

Technical Notes and Correspondence

Delay-Adaptive Predictor Feedback for Systems With Unknown Long Actuator Delay

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Abstract—Stabilization of an unstable system with an unknown actuator delay of substantial length is an important problem that has never been attempted. We present a Lyapunov-based adaptive control design, prove its stability and regulation properties for the plant and actuator states, and present a simulation example inspired by the problem of control of pitch and flight path rates in the unstable X-29 aircraft.

Index Terms—Adaptive control, delay systems, distributed parameter systems.

I. INTRODUCTION

Adaptive control in the presence of actuator delays is challenging. Examples of existing results include [3], [9], [10]. All the existing results deal with unknown parameters but known delay. In this note, we address the more challenging problem where the delay itself is unknown and arbitrarily long.

Consider the system

$$\dot{X}(t) = AX(t) + BU(t - D) \quad (1)$$

where $X \in \mathbb{R}^n$ and where the delay length D is unknown (though constant) and arbitrarily large. We use an actuator delay model given by a *transport PDE*

$$\dot{X}(t) = AX(t) + Bu(0, t) \quad (2)$$

$$Du_t(x, t) = u_x(x, t) \quad (3)$$

$$u(1, t) = U(t) \quad (4)$$

where $u(x, t)$ is the state of the actuator. Instead of a full-state measurement of the actuator state, $u(x, t) = U(t + D(x - 1))$, we employ the state estimate

$$\hat{u}(x, t) = U\left(t + \hat{D}(t)(x - 1)\right) \quad (5)$$

where $\hat{D}(t)$ is the estimate of the unknown delay, obtaining the following *transport equation* representation:

$$\hat{D}(t)\hat{u}_t(x, t) = \hat{u}_x(x, t) + \dot{\hat{D}}(t)(x - 1)\hat{u}_x(x, t) \quad (6)$$

$$\hat{u}(1, t) = U(t). \quad (7)$$

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Note that we do not use the infinite-dimensional observer (6) but only the static estimate (5), where only $\hat{D}(t)$ is updated. Let $\hat{u}(x, t) \triangleq u(x, t) - \hat{u}(x, t)$, which yields $\dot{X}(t) = AX(t) + B\hat{u}(0, t) + B\hat{u}(0, t)$.

We employ an adaptive predictor feedback

$$U(t) = K \left(e^{A\hat{D}(t)} X(t) + \hat{D}(t) \int_0^1 e^{A\hat{D}(t)(1-y)} B \hat{u}(y, t) dy \right) \quad (8)$$

where K is selected to make the matrix $A + BK$ Hurwitz, which means, in particular, that for any $Q = Q^T > 0$, there exists $P = P^T > 0$ such that $P(A + BK) + (A + BK)^T P = -Q$.

A crucial element of our design and analysis is the backstepping transformation

$$\begin{aligned} \hat{w}(x, t) &= \hat{u}(x, t) - \hat{D}(t) \int_0^x K e^{A\hat{D}(t)(x-y)} B \hat{u}(y, t) dy \\ &\quad - K e^{A\hat{D}(t)x} X(t) \end{aligned} \quad (9)$$

$$\begin{aligned} \hat{u}(x, t) &= \hat{w}(x, t) + \hat{D}(t) \int_0^x K e^{(A+BK)\hat{D}(t)(x-y)} B \hat{w}(y, t) dy \\ &\quad + K e^{(A+BK)\hat{D}(t)x} X(t). \end{aligned} \quad (10)$$

Contribution and Organization: In a companion conference paper [6], we established a global adaptive result when $u(x, t)$ is measured. In this note we establish a *local* result when $u(x, t)$ is replaced by the estimate $\hat{u}(x, t)$. The local adaptive result, established in Section III, builds upon robustness of predictor feedback with respect to small errors in D , which is shown in Section II. Simulations for an unstable scalar plant inspired by the X-29 aircraft are shown in Section IV.

II. NONADAPTIVE ROBUSTNESS TO DELAY ERROR

We take $\hat{D}(t)$ as constant and establish the robustness of the predictor feedback to a small error $|D - \hat{D}|$ in an appropriate norm in which the adaptive problem will also be studied in Section III (this is a higher norm than the one in which we established delay-robustness in [4]). We denote $\omega(\theta) = U(\theta)$, $\theta \in [-\max\{D, \hat{D}\}, 0]$ and use $|\cdot|$ for a vector 2-norm.

