pricing, distribution, product improvement, and advertising.

Pricing Strategy

For simplicity we assume that advertising and distribution strategies are fixed. We relax this assumption in the following sections.

Based on the consumer model, equation (3) shows profit for the defending brand, j , before entry of the new brand. We simplify the exposition by suppressing fixed costs which, in no way, affect our analysis.⁴

$$\Pi_b(p) = (p - c)N_b \int_{\alpha_{jj-}}^{\alpha_{jj+}} f(\alpha) d\alpha. \quad (8)$$

Here N_b is the number of consumers before any competitive entry, c is our per unit costs,⁵ p is our price (we are brand j) and $\Pi_b(p)$ is our before entry profit given price p . Note α_{jj+} and α_{jj-} are functions of price.

The new brand can enter the market in a number of ways. (1) If it is

⁴Fixed costs, as usual in economic analysis, do not come into play until one considers barriers to entry and exit. See Schmalensee (1981, 1982), Scherer (1980) and Stigler (1964). Note also that category demand is modeled by N_b . Thus, our explicit model is primarily a market share model. Category demand impacts profit through N_b and (in equation (9)) through N_a . We allow the new product to affect category demand but we do not derive the effect explicitly from our consumer model.

⁵Note the implicit assumption of constant production costs to scale. Most of our theorems can be relaxed for nonconstant costs, but the statements of the theorems become more mathematical and less intuitive.

inefficient, it does not affect our profits. (2) If it exactly matches one of our competitors, our profit is unchanged. (3) If the new brand is not adjacent in the new market, our profits are unchanged. Conditions (1)–(3) are based on the recognition that none of our consumers is lost to the new brand.⁶ Defensive strategy only becomes important under condition (4) $j^+ \geq n \geq j^-$ where n denotes the index number of the new brand and, again, j denotes the defending brand. Since the problem is symmetric with respect to $j^+ \geq n > j$ and $j > n \geq j^-$, we analyze the former case where the new product is efficient and upper adjacent. The latter case follows from transposing the axes.

In case (4), our profits (i.e., the defending brand) after the launch of the competitive new brand, given our price p , are given by

$$\Pi_a(p) = (p - c)N_a \int_{\alpha_{j^-}}^{\alpha_{j^+}} f(\alpha) d\alpha. \quad (9)$$

Here, N_a is the market volume after the competitive entry.

If we lose no customers, the new brand is no threat, thus we are only concerned with the case of $f(\alpha)$ not identically zero in the range α_{j^-} to α_{j^+} . Call this case for an adjacent product, *competitive entry*.

We begin with the intuitive result that we cannot be better off after the new product attacks our market than we could have been by acting optimally before the attack. We state the result formally because it is used to derive later results. (The proofs of Theorem 1, and all subsequent theorems are in the Appendix.)

THEOREM 1. *If total market size does not increase, optimal defensive profits must decrease if the new product is competitive, regardless of the defensive price.*

Theorem 1 illustrates the limits of defensive strategy. Unless the market is increasing at a rate that is rapid enough to offset the loss due to competitive entry, the competitive entry will lower our profit even with the best defensive price. Theorem 1 is true even if the market is growing when growth is from consumers exclusively buying the new brand. Moreover, even if general growth is occurring, provided that our growth is not due to the new brand, it is easy to show that the competitor decreases potential profits.

A key assumption in Theorem 1 is that the defending firm is acting optimally before the new brand enters. However, there exist cases where a new brand awakens a “sleepy” market, the defending firm responds with an active defensive (heavy advertising and lower price), and finds itself with more sales and greater profits. A well-known case of this phenomenon is the reaction by Tylenol to a competitive threat by Datril. Before Datril entered the market actively, Tylenol was a little known, highly priced alternative to aspirin. Tylenol is now the market leader in analgesics. Theorem 1 implies that Tylenol could have done at least as well had it moved optimally before Datril entered.

⁶In a full equilibrium analysis, conditions (1)–(3) are only approximate since nonadjacent brands can affect our profits by causing adjacent brands to reposition or change their price.

Theorem 1 states that no pricing strategy can regain the before-entry profit. Nonetheless, optimal defensive pricing is important. Defensive profits with an optimal defensive price may still be significantly greater than defensive profits with the wrong defensive price.

We begin our study of price response by examining markets which are not highly segmented. In particular, we investigate the market forces present when preferences are uniformly distributed. This line of reasoning is not unlike that of Lancaster (1980) who assumes a different form of uniformity to study competition and product variety.⁷ As Lancaster (1980, p. 283) states: Uniformity provides “a background of regularity against which variations in other features of the system can be studied.”

For our first pricing theorem we introduce an empirically testable market condition which we call *regularity*. Let θ_j be the angle of a ray connecting product j to the origin, $\theta_j = \tan^{-1}(x_{2j}/x_{1j})$. Then a market is said to be regular if product j 's angle, θ_j , lies between the angles, α_{jj-} and α_{jj+} , which define the limits of the tastes of product j 's consumers. Regularity is a reasonable condition which we expect many markets to satisfy, however, as intuitive as regularity seems it is not guaranteed. For example, in the market with products positioned at (2,1,0), (2,2), and (1,10), we find that $\theta_2 > \alpha_{23}$, i.e., $\tan \theta_2 = 1$ and $\tan \alpha_{23} = 1/8$. We present the following theorem for regular markets and then discuss its extension to irregular markets.

THEOREM 2. Defensive Pricing. *If consumer tastes are uniformly distributed and the market is regular, then defensive profits can be increased by decreasing price.*

Theorem 2 provides us with useful insight on defensive pricing; insight that with experience can evolve into very usable “rules of thumb.” For example, when some managers find the market share of their brand decreasing they quickly increase the price in the hopes of increasing lost revenue. Historically, transit managers fall into this class (witness the 1980–1981 fare increases in both Boston and Chicago). Other managers believe an aggressive price decrease is necessary to regain lost share. This strategy is common in package goods. Theorem 2 shows that if consumer tastes are uniform, the price decrease is likely to be optimal in terms of profit. The result itself may not surprise the aggressive package goods managers, but the general applicability of the result to regular markets is interesting. Note that Theorem 2 is true even if the market size is greater after the new entrant.

We now examine what happens if we relax the assumptions of regularity and uniformly distributed tastes.

Irregular Markets. Regularity is sufficient to formally prove Theorem 2, however regularity is not necessary. As the Appendix indicates we use only $\alpha_{jj+} \geq \theta_j$ to prove that profits are decreasing in price. Even these conditions are not necessary. For example, the following theorem is true in both regular and irregular markets. For further results see Theorem 12 in the Appendix.

⁷Lancaster's uniformity assumption is more complex than an assumption of uniform tastes. However, to derive his results he also assumes a form of uniform tastes. See also analyses by Lane (1980), who assumes uniformly distributed tastes.

THEOREM 3. Defensive Pricing. *If the new brand's angle with our brand (α_{jn}) plus the upper adjacent brand's angle with our brand (α_{jj+}) exceeds 90° and if the distribution of consumer tastes is uniform, then the defensive profits will be increased by decreasing price.*

Because we have not been able to develop a general proof of Theorem 2 for irregular markets, we simulated 1,700,000 randomly generated irregular markets. In all cases, profits were found to be decreasing in price. Since the functions in question are reasonably well behaved, such a simulation on a compact set suggests that the theorem also holds for irregular markets. At the very least, violation is extremely rare.

Nonuniform Taste Distributions. We, as the economic literature that has preceded us, have obtained formal results for markets in which tastes are uniformly distributed. Such markets are expected to be reasonably representative of unsegmented markets and thus are a useful beginning to understand price response. We now investigate whether Theorem 2 is robust with respect to nonuniform distributions. Our first result shows that a uniform distribution may be sufficient but not necessary. (We state the result for regular markets, but it is easy to prove that if Theorem 2 is true in irregular markets then so is Theorem 4.)

THEOREM 4. *For an upper adjacent attack in a regular market, if $f'(\alpha) \leq 0$, then defensive profits can be increased by decreasing price from the before-entry optimal. For a lower adjacent attack, the result holds if $f'(\alpha) \geq 0$.*

Theorem 4 suggests that Theorem 2 may hold for consumer tastes, $f(\alpha)$, represented by common probability distributions that are monotonically decreasing (increasing) over their ranges. Such distributions include the triangle distribution and some cases of the exponential and Beta distributions. See Drake (1967) for examples of such distributions. However, all markets which satisfy Theorem 4 are unsegmented by definition.

Consider the following example of highly segmented market. Suppose $f(\alpha)$ is discrete taking on values only at $f(14^\circ) = 1/27$, $f(18.5^\circ) = 15/27$, $f(33.7^\circ) = 10/27$, and $f(84^\circ) = 1/27$. Suppose we are positioned at $(x_{1j}/p, x_{2j}/p) = (11/p, 50/p)$ and our two adjacent competitors are positioned at $(1, 60)$ and $(21, 20)$. If $c = \$9/\text{unit}$ and $N_b = 270,000$ our profit at various price levels is given in Table 2. By inspection, the optimal price is the highest price, \$1.00, that captures both segments 2 and 3. Suppose now that the new brand enters at $(16, 40)$ and that $N_a = N_b$. By inspection of Table 2, the optimal price is the highest price, \$1.03, that captures segment 3. Since the optimal price after entry exceeds that before entry, we have generated an example where it is optimal to *increase* price.

We have shown this result to many marketing managers of both consumer and industrial goods who have found it surprising. Thus, we state the result as a theorem for emphasis.

