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Article *in* Technometrics · May 2012

DOI: 10.1080/00401706.2012.676946

Zhi-Sheng Ye
National University of Singapore

Yan Shen
Xiamen University

Loon Ching Tang
National University of Singapore

Supplementary materials for this article are available online. Please go to http://www.tandfonline.com/r/TECH

Degradation-Based Burn-In Planning Under Competing Risks

Zhi-Sheng Ye, Min Xie, and Loon-Ching Tang

Department of Industrial and Systems Engineering
National University of Singapore
119260 Singapore Q1

Yan Shen

Department of Planning and Statistics
Xiamen University
Xiamen, 361005 P.R. China

Motivated by two real-life examples, this article develops a burn-in planning framework with competing risks. Existing approaches to planning burn-in tests are confined to a single failure mode based on the assumption that this failure mode is subject to infant mortality. Considering the prevalence of competing risks and the high reliability of modern products, our framework differentiates between normal and infant mortality failure modes and recommends degradation-based burn-in approaches. This framework is employed to guide the burn-in planning for an electronic device subject to both a degradation-threshold failure, which is an infant mortality mode and can be modeled by a gamma process with random effect, and a catastrophic mode, which is normal and can be represented with a conventional reliability model. Three degradation-based burn-in models are built and the optimal cutoff degradation levels are derived. Their validity is demonstrated by an electronic device example. We also propose three approaches to deal with uncertainty due to parameter estimation. Algorithmic details and proofs are provided in supplementary material online.

KEY WORDS: Burn-in planning framework; Catastrophic failure; Competing risks; Degradation-threshold failure. Q2

1. INTRODUCTION

Most products may fail in many different ways, known as competing risks. According to the failure mechanism, a failure mode can be either a degradation-threshold (DT) failure or a catastrophic failure. A DT failure, also called a soft failure, occurs when a *measurable* physical degradation reaches a critical threshold level, which is often specified by industrial standards; while a catastrophic failure causes instant product failure. Both kinds of failure modes may be subject to infant mortality. For example, the failure rate of a catastrophic failure mode might be decreasing, indicating that some units will fail very early. Thus, we consider two different classifications of failure modes: DT failure/catastrophic failure and infant mortality failure/normal failure.

To identify and eliminate units with infant mortality, engineers often resort to burn-in by activating all infant mortality failure modes during the test for a certain duration. The purpose of this study is to design a burn-in test that uses both degradation and failure data in a way that will maximize profit, or equivalently minimize the expected cost. We use existing statistical models to describe both reliability and degradation. Burn-in models combine these statistical models with cost information to inform the choice of a burn-in duration and a "scrap" cutoff level, which together define the burn-in test. This test is then applied in production to every manufactured unit before it is shipped.

Although products with competing risks are common in practice, current research on burn-in modeling for such products makes a number of strong assumptions:

- All existing burn-in models pool all failure modes together and model the overall failure rate. However, it would be beneficial to differentiate different failure modes, as it improves estimation accuracy, and allows a burn-in practitioner to understand the failure mechanism and to justify the necessity of burn-in.
- These burn-in models implicitly assume that all failure modes are activated during burn-in. If a normal failure mode can be kept dormant during burn-in, unnecessary product aging due to burn-in would be mitigated. For example, although field use requires full operation of the whole system, we are often able to partially operate a complex system, say, a scanning electron microscope, during testing. If only parts of the system are prone to bad joints during assembly, it would be desirable to burn-in the system by activating these parts only. Another example is that some failure modes are possible only in field use, for example, an ammeter failure due to lightning. These failure modes are normal. They cannot be activated during burn-in, but should be taken into account when making burn-in decisions.
- Most burn-in models deal with systems with binary states, that is, failed or working, and do not make use of any degradation information. Nowadays, many modern products are so well designed and manufactured that they are highly reliable. It may take a very long time for a defective unit to fail even under highly accelerated stresses. Therefore, if a DT failure mode has infant mortality, degradation-based burn-in that bases the screening decision on the prod-

TECHNOMETRICS, MAY 2012, VOL. 54, NO. 2
DOI: 10.1080/00401706.2012.676946

Table 1. Solder/Cu pad interface fracture lifetime data

Sample ID	1	2	3	4	5	6	7	8	9	10
Lifetime (hr)	13,320	17,424	18,600	20,256	23,496	24,000	25,176	27,408	28,776	29,952

uct's degradation level during/after burn-in will be more effective.

1.1 Motivating Examples

A motivating example of this study is from Huang and Askin (2003). An electronic device is subject to two *independent* failure modes, that is, solder/copper (Cu) pad interface fracture, which is regarded as a catastrophic failure, and light intensity degradation, which is a DT failure. The light intensity degradation is measured by the percentage drop from the original light intensity. The device fails if the drop exceeds a prespecified percentage of its original value, or if an interface fracture occurs. The description of the study indicates that these two failure modes can be activated separately during testing. Although Huang and Askin (2003) did not explain how separate activation can be achieved, a possible explanation is that the two failure modes occur in different parts (or modules) of the device, and it is possible to partially operate the device during the test, activating these parts separately. To assess these two failure modes, two *different* tests were conducted under *normal* environmental stresses, each with 10 units. The first test activates the fracture failure mode only, with all the 10 units followed to failure; the second test activates the DT mode only, and each unit is inspected every 500 hr until 4000 hr. Data from these two tests are given in Tables 1 and 2. The degradation data are also displayed in Figure 1.

A simple Weibull plot (not shown) suggests a good fit to the data in Table 1. The estimated shape parameter is greater than 1, indicating that the interface fracture is a normal failure mode. If we apply the restricted least-squares linear regression to the degradation data in Table 2 by fixing the intercept term at 0, the slopes, which are closely related to the degradation rates, range from 0.56% to 2.2% per 1000 hr. If we take into account the slow starters (sample ID 16–20) and use $t = 1000$ hr as the origin, there is still obvious heterogeneity in the slopes from a plot of the truncated degradation paths (not shown). In addition, we find in Section 5.1 that the gamma process with random effects model (Lawless and Crowder 2004) provides a good fit to this dataset, indicating heterogeneous degradation rates.

Burn-in should be used to identify units with high degradation rates so as to enhance field reliability. Because these two modes can be induced *individually*, we are able to activate the light intensity degradation without inducing the catastrophic failures during burn-in. This is desirable as inducing a normal failure mode incurs unnecessary damage to the product. Later in Section 5, we demonstrate that a burn-in test lasting 1134 hr with a scrap threshold of 1.381% degradation leads to a 138% cost reduction, compared with having no burn-in. This is assuming that units with more than 20% degradation are considered failed under the warranty.

However, normal failure modes may have to be activated in some other scenarios. Meeker and Escobar (1998) presented a GaAs laser example of this kind. Most laser devices undergo degradation-based burn-in testing before delivery to customers (Johnson 2006). The degradation of a laser device manifests in an increasing operating current. The device fails when the degradation exceeds a specified threshold, or when a sudden failure occurs (Meeker and Escobar 1998, example 13.5). Possible reasons for the sudden failures include inadvertent shocks and unobserved sudden changes in the device's physical state. Because the laser device is usually inexpensive and of a simple system structure, it is often fully operated during burn-in. Therefore, these catastrophic failures have to be activated at the outset of burn-in.

