Definitions of Voltage Unbalance

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Abstract: This letter reviews three definitions of voltage unbalance developed by NEMA, IEEE, and the power community, respectively. The differing definitions of voltage unbalance are analyzed in order to understand the implications of their use.

Introduction: In a three-phase system, voltage unbalance takes place when the magnitudes of phase or line voltages are different and the phase angles differ from the balanced conditions, or both. This letter addresses three definitions of voltage unbalance from three different communities and provides a numerical example and analysis to compare them. These definitions have important implications when studying for example, the effects of voltage unbalance on the performance of three-phase induction machines.

Definition of Voltage Unbalance: The three definitions of voltage unbalance are stated and analyzed below.

NEMA (National Equipment Manufacturer's Association) Definition: The NEMA definition [1] of voltage unbalance, also known as the line voltage unbalance rate (LVUR), is given by

$$\%\,\text{LVUR} = \frac{\text{max voltage deviation from the avg line voltage}}{\text{avg line voltage}} \cdot 100. \tag{1}$$

The NEMA definition assumes that the average voltage is always equal to the rated value, which is 480 V for the US three-phase systems and since it works only with magnitudes, phase angles are not included.

IEEE Definition: The IEEE definition [2] of voltage unbalance, also known as the phase voltage unbalance rate (PVUR), is given by

$$\%\,\text{PVUR} = \frac{\text{max voltage deviation from the avg phase voltage}}{\text{avg phase voltage}} \cdot 100. \tag{2}$$

The IEEE uses the same definition of voltage unbalance as NEMA, the only difference being that the IEEE uses phase voltages rather than line-to-line voltages. Here again, phase angle information is lost since only magnitudes are considered.

True Definition: The true definition of voltage unbalance is defined as the ratio of the negative sequence voltage component to the positive sequence voltage component [3]. The percentage voltage unbalance factor (% VUF), or the true definition, is given by

$$\%\ \text{VUF} = \frac{\text{negative sequence voltage component}}{\text{positive sequence voltage component}} \cdot 100. \tag{3}$$

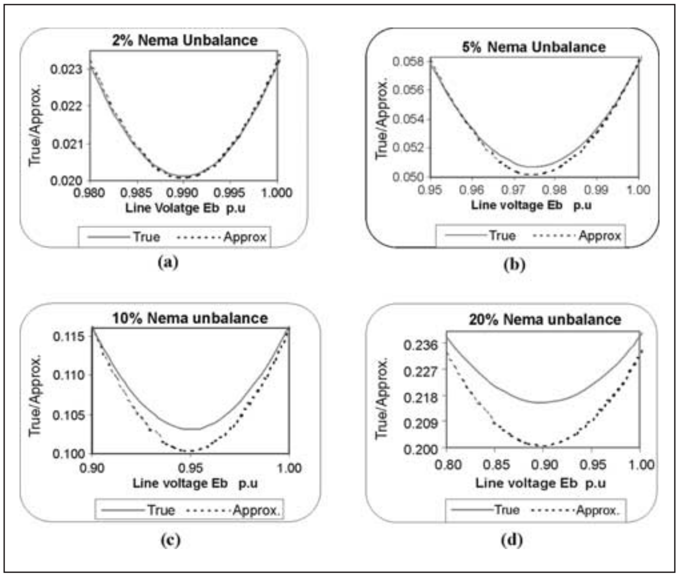

Figure 1. Relationship between the true definition of voltage unbalance and NEMA definition for 2%, 5%, 10%, and 20% values of NEMA unbalance

Table 1. Range of voltage unbalance

NEMA	True definition	Approximation formula
%	%	%
2	2 - 2.3	2 - 2.3
5	5 - 5.8	5 - 5.8
10	10.3 - 11.6	10 - 11.6
20	21 - 23.8	20 - 23.2

The positive and negative sequence voltage components are obtained by resolving three-phase unbalanced line voltages V_{ab}, V_{bc}, and V_{ca} (or phase voltages) into two symmetrical components V_p and V_n (of the line or phase voltages). The two balanced components are given by

$$V_p = \frac{V_{ab} + a \cdot V_{bc} + a^2 \cdot V_{ca}}{3} \tag{4}$$

$$V_n = \frac{V_{ab} + a^2 \cdot V_{bc} + a \cdot V_{ca}}{3} \tag{5}$$

where $a = 1\angle 120°$ and $a^2 = 1\angle 240°$.

The positive and negative sequence voltages can be used when analyzing induction motor behavior under unbalanced conditions. Since the true definition involves both magnitude and angles (complex algebra) when calculating the positive and negative sequence voltage components, a formula given by (6) avoids the use of complex algebra but gives a good approximation to the true definition.

$$\%\ \text{voltage unbalance} = \frac{82 \cdot \sqrt{V^2{}_{abe} + V^2{}_{bce} + V^2{}_{cae}}}{\text{avg line voltage}} \tag{6}$$

where V_{abe} = difference between the line voltage V_{ab} and the average line voltage, etc.

The following example shows how to use the three definitions of voltage unbalance given above.

Suppose three unbalanced line-to-line voltages $V_{ab} = 576\angle 0°$, $V_{bc} = 480\angle 221.4°$, and $V_{ca} = 384\angle 124.2°$ are applied to an induction machine. The average value of the magnitudes will be (576 + 480 + 384)/3 = 480 V and the maximum deviation from average value is (576 - 480) = 96 V. Therefore, the NEMA definition of % voltage unbalance will be 100 · (96/480) = 20%.