THEOREM 5. *There exist distributions of consumer tastes for which the optimal defensive price requires a price increase.*

With careful inspection, the example used to illustrate Theorem 5 is quite

TABLE 2
Example to Demonstrate a Case of the Optimal Defensive Price Requiring a Price Increase (⁰ Indicates Optimal Price before Competitive Entry, * Indicates Optimal Price after Entry)

Price	Before Entry		After Entry	
	Volume	Profit	Volume	Profit
under 0.90		< 0		< 0
0.90	260,000	0	260,000	0
0.91	250,000	2,500	250,000	2,500
0.92	250,000	5,000	250,000	5,000
0.93	250,000	7,500	250,000	7,500
0.94	250,000	10,000	100,000	4,000
0.95	250,000	12,500	100,000	5,000
0.96	250,000	15,000	100,000	6,000
0.97	250,000	17,500	100,000	7,000
0.98	250,000	20,000	100,000	8,000
0.99	250,000	22,500	100,000	9,000
1.00 ⁰	250,000 ⁰	25,000 ⁰	100,000	10,000
1.01	100,000	11,000	100,000	11,000
1.02	100,000	12,000	100,000	12,000
1.03*	100,000	13,000	100,000*	13,000*
1.04	100,000	14,000	0	0
1.05	100,000	15,000	0	0
1.06	100,000	16,000	0	0
1.07	100,000	17,000	0	0
1.08	100,000	18,000	0	0
1.09	0	0	0	0
over 1.09	0	0	0	0

intuitive. The key idea in the example is that there are two dominant segments in the market. Thirty-seven percent (10/27) have a slight preference for attribute 1, $w_2/w_1 = \tan \alpha = 2/3$; 56 percent (15/27) have a stronger preference for attribute 1, $w_2/w_1 = 1/3$. Before the competitive entry, we were selling to both markets. Segment 3 clearly preferred our brand. However, we were forced to lower our price to compete for segment 2 for whom our product is less preferred. It was profitable to do so because of its large size. When the new brand entered, targeted directly at segment 2, we could no longer compete profitably for segment 2. However, since we are still well positioned for segment 3 and they are willing to pay more for our brand, we raise our price to its new optimal profit level. This situation may often occur near the end of a product's life cycle when differentiated products have attracted all but the most satisfied customers. In the social welfare sense, consumers must pay a price for variety. (In Table 2 the price increase is only 3¢, but it is possible to generate examples where the price increase is very large.)

We can, of course, generate necessary and sufficient conditions which identify when a price decrease is optimal (see Theorem 13 in the Appendix for necessary conditions), but such conditions are excessively algebraic and do not appear to provide any useful intuition for the manager. On the other hand, we find Theorems 2, 3, 4, and 5 to be quite usable because they provide

results for interpretable taste distributions. Theorems 2, 3, and 4 suggest that a price decrease is best in unsegmented markets. Theorem 5 suggests that in a highly segmented market, a price increase may be optimal.

Summary. Together Theorems 2, 3, 4, and 5 provide the manager with guidelines to consider in making defensive pricing decisions. Our proven results are specific, but we can generalize with the following propositions which are stated in the form of managerial guidelines:

- If the market is highly segmented and the competitor is attacking one of your segments, examine the situation carefully for a price increase may be optimal.

- If the market is not segmented and consumer tastes are near uniform, a price decrease usually leads to optimal profits. Finally, it is possible to show that if a competitive brand leaves the market and consumer tastes are near uniform, a price increase usually leads to optimal profits.

Distribution Strategy

One strategy to combat a new entrant is for the defending firm to exercise its power in the channel of distribution. Channel power is a complex phenomenon, but to understand defensive distribution strategy we can begin by summarizing the phenomenon with response functions as described in §2.

In response analysis we assume that sales are proportional to a distribution index, D . The index is in turn a function of the effort in dollars, k_d , that we allocate to distribution assuming we spend k_d dollars optimally in the channel.

Under the conditions of response analysis, profit after competitive entry is given by

$$\Pi_a(p, k_d) = (p - c)N_a M_a(p)D - k_d \quad \text{where} \quad M_a(p) = \int_{\alpha_{ij}^-}^{\alpha_{ij}} f(\alpha) d\alpha. \quad (10)$$

That is, $M_a(p)$ is the potential market share of product j as a function of price. We begin by examining the interrelationship between defensive pricing and distribution strategies.

THEOREM 6. Price Decoupling. *The optimal defensive pricing strategy is independent of the optimal defensive distribution strategy, but the optimal distribution strategy depends on the optimal defensive price.*

THEOREM 7. Defensive Distribution Strategy. *If market size does not increase, the optimal defensive distribution strategy is to decrease spending on distribution.*

THEOREM 8. Pre-emptive Distribution. *If market size does not increase, optimal defensive profits must decrease if a new brand enters competitively, regardless of the defensive price and distribution strategy. However, profit might be maintained (less prevention costs), under certain conditions, if the new brand is prevented from entering the market.*

Together Theorems 6, 7, and 8 provide the defensive manager with valuable insights on how much of his budget to allocate to distribution. If he can prevent competitive entry by dominating the channel *and it is legal to do so*, this may be his best strategy. However, in cases when it is illegal to prevent entry, the optimal profit strategy is to *decrease* spending on distribution.

Although decreasing distribution expenditures appears counterintuitive to an active defense, it does make good economic sense. Marketing managers should look at profits as well as sales. The market's potential profitability and rate of return decreases as the new product brings additional competition. Because the market is less profitable, the mathematics tells us that we should spend less of our resources in this market. Instead, we should divert our resources to more profitable ventures. If demand shrinks fast enough, the reward for investment in the mature product decreases until new product development becomes a more attractive alternative.

Just the opposite would be true if a competitor dropped out of the market; we would fight to get a share of its customers. Witness the active competition between the Chicago Tribune and Chicago Sun Times when the Chicago Daily News folded and the active competition by the Boston newspapers when the Record-American folded.

Theorem 6 tells us that price strategy is independent of distribution strategy but not vice versa. Thus the results of Theorems 2, 3, 4, and 5 still hold. If preferences are uniformly distributed the defensive manager should decrease price and decrease distribution spending. In addition, if our response analysis is reasonable, these results imply that a defensive manager should first set price before distribution strategy. This implication suggests that the pricing decision should be made at a higher level in the organization than the distribution level decision. In some sense, the pricing decision is more important because the other decisions depend on it.

Theorem 8 notes that there is no magic in distribution. The market is more competitive and there are less potential profits. Thus the manager should not be misled by sales, but should focus on profits.

Finally, we note that the tactics to implement a distribution strategy include details not addressed by response analysis. Nonetheless, although the details may vary, the basic strategy, as summarized by total spending, is to decrease the distribution budget when defending against a new competitive brand. Moreover, if the brand is sold in several markets, the result is true for each market.

Product Improvement

We now investigate whether or not the defending manager should invest money to improve his physical product. We consider the case where the manager has the option to improve consumer perceptions of his brand by modifying his brand to improve its position. That is, we assume that x_{1j} (or x_{2j}) is an increasing function of c . Let c_b be the optimal production cost before competitive entry and let c_a be the optimal production cost after entry. We further assume that x_{1j} (or x_{2j}) is marginally decreasing (concave) in c .

Profit is then given by

$$\Pi_a(p, k_d, c) = (p - c)N_a M_a(p, c)D(k_d) - k_d \tag{11}$$

where x_{1j} , and hence $M_a(p, c)$, is now an explicit function of the per unit production cost c .

To select the optimal physical product we examine the first order condition implied by equation (11). It is given by

$$\partial M_a(p^*, c^*)/\partial c = M_a(p^*, c^*)/(p^* - c^*). \tag{12}$$

While equation (12) appears to be relatively simple, production cost affects $M_a(p, c)$ in a complex way. We could repeat analyses similar to Theorems 2, 3, 4, and 5 to obtain results for production cost, but such analogous theorems do not provide the same usable intuition. Instead, we gain insight into the solution to equation (12) by examining the marginal forces affecting the optimal production cost, c_b , at which the firm was operating prior to the competitive entry. While this may not guarantee an optimum, it does suggest a directionality for improving profit.

Define a conditional defensive profit function, $\Pi_a(c | 0)$, given by

$$\Pi_a(c | 0) = (p^0 - c)N_b M_a(p^0, c)D(k_d^0) - k_d^0 \tag{13}$$

where p^0 and k_d^0 are the optimal price and distribution expenditures prior to the competitive entry.

The conditional defensive profit function can be interpreted as the profit we obtain after competitive entry if we are only allowed to improve (degrade) our physical product. We now examine how $\Pi_a(c | 0)$ changes as we change c . (Our result is stated for uniformly distributed tastes. We leave it to future research to extend these results as Theorems 3, 4, and 5 extended Theorem 2.)

THEOREM 9. Defensive Product Improvement. *Suppose that consumer tastes are uniformly distributed and the competitive new brand attacks along attribute 2 (i.e., an upper adjacent attack), then at the margin, if product improvement is possible,*

- (a) *Profits are increasing in improvements in attribute 1 (i.e., away from the attack);*
- (b) *Profits are increasing in improvements in attribute 2 (i.e., toward the attack) if*

$$(x_{1j}/p - x_{1n}/p_n)(1 + r_{nj}^{-2}) \leq (x_{1j}/p - x_{1j+}/p_+)(1 + r_{jj+}^{-2})$$

where p_n and p_+ are the prices of the new and upper adjacent brands, respectively.

Symmetric conditions hold for an attack along attribute 1.

The results of Theorem 9 can be stated more simply. Part (a) says reposition by improving product quality to your strength. I.e., the competitive new brand is attacking you on one flank, attribute 2, but you are still positioned better along your strength under this attack, attribute 1. Theorem 9 says that if you can move along attribute 1, do so.