Theorem 1: Consider the system (2)–(4), (6)–(8). There exists $\delta^* > 0$ such that for any $|\hat{D}| < \delta^*$, i.e., for any $\hat{D} \in (D - \delta^*, D + \delta^*)$, the zero solution of the system (X, u, \hat{u}) is exponentially stable, namely, there exist $R, \rho > 0$ such that for all initial conditions satisfying $(X_0, u_0, \hat{u}_0) \in \mathbb{R}^n \times L_2(0, 1) \times H_1(0, 1)$, the following holds:

$$\Gamma(t) \leq R\Gamma(0)e^{-\rho t} \quad (11)$$

$$\Gamma(t) = |X(t)|^2 + \int_0^1 [u(x, t)^2 + \hat{u}(x, t)^2 + \hat{u}_x(x, t)^2] dx. \quad (12)$$

Corollary 2: Consider system (1) with the controller

$$U(t) = K \left(e^{A\hat{D}} X(t) + \int_{t-\hat{D}}^t e^{A(t-\theta)} BU(\theta) d\theta \right).$$

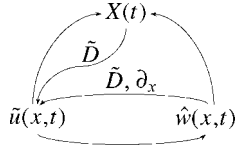


Fig. 1. Interconnections between the different variables.

There exists $\delta^* > 0$ such that, for any $|\tilde{D}| = |D - \hat{D}| < \delta^*$, there exists $R' > 0$ such that for all $X_0 \in R^n$, $\omega \in L_2[-\max\{D, \hat{D}\}, 0] \cap H_1[-\hat{D}, 0]$ the following holds:

$$\begin{aligned} \Pi(t) &\leq R' \Pi(0) e^{-\rho t} \\ \Pi(t) &= |X(t)|^2 + \int_{t-\max\{D, \hat{D}\}}^t U(\theta)^2 d\theta + \int_{t-\hat{D}}^t \dot{U}(\theta)^2 d\theta. \end{aligned}$$

We prove the theorem using the following lemmas.

Lemma 3: The system (2)–(4), (6)–(8) is equivalent to the system in which the X -subsystem is represented as

$$\dot{X}(t) = (A + BK)X(t) + B\hat{w}(0, t) + B\tilde{u}(0, t) \quad (13)$$

the \tilde{u} -subsystem is represented as

$$D\tilde{u}_t(x, t) = \tilde{u}_x(x, t) - \tilde{D}r(x, t) \quad (14)$$

$$\tilde{u}(1, t) = 0 \quad (15)$$

with

$$\begin{aligned} r(x, t) &= Ke^{(A+BK)\hat{D}x} (A + BK)X(t) + \frac{\hat{w}_x(x, t)}{\hat{D}} \\ &\quad + KB\hat{w}(x, t) + \hat{D} \int_0^x K(A + BK)e^{(A+BK)\hat{D}(x-y)} B\hat{w}(y, t) dy \end{aligned}$$

and the \hat{w}/\hat{w}_x -subsystem is represented as

$$\hat{D}\hat{w}_t(x, t) = \hat{w}_x(x, t) - \hat{D}Ke^{A\hat{D}x} B\tilde{u}(0, t) \quad (16)$$

$$\hat{w}(1, t) = 0 \quad (17)$$

$$\hat{D}\hat{w}_{xt}(x, t) = \hat{w}_{xx}(x, t) - \hat{D}^2 Ke^{A\hat{D}x} AB\tilde{u}(0, t) \quad (18)$$

$$\hat{w}_x(1, t) = \hat{D}Ke^{A\hat{D}} \tilde{u}(0, t). \quad (19)$$

The (X, \tilde{u}, \hat{w}) -system is shown in Fig. 1. The \tilde{D} -connections are ‘weak’ and disappear when $\tilde{D} = D$. The exponentially stable cascade connections $\tilde{u} \rightarrow X$ and $\tilde{u} \rightarrow \hat{w} \rightarrow X$ are ‘strong’ and present even when $\tilde{D} = D$. The potentially destabilizing feedback connections through \tilde{D} can be suppressed by making \tilde{D} small. An additional difficulty is that an ‘unbounded’ connection from \hat{w}_x to \tilde{u} exists. We deal with it by including an H_1 norm in the stability analysis.

