Definitions of multipartite nonlocality — Source link

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Article

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Abstract

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Reference

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Definitions of multipartite nonlocality

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In a multipartite setting, it is possible to distinguish quantum states that are genuinely n-way entangled from those that are separable with respect to some bipartition. Similarly, the nonlocal correlations that can arise from measurements on entangled states can be classified into those that are genuinely n-way nonlocal, and those that are local with respect to some bipartition. Svetlichny introduced an inequality intended as a test for genuine tripartite nonlocality. This work introduces two alternative definitions of n-way nonlocality, which we argue are better motivated both from the point of view of the study of nature, and from the point of view of quantum information theory. We show that these definitions are strictly weaker than Svetlichny's, and introduce a series of suitable Bell-type inequalities for the detection of three-way nonlocality. Numerical evidence suggests that all three-way entangled pure quantum states can produce three-way nonlocal correlations.

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Consider two quantum systems, prepared in a joint quantum state $|\psi\rangle$ and located in separate regions of space. Suppose Alice measures one system, obtaining outcome a, and Bob the other, obtaining outcome b. The joint outcome probabilities can be written $P(ab|XY)$, where X denotes Alice's measurement and Y Bob's measurement. If the measurements are performed at spacelike separation, then Bell's condition of *local causality* [1] implies that even if the particles have interacted in the past (or were produced together in the same source), they are now independent. Therefore, even if the quantum state of the two particles is entangled, it ought to be possible to specify a more complete description λ of the joint state of the two particles, such that, given λ, the probabilities can be written in the form

$$P_\lambda(ab|XY) = P_\lambda(a|X)P_\lambda(b|Y). \tag{1}$$

The state λ is conventionally referred to as a *hidden* state, since it is not part of the quantum description of the experiment. Any hidden state λ which satisfies Eq. (1) is *local*. If the observed correlations $P(ab|XY)$ can be explained by a locally causal theory, then they can be written

$$P(ab|XY) = \sum_\lambda q_\lambda P_\lambda(a|X)P_\lambda(b|Y), \tag{2}$$

with $q_\lambda \geqslant 0$ and $\sum_\lambda q_\lambda = 1$. On the other hand, if correlations $P(ab|XY)$ violate a Bell inequality [1], then they cannot be written in this form. Such correlations cannot be explained by a locally causal theory, and are referred to as nonlocal correlations.

Quantum nonlocality is a puzzling aspect of nature, but also an important resource for quantum information processing. An information theoretic interpretation of quantum nonlocality is that two separated parties who wish to simulate the experiment with classical resources cannot do so using only shared random data—they must also communicate with one another. The fact that entangled quantum states can produce nonlocal correlations enables the quantum advantage in communication complexity problems [2], device-independent quantum cryptography [3,4], randomness expansion [5], and measurement-based quantum computation [6,7].

With three or more systems, qualitatively different kinds of nonlocality can be distinguished. For definiteness, consider the tripartite case. If correlations can be written

$$P(abc|XYZ) = \sum_\lambda q_\lambda P_\lambda(a|X)\, P_\lambda(b|Y)\, P_\lambda(c|Z), \tag{3}$$

with $0 \leqslant q_\lambda \leqslant 1$ and $\sum_\lambda q_\lambda = 1$, then they are local. Otherwise they are nonlocal. But, as pointed out by Svetlichny [8], some correlations can be written in the hybrid local-nonlocal form,

$$\begin{aligned} P(abc|XYZ) = &\sum_\lambda q_\lambda\, P_\lambda(ab|XY)\, P_\lambda(c|Z) \\ &+ \sum_\mu q_\mu\, P_\mu(ac|XZ)\, P_\mu(b|Y) \\ &+ \sum_\nu q_\nu\, P_\nu(bc|YZ) P_\nu(a|X), \end{aligned} \tag{4}$$

where $0 \leqslant q_\lambda, q_\mu, q_\nu \leqslant 1$ and $\sum_\lambda q_\lambda + \sum_\mu q_\mu + \sum_\nu q_\nu = 1$. Here, each term in the decomposition factorizes into a product of a probability pertaining to one party's outcome alone, and a joint probability for the two other parties. We say that correlations of the form (4) are S_2 local. If correlations cannot be written in this form, then a term such as $P_\lambda(abc|XYZ)$ must appear somewhere in the decomposition. Such correlations are often said to exhibit *genuine three-way* nonlocality, although we will refer to this as *Svetlichny* nonlocality. Svetlichny introduced an inequality, a violation of which implies Svetlichny nonlocality. Svetlichny's inequality can be violated by appropriate measurements on a Greenberger-Horne-Zeilinger (GHZ) or W state [9].