This article considers the planning of burn-in tests of both kinds, namely ones in which burn-in activates a normal failure mode, such as the GaAs laser example, and ones in which burn-in can selectively activate different failure modes.

Table 2. Light intensity degradation data (in percentage relative to the original measurement)

	Inspection time (hr)								
Sample ID	0	500	1000	1500	2000	2500	3000	3500	4000
11	0	2.5	3.3	4.1	5	5.7	6.5	7.3	8.1
12	0	2.1	2.9	3.7	4.4	5.2	6	6.7	7.5
13	0	2	2.7	3.5	4.3	5	5.8	6.5	7.2
14	0	1.7	2.4	3.2	3.9	4.6	5.4	6.1	6.8
15	0	0.4	1	1.7	2.3	2.9	3.5	4.1	4.7
16	0	0	0.6	1.1	1.7	2.3	2.9	3.4	4
17	0	0	0.5	1.1	1.7	2.2	2.8	3.3	3.9
18	0	0	0.3	0.9	1.5	2	2.6	3.1	3.6
19	0	0	0	0.5	1	1.5	2.1	2.6	3.1
20	0	0	0	0.2	0.7	1.2	1.7	2.2	2.7

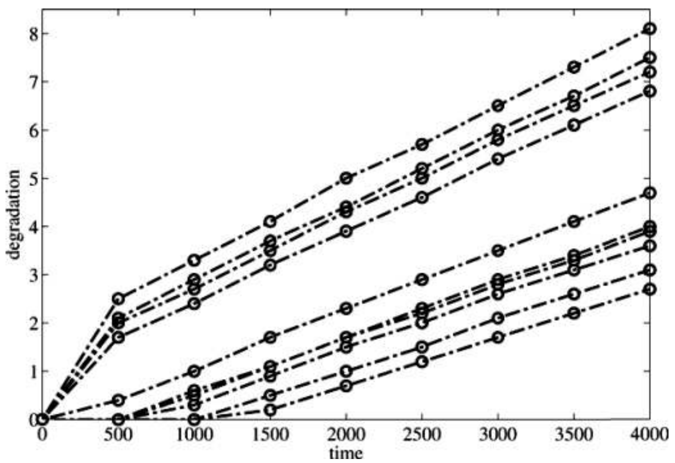

Figure 1. Degradation paths of the 10 test units.

1.2 Related Literature

Traditionally, burn-in is a manufacturing operation intended to fail short-lived units (Nelson 1990, chap. 5.5). It is often conducted under harsh environments that simulate the severest working conditions, for example, a combination of random vibration, thermal cycling, and shock. According to Kececioglu and Sun (1997), for most products, burn-in failures cannot be identified until follow-up functionality testing. Some detailed procedures for implementing burn-in for semiconductor manufacturing were provided in Jula and Leachman (2010). Most burn-in literature focuses on these *failure-based* cases and develops a variety of burn-in models to help decide on the optimal burn-in duration b^*. These burn-in models can be classified as follows:

- Burn-in models that minimize certain cost structures such as joint burn-in and maintenance costs, and joint burn-in and warranty costs, for example, Mi (1996), Wu and Xie (2007), Cha and Finkelstein (2010), and Yuan and Kuo (2010).
- Burn-in models that optimize some performance indices such as survival probability, mean residual life, and percentiles of the residual life, for example, Watson and Wells (1961), Block, Savits, and Singh (2002), and Ye, Tang, and Xie (2011).

These traditional methods are ineffective for highly reliable products, as an extremely long burn-in duration is required. In practice, some measurable quality characteristic of a product usually degrades over time and causes product failure when its degradation exceeds some threshold, that is, a DT failure. The quality characteristic of a defective unit often degrades faster than a normal one. Therefore, a *degradation-based* burn-in test can be adopted, where a unit is scrapped if its degradation level exceeds some degradation cutoff level during or right after burn-in. The cutoff level is often much lower than the failure threshold, making the degradation-based approach much more effective. All existing degradation-based burn-in models are aimed at minimizing the misclassification costs, that is, the costs of the Type I and Type II errors of misclassification. The first degradation-based burn-in model dating back to Tseng and Tang (2001) used a diffusion process to describe the degradation of light-emitting diode (LED) lamps. Tseng, Tang, and Ku (2003), Tseng and Peng (2004), and Tsai, Tseng, and Balakrishnan (2011) extended the analysis by using a transformed Wiener process, an integrated Wiener process, and a gamma process, respectively.

However, most burn-in models, whether failure or degradation-based, commonly assume a single failure mode with infant mortality, notwithstanding the fact that most products fail due to one of a series of failure modes. A good example of competing risks can be found in Meeker, Escobar, and Hong (2009), where a newly designed product has 12 failure modes. Failure mode information can be used to improve the accuracy of both estimation and prediction (Hong and Meeker 2010, p. 150). In fact, there has been considerable research on competing risks. Suzuki, Nakamoto, and Matsuo (2010) reported two competing failure modes, that is, internal and surface cracks, in a load-bending test for brittle materials. Liu and Tang (2010) developed accelerated life test plans for products with independent competing risks. Crowder (2001) provided a book-length treatment on competing risk modeling and estimation.

1.3 Objectives and Outline

The purpose of this work is to develop a burn-in planning framework for products with independent multiple failure modes. Based on this framework, legitimate burn-in strategies for products such as those described in Section 1.1 can be scheduled. Because the trauma failure data were not provided in Meeker and Escobar (1998), we will focus on the electronic device example and build three degradation-based burn-in models. We also propose three methods to cope with parameter uncertainties due to estimation.

The rest of the article is organized as follows. Section 2 develops a general burn-in framework for products with competing risks. Based on this framework, three degradation-based burn-in models are built in Section 3. The cost functions are established and the optimal cutoff levels are derived. Section 4 discusses three methods to deal with statistical uncertainty. In Section 5, validity of our models is demonstrated by the electronic device example. Section 6 concludes the article and points out topics for future research.

2. A BURN-IN PLANNING FRAMEWORK UNDER COMPETING RISKS

Many products are prone to multiple failure modes. We restrict attention to the case where all the failure modes are due to independent causes [see Prentice et al. (1978) for a detailed interpretation of independence]. Behavior of these modes can be accurately assessed through carefully designed life/degradation tests (Hong and Meeker 2010). Test information is collected and analyzed to identify sources of infant mortalities. If some DT failures have infant mortality, degradation-based burn-in should be considered. During burn-in, all infant mortality modes should be activated to identify weak units. On the other hand, we shall try to avoid activating normal failure modes, if possible, to prevent unnecessary system deterioration. Based on these analyses, the burn-in planning framework for products with independent multiple failure modes can be summarized as follows:

- Specify all failure modes and classify them as either DT failures or catastrophic failures. Use degradation/life tests to obtain degradation/failure data. Choose appropriate statistical models, that is, degradation processes and lifetime distributions, to fit the data. Alternatively, the statistical models can be chosen based on previous experience.
- Based on the analysis results of the degradation/life tests data, classify these failure modes into the infant mortality failure class and the normal failure class. Specify all normal failure modes that can be avoided during burn-in and keep them dormant. Activate all infant mortality failure modes during the test.
- If there is any DT failure mode with infant mortality, consider degradation-based burn-in testing. Otherwise, consider failure-based testing. Specify the objective of burn-in, for example, minimize a certain cost or maximize a certain performance index, and build the corresponding burn-in model. This burn-in model should take all normal failure modes into account.
- Parameters in this burn-in model may be directly obtained from previous studies or expert opinions, or estimated from results of degradation/life tests. In the latter scenario, if parameter uncertainty is large, it should be taken into account during model optimization.