Lemma 4: The following holds for (9) and (10):

$$\begin{aligned} \hat{u}_x(x, t) &= \hat{w}_x(x, t) + \hat{D}KB\hat{w}(x, t) \\ &\quad + \hat{D} \int_0^x K(A + BK)\hat{D}e^{(A+BK)\hat{D}(x-y)} B\hat{w}(y, t) dy \\ &\quad + K(A + BK)\hat{D}e^{(A+BK)\hat{D}x} X(t) \end{aligned} \quad (20)$$

$$\begin{aligned} \hat{w}_x(x, t) &= \hat{u}_x(x, t) + \hat{D}KB\hat{u}(x, t) \\ &\quad + \hat{D} \int_0^x KA\hat{D}e^{A\hat{D}(x-y)} B\hat{u}(y, t) dy \\ &\quad + KA\hat{D}e^{A\hat{D}x} X(t). \end{aligned} \quad (21)$$

Lemma 5: The following holds for (9), (10), (20), and (21):

$$\|\hat{u}(t)\|^2 \leq p_1 \|\hat{w}(t)\|^2 + p_2 |X(t)|^2 \quad (22)$$

$$\|\hat{u}_x(t)\|^2 \leq 4 \|\hat{w}_x(t)\|^2 + p_3 \|\hat{w}(t)\|^2 + p_4 |X(t)|^2 \quad (23)$$

$$\|\hat{w}(t)\|^2 \leq q_1 \|\hat{u}(t)\|^2 + q_2 |X(t)|^2 \quad (24)$$

$$\|\hat{w}_x(t)\|^2 \leq 4 \|\hat{u}_x(t)\|^2 + q_3 \|\hat{u}(t)\|^2 + q_4 |X(t)|^2 \quad (25)$$

where

$$\begin{aligned} p_1(\hat{D}) &= 3 \left(1 + \hat{D}^2 |K|^2 e^{2|A+BK|\hat{D}} |B|^2 \right) \\ p_2(\hat{D}) &= 3 |K|^2 e^{2|A+BK|\hat{D}} \\ p_3(\hat{D}) &= 4 \hat{D}^2 |K|^2 |B|^2 \left(1 + \hat{D}^2 |A + BK|^2 e^{2\hat{D}|A+BK|} \right) \\ p_4(\hat{D}) &= 4 |K|^2 \hat{D}^2 |A + BK|^2 e^{2\hat{D}|A+BK|} \\ q_1(\hat{D}) &= 3 \left(1 + \hat{D}^2 |K|^2 e^{2|A|\hat{D}} |B|^2 \right) \\ q_2(\hat{D}) &= 3 |K|^2 e^{2|A|\hat{D}} \\ q_3(\hat{D}) &= 4 \hat{D}^2 |K|^2 |B|^2 \left(1 + \hat{D}^2 |A|^2 e^{2\hat{D}|A|} \right) \\ q_4(\hat{D}) &= 4 |K|^2 \hat{D}^2 |A|^2 e^{2\hat{D}|A|}. \end{aligned}$$

Lemma 6: Consider the Lyapunov function

$$\begin{aligned} V(t) &= X^T(t)PX(t) + b_1 D \int_0^1 (1+x)\tilde{u}(x, t)^2 dx + b_2 \hat{D} \\ &\quad \times \left(\int_0^1 (1+x)\hat{w}(x, t)^2 dx + \int_0^1 (1+x)\hat{w}_x(x, t)^2 dx \right). \end{aligned} \quad (26)$$

There exist positive constants b_1, b_2, ρ , and δ^* such that for any $|\tilde{D}| < \delta^*$, the following holds:

$$\dot{V} \leq -\rho V. \quad (27)$$

Proof: Differentiating (26), along the solutions of (13), (14), (16), (18), and using integration by parts, we obtain

