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Deformation based fatigue failure criterion

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ABSTRACT

It was attempted to establish a deformation based fatigue failure criterion which is independent of the cyclic load history (level, frequency and sequence of load cycles). Based on the analysis and comparison of cyclic and static test results, it was recognaized that the acceleration of fatigue deformations prior to failure starts at the deformation caused by the highest applicable stress in a static test on the same type of specimen. That means, *fatigue failure initiates at the deformation corresponding to the static strength.* This criterion may serve to refine our material models in addition to predict fatigue failure.

INTRODUCTION

Repetitive stresses cause progressive damages in materals. The accumulation of internal damages induces additional deformations and modifications in the stress field that may lead to a complete failure.

The traditional method of predicting fatigue failure is using a relationship between the applied stress ranges or extreme values of the cyclic load and the number of load cycles (generally referred to as Wöhler diagram) which gives an indication on the fatigue life and the endurance limit. The criterion of failure is to apply the corresponding stress level and number of load cycles provided by the Wöhler diagram obtained in previous tests.

An another approach to predict the number of load cycles up to failure is using *damage accumulation rules* such as the Palmgren-Miner rule. Palmgren [1] proposed in 1924 for ball bearings a linear damage accumulation rule which was restated by Miner [2] in 1945 for notched aluminium alloy specimens. According to this rule, the fatigue damage accumulates linearly with the number of load cycles applied at a particular load level. The criterion of failure is to reach 1.0, summing the ratio of the number of cycles related to the maximum number of load cycles producing failure on the various load levels. Based on test results with plain concrete beams of 500 mm length and 100·100 mm section subjected to variable amplitude cyclic loads at midspan, Oh [3] proposed a cubic equation to describe the accumulation of damage instead of a linear one.

All above failure criterions are directly or indirectly based on the cyclic load or stress level and may give inadequate predictions for loads applied with different sequences. There is a need to develop a general rule to predict fatigue failure independent of the load history and may help to reduce costs of expensive fatigue tests.

ANALYSIS OF CYCLIC TEST RESULTS

In previous studies [4] [5] on the fatigue behaviour of bond between steel reinforcing bars and concrete a relationship of deformations in fatigue and static tests was observed. This relationship is supported now with further test results on various fields.

Tests on pull-out specimens

The measurable deformation on pull-out specimens (commonly used for bond tests) is the slip (s) of the bar sections relative to the concrete sections. The slip versus number of load cycles diagram — efficiently used to analyze the fatigue process — may be subdevided into three different phases (Figure 1): *primary phase* (with decreasing deformation rate), *secondary phase* (with constant deformation rate) and *tertiary phase* (with increasing deformation rate). The second linear phase may be reduced to a point of inflexion for high load levels. Both the primary and the secondary phases indicate increasing in-ternal damages but without unforseen failure. Nevertheless, they can be considered as the stabil part of the fatigue process. Reaching the tertiary phase, however, an acceleration of deformation rate takes place leading to failure.

Further cyclic pull-out test results with different load levels ($\tau_{b,max}/\tau_{bu}$), concrete mix, bar diameter (Ø) and bond length (ℓ_b) (Figure 2) resulted in the above three phases as well. Figure 2.e [6] indicates a test result with variable amplitudes applying a cyclic load of steppwise decreasing manner in twenty blocks of 100 000 cycles up to two millions. The registered slip increase is slightly undulating following the load levels, however, its global tendency reflects the above three phases. Figure 2.e indicates the average static bond stress versus slip relationship to all of the cyclic test results in Figure 2 obtained by deformation controlled tests. The slip at the bond strength /s(τ_{bu})/ for that reinforcing bar was found to be 1.5 mm in average.