Remark 1. Although burn-in is intended to identify infant mortality, burn-in models should incorporate normal failure modes, even if they are dormant during burn-in. Ignorance of the normal failure modes would render inferior burn-in decisions with higher costs.

Remark 2. When the infant mortality class includes more than one DT failure mode, each mode should be assigned a cutoff level. If a DT mode is normal, we do not need to monitor its degradation during or after burn-in.

Remark 3. If there is more than one infant mortality mode, it is operationally more convenient to simultaneously activate them and assign to them a common burn-in time, even if they can be activated individually.

Remark 4. To apply the above procedures to the electronic device example, we first identify the two failure modes (i.e., the interface fracture and the light intensity degradation) and use life/degradation tests to quantitatively assess these two modes, as in Huang and Askin (2003). Next, we use the gamma process with random effect to fit the degradation data and the Weibull distribution to fit the fracture data. Based on the results, we identify the light intensity degradation as an infant mortality mode and build the corresponding degradation-based burn-in model. The model will be developed in the next section.

3. DEGRADATION-BASED BURN-IN MODELS

Throughout the article, we shall discuss burn-in under nominal use conditions. If testing is conducted under accelerated stresses, the time scale can be easily transformed into nominal conditions based on the physics of the product (Escobar and Meeker 2006). After preliminaries in Sections 3.1 and 3.2, three degradation-based burn-in models are developed. The first burn-in model considers a single DT failure mode with infant mortality. The second considers additionally a normal failure mode inactive during burn-in and is applicable to the electronic device. The third considers a DT failure mode with infant mortality and an active normal failure mode during burn-in, which is applicable to the GaAs laser. This section focuses more on burn-in modeling. We implicitly assume that all parameters of the degradation-based burn-in models are known. This is true when information about the failure modes is available from previous studies or expert knowledge. The assumption of known parameters will be relaxed in Section 4. The gamma process with random effects introduced by Lawless and Crowder (2004) will form the basis of our degradation-based burn-in models. It is introduced in Section 3.1.

3.1 Preliminaries: Gamma Process With Random Effect

Consider a gamma process $\{Y(t), t \geq 0\}$ with random effect θ. Given θ, the process has independent and gamma-distributed increments, that is, for $0 \leq u < t$, $Y(t) - Y(u)$ follows $\text{Gamma}(\eta_t - \eta_u, \theta)$ with probability density function (PDF)

$$c_f \cdot (1 - P_3(b, \tau)) - K \cdot P_3(b, \tau), \tag{1}$$

where $\eta_t = \eta(t)$ is a given, monotone increasing and differentiable function of t with $\eta_0 = 0$. A mathematically tractable model results when $\theta \sim \text{Gamma}(k, \lambda)$. The unconditional PDF of $Y(t) - Y(u)$ can then be obtained by integrating θ out of (1), which yields

$$\frac{k[Y(t) - Y(u)]}{\lambda(\eta_t - \eta_u)} \sim F_{2(\eta_t - \eta_u), 2k}, \tag{2}$$

where $F_{m,n}$ is the F-distribution with degrees of freedom (m, n).

The random effect θ is unknown but fixed for a unit. Given the degradation level $Y(b) = y_b$ at time b, it can be shown that the conditional distribution of the random effect θ follows $\text{Gamma}(\eta_b + k, \lambda + y_b)$. This relation implies that given $Y(b) = y_b$,

$$\left(\frac{\eta_b + k}{\eta_t - \eta_b}\right)\left(\frac{Y(t) - Y(b)}{Y(b) + \lambda}\right) \sim F_{2(\eta_t - \eta_b), 2\eta_b + 2k}. \tag{3}$$

For more details about this process, see Lawless and Crowder (2004).

3.2 Problem Formulation

Consider a nonrepairable product sold with a preset mission time, for example, a warranty period, of duration τ. Degradation of its key quality characteristic $\{Y(t), t \geq 0\}$ follows a gamma process with a gamma-distributed random effect θ, $\theta \sim \text{Gamma}(k, \lambda)$; Y_f is a fixed degradation threshold for this process. We assume that this DT mode is subject to infant mortality. In addition to this mode, the product is also prone to a catastrophic failure with cumulative distribution function (CDF) $G(\cdot)$ and survival function (SF) $\bar{G}(\cdot)$, which is a normal failure mode.

Burn-in is used to identify and eliminate units with high degradation rates. Functionality of a unit is not monitored during burn-in. The screening rule is as follows: After burn-in with a duration of b, the degradation of a unit is nondestructively measured; if the degradation exceeds a predetermined cutoff level ξ_b, the unit is scrapped. The per-unit burn-in cost includes setup cost c_s and the burn-in operational cost c_0 per time unit of burn-in. If a unit has failed at the end of the burn-in period (e.g., due to the catastrophic failure), it is scrapped with cost c_p. Otherwise, its degradation is measured with measurement cost c_{mea}. If the degradation exceeds the cutoff level ξ_b, the unit is rejected with disposal cost c_d (e.g., reworked or sold at a lower price). An accepted unit will be put into field operation. If it fails within the mission time τ, some handling and administrative cost c_f is incurred. Otherwise, a gain of K is generated.

3.3 Degradation-Based Burn-In Model With Single Failure Mode

To start, we shall build a burn-in model for products with only a single failure mode, that is, the DT failure. The purpose of burn-in is simply to identify units with high degradation rates. Denote $\xi_{1,b}$ as the cutoff degradation level with burn-in duration b. Because there is no catastrophic failure, all units will not fail during burn-in and thus should be measured after the test. Therefore, the expected burn-in cost can be expressed as

$$c_0 b + (c_s + c_{\text{mea}}) + c_d \cdot \Pr(Y(b) \geq \xi_{1,b}),$$

where the screening probability can be obtained based on (2) as

$$\Pr(Y(b) \geq \xi_{1,b}) = 1 - F_{2\eta_b,2k}\left(\frac{k\xi_{1,b}}{\lambda\eta_b}\right), \tag{4}$$

where $F_{m,n}(t)$ is the CDF of the F-distribution with degrees of freedom (m, n). With probability $\Pr(Y(b) < \xi_{1,b})$, a burnt-in unit is accepted and put into field operation. The field operation cost of this unit can be expressed as