$$\begin{aligned} \dot{V} &= -X^T(t)QX(t) + 2X^T(t)PB(\hat{w}(0, t) + \tilde{u}(0, t)) \\ &\quad + 2b_1 D \int_0^1 (1+x)\tilde{u}(x, t)\tilde{u}_t(x, t) dx \\ &\quad + 2b_2 \hat{D} \left(\int_0^1 (1+x)\hat{w}(x, t)\hat{w}_t(x, t) dx \right. \\ &\quad \left. + \int_0^1 (1+x)\hat{w}_x(x, t)\hat{w}_{xt}(x, t) dx \right) \\ &= -X^T QX + 2X^T PB(\hat{w}(0, t) + \tilde{u}(0, t)) \\ &\quad - 2b_1 \left(\frac{\tilde{u}(0, t)^2}{2} + \frac{\|\tilde{u}(t)\|^2}{2} \right. \\ &\quad \left. + \hat{D} \int_0^1 (1+x)\tilde{u}(x, t)r(x, t) dx \right) \\ &\quad - 2b_2 \left(\frac{\hat{w}(0, t)^2}{2} + \frac{\|\hat{w}(t)\|^2}{2} \right. \\ &\quad \left. + \hat{D}K \int_0^1 (1+x)\hat{w}(x, t)e^{A\hat{D}x}\tilde{u}(0, t) dx \right) \end{aligned}$$

$$-2b_2 \left(\frac{\hat{w}_x(0,t)^2}{2} - \hat{w}_x(1,t)^2 + \frac{\|\hat{w}_x(t)\|^2}{2} + \hat{D}^2 K A \int_0^1 (1+x) \hat{w}_x(x,t) e^{A\hat{D}x} B \tilde{u}(0,t) dx \right).$$

Let us define the following constants, $c_1 = K\|B\|$, $c_2 = \hat{D}K\|A + BK\|e^{\|A+BK\|\hat{D}}\|B\|$, $c_3 = c_2/(\hat{D}\|B\|)$, $c_4 = 2\hat{D}K e^{\|A\|\hat{D}}$, $c_5 = c_4/2$, $c_6 = 2\hat{D}^2 K\|AB\|e^{2\|A\|\hat{D}}$, and choose $b_2 \geq 8|PB|/\lambda_{\min}(Q)$. Using the Cauchy-Schwartz and Young inequalities, we have

$$\begin{aligned} \dot{V}(t) \leq & -\frac{\lambda_{\min}(Q)}{2} |X(t)|^2 + \frac{b_2}{2} (\hat{w}(0,t)^2 + \tilde{u}(0,t)^2) \\ & - 2b_1 \left(\frac{\tilde{u}(0,t)^2}{2} + \frac{\|\tilde{u}(t)\|^2}{2} - \frac{|\hat{D}|}{\hat{D}} \|\tilde{u}(t)\|^2 \right. \\ & \quad - \frac{|\hat{D}|}{\hat{D}} \|\hat{w}_x(t)\|^2 - |\hat{D}|c_1^2 \|\tilde{u}(t)\|^2 - |\hat{D}| \|\hat{w}(t)\|^2 \\ & \quad - |\hat{D}| (c_2^2 \|\tilde{u}(t)\|^2 + \|\hat{w}(t)\|^2) \\ & \quad \left. - |\hat{D}| (|X(t)|^2 + c_3^2 \|\tilde{u}(t)\|^2) \right) \\ & - 2b_2 \left(\frac{\hat{w}(0,t)^2}{2} + \frac{\|\hat{w}(t)\|^2}{2} - c_4^2 \tilde{u}(0,t)^2 - \frac{\|\hat{w}(t)\|^2}{4} \right) \\ & - 2b_2 \left(\frac{\hat{w}_x(0,t)^2}{2} + \frac{\|\hat{w}_x(t)\|^2}{2} - c_5^2 \tilde{u}(0,t)^2 \right. \\ & \quad \left. - \frac{\|\hat{w}_x(t)\|^2}{4} - c_6^2 \tilde{u}(0,t)^2 \right). \end{aligned}$$

Grouping the like terms, we obtain

$$\begin{aligned} \dot{V}(t) \leq & - \left(\frac{\lambda_{\min}(Q)}{2} - 2|\hat{D}|b_1 \right) |X(t)|^2 \\ & - b_1 \left(1 - 2|\hat{D}| \left(\frac{1}{\hat{D}} + c_1^2 + c_2^2 + c_3^2 \right) \right) \|\tilde{u}(t)\|^2 \\ & - \left(b_1 - 2b_2 \left(\frac{1}{4} + c_4^2 + c_5^2 + c_6^2 \right) \right) \tilde{u}(0,t)^2 \\ & - \left(b_2 - \frac{2|\hat{D}|}{\hat{D}} b_1 \right) \|\hat{w}_x(t)\|^2 \\ & - \left(\frac{b_2}{2} - 2b_1|\hat{D}| \right) \|\hat{w}(t)\|^2 \\ & - \frac{b_2}{2} \hat{w}(0)^2 - b_2 \hat{w}_x(0,t)^2 \end{aligned} \quad (28)$$