Part (b) says that improving product quality in the direction of your competitor's strength, attribute 2, is not automatic. The testable condition given in Theorem 9 must be checked. (§5 of this paper provides a practical procedure to estimate the new brand's position which is the data necessary to check the condition of Theorem 9.) Movement along attribute 2 is more complex because there are two conflicting effects. First, sales potential may be low in the direction of the new brand's competitive strength and, hence, we seek to move away from our competitor. However, a second effect is also present. If the competitor can be easily dominated on attribute 2, then we will seek to regain lost sales by attacking our competitor's strength. The first effect will dominate when our competitor is very strong, i.e., $r_{jj+} \gg r_{jn}$. In this case, Theorem 9(b) directs us not to counterattack on the competitor's strength. The second effect will dominate when the competitor attacks strongly on x_{1j} as well as on x_{2j} , i.e., when $x_{1j}/p \cong x_{1n}/p_n$. In this case, Theorem 9(b) directs us to counterattack on our competitor's strength in addition to moving to our strength. The opposite results will hold for the deletion of an adjacent product.

Advertising Strategy

There are two components to advertising strategy. One goal of advertising is to entice consumers into introducing our brand into their evoked sets, or in a dynamic framework, to keep consumers from forgetting our brand. We refer to this form of advertising, as awareness advertising, and handle it with response analysis much like we handled distribution. (Sales are proportional to the number of consumers aware of our product.) In some categories, another goal of advertising is to reposition our brand.⁸ For example, if we are under attack by a new liquid dishwashing detergent stressing mildness, we may wish to advertise to increase the perceived mildness (or efficacy) of our brand. If this form of advertising is possible, we invest advertising dollars to increase x_{1j} or x_{2j} . Since repositioning advertising affects α_{nj} and α_{jj-} , it is more complex than awareness advertising. In the following analysis we consider expenditures on awareness advertising, k_a , and repositioning, k_r , separately. In other words, our results enable the defensive manager to decide both on the overall advertising budget and on how to allocate it among awareness and repositioning.

⁸While repositioning without changing the physical product is common in image sensitive product categories such as softdrinks, perfumes, and some beers, it may not be possible in many categories. In which case, we combine repositioning with physical product improvement. As is clear in Theorems 9 and 11, the results are in the same direction.

We begin by considering awareness advertising. Let A , a function of k_a , be the probability a consumer is aware of our brand.⁹ Brands must continue spending in order to maintain awareness levels. Equation (14) expresses our profit after the new brand enters:

$$\Pi_a(p, k_d, k_a) = (p - c)N_a M_a(p, c)AD - k_a - k_d. \tag{14}$$

$M_a(p, c)$ was defined in equations (10) and (11).

THEOREM 10. *Defensive Strategy for Awareness Advertising. If market size does not increase, the optimal defensive strategy for advertising includes decreasing the budget for awareness advertising.*

Theorem 10 is not surprising because awareness advertising is modeled with a response function that has properties similar to the response function for distribution. Thus all of our comments (and an analogy to the preemptive distribution theorem) apply to awareness advertising.

Now consider repositioning advertising. Following arguments similar to those for physical product improvement, we handle repositioning by a form of response analysis where x_{1j} (or x_{2j}) is an increasing but marginally decreasing (concave) function of the investment, k_r , in repositioning. Profit is given by

$$\Pi_a(p, k_d, k_a, k_r) = (p - c)N_a M_a(p, k_r, c)AD - k_d - k_a - k_r. \tag{15}$$

Following our previous analysis of product improvement, we define a conditional profit function to represent the profit attainable if we are only allowed to adjust k_r . Based on such a definition (analogous to equation (13)) we obtain the following theorem.

THEOREM 11. *Repositioning by Advertising. Suppose consumer tastes are uniformly distributed and the new competitive brand attacks along attribute 2 (i.e., an upper adjacent attack), then at the margin if repositioning is possible,*

(a) *profits are increasing in repositioning spending along attribute 1 (i.e., away from the attack);*

(b) *profits are increasing in repositioning spending along attribute 2 (i.e., toward the attack) if and only if*

$$(x_{1j}/p - x_{1n}/p_n)(1 + r_{nj}^{-2}) < (x_{1j}/p - x_{1j+}/p_+)(1 + r_{jj+}^{-2}),$$

where p_n and p_+ are the prices of the new and upper adjacent brands, respectively.

Symmetric conditions hold for an attack along attribute 1.

⁹Our use of the letter 'A' in the following discussion should not be confused with our use of the letter 'A' to denote choice sets.

Theorem 11 is a conditional result at the old optimal. But coupled with Theorem 10 it provides very usable insight into defensive advertising strategy. Theorem 10 says *decrease awareness advertising*. Theorem 11 says that at least in the case of uniform consumer tastes, marginal gains are possible if we *increase repositioning advertising along our strength*. Together these results suggest that the defensive manager (facing uniformly distributed tastes) should reallocate advertising from an awareness function to a repositioning function. For example, he might want to select copy that stresses “more effective in cleaning hard-to-clean dishes” over copy that simply gets attention for his brand name.

Finally, we caution the reader that in many categories repositioning by advertising may not be possible without corresponding improvements in the physical product. Since Theorems 9 and 11 suggest the same directional adjustment of advertising and product improvement, we feel such changes should be made together.

Summary

Our goal in this section was to provide insight on defensive strategy, i.e., rules of thumb that rely on believable abstractions that model major components of market response. Toward this end we searched for directional guidelines that help the manager understand qualitatively how to modify his marketing expenditures in response to a competitive new product. In many situations, especially where data are hard to obtain or extremely noisy, such qualitative results may be more usable than specific quantitative results. If good data are available the qualitative results may help the manager understand and accept more specific optimization results.

In particular, our theorems show in general that:

- distribution expenditures should be decreased, unless the new brand can be prevented from entering the market,
- awareness advertising should be decreased, and
- profit is always decreased by a competitive new brand. More specifically, if consumer tastes are uniformly distributed,
 - profits will increase if price is decreased,
 - (at the margin) the brand should be improved in the direction of the defending brand’s strength,
 - (at the margin) advertising for repositioning should be increased in the direction of the defending brand’s strength.

We have also shown that there exist highly segmented taste distributions for which a price increase may be optimal.

We caution the manager that like any mathematical scientific theory, the above results are based on assumptions. Our results are true to the extent that the market which the defensive manager faces can be approximated by our model of the market. Since our model is based on previously tested assumptions about consumer behavior, we believe there will be many situations that can be approximated by our model. At the very least we believe that Theorems 1 through 13 provide the foundations of a theory of defensive

strategy that can be subjected to empirical testing and theoretical modification. (As referenced earlier, Theorems 12 and 13 are generalizations of the pricing theorems. See Appendix.) In the long run it will be the interplay of empirics and theory that will provide greater understanding of defensive strategy. See §7 for a discussion of validation issues and §8 for a discussion of future research.

4. Estimation of the Consumer Taste Distribution

The defensive pricing and repositioning theorems depend upon the distribution of consumer tastes. In some markets this information will be collected through standard techniques. See the review by Green and Srinivasan (1978). In other cases, the defensive manager may only have sales and positioning data available. We now describe a technique that provides an estimate of the taste distribution from such data.

We begin with a technique to estimate consumer tastes when everyone has the same evoked set. We then extend this technique to the case where evoked sets vary.

Homogeneous Evoked Sets

We return to the notation of §2. Let $m_{j|l} = \text{Prob}[\alpha_{jj-} < \tilde{\alpha} \leq \alpha_{jj+} \text{ for } j \in A_l]$ and let $M_{j|l}$ be observed market share of product j for consumers who evoke A_l . For this subsection we are concerned only with consumers who evoke A_l , thus for notational simplicity, drop the argument l . Our problem is then to select $f(\alpha)$ such that $m_j = M_j$ for all $j \in A_l$.

One solution is to select a parameterized family of functions, say a Beta distribution, $f_\beta(\alpha | \sigma, \delta)$, and select σ and δ with maximum likelihood techniques. We feel most common distributions (Beta, Gamma, Normal, Lognormal, Exponential, Laplace, etc.) are not flexible enough to analyze defensive strategy because they restrict the possible shape of the taste distribution. This is particularly important because the directionality of defensive pricing strategy can be dependent on the detailed shape of the taste distribution.

An alternate solution is illustrated in Figure 6. We approximate $f(\alpha)$ with a series of uniform distributions. This procedure can approximate any $f(\alpha)$ and, in the limit, as the number of line segments gets large, the procedure converges to the true $f(\alpha)$. The problem now becomes how to select the endpoints and the heights of the uniform distributions. If our approximation is a good one, the area under the approximate curve should be close to the area under the actual curve. In other words, if we calculate the area under the actual curve between any two angles, that area should roughly equal the area under the curve formed by the uniform approximation. Now, if we take the area between the lower and upper adjacent angles for any brand, that area should equal the brand's market share. For example, in Figure 6, if "a" and "d" are the lower and upper adjacent angles for the brand, the area of the rectangle $a - b - c - d$ should equal the market share for the brand. Moreover, if we choose the end points to be the adjacent angles, the brand market shares

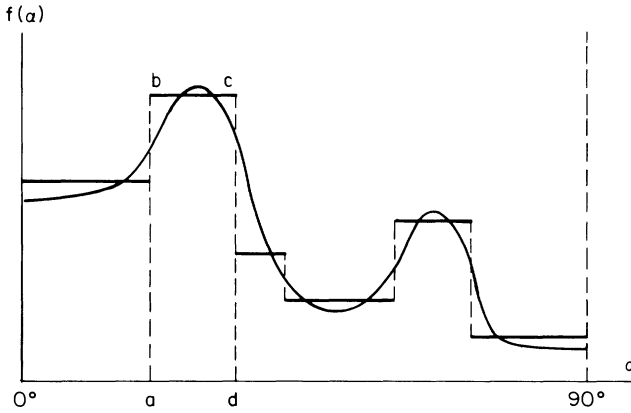


FIGURE 6. Approximating the Consumer Taste Distribution with a Series of Uniform Distributions.

become the estimates of the respective areas. Then the area of the *j*th rectangle is m_j and the height, h_j , of the *j*th uniform distribution becomes

$$m_j / (\alpha_{jj+} - \alpha_{jj-}).$$

Given this approximation, we can exactly satisfy the observed shares if $m_j = M_j$. Figure 7 illustrates this approximation for a six-brand choice set. This procedure provides one approximation to $f(\alpha)$ that we believe is sufficient to distinguish among alternative defensive pricing strategies.