$$c_f - (c_f + K) \cdot \Pr(Y(b+\tau) \leq Y_f | Y(b) \leq \xi_{1,b}).$$

The conditional probability is the probability that this unit survives the mission time τ, which is given by

$$\Pr(Y(b+\tau) \leq Y_f | Y(b) \leq \xi_{1,b}) = \frac{1}{\Pr(Y(b) < \xi_{1,b})} \times \int_0^{\xi_{1,b}} \Pr(\Delta Y_b \leq Y_f - u | Y(b) = u) f_{Y(b)}(u)du, \tag{5}$$

where $\Delta Y_b = Y(b+\tau) - Y(b)$. Based on (3), (5) can be expressed as

$$\Pr(Y(b+\tau) \leq Y_f | Y(b) < \xi_{1,b}) = \frac{1}{\Pr(Y(b) < \xi_{1,b})} \times \int_0^{\xi_{1,b}} F_{2\Delta\eta_b,2\eta_b+2k}\left(\left(\frac{\eta_b + k}{\Delta\eta_b}\right)\left(\frac{Y_f - u}{u + \lambda}\right)\right) f_{Y(b)}(u)\,du, \tag{6}$$

where $\Delta\eta_b = \eta_{b+\tau} - \eta_b$. Summing up the mean burn-in cost and field operation cost, the expected total cost $E[C(b, \xi_{1,b})]$ for a unit is given by

$$\begin{aligned} E[C(b, \xi_{1,b})] = {} & c_0 b + (c_s + c_{\text{mea}}) + c_d \cdot \Pr(Y(b) \geq \xi_{1,b}) \\ & + \Pr(Y(b) < \xi_{1,b})[c_f - (c_f + K) \\ & \cdot \Pr(Y(b+\tau) \leq Y_f | Y(b) \leq \xi_{1,b})]. \end{aligned} \tag{7}$$

The optimal cutoff level $\xi^*_{1,b}$ can be obtained by minimizing (7) over $\xi_{1,b}$ with a fixed b. A first derivative test reveals that the minimum is achieved when $\frac{\partial}{\partial \xi_{1,b}} E[C(b, \xi_{1,b})] = 0$. The result is summarized in Theorem 1. To simplify notation, define

$$\Lambda = (c_f - c_d)/(c_f + K). \tag{8}$$

Theorem 1. Suppose that degradation path of a product follows a gamma process with random effect and the total cost function is given by (7). For a fixed burn-in duration b, we have the following:

(a) If $0 \leq \Lambda \leq 1$, the optimal cutoff level $\xi^*_{1,b}$ is

$$\xi^*_{1,b} = \frac{(\eta_b + k)Y_f - \Delta\eta_b \lambda F^{-1}_{2\Delta\eta_b,2\eta_b+2k}(\Lambda)}{(\eta_b + k) + \Delta\eta_b F^{-1}_{2\Delta\eta_b,2\eta_b+2k}(\Lambda)}, \tag{9}$$

where $F^{-1}_{m,n}(\cdot)$ is the percentile function of the F-distribution with degrees of freedom (m, n). In addition, if $\eta(\cdot)$ is concave in t, $\xi^*_{1,b}$ is increasing in b.

(b) If $\Lambda < 0$, the optimal cutoff degradation level is $\xi^*_{1,b} = \infty$.

(c) If $\Lambda > 1$, the optimal cutoff degradation level is $\xi^*_{1,b} = 0$.

The proof of this theorem is given in the supplementary material. The condition that $\eta(\cdot)$ is concave is necessary for $\xi^*_{1,b}$ to be increasing in b. For example, if $\eta(\cdot)$ adopts an exponential form, we find that $\xi^*_{1,b}$ may not be monotonically increasing in b. After $\xi^*_{1,b}$ is determined, the optimal burn-in duration b^* can be obtained by minimizing (7) with $\xi_{1,b}$ fixed at $\xi^*_{1,b}$.

It can be seen from Theorem 1 that the optimal cutoff levels do not depend on the cost parameters of burn-in operation, that is, c_0, c_s, and c_{mea}. This is because at the time of making the screening decision, the burn-in operational cost can be regarded as a sunk cost. It is also interesting to see that Λ serves as a normalized risk measure. When Λ is large (e.g., a large c_f and a small K), $\xi^*_{1,b}$ would be small, indicating a stringent criterion under which more units will be scrapped. Conversely, small Λ leads to a looser criterion.

3.4 Two Failure Modes With Normal Failures Inactive During Burn-In

In this section, we consider the scenario where there is a normal failure mode but only the DT mode is activated during burn-in. This scenario fits into the electronic device example, as the interface fracture is normal, and can be avoided during burn-in. Define $\xi_{2,b}$ as the cutoff degradation level with burn-in duration b. Because the normal mode is inactive during burn-in, all units will not fail during burn-in and thus should be measured after the test. The expected burn-in cost is

$$c_0 b + c_s + c_{\text{mea}} + c_d \cdot \Pr(Y(b) \geq \xi_{2,b}).$$

If $Y(b) < \xi_{2,b}$, a burnt-in unit is put into field use. Denote $P_2(b, \tau)$ as the probability that this unit survives the mission time. It should be noted that the normal failure mode is active during field use. Therefore, this probability is given by

$$P_2(b, \tau) = \Pr(Y(b+\tau) \leq Y_f | Y(b) < \xi_{2,b}) \cdot \bar{G}(\tau).$$

The expected field operation cost is

$$c_f \cdot (1 - P_2(b,\tau)) - K \cdot P_2(b,\tau).$$

The mean total cost $E[C(b,\xi_{2,b})]$ per unit can thus be expressed as

$$\begin{aligned} E[C(b,\xi_{2,b})] &= c_0 b + c_s + c_{\text{mea}} + c_d \cdot \Pr(Y(b) \ge \xi_{2,b}) \\ &\quad + \Pr(Y(b) < \xi_{2,b})[c_f \cdot (1 - P_2(b,\tau)) - K \cdot P_2(b,\tau)]. \end{aligned} \quad (10)$$

The optimal cutoff level $\xi^*_{2,b}$ for each burn-in time b can be obtained by minimizing (10) over $\xi_{2,b}$ with a fixed b. The result is encapsulated in Theorem 2. Its proof is given in the supplementary material.

Theorem 2. Suppose that in addition to the DT failure, there is a normal failure mode that can be avoided during burn-in. When the expected cost function is given by (10), we have the following:

(a) If $0 \le \Lambda \le \bar{G}(\tau)$, the optimal cutoff level with a fixed burn-in duration b is

$$\xi^*_{2,b} = \frac{(\eta_b + k)Y_f - \Delta\eta_b \lambda F^{-1}_{2\Delta\eta_b, 2\eta_b+2k}(\Lambda/\bar{G}(\tau))}{(\eta_b + k) + \Delta\eta_b F^{-1}_{2\Delta\eta_b, 2\eta_b+2k}(\Lambda/\bar{G}(\tau))}. \quad (11)$$

In addition, if $\eta(t)$ is concave in t, $\xi^*_{2,b}$ is increasing in b.