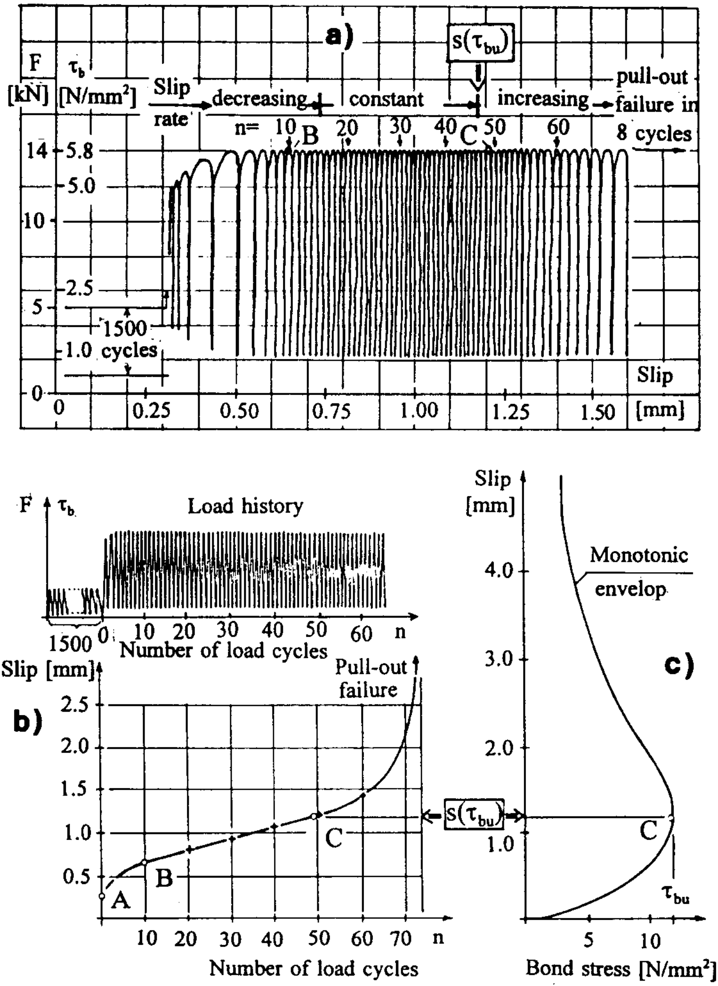

Figure 1: Fatigue of bond, Ø8, f_y=400 N/mm², ℓ_b=12Ø, f_c=16 N/mm²
a) τ_b-s **b)** s-n **c)** τ_b-n ; Balázs [5]

Tests on plain concrete

Cyclic test results on plain concrete specimens either with compressive loads (Holmen [9] and Sparks [10]) or tensile loads (Cornelissen [11]) resulted in strain versus number of load cycles diagrams showing the above three phases including an intermediate linear portion (Figure 3). The only difference is that the tertiary phase for the repeated tensile load is considerably shorter than that of a repeated compressive load. Petković observed [12] that the second phase is longer and more pronounced for high strength concrete. The above three phases of the failure process of concrete were also distinguished in the failure mechanichs approach of Mihashi and Izumi [13].

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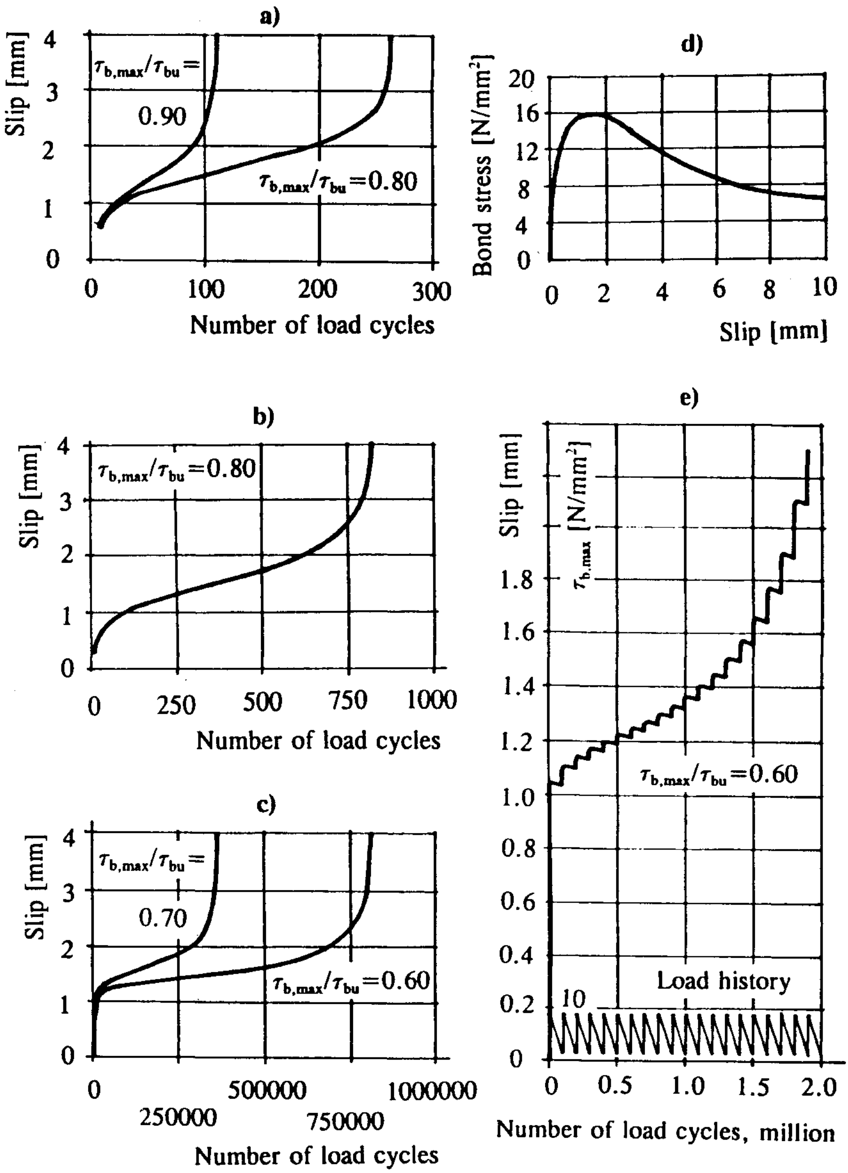