Summarizing, $\hat{f}(\alpha)$ estimates $f(\alpha)$ and is given

$$\hat{f}(\alpha) = M_j / (\alpha_{jj+} - \alpha_{jj-}) \quad \text{where } \alpha_{jj+} \geq \alpha > \alpha_{jj-}. \tag{16}$$

Heterogeneous Evoked Sets

The extension to heterogeneous evoked sets is deceptively simple. Since α_{jj-} and α_{jj+} are dependent on the evoked set, A_l , we can obtain an aggregated $\hat{f}(\alpha)$ by a weighted sum of $f(\alpha)$ given A_l , denoted $\hat{f}(\alpha | A_l)$. That is,

$$\hat{f}(\alpha) = \sum_{l=1}^L S_l \hat{f}(\alpha | A_l) \tag{17}$$

where $\hat{f}(\alpha)$ estimates $f(\alpha)$, $\hat{f}(\alpha | A_l)$ estimates $f(\alpha | A_l)$ and S_l is the proportion of consumers who evoke A_l . Estimates for S_l are discussed in the next section. For prediction we must work with the set of disaggregated $\hat{f}(\alpha | A_l)$'s, unless we can insure that evoked sets will change appropriately when we change positions according to the aggregated $\hat{f}(\alpha)$.

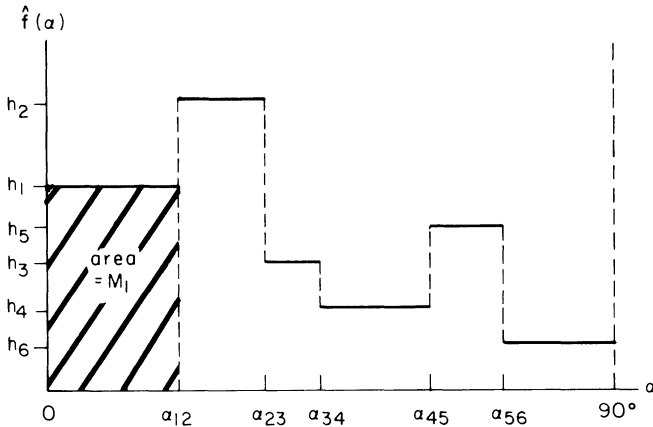


FIGURE 7. Estimation of Consumer Tastes for a Hypothetical Six-Product Choice Set.

Summary

In general, we posit that (17) will provide a good approximation if the number of brands, J , is somewhat larger than the number of perceptual dimensions.

5. Estimation of the New Product's Position

Most of the theorems of §3 are independent of the new product's position (as long as it's competitive), but the repositioning theorems (Theorems 9 and 11) depend explicitly upon the new product's specific position. In some cases, a defending manager will commission a positioning study to identify that position. But such studies are expensive (\$50–100 thousand).

In this section we provide alternative procedures that depend upon only (1) the positioning map *prior* to competitive entry and (2) sales after competitive entry. In this way a manager can select the technique appropriate to his situation. We close this section with a technique to estimate the probability that the new product enters each evoked set.

Maximum Likelihood Estimates

Suppose the new brand, n , enters the evoked set A_l^* with the corresponding probability, S_l^* . ($A_l^* = A_l \cup \{n\}$.) In other words, S_l^* is the probability that a random consumer evokes set l and is aware of brand n . Then the market share of the new brand as well as after-entry market shares for previous brands can be computed with the consumer model in §2 if the new brand's position, $(x_{1n}/p_n, x_{2n}/p_n)$, is known. Thus, given the new brand's position, we can obtain

$$m_j^* = \sum_{l=1}^L m_{j|l}(S_l - S_l^*) + \sum_{l=1}^L m_{j|l}^* S_l^*, \tag{18}$$

$$m_n^* = \sum_{l=1}^L m_{n|l}^* S_l^*, \quad \text{where} \tag{19}$$

m_j^* = the post-entry market share for brand j given the new brand enters at \mathbf{x}_n .

$m_{j|I}^*$ = the post-entry market share for brand j among those with evoked set A_I given the new brand is positioned at \mathbf{x}_n .

m_n^* = the market share of the new brand given \mathbf{x}_n and A_I^* .

If we assume consumers are drawn at random to form an estimation sample then the log-likelihood function, $L(\mathbf{x}_n)$, is given by

$$L(\mathbf{x}_n) = \sum_{j=1}^J M_j \log m_j^*(\mathbf{x}_n) + M_n \log m_n^*(\mathbf{x}_n) \quad \text{where} \quad (20)$$

$L(\mathbf{x}_n)$ = the log-likelihood function evaluated at $\mathbf{x}_n = (x_{1n}, x_{2n})$, the position of the new brand.

$m_j^*(\mathbf{x}_n)$ = m_j^* evaluated at $\mathbf{x}_n = (x_{1n}, x_{2n})$,

$m_n^*(\mathbf{x}_n)$ = m_n^* evaluated at $\mathbf{x}_n = (x_{1n}, x_{2n})$,

M_j = post-entry market share of brand j .

The maximum likelihood estimator of the new brand's position is then the value of \mathbf{x}_n that maximizes $L(\mathbf{x}_n)$.

Equation (20) is easy to derive but difficult to use. The key terms, $m_j^*(\mathbf{x}_n)$ and $m_n^*(\mathbf{x}_n)$, are highly nonlinear in \mathbf{x}_n even for a uniform distribution. Thus the optimization implied by (20) is solvable in theory, but extremely difficult in practice. Fortunately, there is a more practical technique for obtaining estimates of \mathbf{x}_n .

Bayes Estimates

In discussing defensive strategies with product managers in both consumer and industrial products, we discovered that most defending managers have reasonable hypotheses about how the competitive brand is positioned. For example, when Colgate-Palmolive launched *Dermassage* liquid dishwashing detergent with the advertising message: "Dermassage actually improves dry, irritated detergent hands and cuts even the toughest grease," the experienced brand managers at Procter and Gamble, Lever Bros. and Purex could be expected to make informed estimates of *Dermassage's* position in perceptual space. The existence of such prior estimates suggests a Bayesian solution.

Suppose that the defending manager provides a prior estimate, $f_x(\mathbf{x}_n)$, of the distribution of the new product's position, $\tilde{\mathbf{x}}_n$. (Note that $\tilde{\mathbf{x}}_n$ denotes \mathbf{x}_n expressed as a random variable.) For tractability we discretize the prior distribution. We then use the consumer model in §2 to derive $m_j^*[\mathbf{x}_n(\beta)]$ and $m_n^*[\mathbf{x}_n(\beta)]$ where $\mathbf{x}_n(\beta)$ for $\beta = 1, 2, \dots, T$ are the possible positions for the new product for each potential new product position, where β indexes the discretized new brand positions. See (18) and (19). If consumers are drawn at random for the estimation sample, then the Bayesian posterior distribution,

$f_x(\mathbf{x}_n | \mathbf{M})$, is given by

$$f_x(\mathbf{x}_n(\beta) | \mathbf{M}) = f_x[\mathbf{x}_n(\beta)]K(\mathbf{M} | \mathbf{m}_\beta) / \sum_{\Upsilon} f_x[\mathbf{x}_n(\Upsilon)]K(\mathbf{M} | \mathbf{m}_\Upsilon) \quad \text{where} \tag{21}$$

$$K(\mathbf{M} | \mathbf{m}_\beta) = m_n[\mathbf{x}_n(\beta)]^{M_n \eta} \cdot \prod_{j=1}^J \{m_j[\mathbf{x}_n(\beta)]\}^{M_j \eta},$$

\mathbf{M} is the vector of post-entry market shares, $K(\mathbf{M} | \mathbf{m}_\beta)$ is the kernel of the sampling distribution for η consumers drawn at random from a population defined by the multinomial probabilities, and $\mathbf{m}_\beta = \{m_n[\mathbf{x}_n(\beta)] \text{ and } m_j[\mathbf{x}_n(\beta)] \text{ for all } j\}$. The use of (21) to update a managerial prior is shown in Figure 8.

Equation (21) looks complicated but it is relatively easy to use. First, the manager specifies his prior beliefs about the new brand’s position in the form of a discrete set of points $\mathbf{x}_n(\beta)$, and the probabilities, $f_x[\mathbf{x}_n(\beta)]$, that each point is the new position. For example, he may believe that points, (3.5, 4.0), (4.0, 3.5), (4.0, 4.0), are the potential, equally likely, positions of the new brand, Dermassage.¹⁰ As shown in Figure 9, the manager has a general idea of where Dermassage is positioned but does not know the exact position. We first use (18) and (19) to compute the predicted market shares, $m_j[\mathbf{x}_n(\beta)]$, for each of the three potential new brand positions. For this example, assume that tastes are uniform and everyone evokes all brands. This case is shown in Table 3. Now suppose that we sample 50 consumers and find that, in our sample, the observed market shares are Ajax (20%), Dermassage (30%), Joy (20%), and Ivory (30%). We use (21) to compute the kernel of the sampling distribution, e.g.,

$$K(\mathbf{M} | \mathbf{m}_{\beta=1}) = (0.30)^{10}(0.10)^{15}(0.30)^{10}(0.30)^{15} \quad \text{for } \beta = 1.$$

We can then compute the posterior probabilities as given in Table 3.