(b) If $\Lambda < 0$, the optimal cutoff degradation level is $\xi^*_{2,b} = \infty$.

(c) If $\Lambda > \bar{G}(\tau)$, the optimal cutoff degradation level is $\xi^*_{2,b} = 0$.

Similarly, the optimal cutoff level $\xi^*_{2,b}$ does not depend on the burn-in operational cost (excluding the disposal cost c_d). When there is a normal failure mode, $\xi^*_{2,b}$ is smaller than $\xi^*_{1,b}$. This means that when the product deteriorates due to some other mechanisms, for example, some normal failure modes, the only way we can enhance the reliability is to adopt a more stringent criterion for the infant mortality modes.

3.5 Two Failure Modes With Normal Failures Active During Burn-In

We further consider the case where there is a catastrophic failure mode that is normal but has to be activated during burn-in. This burn-in model is fit for the GaAs laser example. After burn-in with duration b, some units would fail due to catastrophic failures. The proportion is $G(b)$ and thus, the expected scrapping cost is $c_p G(b)$. The degradation level of a functioning unit is measured. With probability $\Pr(Y(b) \ge \xi_{3,b})$, its degradation would exceed the cutoff degradation level $\xi_{3,b}$, and the unit would be rejected. Otherwise, the unit is accepted and put into field use with the expected field operation cost

$$c_f \cdot (1 - P_3(b,\tau)) - K \cdot P_3(b,\tau),$$

where $P_3(b,\tau)$ is the probability of fulfilling the mission:

$$P_3(b,\tau) = \Pr(Y(b+\tau) \le Y_f | Y(b) < \xi_{3,b}) \cdot \bar{G}(b+\tau)/\bar{G}(b).$$

Therefore, the expected cost function $E[C(b,\xi_{3,b})]$ is given by

$$\begin{aligned} E[C(b,\xi_{3,b})] &= c_s + c_p G(b) + c_{\text{mea}} \bar{G}(b) \\ &\quad + c_d \cdot \bar{G}(b) \Pr(Y(b) \ge \xi_{3,b}) \bar{G}(b) \times \Pr(Y(b) < \xi_{3,b}) \\ &\quad \times [c_f - (c_f + K) P_3(b,\tau)]. \end{aligned} \quad (12)$$

As in Sections 3.3 and 3.4, we can determine $\xi^*_{3,b}$ through minimizing (12) over $\xi_{3,b}$ with a fixed b. The result is summarized in Theorem 3.

Theorem 3. Suppose that in addition to the DT failure, there is a normal failure mode that has to be activated during burn-in. When the mean cost function is given by (12), we have the following:

(a) If $0 \le \Lambda \le \bar{G}(b+\tau)/\bar{G}(b)$, the optimal cutoff degradation level is

$$\xi^*_{3,b} = \frac{(\eta_b + k)Y_f - \Delta\eta_b \lambda F^{-1}_{2\Delta\eta_b, 2\eta_b+2k}(\Lambda \cdot \bar{G}(b)/\bar{G}(b+\tau))}{(\eta_b + k) + \Delta\eta_b F^{-1}_{2\Delta\eta_b, 2\eta_b+2k}(\Lambda \cdot \bar{G}(b)/\bar{G}(b+\tau))}. \quad (13)$$

(b) If $\Lambda < 0$, the optimal cutoff degradation level is $\xi^*_{3,b} = \infty$.

(c) If $\Lambda > \bar{G}(b+\tau)/\bar{G}(b)$, the optimal cutoff degradation level is $\xi^*_{3,b} = 0$.

The proof is similar to those of Theorems 1 and 2, and thus is not presented. Both the models in this section and in Section 3.4 can be readily generalized to the cases of multiple normal failure modes.

4. OPTIMIZATION UNDER PARAMETER UNCERTAINTY

Usually, the process/distribution parameters have to be estimated from testing data (e.g., the electronic device example), and thus are subject to estimation uncertainties. Denote $\boldsymbol{\Upsilon}$ as the vector of parameters to estimate. This section considers three approaches to take this risk into consideration.

4.1 The Plug-In Method

A traditional approach to cope with this issue is to simply take the maximum likelihood (ML) estimate $\hat{\Upsilon}$ and substitute it into the models in (7), (10), and (12). Optimal burn-in settings can then be determined through optimizing the cost functions by using Theorems 1–3. This approach is appropriate when sufficient data are available to ensure small estimation error. When the uncertainty issue is severe, however, $\hat{\Upsilon}$ may take values significantly different from $\boldsymbol{\Upsilon}$, and the solution found by using this approach may be far from optimal. This method needs modification to take into account parameter uncertainties, especially when the dataset size is small.

4.2 Averaging Over Uncertainty in Parameter Estimates

The three models built in Section 3 rely on the mean costs, as both the degradation process and the catastrophic failures are stochastic. The estimated parameters $\hat{\Upsilon}$ are subject to uncertainties and can also be treated as random variables, conditional on which the cost functions take the forms of (7), (10), and (12), respectively. In order to obtain the unconditional mean cost, we need to take the expectation of the cost functions over these estimated parameters. Denote $E_{\hat{\Upsilon}}[C(b, \xi_{i,b})|\hat{\Upsilon}]$, $i = 1, 2, 3$, as the conditional mean cost per unit. It is noted that Theorems 1–3 are no longer applicable here. It is extremely difficult, if not impossible, to derive closed-form expressions for the unconditional mean cost, as the distribution of $\hat{\boldsymbol{\Upsilon}}$ is complicated. We recommend using the bootstrap to generate N sample estimates, computing the conditional mean cost for each estimate, and then averaging over the costs to approximate the unconditional cost. Note that $N = 1000$ is used in this study. The simultaneous perturbation stochastic approximation (SPSA) algorithm, an algorithmic optimization method for functions that cannot be directly computed, can be used to locate the optimum. The theory and effectiveness of the SPSA algorithm have been well established (Spall 2003). Some MATLAB codes are available on the SPSA website (*www.jhuapl.edu/spsa*). A two-dimensional contour plot is also helpful in visualizing the optimal settings.

4.3 Chance Constraint

The expectation-based method may not be a good choice for a risk-averse manufacturer, as the realized cost is often higher than the expected value. A risk-averse manufacturer may want to put greater funds in reserve to protect against possible future losses. A justifiable means is to plan the study such that an upper bound of the resulting cost would be controllable with high probability. This method also avoids the overconservatism issue faced with the worst-case analysis. In this study, the chance constraint method essentially minimizes the upper α quantile of the costs as follows:

$$\begin{aligned} &\underset{b,\xi_{i,b}\geq 0}{\text{minimize}}\ y \\ &\text{subject to } \Pr_{\hat{\Upsilon}}\{E_{\hat{\Upsilon}}[C(b, \xi_{i,b})|\hat{\Upsilon}] \leq y\} \geq 1 - \alpha. \end{aligned} \tag{14}$$

The optimal burn-in setting suggested by (14) gives a $1 - \alpha$ guarantee that the total cost will be less than y^*, the optimal value of (14). Since the cost function adopts a complex form and the distribution for $\hat{\Upsilon}$ is unknown, this problem cannot be solved analytically. However, the upper α quantile can be estimated by the bootstrap, and the optimization can be done by using algorithms that are derivative free or that use numerical gradients, for example, the mesh adaptive direct search (MADS) algorithm (Audet and Dennis 2006). As with the expectation approach, we use $N = 1000$ bootstrap samples. A detailed procedure to solve (14) is given in the supplementary material.