and, with some further majorizations (for $|\hat{D}| < D$), we get

$$\begin{aligned} \dot{V}(t) \leq & - \left(\frac{\lambda_{\min}(Q)}{2} - 2|\hat{D}|b_1 \right) |X(t)|^2 \\ & - b_1 \left(1 - 2\frac{|\hat{D}|}{D} \left(\frac{1+D(c_1^2+c_2^2+c_3^2)}{D-|\hat{D}|} \right) \right) \|\tilde{u}(t)\|^2 \\ & - \left(b_1 - 2b_2 \left(\frac{1}{4} + c_4^2 + c_5^2 + c_6^2 \right) \right) \tilde{u}(0,t)^2 \\ & - \left(b_2 - \frac{2|\hat{D}|}{D-|\hat{D}|} b_1 \right) \|\hat{w}_x(t)\|^2 \\ & - \left(\frac{b_2}{2} - 2b_1|\hat{D}| \right) \|\hat{w}(t)\|^2 \\ & - \frac{b_2}{2} \hat{w}(0,t)^2 - b_2 \hat{w}_x(0)^2. \end{aligned}$$

Assuming that $|\hat{D}| < \delta^*$, where

$$\delta^* = \min \left\{ \frac{D}{3+2(c_1^2+c_2^2+c_3^2)}, \frac{Db_2}{4b_1+b_2}, \frac{b_2}{4b_1}, \frac{\lambda_{\min}(Q)D}{4b_1} \right\}$$

and $b_1 > (2/(1/4+c_4^2+c_5^2+c_6^2))b_2$, from (28), we obtain

$$\begin{aligned} \dot{V} & \leq -\eta (|X(t)|^2 + \|\tilde{u}(t)\|^2 + \tilde{u}(0,t)^2 + \|\hat{w}_x(t)\|^2 \\ & \quad + \|\hat{w}(t)\|^2 + \hat{w}(0,t)^2 + \hat{w}_x(0,t)^2) \\ & \leq -\eta\Gamma_0 \end{aligned} \quad (29)$$

where $\Gamma_0(t) = |X(t)|^2 + \|\tilde{u}(t)\|^2 + \|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2$, and $\eta = \min\{(\lambda_{\min}(Q)/2) - 2|\hat{D}|b_1, b_1(1 - 2\hat{D}((1/\hat{D}) + c_1^2 + c_2^2 + c_3^2)), b_1 - 2b_2((1/4) + c_4^2 + c_5^2 + c_6^2), b_2 - (2|\hat{D}|/\hat{D})b_1, (b_2/2) - 2b_1|\hat{D}|, (b_2/2)\}$ is positive. Having obtained (30), to complete the proof of (27), we first obtain the following inequalities from (26):

$$\begin{aligned} V(t) & \geq \lambda_{\min}(P) |X(t)|^2 \\ & \quad + b_1 D \|\tilde{u}(t)\|^2 + b_2 \hat{D} (\|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2) \\ & \geq \min \left\{ \lambda_{\min}(P), b_1 D, b_2 \hat{D} \right\} \Gamma_0(t). \end{aligned} \quad (31)$$

From (30) and (31), we complete the proof of (27) with $\rho = \eta / \min\{\lambda_{\min}(P), b_1 D, b_2 \hat{D}\}$. ■

Lemma 7: $\exists d_1, d_2 > 0$ such that $d_1\Gamma(t) \leq V(t) \leq d_2\Gamma(t)$.