As we have chosen the data, the market shares from 50 consumers clearly identify (4.0, 4.0) as the most likely position for Dermassage. The point, (4.0, 3.5), still has an 11% chance of being the actual position; the point, (3.5, 4.0), is all but ruled out. Not all applications will be so dramatic in identifying the new brand’s position with such small sample sizes, but all applications will follow the same conceptual framework.

Evoked Set Probabilities

Equations (17) through (21) depend on the evoked set probabilities, S_i and S_i^* . In §2 we assume that S_i is known prior to the new product’s launch. If the manager collects data on the evoked sets after the launch of the new brand, (17) through (21) can be used directly.

¹⁰This hypothetical example does not necessarily reflect true market shares and market positions.

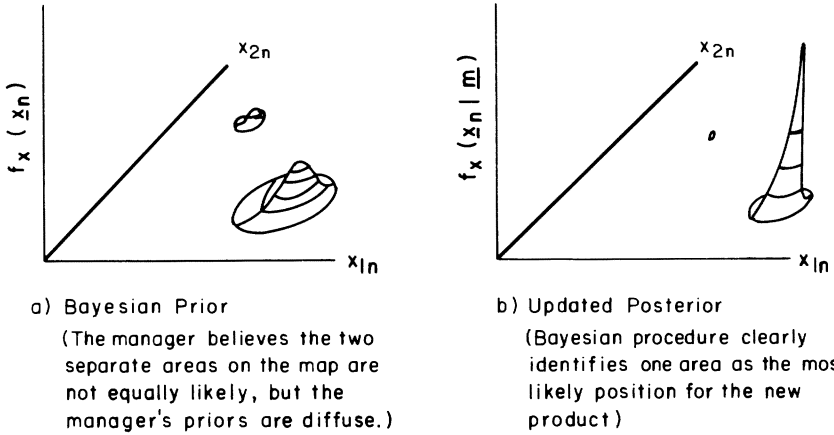


FIGURE 8. Bayesian Updating Procedure for a Hypothetical Market. (The Bayesian Prior (a) Is Based on Managerial Judgment. The Bayesian Posterior (b) Is the Result of Using Observed Sales Data to Update Managerial Judgment. See Equation 21.)

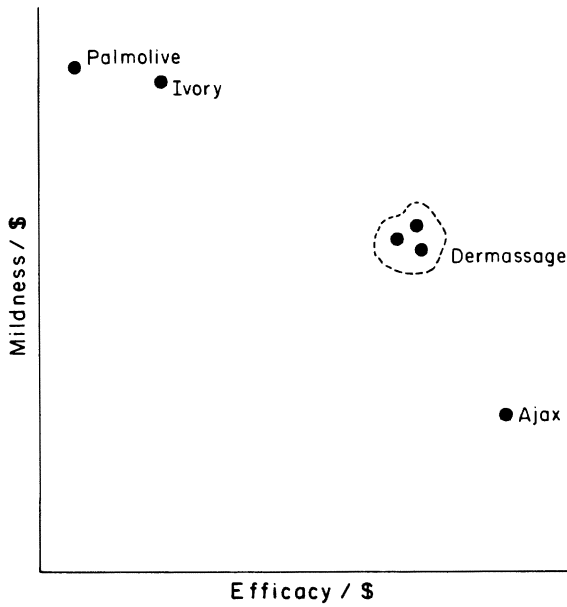


FIGURE 9. Example Managerial Priors for the New Product's Position (Three Equally Likely Points for Dermassage).

TABLE 3
Calculations for Bayesian Updating Procedure

Position of Dermassage	$\beta = 1$ (3.5, 4.0)	$\beta = 2$ (4.0, 3.5)	$\beta = 3$ (4.0, 4.0)	Observed Share
Market Shares				
Ajax	0.30	0.24	0.20	0.20
Dermassage	0.10	0.19	0.30	0.30
Joy	0.30	0.27	0.20	0.20
Ivory	0.30	0.30	0.30	0.30
Prior Probability	0.33	0.33	0.33	—
Posterior Probability	< 0.01	0.11	0.88	—

In many cases evoked set information will be too expensive to collect after the new brand is launched. Equations (17) through (21) can still be used if we assume either that (1) the new brand enters evoked sets randomly or (2) that the new brand enters sets in proportion to the probability that it would be selected if it were in that evoked set.

Case 1 is based on the assumption that awareness and availability are independent of preference. In case 1, (18) and (19) reduce to

$$m_j^*(x_n) = \sum_l [(1 - W)m_{j|l} + Wm_{j|l}^*]S_l, \tag{22}$$

$$m_n^*(x_n) = W \sum_l m_{n|l}^* S_l, \tag{23}$$

respectively, where W is the aggregate percent of consumers who are aware of the new brand and find it available.

Case 2 is based on the assumption that evoking is functionally dependent on preference. In other words, case 2 assumes that consumers are more likely to become aware of brands that closely match their preferences. In case 2, equations (18) and (19) reduce to

$$m_j^*(x_n) = \sum_l m_{j|l} S_l - \left[W \sum_l (m_{j|l} - m_{j|l}^*) m_{n|l}^* S_l \right] / \left[\sum_l m_{n|l}^* S_l \right], \tag{24}$$

$$m_n^*(x_n) = W \sum_l (m_{n|l}^*)^2 S_l / \sum_l m_{n|l}^* S_l, \tag{25}$$

respectively. The derivations of (24) and (25) are given as Theorem 14 in the Appendix.

Equations (22) through (25) are useful computational results since together they provide flexibility in modeling how a new brand enters evoked sets. More importantly each can be applied if we know only the aggregate percent of consumers who evoke the new brand.

Summary

This section has provided a practical means to estimate the new brand's position and its impact on sales. The only data on the new market that are required are (1) sales of each brand and (2) aggregate evoking of the new brand. (We assume, of course, that the old product positions, the old evoked set probabilities, and the taste distribution are known.)

6. Diagnostics on Competitive Response

Defensive strategy is how to react to a competitive new product. However, the concepts developed in §§2 and 3 are useful for representing competitive threats and, in particular, for representing how our competitors might react to the new brand. In this section we briefly outline these heuristic concepts.

Angle of Attack

The general equilibrium for a market depends on the cost structures faced by each firm as well as the distribution of consumer tastes. However, in talking with managers we have found it useful to represent the competitive threats posed by each firm as that portion of the taste spectrum that that firm is capturing. In particular we define the "angle of attack," Λ_j , to represent the average tastes of product j 's consumers, i.e.:

$$\Lambda_j = (\alpha_{j+} + \alpha_{j-})/2. \quad (26)$$

With this definition we should be most concerned with products that have angles of attack that are close to ours. In defensive strategy formulation we believe it is useful to superimpose the angle of attack of the new product on the preference distribution to determine how competitive the new product is and to determine which of our competitors is most likely to engage in an active defense. The angle of attack does not capture all of the richness of the analytic model, but does provide a useful visual interpretation of our model that serves as a stimulus to discussion.

Price Diagnostics

Theorems 2, 3, 4, 5, 12, and 13 tell us how to change our price in order to react to a competitive new product, but they can just as easily be applied to predict how our competitors should react to the new product if they too are maximizing profit according to our model.

We can also gain some insight on their response by examining the multiattributed space of product attributes. See Figure 10. Such a map tells us the maximum price that our competitor (in this case Ajax) can charge and still remain efficient. This point is marked "upper bound" in Figure 10. The map also indicates the point at which Ajax's price forces the new product to lower its price to remain efficient. Beyond this point, labeled "lower bound" in Figure 10, Ajax and the new product enter a (possibly escalating) price war. We cannot be more specific in our predictions of Ajax's behavior without

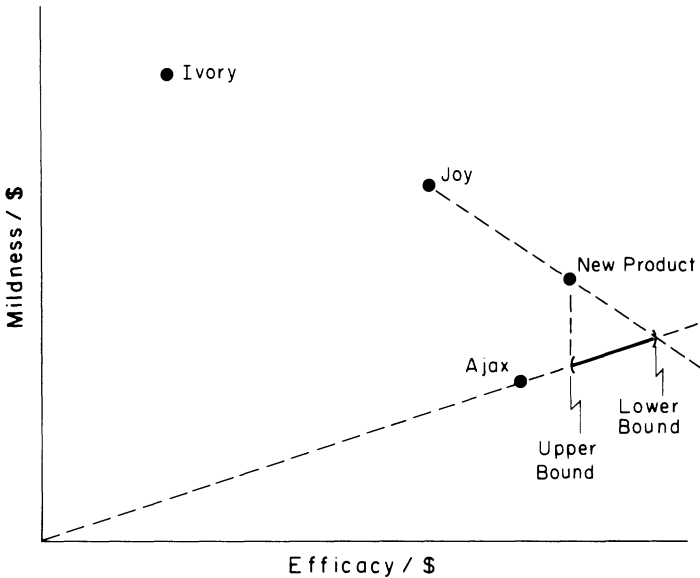


FIGURE 10. Some Insights on Competitive Price Response to the New Product.

knowing the detailed cost structure of the market, but Figure 10 does serve as a useful first cut at visualizing Ajax’s first response to the new product.

We leave the exact prediction of the competitor’s price to future equilibrium analyses.

7. Validation Issues

There are at least two validation issues relevant to deductive theory. The first is a test of the assumptions and the sensitivity of the theorems to the assumptions. The second is a test of whether or not managers already appear to intuitively follow the predictions of the theorems. We discuss each issue in turn.