4.4 Additional Remarks

Calibration of the naïve approach, that is, the plug-in method, can also be done by asymptotic expansions instead of simulation (Barndorff-Nielsen and Cox 1996). But due to the complexity, this method is not discussed here.

When parameter uncertainty is large, or when the manufacturer is unsure how bad the parameter uncertainty is, the expectation approach is recommended when the manufacturer is risk neutral, and the chance constraint approach when risk averse. On the other hand, the plug-in method is applicable when the parameters are known from other sources (e.g., a previous study), or when enough data from burn-in and in-operation are collected. The latter case is appropriate for the electronic device example. These in-operation data update the ML estimates, whose consistency ensures minor uncertainty with a large dataset size. In addition, the plug-in method is appropriate if the manufacturer is not concerned about parameter uncertainty, or if it wants a fast answer. Although we recommend the plug-in approach for the electronic device example, we consider all the three approaches in Section 5.

5. AN ILLUSTRATIVE EXAMPLE

The burn-in model developed in Section 3.4, with a DT mode and a catastrophic failure mode that is inactive during burn-in, is applied to the electronic device example. The optimization approaches presented in Section 4 are applied to determine the optimal burn-in settings. Although Huang and Askin (2003) used a degradation threshold of $Y_f = 40$, we use $Y_f = 25$ throughout this section for a better illustration of our model. The following cost profile is adopted:

- burn-in operational cost $c_0 = \$0.01$/hr,
- burn-in setup cost $c_s = \$0.1$,
- measurement cost $c_{\text{mea}} = \$0.1$,
- disposal cost $c_d = -\$40$,
- warranty period $\tau = 1.5$ years,
- within-warranty failure cost $c_f = \$1000$,
- profit $K = \$500$.

Here, a negative disposal cost means that the manufacturer is able to sell a unit at a lower price without any penalty cost if it deems that the unit's quality is not high enough. A high c_f is used because a warranty failure incurs not only repair/replacement cost but also reputation losses. For example, Toyota uses a multiple of six times of the repair cost for a field failure to measure the reputation cost, while the Westinghouse uses a multiple of four (Balachandran and Radhakrishnan 2005). Before proceeding to the burn-in model optimization, the distribution/degradation parameters need to be estimated from testing data.

5.1 Data Analysis

Consider the degradation data in Table 2. Not all units start degradation from time 0, meaning that there are some slow starters. This may be due to limitations in measurement precision. Because the gamma process always has a positive increment, when the degradation values are zero, we treat them as missing data. Figure 1 in Section 1 shows that the degradation paths are approximately linear when $t > 500$. In addition, the degradation rates, indicated by the slopes of these paths, vary from unit to unit. A gamma process with random effect may be appropriate to fit the data. We assume that $\eta(t) = \beta t$, where

Table 3. Estimates for the degradation parameters

Parameters	β	λ	k
ML estimate	0.00812	1.3052	9.1778
Standard error	(0.0015)	(0.8362)	(5.1833)

β is a parameter to estimate. The likelihood function has been derived in Lawless and Crowder (2004), and is briefly described here.

Assume that n units are under test. For the ith unit, $\mathbf{t}_i = \{t_{i:0} = 0, t_{i:1}, \ldots, t_{i:m_i}\}$ is the set of checkpoints. Define $\Delta Y_{i:j} = Y_i(t_{i:j}) - Y_i(t_{i:j-1})$ for $j = 1, 2, \ldots, m_i$. The joint density for $\Delta Y_{i:1}, \ldots, \Delta Y_{i:m_i}$ is

$$f_{\Delta Y_{i:1},\ldots,\Delta Y_{i:m_i}}(\Delta y_{i:1}, \ldots, \Delta y_{i:m_i}) = \frac{\lambda^k \Gamma(\eta(t_{i:m_i}) + k)}{(\lambda + y_i(t_{i:m_i}))^{\eta(t_{i:m_i})+k}\Gamma(k)} \prod_{j=1}^{m_i} \frac{(\Delta y_{i:j})^{\Delta\eta_{i:j}-1}}{\Gamma(\Delta\eta_{i:j})},$$

where $\Delta\eta_{i:j} = \eta(t_{i:j}) - \eta(t_{i:j-1})$. The log-likelihood function can thus be expressed as

$$l(\theta, k, \lambda) = \sum_{i=1}^{n} \log f_{\Delta Y_{i:1},\ldots,\Delta Y_{i:m_i}}(\Delta y_{i:1}, \ldots, \Delta y_{i:m_i}).$$

ML estimates of this process are listed in Table 3.

For comparison, the estimated CDF for the time to threshold-defined failures from the method of Huang and Askin (2003) (H-A method), the gamma process with random effect (Gamma method), and the Kaplan–Meier (KM) estimates are depicted in Figure 2. To examine the sensitivity of the estimation results to the threshold, different values of Y_f, that is, 25, 30, and 40, are used. The KM estimation uses pseudo failure times obtained by fitting each degradation path by restricted linear regression and extrapolating to the threshold (Meeker and Escobar 1998, p. 339). Compared with the H-A method, the estimated CDF based on the gamma process lies within the 95% pointwise confidence bound of the KM estimates. Therefore, the gamma process presents an attractive alternative to describe the degradation.

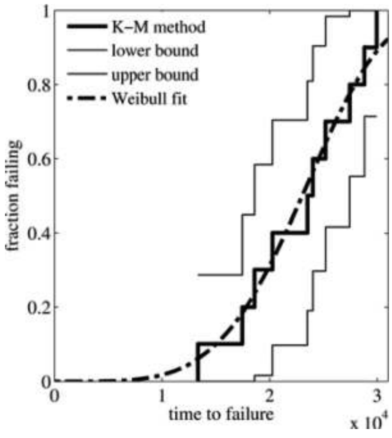

Figure 3. Weibull distribution fit to the catastrophic failure data.