Proof: From (22)–(25), we get

$$\begin{aligned} \Gamma(t) & \leq |X(t)|^2 + 2 (\|\tilde{u}(t)\|^2 + \|\tilde{u}_x(t)\|^2) \\ & \quad + \|\hat{u}(t)\|^2 + \|\hat{u}_x(t)\|^2 \\ & \leq |X(t)|^2 + 2\|\tilde{u}(t)\|^2 + 3(p_1 \|\hat{w}(t)\|^2 + p_2 |X(t)|^2) \\ & \quad + 4\|\hat{w}_x(t)\|^2 + p_3 \|\hat{w}(t)\|^2 + p_4 |X(t)|^2 \\ & \leq \max\{1 + 3p_2 + p_4, 2, 3p_1 + p_3, 4\} \Gamma_0(t) \end{aligned}$$

and

$$\begin{aligned} V(t) & \leq \lambda_{\max}(P) |X(t)|^2 + 2 (\|\tilde{u}(t)\|^2 + \|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2) \\ & \leq \lambda_{\max}(P) |X(t)|^2 + 4 (\|\tilde{u}(t)\|^2 + \|\hat{u}(t)\|^2) \\ & \quad + 2 (q_1 \|\hat{u}(t)\|^2 + q_2 |X(t)|^2) \\ & \quad + 2 (4\|\hat{u}_x(t)\|^2 + q_3 \|\hat{u}(t)\|^2 + q_4 |X(t)|^2) \end{aligned}$$

so $d_1 = (\max\{1 + 3p_2 + p_4, 3p_1 + p_3\}) / \min\{\lambda_{\min}(P), b_1 D, b_2 \hat{D}\}$ and $d_2 = \max\{\lambda_{\max}(P) + 2q_2 + 2q_4, 4 + 2q_1 + 2q_3\}$. ■

We now complete the proof of Theorem 1. From Lemma 6, it follows that $V(t) \leq V(0)e^{-\rho t}$. From Lemma 7, we get $\Gamma(t) \leq (d_2/d_1)\Gamma(0)e^{-\rho t}$. So $R = d_2/d_1$, which completes the proof of Theorem 1. Next, we prove Corollary 2.

Lemma 8: $\exists d_3, d_4 > 0$ such that $d_3\Gamma(t) \leq \Pi(t) \leq d_4\Gamma(t)$.

Proof: By substituting $u(x,t) = U(t + D(x-1))$ into (12), we get

$$\Gamma(t) = |X(t)|^2 + \frac{1}{D} \int_{t-D}^t U(\theta)^2 d\theta + \frac{1}{\hat{D}} \int_{t-\hat{D}}^t U(\theta)^2 d\theta + \hat{D} \int_{t-\hat{D}}^t \dot{U}(\theta)^2 d\theta.$$

Thus, $d_3 = 1/2 \max\{1, (1/D), (1/\hat{D}), \hat{D}\}$, $d_4 = \max\{1, D, \hat{D}, (1/\hat{D})\}$. ■

By combining Theorem 1 with Lemma 8, we complete the proof of Corollary 2 with $R' = (d_4/d_3)R$.

III. ADAPTIVE CONTROL WITH ESTIMATION OF THE TRANSPORT PDE STATE

Now we establish *local* stability for an *adaptive* controller. A global result is not obtainable because the solution $u(x, t) = U(t + D(x - 1))$ is not linearly parametrizable in D .

Assumption 1: A lower bound $\underline{D} > 0$ and an upper bound $\bar{D} \geq \underline{D}$ on the unknown delay D are known.

The update law for \hat{D} is chosen as

$$\dot{\hat{D}}(t) = \gamma \text{Proj}_{[\underline{D}, \bar{D}]} \{\tau(t)\}, \quad \gamma > 0 \quad (32)$$

$$\begin{aligned} \tau(t) = & - \int_0^1 (1+x) \hat{w}(x, t) K e^{A\hat{D}(t)x} dx \\ & \times (AX(t) + B\hat{u}(0, t)) \end{aligned} \quad (33)$$

where the standard projection operator is given by

$$\text{Proj}_{[\underline{D}, \bar{D}]} \{\tau\} = \begin{cases} 0, & \hat{D} = \underline{D} \text{ and } \tau < 0 \\ 0, & \hat{D} = \bar{D} \text{ and } \tau > 0 \\ \tau, & \text{else} \end{cases}$$