Assumptions

Our theorems analyze aggregate response, thus, any test of our assumptions should test their validity as a model of aggregate market behavior. We chose assumptions that are well documented in the marketing literature. See the detailed discussion in §2. As an aggregate market model, response analysis and utility maximization in a multiattributed space appear to be reasonable. Linearity is a useful first order analysis which encompasses a surprisingly broad range of commonly used utility functions including the ideal point models, the Cobb–Douglas product form, and multiplicative conjoint models. Our price-scaling assumption, i.e., the assumption that the consumer chooses the product that maximizes u/p where u is the linearized utility function and p is price, follows directly from economic theory (e.g., see Debreu 1959). The assumption has been used in marketing (see Hauser and Simmie 1981, Ratchford 1975). Only our pricing theorems use this assumption.

An alternative approach to testing our assumptions is to test the robustness of our theorems to relaxations in the assumptions. For example, some form of the pricing theorems may also hold for utility that is simply decreasing in price.

Behavior of Firms

Our theorems suggest how managers *should* behave. We can provide a weak test of our results by observing how managers *do* behave. If they behave as predicted, then our model escapes falsification, but it is not yet validated. If they do not behave as predicted, then either (1) the assumptions are wrong, (2) the model is not sufficiently rich, or (3) they are acting suboptimally. Unfortunately we cannot distinguish among these three options without further study.

Because defensive marketing strategy is a new subject of inquiry, there is little empirical evidence on defensive response. One empirical study of which we are aware is a study by Beshel (1982) of the prescription analgesics market. Before November 1980, the market was dominated by Tylenol with Codeine (McNeil Laboratories), Empirin with Codeine (Burroughs Wellcome), and Darvocet N-100 (Eli Lilly). Tylenol w/Codeine and Empirin w/Codeine were positioned as more effective; Darvocet N-100 was positioned as gentler (fewer side effects). In November 1980, McNeil introduced Zomax which was positioned as gentler than Tylenol w/Codeine and Empirin w/Codeine and more effective than Darvocet N-100. Since Zomax was attacking all these brands, Theorems 7 and 10 suggest that all these brands should reduce their distribution and promotion expenditures. Beshel (1982, pp. 91–93) presents evidence that, that is probably what happened. Beshel's study is indicative of the types of descriptive analyses that are necessary to further develop a body of defensive marketing theory.

Another empirical study is also consistent with our results. We would suspect that near the end of a product's life cycle in highly segmented markets, the ultimate degree of product differentiation has occurred. Hence, brands would have been launched targeted at each segment. At this point, conditions for Theorem 5 would frequently be met within each segment. Hise and McGinnis (1975) found that near the end of a product's life cycle, price increases were more common than price decreases. Of course, numerous brand deletions occur during this period which complicate the findings.

Both studies are indicative of the types of descriptive analyses that are necessary to further develop a body of defensive marketing theory.

8. Limitations and Future Research

Theorems 1 through 14 represent a beginning, but they can be improved through further research.

Competitive Issues

An explicit assumption of all of our analyses is that of a two-product equilibrium. We react as if no other product besides us reacts to the new product.

Consider pricing under uniformly distributed tastes. If we assume the product enters with perfect foresight and an equilibrium exists, then in a two-product market our results are the full equilibrium. In a three-product market with the new product attacking both existing products, then, if our competitor reacts by reducing his price, his effect on us, if he has an effect, is to cause us to reduce our price. This response is in the same direction as the response caused by the new product. Thus, intuitively, it would seem that our pricing results would hold for a full equilibrium in a three-product market. We can create similar intuitive arguments for multiproduct markets, for nonadjacent attacks, and for other marketing variables, but such arguments are not rigorous and do not prove our results for a full equilibrium.

Full equilibrium analyses are difficult but potentially fruitful directions for future study. One can begin to study this issue by simulation or by examining the equilibrium that results when each existing firm follows a two-product equilibrium strategy. After such results are obtained, improved foresight can be incorporated in the model and the model can be extended to product line decisions.

Pricing Theorems

Our pricing theorems assume price scaling. It would be interesting to examine their sensitivity to alternative models of consumer price response. Furthermore, one can examine pricing strategy under different distributional assumptions about consumer tastes. For example, is it always optimal to decrease price if tastes are approximately normally distributed (truncated at 0° and 90°)? Theorem 13 provides a starting point for such analyses.

9. Summary

Among the key results of this paper are the theorems which investigate defensive marketing strategy. These theorems are the logical consequences of a consumer model based on the assumptions that consumers are heterogeneous and choose within a product category by maximizing a weighted sum of perceived product attributes. Since this consumer model is based on empirical marketing research we feel that it is a good starting point with which to analyze defensive marketing strategies.

We feel that the theorems provide usable managerial guidelines. When the appropriate data are available normative optimization models may be the best way to proceed, but, by their very nature, defensive marketing strategies are often made quickly and without extensive data collection. The theorems tell the manager that as the result of a competitive entrant (1) the defender's profit will decrease, (2) if entry cannot be prevented, budgets for distribution and awareness advertising should be decreased, and (3) the defender should carefully compare the competitor's angle of attack to his position and the distribution of consumer tastes. If tastes are segmented and if the competitors clearly out-position his product in one of his consumer segments, a price increase may be optimal. If consumer tastes are uniformly distributed across the spectrum, the defender should decrease price, improve product quality to his strength, and advertise to support changes. §§4, 5, and 6 provide practical

procedures to identify the consumer taste distribution, the new brand's position and its entry into evoked sets. These data allow us to decide when each theorem is appropriate. Because these results are robust with respect to many details of the model they are good managerial rules of thumb even when the data are extremely noisy.

We feel that the theorems are a good beginning to guide the development of empirical models and to encourage the development of a generalizable managerial theory of how to respond to competitive new products.

Glossary

- Adjacent A lower (upper) adjacent brand is the next efficient brand in the choice set as we proceed clockwise (counter-clockwise) around the efficient frontier.
- Competitive A new product represents a competitive attack if it is adjacent and there are at least some consumers whose tastes cause them to prefer the new product, i.e., $f(\alpha) \neq 0$ for all α in the range $\alpha_{jn} \leq \alpha \leq \alpha_{jj+}$.
- Efficient A product is efficient in a multiattribute space if there is no other product or linear combination of products in the space that dominates the efficient product on all attributes.
- Regular A market is regular if, for every product in that market, the angle of the ray connecting the product's position to the origin lies between the angle which defines the limits of the tastes of that product's consumers, i.e., $\alpha_{jj-} \leq \theta_j \leq \alpha_{jj+}$.

Appendix

LEMMA 1. *Sales under uniformly distributed tastes are decreasing in price.*

PROOF. Sales under the condition of uniform tastes is given by $N_a(\alpha_{nj} - \alpha_{jj-})$. The result holds if $\alpha'_{nj} - \alpha'_{jj-} < 0$ where the derivative is with respect to price, p_j . By direct computation,

$$\alpha'_{nj} = p_n(x_{2j}x_{1n} - x_{1j}x_{2n}) / [(p_jx_{2n} - p_nx_{2j})^2 + (p_nx_{1j} - p_jx_{1n})^2].$$

Since the denominator is positive we have $\alpha'_{nj} < 0$ if $x_{2j}x_{1n} < x_{1j}x_{2n}$.

Since n is now upper adjacent to j , $x_{2j} \leq x_{2n}$ and $x_{1n} \leq x_{1j}$ where at least one inequality is strict. Thus $\alpha'_{nj} < 0$. By symmetry $\alpha'_{jj-} > 0$. By the same arguments $\alpha'_{jj+} < 0$, thus the result holds both before and after entry.

LEMMA 2. *$M_b(p)$ is a nonincreasing function of price. $M_b(p)$ is a decreasing function of price for $f(\alpha_{jj+})$ or $f(\alpha_{jj-}) > 0$, where $M_b(p) = \int_{\alpha_{jj-}}^{\alpha_{jj+}} f(\alpha) d\alpha$. Similarly, let*

$$M_a(p) = \int_{\alpha_{jj-}}^{\alpha_n} f(\alpha) d\alpha.$$

PROOF. Let (\cdot) denote the derivative with respect to price, p_j . $M'_b(p) = f(\alpha_{jj+})\alpha'_{jj+} - f(\alpha_{jj-})\alpha'_{jj-}$. Following Lemma 1, we see that $\alpha'_{jj+} < 0$ and $\alpha'_{jj-} > 0$. Since $f(\alpha) \geq 0$, it follows that $M'_b(p) \leq 0$. If $f(\alpha_{jj+})$ or $f(\alpha_{jj-}) > 0$, $M'_b(p) < 0$.

LEMMA 3. *$\alpha'_{jn} < \alpha'_{jj+}$ implies $\Pi'_a(p_0) < 0$ whenever $f'(\alpha) \leq 0$.*

PROOF. By assumption and Lemma 1, $\alpha'_{jn} < \alpha'_{jj+} < 0$. Because $\alpha_{jn} < \alpha_{jj+}$ by the definition of competitive, $f(\alpha_{jn})\alpha'_{jn} < f(\alpha_{jj+})\alpha'_{jj+} < 0$ whenever $f'(\alpha) \leq 0$. Subtracting a positive constant from each side yields

$$f(\alpha_{jn})\alpha'_{jn} - f(\alpha_{jj-})\alpha'_{jj-} < f(\alpha_{jj+})\alpha'_{jj+} - f(\alpha_{jj-})\alpha'_{jj-}.$$

Hence we find, by definition $M'_a < M'_b < 0$. So because $p^0 - c > 0$, we know

$$(p^0 - c)M'_a < (p^0 - c)M'_b < 0,$$

but

$$\Pi'_b(p^0) = N_b(M_b + (p^0 - c)M'_b) = 0,$$

hence

$$M_b + (p^0 - c)M'_b = 0$$

so

$$M_a + (p^0 - c)M'_a < 0 \quad \text{and} \quad N_a[M_a + (p^0 - c)M'_a] < 0$$

for any $N_a > 0$. Finally, by the definition of $\Pi'_a(p^0)$, we have shown $\Pi'_a(p^0) < 0$.