Figure 2: Fatigue of bond, Ø16, f_y=500 N/mm², ℓ_b=5Ø, f_{ck}=25 N/mm²
a) b) c) under constant amplitude cyclic loads; Koch-Balázs [8]
d) static bond stress vs. slip rel. (av. of 3 results); Koch-Balázs [7]
e) under linearly decreasing ampl. of 20 blocks; Balázs-Koch [6]

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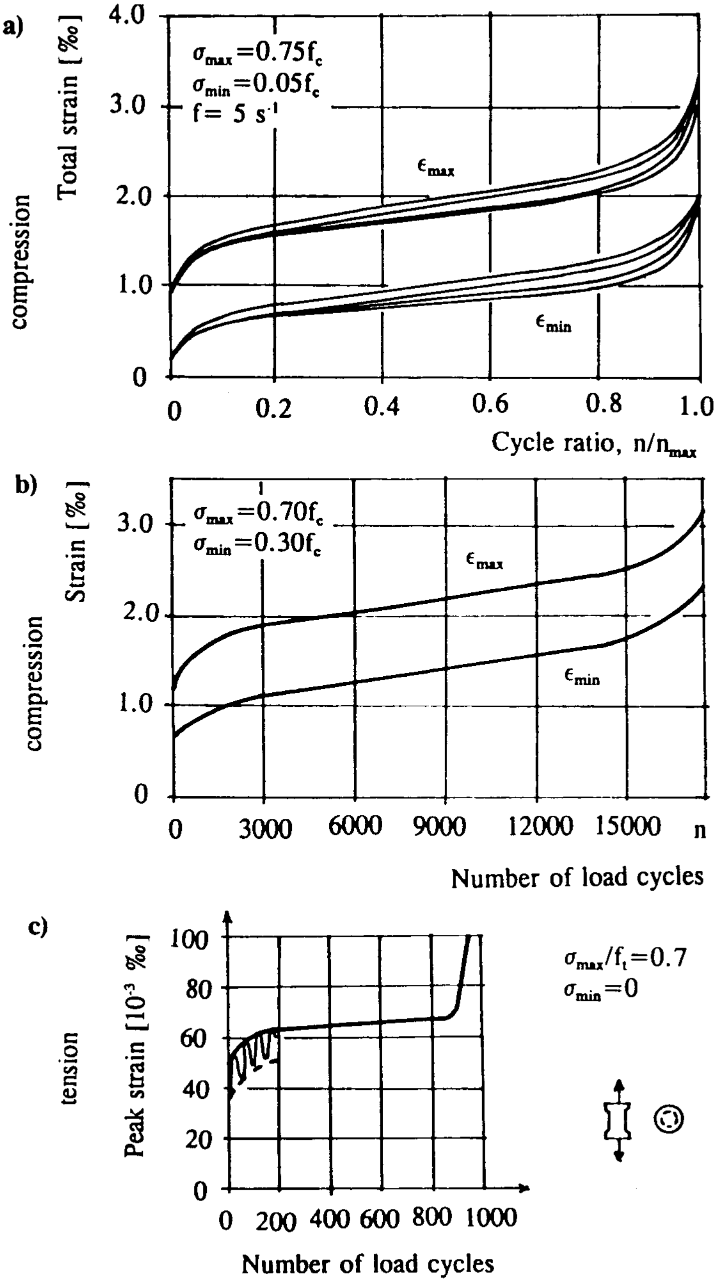