In further work, Seevinck and Svetlichny [10] and, independently, Collins *et al.* [11] generalized the tripartite notion of Svetlichny nonlocality to n parties. In both Refs. [10] and [11], an inequality is derived that detects n-partite Svetlichny nonlocality. See also Refs. [9,12,13].

The present Brief Report considers two alternative definitions of genuine multipartite nonlocality, which are different from Svetlichny's. We argue that these definitions are better motivated, both physically and from the point of view of information theory. We show that the alternative definitions are strictly weaker than Svetlichny's and describe a Bell inequality such that its violation is sufficient for genuine three-way nonlocality according to both alternative definitions. Numerical evidence suggests that any pure, three-way entangled quantum state can produce correlations that violate this inequality. On the other hand, there exist pure, three-way entangled quantum states for which we have not been able to find any measurements giving rise to Svetlichny nonlocality.

Different kinds of nonlocality. Consider again the case of bipartite correlations. There are various ways in which a hidden state λ might fail to be local. Let $P_\lambda(a|XY) = \sum_b P_\lambda(ab|XY)$ be the marginal probability for Alice to obtain outcome a when the measurement choices are X and Y, and similarly let $P_\lambda(b|XY) = \sum_a P_\lambda(ab|XY)$ be the probability for Bob to obtain b. Suppose that λ satisfies

$$P_\lambda(a|XY) = P_\lambda(a|XY') \quad \forall a,X,Y,Y', \tag{5}$$

$$P_\lambda(b|XY) = P_\lambda(b|X'Y) \quad \forall b,Y,X,X'. \tag{6}$$

In this case, if Alice and Bob are in possession of two particles, which they know to be in the hidden state λ, then even if λ is nonlocal, observing her own outcome gives Alice no information about Bob's measurement choice. This is because the marginal probabilities for a are independent of Bob's choice. Hence Bob cannot send signals to Alice by varying his measurement choice. Similarly, Alice cannot send signals to Bob. Such a λ is *nonsignaling*.

If Eq. (5) is satisfied but Eq. (6) is violated, then Bob's outcome gives him at least some information about Alice's measurement choice, hence Alice can send signals to Bob. The hidden state λ is *one-way signaling*. Similarly, if Eq. (6) is satisfied but Eq. (5) is violated. If Eqs. (5) and (6) are both violated then λ is *two-way signaling*.

So far, this discussion has followed many treatments of quantum nonlocality in that no attention has been given to the timing of Alice's and Bob's measurements. It has been assumed—naively—that the measurements can unproblematically be regarded as simultaneous, or alternatively that the probabilities $P_\lambda(ab|XY)$ are independent of the timing of the measurements. With one-way and two-way signaling states, this can quickly cause problems. Suppose that a hidden state λ is one-way signaling from Alice to Bob. Then the outcome probabilities for a measurement of Bob's depend on which measurement setting Alice chooses. If Bob obtains his measurement outcome before Alice chooses her setting (with respect to some frame) then this implies some kind of backwards causality (with respect to that frame). Worse, Fig. 1 shows how signaling hidden states can lead to grandfather-style paradoxes, where no consistent assignment of probabilities to outcomes is possible.

One solution to these problems would be to restrict attention to models that involve only nonsignaling hidden states. But a more general solution is to introduce a notion of a hidden

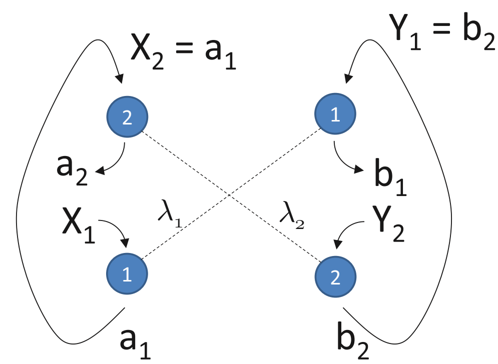

FIG. 1. (Color online) Let $X,Y,a,b \in \{0,1\}$. The particle pair labeled 1 is independent from the pair labeled 2. The joint state λ_1 is such that if $a_1 = Y_1$, then $P_{\lambda_1}(a_1 b_1|X_1 Y_1) = 0$, whereas λ_2 is such that if $b_2 \neq X_2$, then $P_{\lambda_2}(a_2 b_2|X_2 Y_2) = 0$. Consistent predictions are impossible if measurement choices are as shown.

