DEGREE OF NEGATION OF AN AXIOM

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In this article we present the two *classical negations* of Euclid's Fifth Postulate (done by Lobachevski-Bolyai-Gauss, and respectively by Riemann), and in addition of these we propose a *partial negation* (or a degree of negation) of an axiom in geometry.

The most important contribution of this article is the introduction of the degree of negation (or partial negation) of an axiom and, more general, of a scientific or humanistic proposition (theorem, lemma, etc.) in any field - which works somehow like the negation in fuzzy logic (with a degree of truth, and a degree of falsehood) or like the negation in neutrosophic logic [with a degree of truth, a degree of falsehood, and a degree of neutrality (i.e. neither truth nor falsehood, but unknown, ambiguous, indeterminate)].

The Euclid's Fifth postulate is formulated as follows: if a straight line, which intersects two straight lines, form interior angles on the same side, smaller than two right angles, then these straight lines, extended to infinite, will intersect on the side where the interior angles are less than two right angles.

This postulate is better known under the following formulation: through an exterior point of a straight line one can construct one and only one parallel to the given straight line.

The Euclid's V postulate (323 BC - 283 BC) is worldwide known, logically consistent in itself, but also along with other four postulates with which to form a consistent axiomatic system.

The question, which has been posted since antiquity, is if the fifth postulate is dependent of the first four?

An axiomatic system, in a classical vision, must be:

1) **Consistent** (the axioms should not contradict each other: that is some of them to affirm something, and others the opposite);
2) **Independent** (an axiom must not be a consequence of the others by applying certain rules, theorems, lemmas, methods valid in that system; if an axiom is proved to be dependent (results) of the others, it is eliminated from that system; the system must be minimal);
3) **Complete** (the axioms must develop the complete theory, not only parts of it).

The geometers thought that the V postulate (= axiom) is a consequence of the Euclid's first four postulates. Euclid himself invited others in this research. Therefore, the system proposed by Euclid, which created the foundation of classical geometry, seemed not be independent.

In this case, the V postulate could be eliminated, without disturbing at all the geometry's development.

There were numerous tentative to "proof" this "dependency", obviously unsuccessful. Therefore, the V postulate has a historic significance because many mathematicians studied it.

Then, ideas revolved around negating the V postulate, and the construction of an axiomatic system from the first four unchanged Euclidean postulates plus the negation of the fifth postulate. It has been observed that there could be obtained different geometries which are bizarre, strange, and apparently not connected with the reality.

a) Lobachevski (1793-1856), Russian mathematician, was first to negate as follows: "Through an exterior point to a straight line we can construct an infinite number of parallels to that straight line", and it has been named **Lobachevski's geometry** or hyperbolic geometry. This negation is 100%.

After him, independently, the same thing was done by Bolyai (1802-1860), Hungarian from Transylvania, and Gauss (1777-1855), German. But Lobachevski was first to publish his article.

Beltrami (1835-1900), Italian, found a model (= geometric construction and conventions in defining the notions of space, straight line, parallelism) of the hyperbolic geometry, that constituted a progress and assigning an important role to it. Analogously, the French mathematician Poincaré (1854-1912).

b) Riemann (1826-1866), German, formulated another negation: "Through an exterior point of a straight line one cannot construct any parallel to the given straight line", which has been named **Riemannian geometry** or elliptic geometry. This negation is also 100%.

c) Smarandache (b. 1954) partially negated the V postulate: "There exist straight lines and exterior points to them such that from those exterior points one can construct to the given straight lines:

1. only one parallel – in a certain zone of the geometric space [therefore, here functions the Euclidean geometry];
2. more parallels, but in a finite number – in another space zone;
3. an infinite number of parallels, but numerable – in another zone of the space;
4. an infinite number of parallels, but non-numerable – in another zone of the space [therefore, here functions Lobachevski's geometry];
5. no parallel – in another zone of the space [therefore, here functions the Riemannian geometry].

Therefore, the whole space is divided in five regions (zones), and each zone functions differently. This negation is not 100% as the previous ones.

I was a student at that time; the idea came to me in 1969. Why? Because I observed that in practice the spaces are not pure, homogeneous, but a mixture of different structures. In this way I united the three (Euclidean, hyperbolic, and elliptic) geometries connected by the V postulate, and I even extended them (with other two adjacent zones).

The problem was: how to connect a point from one zone, with a point from another different zone (how crossing the "frontiers")?

In "Bulletin of Pure and Applied Science" (Delhi, India), then in the prestigious German magazine which reviews articles of mathematics "Zentralblatt für Mathematik" (Berlin) there exist four variants of Smarandache Non-Euclidean Geometries [following the tradition: Euclid's (classical, traditional) geometry, Lobachevski's geometry, Riemannian geometry, Smarandache geometries].

The most important contribution of **Smarandache geometries** was the introduction of the **degree of negation of an axiom** (and more general the degree of negation of a theorem, lemma, scientific or humanistic proposition) which works somehow like the negation in fuzzy logic (with a degree of truth, and a degree of falsehood) or more general like the negation in neutrosophic logic (with a degree of truth, a degree of falsehood, and a degree of neutrality (neither true nor false, but unknown, ambiguous, indeterminate) [not only Enclid's geometrical axioms, but any scientific or humanistic proposition in any field] or partial negation of an axiom (and, in general, partial negation of a scientific or humanistic proposition in any field).

These geometries connect many geometrical spaces with different structures into a heterogeneous multi-space.

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