Figure 3: Fatigue behaviour of plain concrete
a) compressed cylinder (150·300 mm, $f_{ck}=40$ N/mm^2); Holmen [9]
b) compressed prism (102·102·203 mm); Sparks [10]
c) tensioned cyl. (120·254 mm, $f_t=2,9$ N/mm^2); Cornelissen [11]

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DICUSSION OF THE FAILURE CRITERION

By comparing the presented cyclic test results to the results of deformation controlled tests, it was recognaized that the acceleration of the fatigue deformations (at the beginning of the tertiary phase of deformation versus number of load cycles diagrams) initiates at the deformation corresponding to the strength obtained by the static test. It was found to be valid independent-ly of the load history (i.e. independently of the level, frequency and sequence of load cycles) and type of specimen. Based on these test results, a deformation based fatigue failure criterion seems to be reasonable that: ***fatigue failure initiates at the deformation corresponding to the static strength (Figure 4).*** This criterion gives a guidance to predict fatigue failure, i.e., gives a limit of deformations below that the fatigue failure is practically excluded. The type of failure (tension, compression, shear, splitting, etc.) and the direction of load, however, should be the same for both the fatigue and the static tests.

Considering the inevitable scatter of test results, it is reasonable to consider a range of governing deformations incorporating the specific value of deformation ($\epsilon(\sigma_u)$) rather than just $\epsilon(\sigma_u)$. As an average behaviour, however, the hypothesis is still valid for $\epsilon(\sigma_u)$.

These comparisons *do not support the classification of* fatigue behaviour into *low and high cycles fatigue.* The deformation versus number of load cycles diagrams in both cases indicate similar fatigue processes. The only difference is that the length of the failure phase related to the entire fatigue life is longer for low cycles fatigue and reaches 50 % in absence of the secondary linear phase. Even if the number of load cycles to failure may differ considerably, the failure phase initiates at $\epsilon(\sigma_u)$.

Further advantages of the hypothesis

The limit deformation (i.e., the deformation at static strength) used to define the beginning of the failure phase in fatigue is often available or at least relatively *easy to obtain by tests.* This criterion may *refine our material models* and may allow *extrapolation for the fatigue life* if the failure was not yet reached, *it may reduce the costs* of sometimes extremely expensive fatigue tests. Whenever the secondary linear phase has already been reached, the beginning of the tertiary failure phase may be linearly extrapolated using the tangent of the deformation vs. number of load cycles diagram (which is, however, dependent on the applied load level) up to $\epsilon(\sigma_u)$ (Figure 4). It may help to predict the end of the test.

CONCLUSIONS

Various fields of test results indicate that the deformation versus number of

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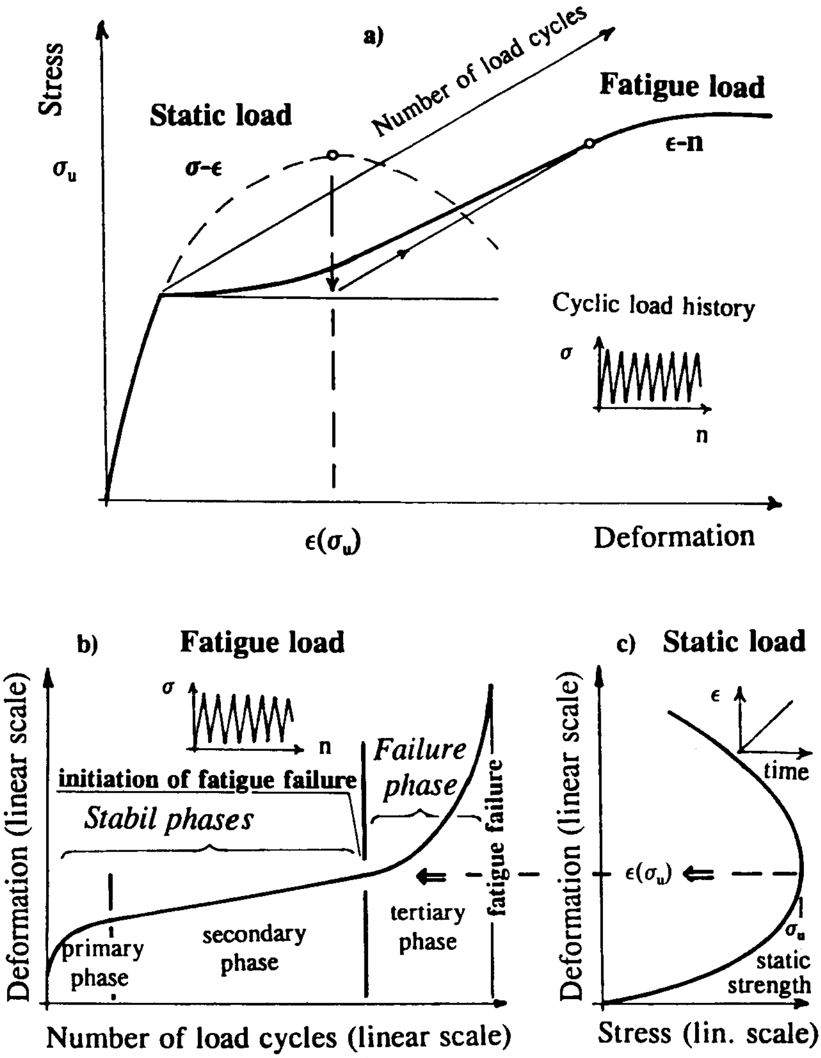