state, according to which the outcome probabilities can vary according to the timing of the measurements. In a fully general treatment, λ will be time dependent, or alternatively, λ will refer to the state of the particles at some fixed time (perhaps just after creation) in some fixed frame, and the probabilities for outcomes depend on the exact timing of the measurements.

For now, let us keep things simple. Consider a hidden state λ such that the probabilities do not depend on the exact timing of measurements, but do depend on the time *ordering*, where this ordering is determined with respect to a fixed background frame. If Alice performs X before Bob performs Y, the probabilities are given by

$$P_\lambda^{A<B}(ab|XY). \tag{7}$$

If Bob performs Y before Alice performs X, the correlations may be different with probabilities given by

$$P_\lambda^{B<A}(ab|XY). \tag{8}$$

Paradoxes such as that of Fig. 1 are avoided if (1) the fixed background frame determining the time ordering of measurements is the same for all particle pairs, and (2) the correlations $P_\lambda^{A<B}(ab|XY)$ and $P_\lambda^{B<A}(ab|XY)$ are at most one-way signaling, with $P_\lambda^{A<B}(ab|XY)$ satisfying Eq. (5) and $P_\lambda^{B<A}(ab|XY)$ satisfying Eq. (6). An explicit model depending on the time ordering of measurements and satisfying the above two conditions is given by Bohm's theory [14].

Given a set of bipartite quantum correlations $P(ab|XY)$, these considerations about the time ordering of measurements do not make any difference to the basic question of whether or not $P(ab|XY)$ is nonlocal, which is perhaps why time ordering is not often emphasized. In the case of three or more observers, however, it makes an important difference to the classification of different kinds of multipartite nonlocality.

Genuine three-way nonlocality. In Eq. (4), the probabilities are assumed to be independent of the time ordering of measurements, and no constraint is placed on the bipartite correlations appearing in each term. So $P_\lambda(ab|XY)$, for example, can be one-way or two-way signaling. But, as shown above, problems can arise with signaling hidden states, including paradoxes that can result if measurement outcomes can be used to determine measurement choices on other particles. One remedy is to consider only nonsignaling hidden states. This suggests the following definition of genuine tripartite nonlocality.

Definition 1. Suppose that $P(abc|XYZ)$ can be written in the form

$$P(abc|xyz) = \sum_\lambda q_\lambda P_\lambda(ab|xy)P_\lambda(c|z) + \sum_\mu q_\mu P_\mu(ac|xz)P_\mu(b|y) + \sum_\nu q_\nu P_\nu(bc|yz)P_\nu(a|x), \quad (9)$$

where the bipartite terms are nonsignaling, satisfying conditions of the form (5) and (6). Then the correlations are NS_2 local. Otherwise, we say that they are genuinely three-way NS nonlocal.

As we have seen, however, a more general remedy is to define hidden states in such a way that correlations can depend on the time ordering of the measurements. It is convenient to write $P_\lambda^{T_{AB}}(ab|XY)$ for a set of time-order-dependent correlations, so that $P_\lambda^{T_{AB}}(ab|XY) = P_\lambda^{A<B}(ab|XY)$ when Alice measures before Bob and $P_\lambda^{T_{AB}}(ab|XY) = P_\lambda^{B<A}(ab|XY)$ when Bob measures before Alice. As always, assume that $P_\lambda^{A<B}(ab|XY)$ and $P_\lambda^{B<A}(ab|XY)$ are at most one-way signaling, satisfying Eqs. (5) and (6), respectively.