Theorem 9: Consider the closed loop consisting of the plant (2)–(4), (6), (7), the control law (8), and the update law defined by (32), (9). Let Assumption 1 hold and let

$$\begin{aligned} \Upsilon(t) = & |X(t)|^2 + \int_0^1 u(x, t)^2 dx + \int_0^1 \hat{u}(x, t)^2 dx \\ & + \int_0^1 \hat{u}_x(x, t)^2 dx + \tilde{D}(t)^2 \end{aligned}$$

denote the norm of the overall state of the closed-loop system. There exist positive constants ρ and R such that if the initial state $(X_0, u_0, \hat{u}_0, \hat{D}_0)$ is such that $\Upsilon(0) < \rho$, then

$$\begin{aligned} \Upsilon(t) & \leq R\Upsilon(0), \\ \lim_{t \rightarrow \infty} X(t) & = 0, \quad \lim_{t \rightarrow \infty} U(t) = 0. \end{aligned}$$

Proof: Mimicking the (omitted) proof of Lemma 3, we obtain the (X, \tilde{u}, \hat{w}) -system as

$$\dot{X}(t) = (A + BK)X(t) + B\hat{w}(0, t) + B\tilde{u}(0, t) \quad (34)$$

$$\begin{aligned} D\dot{\tilde{u}}_t(x, t) = & \tilde{u}_x(x, t) - \tilde{D}(t)r(x, t) \\ & - D\dot{\tilde{D}}(t)(x-1)r(x, t) \end{aligned} \quad (35)$$

$$\tilde{u}(1, t) = 0 \quad (36)$$

$$\begin{aligned} \hat{D}(t)\dot{\hat{w}}_t(x, t) = & \hat{w}_x(x, t) - \hat{D}(t)\dot{\hat{D}}(t)s(x, t) \\ & - \hat{D}(t)K e^{A\hat{D}(t)x} B\tilde{u}(0, t) \end{aligned} \quad (37)$$

$$\hat{w}(1, t) = 0 \quad (38)$$

where

$$\begin{aligned} r(x, t) = & \frac{\hat{w}_x(x, t)}{\hat{D}(t)} + KB\hat{w}(x, t) \\ & + \int_0^x K(A + BK)\hat{D}(t) \\ & \times e^{(A+BK)\hat{D}(t)(x-y)} B\hat{w}(y, t) dy \\ & + K(A + BK)e^{(A+BK)\hat{D}(t)x} X(t) \end{aligned} \quad (39)$$

$$\begin{aligned} s(x, t) = & (1 - x) \\ & \times \left(\frac{\hat{w}_x(x, t)}{\hat{D}(t)} + K e^{A\hat{D}(t)x} B (KX(t) + \hat{w}(0, t)) \right) \\ & + \int_0^x \hat{w}(y, t) \\ & \times \left[K \left(I + A\hat{D}(t)(x-y) \right) e^{A\hat{D}(t)(x-y)} B \right. \\ & \left. + \hat{D}(t) \int_y^x K \left(I + A\hat{D}(t)(x-\xi) \right) \right. \\ & \left. \times e^{A\hat{D}(t)(x-\xi)} BK \right. \\ & \left. \times e^{(A+BK)\hat{D}(t)(\xi-y)} B d\xi \right] dy \\ & + \left(KAe^{A\hat{D}(t)x} + \int_0^x K \left(I + A\hat{D}(t)(x-y) \right) \right. \\ & \left. \times e^{A\hat{D}(t)(x-y)} BK \right. \\ & \left. \times e^{(A+BK)\hat{D}(t)y} dy \right) X(t). \end{aligned} \quad (40)$$

Since our Lyapunov analysis will involve an H_1 norm of \hat{w} , we also need the governing equations of the \hat{w}_x -system

$$\begin{aligned} \hat{D}(t)\dot{\hat{w}}_{xt}(x, t) = & \hat{w}_{xx}(x, t) - \hat{D}(t)\dot{\hat{D}}(t)s_x(x, t) \\ & - KA\hat{D}^2 e^{A\hat{D}(t)x} B\tilde{u}(0, t) \end{aligned} \quad (41)$$

$$\begin{aligned} \hat{w}_x(1, t) = & \hat{D}(t)\dot{\hat{D}}(t) \\ & \times \left(\left[KAe^{A\hat{D}(t)} + \int_0^1 K \left(I + A\hat{D}(t)(1-y) \right) \right. \right. \\ & \left. \left. \times e^{A\hat{D}(t)(1-y)} BK \right. \right. \\ & \left. \left. \times e^{(A+BK)\hat{D}(t)y} dy \right] X(t) \right. \\ & \left. + \int_0^1 \hat{w}(y, t) \right. \\ & \left. \times \left[K \left(I + A\hat{D}(t)(1-y) \right) e^{A\hat{D}(t)(1-y)} \right. \right. \\ & \left. \left. + \int_y^1 K \left(I + A\hat{D}(t)(1-\xi) \right) e^{A\hat{D}(t)(1-\xi)} \right. \right. \\ & \left. \left. \times BK e^{(A+BK)\hat{D}(t)(\xi-y)} d\xi \right] B dy \right) \\ & + \hat{D}(t)K e^{A\hat{D}(t)} B\tilde{u}(0, t) \end{aligned} \quad (42)$$