The positive sequence voltage is $V_p = 473.1\angle -5.04°$ and the negative sequence voltage is $V_n = 112.6\angle 21.74°$ for the above three unbalanced voltages. The true definition of % voltage unbalance will be 100 · (112.6/473.1) = 23.8%

Applying the approximate formula to the above example results in $V_{abe} = (576 - 480) = 96$, $V_{bce} = (480 - 480) = 0$, and $V_{cae} = (480 - 384) = 96$, therefore % voltage unbalance will be 82 · (103.5/380) = 23.2%. This value is close to the true value of 23.8%. The induction machine will respond to the true value of voltage unbalance, but NEMA will be assuming 20% unbalance for the same set of voltages.

In order to understand the implications of using these definitions of voltage unbalance, an analysis was conducted. Since the IEEE and NEMA definitions are similar and the derating curve is based on NEMA, it was decided to compare the NEMA definition with the true definition of voltage unbalance.

Analysis: Suppose three unbalanced line voltages are given by

$$\overline{E}_{ab} = E_{ab}\angle 0°,\ \overline{E}_{bc} = E_{bc}\angle\theta_{bc},\ \text{and}\ \overline{E}_{ca} = E_{ca}\angle\theta_{ca}$$

for a given percentage of voltage unbalance based on the NEMA definition, say 5%, assuming an average voltage of 460 V and naming the line voltage with the largest deviation from the average, E_{ab}. The following calculations are made:

$$\%\text{LVUR} = \frac{E_{ab} - 460}{460} = 0.05$$

$$E_{ab} - 460 = 0.05 \cdot 460 = 23 \qquad \therefore E_{ab} = 483$$

$$\text{The avg voltage} = \frac{E_{ab} + E_{bc} + E_{ca}}{3} = 460$$

$$\therefore E_{bc} + E_{ca} = 897 \text{ and } E_{ca} = 897 - E_{bc}.$$

E_{bc} and E_{ca} can be written as $|E_{bc} - 460| < 23$ and $|E_{ca} - 460| < 23$, respectively. This is so because E_{ab} has the largest deviation from average voltage and the average value should be 460. Hence,

$$437 < E_{bc} < 460 \text{ and } 437 < E_{ca} < 460.$$

The vector sum of the line voltages is $\overline{E}_{ab} + \overline{E}_{bc} + \overline{E}_{ca} = 0$, since the zero sequence voltage must be zero in the absence of a fault. This equation can be resolved as follows:

$$E_{ab}\angle 0^\circ + E_{bc}\angle\theta_{bc} + E_{ca}\angle\theta_{ca} = 0$$

$$483 + E_{bc} \cdot \cos\theta_{bc} + j\,E_{bc} \cdot \sin\theta_{bc} + (897 - E_{bc}) \cdot \cos\theta_{ca} + j(897 - E_{bc}) \cdot \sin\theta_{ca} = 0. \tag{7}$$

So for a given E_{bc}, angle θ_{bc} and angle θ_{ca} can be obtained from (7) by separating it into two parts, real and imaginary, and solving the two equations.

From the above calculations, the true definition of voltage unbalance will be

$$\%\text{ VUF} = \frac{483\angle 0^\circ + a^2 \cdot E_{bc}\angle\theta_{bc} + a \cdot (897 - E_{bc})\angle\theta_{ca}}{483\angle 0^\circ + a \cdot E_{bc}\angle\theta_{bc} + a^2 \cdot (897 - E_{bc})\angle\theta_{ca}} \cdot 100 \tag{8}$$

where $a = 1\angle 120^\circ$ and $a^2 = 1\angle 240^\circ$. The approximation formula of the true definition will given by

$$\%\text{ voltage unbalance} = \frac{82 \cdot \sqrt{(483 - 460)^2 + (E_b - 460)^2 + ((897 - E_b) - 460)^2}}{460}. \tag{9}$$

From this analysis, it is found that for a given value of % unbalance, based on the NEMA definition, there is range of % unbalance, based on the true definition and also using the approximation formula. This is shown in Figure 1 for 2%, 5%, 10%, and 20% NEMA definition of voltage unbalance. The solid line represents the true definition and dotted line represents the approximation formula.

Figure 1(a) shows that for a 2% NEMA unbalance, the true definition and the approximation formula agree very closely. For 5% NEMA unbalance shown in Figure 1(b) the approximation formula starts to deviate slightly from the true definition. Figures 1(c) and (d) show that as the % NEMA unbalance increases, the approximation formula deviates even more from the true definition.

The difference between the NEMA definition and the true definition can differ substantially when the voltage unbalance is extremely high, as shown in Figure 1(d). Table 1 shows the range of the true definition and the approximate formula of % unbalance obtained at 2%, 5%, 10%, and 20% NEMA unbalance.

Below 5% NEMA unbalance, the maximum difference between the NEMA and the true definition is 0.8%. This difference may not be significant in determining the derating of induction machines, for example. Above that, say 20%, the difference can be as high as 3.8%. The motor will respond to the true value of 23.8%, but NEMA will be assuming a 20% unbalance.

Conclusions: This letter has addressed three definitions of voltage unbalance. An analysis was done to show how these definitions are related. It was found that for a given NEMA unbalance, there is a range of unbalance based on the true definition and the approximation formula. Below 5% unbalance, the difference between the NEMA definition and the true definition is very small (0.8%) and this may have an insignificant effect on motor derating. The difference is high for extreme values of % unbalance based on the NEMA definition.

The approximation formula avoids complex algebra and gives a good approximation to the true definition at low value of % voltage unbalance.

References:

[1] *Motors and Generators*, ANSI/NEMA Standard MG1-1993.

[2] *IEEE Standard Test Procedure for Polyphase Induction Motors and Generators,* IEEE Standard 112, 1991.

[3] R.C. Dugan, M.F. McGranaghan, and H.W. Beaty, *Electrical Power Systems Quality*. New York: McGraw-Hill, 1996.

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