Figure 3 displays estimates of time to failure for the catastrophic failure mode, using the KM method and the Weibull model. The Weibull distribution provides a good fit to the traumatic failure data. The ML estimate (standard error) for the shape parameter is 4.4012 (1.5176) and the scale parameter is 25,023 (1898.0). Therefore, the SF for the catastrophic failure time is

$$\bar{G}(t) = \exp\left[-\left(\frac{t}{25,023}\right)^{4.4012}\right]. \quad (15)$$

5.2 The Plug-In Approach

Of the two failure modes for this device, the catastrophic mode can be avoided during burn-in. The cost function (10) can be applied to identify weak units. We first ignore parameter uncertainties and apply the plug-in approach in Section 4.1. The optimal cutoff level for each burn-in time can be determined by (11), after which the optimal burn-in duration can be determined by simple search methods. The optimal burn-in scheme is to burn-in a unit for $b^* = 487$ hr with a cutoff level $\xi^*_{2,b^*} = 1.076$, leading to the optimal cost of –\$268.6. The total cost without

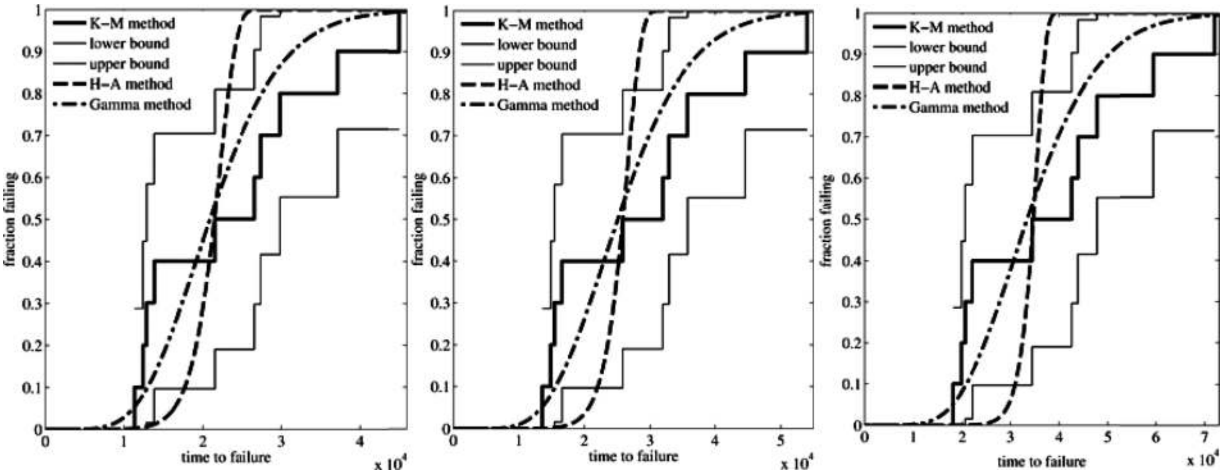

Figure 2. Comparison of the gamma process and the H-A method in estimating CDF of the time to threshold-defined failure. From the left panel to the right, $Y_f = 25$, 30, and 40, respectively.

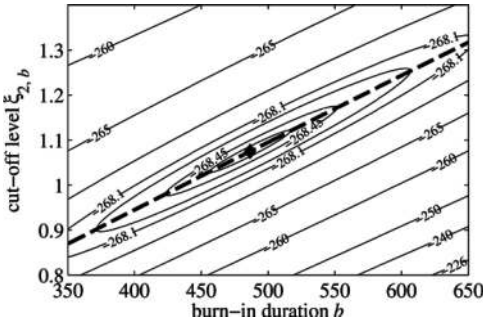

Figure 4. The expected total cost obtained by treating the ML estimates as the true values. The dashed line is the optimal cutoff level determined by Theorem 2 and the diamond point is the optimal burn-in scheme.

burn-in is –\$264.8. Burn-in reduces the cost by 1.44%. In addition, burn-in improves the field reliability. Originally, 10.2% of the product would fail due to DT mode within τ. This proportion reduces to 5.50% with burn-in. Figure 4 gives the contour of the expected total cost with respect to b and $\xi_{2,b}$. To verify Theorem 2, Figure 4 also shows the optimal cutoff levels determined by (11), depicted by the dashed line. From the contour, we can see that for each fixed b, the cost decreases when $\xi_{2,b}$ moves toward $\xi^*_{2,b}$. This supports that $\xi^*_{2,b}$ is the optimal cutoff level for each fixed b. In addition, Figure 4 indicates that $\xi^*_{2,b}$ is increasing in b, which is concordant with Theorem 2 because $\eta(t)$ is linear and thus is concave.

Throughout this section we use $Y_f = 25$. However, Huang and Askin (2003) observed considerable sensitivity of the estimation to the threshold Y_f. We therefore conduct a sensitivity analysis to examine the optimal burn-in settings against different values of Y_f. We find that when Y_f is set at 20, the optimal burn-in time increases to 1134 hr while the optimal cutoff level is 1.381, leading to an optimal cost of –\$148.0. The total cost without burn-in is –\$62.2. Burn-in reduces the cost by 138%. This is because a smaller Y_f results in more warranty failures; a stringent screening criterion is thus needed to mitigate the risk of DT failures. If $Y_f = 40$, then burn-in is not necessary because most units will not fail due to the DT mode within τ. These results of sensitivity analysis justify the purpose of burn-in, that is, to screen units that may fail within warranty with high probability.

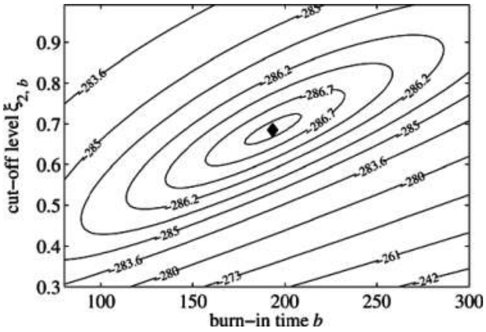

Figure 5. The unconditional expected total cost by treating the ML estimates as random variables. The diamond point is the optimal burn-in scheme.

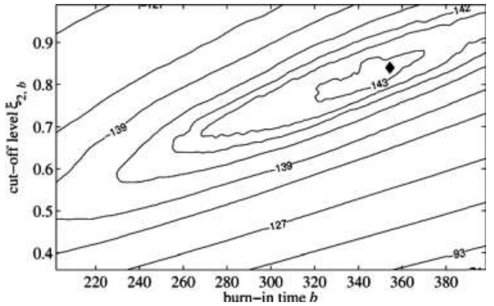

Figure 6. The approximate 90% upper quantile of the total cost with burn-in by using the bootstrap.

5.3 Averaging Over Uncertainty in Parameter Estimates

In view of the fact that the ML estimates themselves are random variables, we can average over them to obtain the unconditional expected cost. To use the SPSA algorithm, we follow the implementation guidance provided by Spall (2003, chap. 7.5). The optimal burn-in duration is 193 hr with an optimal cutoff level of 0.685. The associated optimal cost is –\$286.9. Without burn-in, the unconditional expected cost is –\$275.7. Again, products subjected to burn-in enjoy a relative cost reduction of 4.07%. A contour plot is provided in Figure 5. This figure suggests that this local optimum is indeed globally optimal.

5.4 Using Chance Constraint

To suggest an optimal burn-in scheme for a risk-averse manufacturer, the chance constraint method is employed. We set $\alpha = 0.10$ and apply the bootstrap method in conjunction with the MADS algorithm, as described in the supplementary material, to solve (14). The optimal burn-in duration and cutoff level are 355 and 0.840, respectively. The associated minimal cost is –\$143.4. Without burn-in, the cost is –\$102.4, which is 40.0% higher than the optimum. Using the same bootstrap samples, the contour plot is given in Figure 6. It is noted that the contour is not very smooth because of the bootstrap approximation (Hall 1997, appendix I).