THEOREM 1. *If total market size does not increase, optimal defensive profits must decrease if the new product is competitive, regardless of the defensive price.*

PROOF. Let p^0 be our optimal price before entry and let p^* be our optimal price after entry. First,

$$\Pi_b(p^0) = (p^0 - c)N_bM_b(p^0) \geq (p^* - c)N_bM_b(p^*)$$

by the definition of optimality at p^0 . $N_b \geq N_a$ by assumption. $M_b(p) > M_a(p)$ since $\alpha_{jj+} > \alpha_{jn}$ and $f(\alpha)$ not identically zero in the range, $[\alpha_{jj+}, \alpha_{jn}]$, by the definition of competitive. Thus,

$$(p^* - c)N_bM_b(p^*) > (p^* - c)N_aM_a(p^*) = \Pi_a(p^*).$$

We have proven $\Pi_b(p^0) > \Pi_a(p^*)$ which is the result.

THEOREM 2. Defensive Pricing. *If consumer tastes are uniformly distributed and the market is regular, then defensive profits can be increased by decreasing price.*

PROOF. To show that a price decrease increases profit we must show that $\Pi'_a(p^0) < 0$.

From regularity we know $\theta_j < \alpha_{jj+}$ hence $\theta_j < \alpha_{jj+} + \alpha_{nj}$ given $\alpha_{nj} > 0$. If $\alpha_{jj+} + \alpha_{nj} < 90^\circ$, then

$$\tan \theta_j < \tan(\alpha_{jj+} + \alpha_{nj}),$$

$$\tan \theta_j < (\tan \alpha_{jj+} + \tan \alpha_{nj}) / (1 - \tan \alpha_{jj+} \tan \alpha_{nj})$$

and by definition

$$x_{2j} / x_{1j} < (r_{jj+} + r_{jn}) / (1 - r_{jj+} r_{jn}),$$

where $1 - r_{jj+} r_{jn} > 0$, which implies $x_{2j}(1 - r_{jj+} r_{jn}) < x_{1j}(r_{jj+} + r_{jn})$. Moreover, if $\alpha_{jj+} + \alpha_{nj} > 90^\circ$, then $\tan \theta_j > \tan(\alpha_{jj+} + \alpha_{nj})$ because $\tan(\alpha_{jj+} + \alpha_{nj}) < 0$. However, $\tan \theta_j > \tan(\alpha_{jj+} + \alpha_{nj})$ implies

$$x_{2j} / x_{1j} > (r_{jj+} + r_{jn}) / (1 - r_{jj+} r_{jn})$$

where $1 - r_{jj+} r_{jn} < 0$. Therefore, in both cases, we find $x_{2j}(1 - r_{jj+} r_{jn}) < x_{1j}(r_{jj+} + r_{jn})$. Multiplying by $(r_{jj+} - r_{jn})$ and rearranging terms we find that

$$(x_{1j} + x_{2j}r_{jn})(1 + r_{jj+}^2) > (x_{1j} + x_{2j}r_{jj+})(1 + r_{jn}^2)$$

which is algebraically equivalent to (note that both sides of the inequality are positive):

$$-(x_{2j} + p_{j+}^{-1} - x_{2j}p^{-1})r'_{jj+}(1 + r_{jn}^2) < -(x_{2n}p_n^{-1} - x_{2j}p^{-1})r'_{jn}(1 + r_{jj+}^2)$$

but $(x_{2j} + p_{j+}^{-1} - x_{2j}p^{-1}) > (x_{2n}p_n^{-1} - x_{2j}p^{-1}) > 0$, hence it must be the case that

$$-r'_{jj+}(1 + r_{jn}^2) < -r'_{jn}(1 + r_{jj+}^2),$$

dividing through

$$-r'_{jj+} / (1 + r_{jj+}^2) < -r'_{jn} / (1 + r_{jn}^2),$$

therefore by definition $-\alpha'_{jj+} < -\alpha'_{jn}$ or $\alpha'_{jj+} > \alpha'_{jn}$ and Lemma 3 implies that $\Pi'_a(p_0) < 0$ since $f'(\alpha) = 0$.

THEOREM 3. Defensive Pricing. *If the new brand's angle (α_n) plus the upper adjacent brand's angle (α_{jj+}) exceeds 90° and if the distribution of consumer tastes is uniformly distributed then defensive profits can be increased by decreasing price from the before-entry optimal.*

PROOF. If $\alpha_{jj+} + \alpha_{nj} > 90^\circ$ then

$$\tan \theta_j > \tan(\alpha_{jj+} + \alpha_{nj})$$

because $\tan(\alpha_{jj+} + \alpha_{nj}) < 0$. Hence

$$x_{2j}/x_{1j} > (r_{jj+} + r_{jn})/(1 - r_{jj+} r_{jn})$$

and the proof follows from that point in the proof for Theorem 2.

THEOREM 4. *For an upper adjacent attack in a regular market, if $f'(\alpha) \leq 0$ the defensive profits can be increased by decreasing price from the before-entry optimal. For a lower adjacent attack, the result holds if $f'(\alpha) \geq 0$.*

PROOF. The proof to Theorem 2 shows that $\alpha'_n < \alpha'_{jj+}$. Lemma 3 then proves the result for $f'(\alpha) \leq 0$. The result for a lower adjacent attack follows by symmetry.

THEOREM 5. *There exist distributions of consumer tastes for which the optimal defensive price requires a price increase.*

PROOF (by construction). See example in text.

THEOREM 6. Price Decoupling. *The optimal defensive pricing strategy is independent of the optimal defensive distribution strategy, but the optimal distribution strategy depends on the optimal defensive price.*

PROOF. Differentiating $\Pi_a(p, k_d)$ with respect to p and setting the derivative to zero yields $[M_a(p) + (p - c)dM_a(p)/dp]N_a D = 0$. Since $N_a D > 0$ for the optimal price, the solution, p^* , of the first order conditions is independent of k_d . Differentiating $\Pi_a(p, k_d)$ with respect to k_d and setting the derivative to zero yields $(p - c)N_a M_a(p)(dD/dk_d) = 1$. The solution to this equation, k_d^* , is clearly a function of p .

Since the first order conditions for p are independent of k_d , so are the second order conditions. For k_d^* dependent on p it is sufficient to examine first order conditions.

THEOREM 7. Defensive Distribution Strategy. *If the market size does not increase then the optimal defensive distribution strategy to a competitive entry is to decrease spending on distribution.*

PROOF. Differentiating $\Pi(p, k_d)$ with respect to k_d and recognizing that $\Pi_a(p) = (p - c)N_a M_a(p)$ as given by (9) and (10) yields the first order condition for the optimal k_d^* as $\Pi_a(p^*)[dD(k_d^*)/dk_d] = 1$. Similarly we can show that the first order condition for the optimal before entry spending, k_d^0 , is $\Pi_b(p^0)[dD(k_d^0)/dk_d] = 1$. Since $\Pi_b(p^0) > \Pi_a(p^*)$ by Theorem 1, we have $dD(k_d^*)/dk_d > dD(k_d^0)/dk_d$. But $dD(k_d)/dk_d$ is decreasing in k_d , hence $k_d^* < k_d^0$. The second order conditions are satisfied if $(p^* - c) > 0$ since $d^2D(k_d^*)/dk_d^2 < 0$ by assumption. If $(p^* - c) \leq 0$ then the optimal k_d must be zero, i.e., no production.

THEOREM 8. Preemptive Distribution. *If market size does not increase, optimal defense profits must decrease if a new brand enters competitively, regardless of the defense price and distribution strategy. However, profit might be maintained (less prevention costs), under certain conditions, if the new brand is prevented from entering the market.*

PROOF. Define $\Pi_b(p, k_d)$ analogously to equation (10). Note that $\Pi_a(p, k_d) = \Pi_a(p)D(k_d) - k_d$ where $\Pi_a(p)$ is defined by (8). Then

$$\begin{aligned} \Pi_b(p^0, k_d^0) &\geq \Pi_b(p^0, k_d^*) = \Pi_b(p^0)D(k_d^*) - k_d^* \\ &> \Pi_a(p^*)D(k_d^*) - k_d^* = \Pi_a(p^*, k_d^*). \end{aligned}$$

The first step is by the definition of optimality; the second step is by definition, the third step is true by Theorem 1, and the fourth step is by definition. The last statement of the theorem follows from the recognition that it is potentially possible to construct situations where the cost of preventing entry is smaller than $\Pi_b(p^0, k_d^0) - \Pi_a(p^*, k_d^*)$.

THEOREM 9. Defensive Product Improvement. *Suppose that consumer tastes are uniformly distributed and the competitive new brand attacks along attribute 2 (i.e., an upper adjacent attack), then at the margin if product improvement is possible,*

- (a) Profits are increasing in improvements in attribute 1 (i.e., away from the attack);
- (b) Profits are increasing in improvements in attribute 2 (i.e., toward the attack) if

$$(x_{1j}/p - x_{1n}/p_n)(1 + r_{nj}^{-2}) < (x_{1j}/p - x_{1j+}/p_+)(1 + r_{jj+}^{-2}).$$

Symmetric conditions hold for an attack along attribute 1.