Figure 4: Deformation based fatigue failure criterion
a) three dimensional representation: σ-ϵ-n
b) max. def. vs. number of load cycles diagram in a fatigue test
c) stress versus deformation diagram in a static test

load cycles diagrams up to failure may be subdevided into three different phases as primary phase (the deformation rate is decreasing), secondary phase (the deformation rate is constant) and tertiary phase (the deformation rate is rapidly increasing). It is valid for both for low and high cycles fatigue, however, the second linear portion may be reduced to a point of inflexion for

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low cycles fatigue. In case of variable amplitude loads, the three portions of the deformation versus number of load cycles diagram can only be recognized on the overall tendency.

Comparing the cyclic and static test results, a relationship is observed independently of the cyclic load history. The acceleration of the fatigue deformations prior to failure starts reaching the deformation at the highest applicable stress on the same type of speciemen in a static test. It may be briefly summarized as: *fatigue failure initiates at the deformation corresponding to the static strength.* This criterion may serve to refine our material models in addition to predict fatigue failure.

REFERENCES

1. Palmgren, A. 'Die Lebensdauer von Kugellagern' *Zeitschrift des Vereines* deutsche Ingenieure, April, pp.339-341, 1924
2. Miner, M.A. 'Cumulative Damage in Fatigue' *Journal of the Applied Mechanics*, Sept. pp.159-164, 1945
3. Oh, B.H. 'Cumulative Damage Theory of Concrete under Variable Fatigue Loadings' *ACI Materials Journal*, Jan.-Febr. pp.41-48, 1991
4. Balázs G.L. 'Bond Behaviour Under Repeated Load' *Studi e Ricerche*, Politecnico di Milano, Italia, Vol.8, pp.395-430, 1986
5. Balázs G.L. 'Fatigue of Bond' *ACI Mat. Journ.*, Vol.88. pp.620-629, 1991
6. Balázs G.L. and Koch, R. 'Influence of Load History on Bond Behaviour' pp.7-1 to 7-10, *Proceedings of the Int. Conf. on Bond in Concrete*, Riga 1992, TU Riga 1992
7. Koch, R. and Balázs G.L. 'Influence of Cyclic Loading on Bond Strength and Related Slip' pp.7-11 to 7-20, *Proceedings of the Int. Conf. on Bond in Concrete*, Riga, 1992, TU Riga 1992
8. Koch, R. and Balázs G.L. 'Slip increase under long term and cyclic loads' *Otto-Graf-Journal* (journal of the Otto-Graf-Institute, Stuttgart) Vol.4 1993
9. Holmen, J.O. 'Fatigue of Concrete by Constant and Variable Amplitude Loading' *Report of the University of Trondheim*, No.79-1. 218 p, 1979
10. Sparks, P.R. 'The Influence of Rate of Loading and Material Variability on the Fatigue Characteristics of Concrete' *Fatigue of Concrete Structures* (ed. S.P.Shah), ACI SP-75, pp.331-341, 1982
11. Cornelissen,H.A.W. 'Fatigue of concrete in tension' *HERON Publications* Delft University of Technology, Nº4, pp.1-67, 1984
12. Petković, G. 'Properties of Concrete Related to Fatigue Damage with Emphasis on High Strength Concrete' *BK-Report*, Un. of Trondheim, 1991:2
13. Mihashi, H., Izumi, M. 'Damage mechanism and mechanical behaviour of concrete under cyclic loads' contribution of the transaction of the *Architectural Institute of Japan*, April, pp.1-7, 1984