6. CONCLUSIONS

This study developed a general burn-in planning framework for products with independent competing risks. This framework suggested identifying all failure modes, classifying them into the right classes, activating all infant mortality modes during burn-in, and trying to keep the normal modes dormant. In addition, a degradation-based burn-in approach was recommended when some DT modes have infant mortality. In view of the prevalence of multiple failure modes, this framework furnishes a good guide to burn-in for practitioners. Based on this framework, three degradation-based burn-in models were developed, one of which was applied to the electronic device example. In addition, three approaches were proposed to account for parameter uncertainties.

There are several directions for future research. First, this article studies burn-in planning under independent competing risks. The scenario of dependent competing risks deserves further investigation. Second, we consider the case where degradation level is measured only after burn-in, that is, a single inspection point. When the measurement cost is low and the burn-in operational cost c_0 is high, it might be more cost effective to consider multiple inspection points, each associated with a cutoff level.

SUPPLEMENTARY MATERIAL

Supplementary Information: In this document, we provide the proofs of Theorems 1 and 2, as well as a detailed description of the procedure to solve the chance constraint problem (14) in Section 4.3.

ACKNOWLEDGMENTS

The authors thank the editor, the associate editor, and three reviewers for their critical and constructive comments that have greatly helped in the revision of an earlier version of the article.

[Received August 2010. Revised February 2012.]

REFERENCES

Audet, C., and Dennis, J. E. (2006), "Mesh Adaptive Direct Search Algorithms for Constrained Optimization," *SIAM Journal on Optimization*, 1, 188–217. [7]

Balachandran, K. R., and Radhakrishnan, S. (2005), "Quality Implications of Warranties in a Supply Chain," *Management Science*, 51, 1266–1277. [7]

Barndorff-Nielsen, O. E., and Cox, D. R. (1996), "Prediction and Asymptotics," *Bernoulli*, 2, 319–340. [7]

Block, H. W., Savits, T. H., and Singh, H. (2002), "A Criterion for Burn-in That Balances Mean Residual Life and Residual Variance," *Operations Research*, 50, 290–296. [3]

Cha, J. H., and Finkelstein, M. (2010), "Burn-in by Environmental Shocks for Two Ordered Subpopulations," *European Journal of Operational Research*, 206, 111–117. [3]

Crowder, M. J. (2001), *Classical Competing Risks*, Boca Raton, FL: Chapman and Hall/CRC Press. [3]

Escobar, L. A., and Meeker, W. Q. (2006), "A Review of Accelerated Test Models," *Statistical Science*, 21, 552–577. [4]

Hall, P. (1997), *The Bootstrap and Edgeworth Expansion*, New York: Springer-Verlag. [9]

Hong, Y. L., and Meeker, W. Q. (2010), "Field-Failure and Warranty Prediction Based on Auxiliary Use-Rate Information," *Technometrics*, 52, 148–159. [3]

Huang, W., and Askin, R. G. (2003), "Reliability Analysis of Electronic Devices With Multiple Competing Failure Modes Involving Performance Aging Degradation," *Quality and Reliability Engineering International*, 19, 241–254. [2,4,7,8,9]

Johnson, L. A. (2006), "Laser Diode Burn-in and Reliability Testing," *IEEE Communications Magazine*, 44, 7–10. [2]

Jula, P., and Leachman, R. C. (2010), "Coordinated Multistage Scheduling of Parallel Batch-Processing Machines Under Multiresource Constraints," *Operations Research*, 58, 933–947. [3]

Kececioglu, D., and Sun, F. B. (1997), *Burn-in Testing: Its Quantification and Optimization*, Upper Saddle River, NJ: Prentice Hall. [3]

Lawless, J., and Crowder, M. (2004), "Covariates and Random Effects in a Gamma Process Model With Application to Degradation and Failure," *Lifetime Data Analysis*, 10, 213–227. [2,4,7]

Liu, X., and Tang, L. C. (2010), "Accelerated Life Test Plans for Repairable Systems With Multiple Independent Risks," *IEEE Transactions on Reliability*, 59, 115–127. [3]

Meeker, W. Q., and Escobar, L. A. (1998), *Statistical Methods for Reliability Data*, New York: Wiley. [2,3,8]

Meeker, W. Q., Escobar, L. A., and Hong, Y. L. (2009), "Using Accelerated Life Tests Results to Predict Product Field Reliability," *Technometrics*, 51, 146–161. [3]

Mi, J. (1996), "Minimizing Some Cost Functions Related to Both Burn-in and Field Use," *Operations Research*, 44, 497–500. [3]

Nelson, W. (1990), *Accelerated Testing: Statistical Models, Test Plans and Data Analyses*, Hoboken, NJ: Wiley. [3]

Prentice, R. L., Kalbfleisch, J. D., Peterson, A. V., Jr., Flournoy, N., Farewell, V., and Breslow, N. (1978), "The Analysis of Failure Times in the Presence of Competing Risks," *Biometrics*, 34, 541–554. [3]

Spall, J. C. (2003), *Introduction to Stochastic Search and Optimization: Estimation, Simulation, and Control*, Hoboken, NJ: Wiley. [6,9]

Suzuki, K., Nakamoto, T., and Matsuo, Y. (2010), "Optimum Specimen Sizes and Sample Allocation for Estimating Weibull Shape Parameters for Two Competing Failure Modes," *Technometrics*, 52, 209–220. [3]

Tsai, C. C., Tseng, S. T., and Balakrishnan, N. (2011), "Optimal Burn-in Policy for Highly Reliable Products Using Gamma Degradation Process," *IEEE Transactions on Reliability*, 60, 234–245. [3]

Tseng, S. T., and Peng, C. Y. (2004), "Optimal Burn-in Policy by Using an Integrated Wiener Process," *IIE Transactions*, 36, 1161–1170. [3]

Tseng, S. T., and Tang, J. (2001), "Optimal Burn-in Time for Highly Reliable Products," *International Journal of Industrial Engineering*, 8, 329–338. [3]

Tseng, S. T., Tang, J., and Ku, I. H. (2003), "Determination of Burn in Parameters and Residual Life for Highly Reliable Products," *Naval Research Logistics*, 50, 1–14. [3]

Watson, G. S., and Wells, W. T. (1961), "On the Possibility of Improving the Mean Useful Life of Items by Eliminating Those With Short Lives," *Technometrics*, 3, 281–298. [3]

Wu, S., and Xie, M. (2007), "Classifying Weak and Strong Components Using ROC Analysis With Application to Burn-In," *IEEE Transactions on Reliability*, 56, 552–561. [3]

Ye, Z. S., Tang, L. C., and Xie, M. (2011), "A Burn-in Scheme Based on Percentiles of the Residual Life," *Journal of Quality Technology*, 43, 334–345. [3]

Yuan, T., and Kuo, Y. (2010), "Bayesian Analysis of Hazard Rate, Change Point, and Cost-Optimal Burn-in Time for Electronic Devices," *IEEE Transactions on Reliability*, 59, 132–138. [3]