PROOF. We first recognize that for uniformly distributed tastes, before-entry profits, $\Pi_b(c)$, are given by

$$\Pi_b(c) = \Pi_a(c | 0) + H(p^0 - c)(\alpha_{jj+} - \alpha_{jn})$$

where $H = N_b D(k_d^0)/90^\circ$. H is a positive constant independent of c . Let $z_j = x_{1j}/p$ and let $y_j = x_{2j}/p$. Differentiating $\Pi_b(c)$ with respect to c yields

$$\Pi'_b(c) = \Pi'_a(c | 0) + H[(p^0 - c)(\alpha'_{jj+} - \alpha'_{jn}) - (\alpha_{jj+} - \alpha_{jn})]$$

where (\cdot) denotes the derivative with respect to c . By the definition of c_b being optimal, $\Pi'_b(c) = 0$ at c_b . Since $H > 0$, $\Pi'_a(c | 0) > 0$ if the term in brackets is negative. Since $\alpha_{jj+} - \alpha_{jn} > 0$ by the definition of competitive and since $(p^0 - c_b) \geq 0$ at the before-entry optimal, the term in brackets is negative if $\alpha'_{jj+} - \alpha'_{jn} < 0$.

Part (a). Consider spending with respect to attribute 1. Clearly $dx_{1j}/dc > 0$, thus $dz_j/dc > 0$. Since

$$d(\alpha_{jj+} - \alpha_{jn})/dc = [d(\alpha_{jj+} - \alpha_{jn})/dz_j][dz_j/dc]$$

for spending along attribute 1, we need only show $d(\alpha_{jj+} - \alpha_{jn})/dz_j \leq 0$.

Redefine (\cdot) to denote differentiation with respect to z_j . Then we must show $\alpha'_{jj+} < \alpha'_{jn}$. Since $r_{jj+} = (z_j - z_{j+})/(y_{j+} - y_j)$, $r'_{jj+} = (y_{j+} - y_j)^{-1}$. Similarly, $r'_{jn} = (y_n - y_j)^{-1}$. Hence $0 < r'_{jj+} < r'_{jn}$. Now $r_{jj+} > r_{jn}$, hence

$$0 < (1 + r_{jj+}^2)^{-1} < (1 + r_{jn}^2)^{-1}.$$

Putting these results together yields

$$r'_{jj+} (1 + r_{jj+}^2)^{-1} < r'_{jn} (1 + r_{jn}^2)^{-1}.$$

Finally, $\alpha = \tan^{-1} r$, $\alpha' = r'(1 + r^2)^{-1}$. Thus $\alpha'_{jj+} < \alpha'_{jn}$ and the result follows.

Part (b). Analogously, we must show $d(\alpha_{jj+} - \alpha_{jn})/dy_j \leq 0$ to establish the result in part (b). Redefine (\cdot) to denote differentiation with respect to y_j . We derive conditions for $\alpha'_{jj+} \leq \alpha'_{jn}$. As shown above this is equivalent to

$$r'_{jj+} (1 + r_{jj+}^2)^{-1} \leq r'_{jn} (1 + r_{jn}^2)^{-1}.$$

Since $r_{jj+} = (z_j - z_{j+})/(y_{j+} - y_j)$,

$$r'_{jj+} = (z_j - z_{j+})/(y_{j+} - y_j)^2 = r_{jj+}^2/(z_j - z_{j+}).$$

Similarly $r'_{jn} = r_{jn}^2/(z_j - z_n)$. Substituting yields the condition in part (b). Finally, the last statement of the theorem follows by symmetry.

THEOREM 10. Defensive Strategy for Awareness Advertising. *If market size does not increase, then the optimal defensive strategy for advertising includes decreasing the budget for awareness advertising.*

PROOF. Differentiating $\Pi_a(p^*, k_d^*, k_a)$ with respect to k_a yields the equation $[\Pi_a(p^*, k_d^*) + k_d^*][dA(k_d^*)/dk_d] = 1$. Similarly, we get $[\Pi_b(p^0, k_d^0) + k_d^0][dA(k_d^0)/dk_d] = 1$. The result follows analogously to the proof of Theorem 7 since $\Pi_b(p^0, k_d^0) > \Pi_a(p^*, k_d^*)$, $k_d^0 > k_d^*$, and A is marginally decreasing in k_a . Second order conditions follow from $d^2A(k_d^*)/dk_d^2 < 0$.

THEOREM 11. Repositioning by Advertising. *Suppose consumer tastes are uniformly distributed and the new competitive brand attacks along attribute 2 (i.e., an upper adjacent attack), then at the margin if repositioning is possible,*

- (a) profits are increasing in repositioning spending along attribute 1 (i.e., away from the attack);
- (b) profits are increasing in repositioning spending along attribute 2 (i.e., toward the attack) if and only if

$$(x_{1j}/p - x_{1n}/p_n)(1 + r_{nj}^{-2}) < (x_{1j}/p - x_{1j+}/p_+)(1 + r_{jj+}^{-2}).$$

Symmetric conditions hold for an attack along attribute 1.

PROOF. Define a conditional profit function,

$$\Pi_a(k_r | 0) = (p^0 - c_b)N_b M_b(p^0, k_r, c_b)D(k_d^0)A - k_d^0 - k_a^0 - k_r$$

and recognize that under uniformly distributed tastes, the before-entry profit, $\Pi_b(k_r)$, is given by

$$\Pi_b(k_r) = \Pi_a(k_r | 0) + G(\alpha_{jj+} - \alpha'_{jn})$$

where $G = (p^0 - c_b)N_b D(k_a^0)A(k_a^0)/90$ is a positive constant independent of repositioning spending, k_r .

Let $z_j = x_{1j}/p$ and let $y_j = x_{2j}/p$. Let k_{ri} be repositioning spending along x_{ij} , for $i = 1, 2$. Differentiating with respect to k_{ri} yields

$$\Pi'_b(k_r) = \Pi'_a(k_r | 0) + (G/90)(\alpha'_{jj+} - \alpha'_{jn}) = 0$$

where (') denotes the derivative with respect to k_{ri} . Because $G > 0$, $\Pi'_a(k_r | 0) > 0$ if $\alpha'_{jj+} < \alpha'_{jn}$. Since z_j is proportional to x_{1j} and $x'_{1j} > 0$, part (a) follows if $(d\alpha_{jj+}/dz_j) < (d\alpha_{jn}/dz_j)$. Similarly, part (b) follows if $(d\alpha_{jj+}/dy_j) < (d\alpha_{jn}/dy_j)$. These are exactly the conditions we examined in the proof of Theorem 9. (Note $x'_{ij} > 0$ is equivalent to the statement, "if repositioning is possible.")

Note that in Theorem 9, condition (b) is sufficient but not necessary but equality is allowed. In Theorem 11, condition (b) is necessary and sufficient but equality is not allowed.

THEOREM 12. Defensive Pricing in Irregular Markets. *If consumer tastes are uniformly distributed and $\alpha_{jj+} + \alpha_{jn} > \theta_j$, which is implied by regularity, then the optimal defensive price strategy is to decrease price.*

PROOF. The proof follows from line 2 of the proof to Theorem 2. Note that we could also prove the first order conditions for $f'(\alpha) \leq 0$ by Lemma 3.

THEOREM 13. *If the size of the market remains constant, defensive profits are decreasing in price at the before-entry optimal if and only if the distribution of consumer tastes, $f(\alpha)$, satisfies the following equation:*

$$\begin{aligned} \int_{\alpha_{jn}}^{\alpha_{jj+}} f(\alpha) d\alpha + (p - c) \{ & f(\alpha_{jj+})p_j + (x_{2j}x_{1j+} - x_{1j}x_{2j+}) \\ & + [(p_jx_{2j+} - p_j + x_{2j})^2 + (p_j + x_{1j} - p_jx_{1j+})^2] \\ & - f(\alpha_{jn})p_n(x_{2j}x_{1n} - x_{1j}x_{2n}) \\ & + [(p_jx_{2n} - p_nx_{2j})^2 + (p_nx_{1j} - p_jx_{1n})^2] \} > 0. \end{aligned}$$

PROOF. $\Pi_b(p^0) = \Pi_a(p^0) + Z(p^0)$ where $Z(p) = N_a(p - c) \int_{\alpha_{jn}}^{\alpha_{jj+}} f(\alpha) d\alpha$. Since $\Pi'_b(p^0) = 0$ we need only show that $Z'(p^0) > 0$ to establish that $\Pi'_a(p^0) < 0$. (') denotes the derivative with respect to price at p . Taking derivatives gives us

$$Z'(p) = N_a \int_{\alpha_{jn}}^{\alpha_{jj+}} f(\alpha) d\alpha + N_a(p - c)[f(\alpha_{jj+})\alpha'_{jj+} - f(\alpha_{jn})\alpha'_{jn}] > 0.$$

Finally substituting the explicit terms for α'_{jj+} and α'_{jn} yields the result.

THEOREM 14. *If evoking is proportional to the probability that the new product will be chosen if it is in the evoked set, then the forecasted market shares are given by (24) (25) in the text.*

PROOF. By (19),

$$\begin{aligned} m_n^*(x_n) &= \sum_l m_{n|l}^* S_l^* = \left(\sum_l m_{n|l}^* S_l^* \right) \left(\sum_k m_{n|k}^* S_k \right) / \left(\sum_k m_{n|k}^* | S_k \right) \\ &= \sum_l \sum_k m_{n|l}^* m_{n|k}^* S_l^* S_k (S_k^* S_k^{-1}) / \left(\sum_k m_{n|k}^* S_k \right). \end{aligned}$$

By assumption, S_k^* is proportional to $S_k m_{n|k}^*$. Substituting and rearranging terms yields

$$m_n^*(x_n) = \sum_l \sum_k m_{n|l}^* m_{n|k}^* S_l m_{n|l}^* S_k S_k^{-1} m_{n|k}^{-1} / \left(\sum_k m_{n|k}^* S_k \right).$$

Recognizing $W = \sum_k S_k^*$ yields the equation for $m_n^*(x_n)$ in the text. Using this result and similar substitutions yields (25).

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