Definition 2. Suppose that $P(abc|XYZ)$ can be written in the form

$$P(abc|xyz) = \sum_\lambda q_\lambda P_\lambda^{T_{AB}}(ab|xy)P_\lambda(c|z) + \sum_\mu q_\mu P_\mu^{T_{AC}}(ac|xz)P_\lambda(b|y) + \sum_\nu q_\nu P_\nu^{T_{BC}}(bc|yz)P_\lambda(a|x). \quad (10)$$

Then the correlations are T_2 local. Otherwise they are genuinely three-way nonlocal.

Interpretation from the point of view of quantum information. It is useful to contrast Definition 2 and Svetlichny's one from the perspective of classical simulations of quantum correlations in term of shared random data and communication (for examples of such a model, see Ref. [15]). Svetlichny models naturally correspond to simulation models where all parties receive their input (the measurement they are to simulate) at the same time, then there are several rounds of communication between subsets of the parties, and, finally, all parties produce an output (the measurement outcome). Models of the form (10), on the other hand, correspond to simulation models where inputs are given to the parties in a sequence, where the order in the sequence is arbitrary and not fixed in advance. On receiving an input, a party must produce an output immediately and may send a communication to a subset of the other parties. This means that although a party's output can depend on communications already received, it cannot depend on communications from parties later in the sequence.

The distinction between both types of models is crucial for the simulation of quantum correlations in applications such as measurement-based computation where measurements are performed adaptively, that is, where the choice of which measurement to perform on a particular system may depend on the measurement outcome that was obtained from another system. In this context, Svetlichny-type simulation models in which all inputs are given at the same time are not relevant.

Finally, models based on the definition (9) can be interpreted as simulation models where classical communication is replaced by no-signaling resources [16] [such as Popescu-Rohrlich (PR) boxes [17]]. They are well adapted to the characterization of nonlocality for cryptographic applications secure against postquantum adversaries [3].

Characterization and detection of three-way nonlocality. Let NS_2 be the set of all tripartite correlations that are NS_2 local, according to Definition 1. Similarly, let T_2 be the set of correlations that are T_2 local according to Definition 2, and S_2 the set of S_2 local correlations according to the Svetlichny definition. Given these sets, we have the following results (see details in the Supplemental Material [18]). First, the different definitions of multipartite nonlocality are inequivalent, as one can show the following theorem:

Theorem 1. $\mathrm{NS}_2 \subset T_2 \subset S_2$ where the inclusion is strict.

Proof. See Appendix C in the Supplemental Material [18].

Note that, contrarily to S_2 models, both NS_2 and T_2 models can only reproduce no-signaling correlations (this is true on average for T_2 models even though they may involve one-way signaling between the parties at the hidden level; see Appendix B in the Supplemental Material [18]). Second, the NS_2, T_2 and S_2 sets can be characterized efficiently:

Theorem 2. Given correlations $P(abc|XYZ)$ with a finite number of measurement settings and outputs, it is a linear programming problem to determine whether they belong to the sets NS_2, T_2 or S_2.

Proof. See Appendix A in the Supplemental Material [18].

Furthermore, we have an inequality for the NS_2 and T_2 sets:

Theorem 3. If correlations $P(abc|XYZ)$ are NS_2 or T_2 local, then

$$\begin{aligned} I = &-2P(A_1B_1) - 2P(B_1C_1) - 2P(A_1C_1) \\ &- P(A_0B_0C_1) - P(A_0B_1C_0) - P(A_1B_0C_0) \\ &+ 2P(A_1B_1C_0) + 2P(A_1B_0C_1) + 2P(A_0B_1C_1) \\ &+ 2P(A_1B_1C_1) \leqslant 0, \end{aligned} \quad (11)$$

where we have introduced the notation $P(A_iB_j) \equiv P(a=0,b=0|X=i,Y=j)$, $P(A_iB_jC_k) \equiv P(a=0,b=0,c=0|X=i,Y=j,Z=k)$.

Proof. See Appendix B in the Supplemental Material [18].

Just like Svetlichny introduced an inequality, violation of which implies Svetlichny nonlocality, Eq. (11) is a Bell-type inequality, a violation of which implies that correlations are genuinely three-way nonlocal (hence also three-way NS nonlocal). In Appendix D (see Supplemental Material [18]) we provide also a complete characterization of the NS_2 polytope in the presence of binary inputs and outputs. Inequality (11) belongs to the family number 6 in this list, and is thus a tight constraint on the NS_2 as well as the T_2 sets.

Multipartite nonlocality and noisy quantum states. It is interesting to investigate the extent to which different quantum states can produce each type of multipartite nonlocality. Consider an experiment in which measurements are performed

TABLE I. Minimum values of the p parameter required for the quantum states ρ_{GHZ} and ρ_W to exhibit genuine multipartite nonlocality. These values were found by numerical optimization. It is assumed that three parties each have two possible measurement settings, each with two outcomes. If $p > p_{\mathrm{NS}}$ then correlations can be produced which are three-way NS nonlocal (see Definition 1). If $p > p_T$, then correlations can be produced which are three-way nonlocal (Definition 2). If $p > p_S$, then correlations can be produced which are Svetlichny nonlocal. Inequalities demonstrating the different notions of nonlocality of ρ_W for values of p higher than the above thresholds are described in Appendix D in the Supplemental Material [18].

State	p_{NS}	p_T	p_S
ρ_{GHZ}	$1/\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$
ρ_W	0.8	0.82	0.92

on a tripartite quantum state, with each party having a choice of two measurement settings, and each measurement having two possible outcomes. Let

$$|\mathrm{GHZ}\rangle = 1/\sqrt{2}(|000\rangle + |111\rangle), \tag{12}$$

$$|W\rangle = 1/\sqrt{3}(|001\rangle + |010\rangle + |100\rangle), \tag{13}$$

$$\rho_{\mathrm{GHZ}} = p\,|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| + (1-p)I/8, \tag{14}$$

$$\rho_W = p\,|W\rangle\langle W| + (1-p)I/8, \tag{15}$$

where I is the identity and $0 \leqslant p \leqslant 1$. We have determined using linear programming the minimum values of p for which the states ρ_{GHZ} and ρ_W will exhibit each kind of multipartite nonlocality. Results are summarized in Table I.

For the noisy GHZ state, it makes no difference which definition is employed—three-way NS nonlocal, three-way nonlocal, and Svetlichny nonlocal correlations can be generated whenever $p > 1/\sqrt{2}$. In the case of the noisy W state, there is a range of values of p for which the state is too noisy to exhibit Svetlichny nonlocality, but can still produce correlations which are three-way nonlocal, and similarly a range of values of p for which the state is too noisy to exhibit three-way nonlocality, but can still produce correlations which are three-way NS nonlocal. This again demonstrates that the different definitions of multipartite nonlocality are strictly inequivalent.

Multipartite nonlocality and tripartite entanglement. Finally, we conclude by presenting numerical results that suggest that all pure tripartite entangled states are three-way nonlocal. An arbitrary pure state of three qubits that is genuinely tripartite entangled can always be written in the form [19] $|\psi\rangle = \lambda_0|000\rangle + \lambda_1 e^{\phi}|100\rangle + \lambda_2|101\rangle + \lambda_3|110\rangle + \lambda_4|111\rangle$, with $\phi \in [0,\pi]$, $\lambda_i \geqslant 0$, $\sum_i \lambda_i^2 = 1$, $\lambda_0 \neq 0$, $\lambda_2 + \lambda_4 \neq 0$, and $\lambda_3 + \lambda_4 \neq 0$. We tested inequality (11) for $8^5 = 32\,768$ states of this form obtained by considering eight possible values for five independent variables parametrizing these states. After numerical optimization of the measurement settings, a violation was found in each case. We thus have the following conjecture:

Conjecture 1. All genuinely tripartite entangled states can, with a suitable choice of measurements, generate genuinely three-way nonlocal correlations.

Note, however, that we were not able to find any violation of the Svetlichny type for the following tripartite entangled state, $|\psi\rangle = \frac{\sqrt{3}}{2}|000\rangle + \frac{\sqrt{3}}{4}|110\rangle + \frac{1}{4}|111\rangle$ [though it violates inequality (11)]. Our search included the 1087 different Svetlichny inequalities introduced in Ref. [20], as well as a linear programming search over the Svetlichny polytope with two measurements settings per party.

Note added in proof. While the present Brief Report formally makes public the alternative definitions of multipartite nonlocality presented here, they have already been communicated privately to close collaborators. In particular, Definitions 1 and 2 were used in Refs. [21–23]. Note also the independent work [24] where Definition 2 is introduced and motivated from a different (though related) perspective.

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