where we get the (43), as shown at the bottom of the next page. We now start our Lyapunov analysis by introducing

$$\begin{aligned} V(t) = & X^T(t)PX(t) + b_1 D \int_0^1 (1+x)\tilde{u}(x, t)^2 dx \\ & + b_2 \hat{D}(t) \int_0^1 (1+x) (\hat{w}(x, t)^2 + \hat{w}_x(x, t)^2) dx + b_3 \tilde{D}(t)^2. \end{aligned}$$

With (34)–(43), we get

$$\begin{aligned}
\dot{V}(t) = & -X^T(t)QX(t) + 2X^T(t)PB(\hat{w}(0,t) + \tilde{u}(0,t)) \\
& + b_1(-\tilde{u}(0,t)^2 - \|\tilde{u}(t)\|^2 \\
& - 2\dot{D}(t) \int_0^1 (1+x)\tilde{u}(x,t)r(x,t)dx \\
& - 2D\dot{D}(t) \int_0^1 (x^2-1)\tilde{u}(x,t)r(x,t)dx) \\
& + b_2(-\hat{w}(0,t)^2 - \|\hat{w}(t)\|^2 \\
& - 2\dot{D}(t)\dot{D}(t) \int_0^1 (1+x)\hat{w}(x,t)s(x,t)dx \\
& - 2\hat{D}(t) \int_0^1 (1+x)Ke^{A\hat{D}(t)x}B\tilde{u}(0,t)\hat{w}(x,t)dx) \\
& + b_2(2\hat{w}_x(1,t)^2 - \hat{w}_x(0,t)^2 - \|\hat{w}_x(t)\|^2 \\
& - 2\dot{D}(t)\dot{D}(t) \int_0^1 (1+x)\hat{w}_x(x,t)s_x(x,t)dx \\
& - 2\hat{D}(t)^2 \int_0^1 (1+x)AKe^{A\hat{D}(t)x} \\
& \quad \times B\tilde{u}(0,t)\hat{w}_x(x,t)dx) \\
& + \dot{D}(t)b_2 \left(\int_0^1 (1+x)\hat{w}(x,t)^2 dx \right. \\
& \quad \left. + \int_0^1 (1+x)\hat{w}_x(x,t)^2 dx \right) \\
& - 2b_3\dot{D}(t)\dot{D}(t).
\end{aligned}$$

Using (32), (39), (40), (42), (43), the properties of the projection operator, and Agmon's inequality $\hat{w}(0,t)^2 \leq 4\|\hat{w}_x(t)\|^2$ (with the fact that $\hat{w}(1,t) = 0$), we find constants M_1, M_2, \dots, M_8 (independent of initial conditions) such that

$$\begin{aligned}
& \left| \int_0^1 (1+x)\tilde{u}(x,t)r(x,t)dx \right| \\
& \leq M_1 (\|\tilde{u}(t)\|^2 + \|\hat{w}_x(t)\|^2 + \|\hat{w}(t)\|^2 + |X(t)|^2)
\end{aligned}$$

$$\begin{aligned}
& \left| \int_0^1 (x^2-1)\tilde{u}(x,t)r(x,t)dx \right| \\
& \leq M_1 (\|\tilde{u}(t)\|^2 + \|\hat{w}_x(t)\|^2 + \|\hat{w}(t)\|^2 + |X(t)|^2) \\
& \left| 2\dot{D}(t) \int_0^1 (1+x)\hat{w}(x,t)s(x,t)dx \right| \\
& \leq 4M_2 (\|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2 + |X(t)|^2) \\
& \left| 2\dot{D}(t) \int_0^1 (1+x)Ke^{A\hat{D}(t)x}B\tilde{u}(0,t)\hat{w}(x,t)dx \right| \\
& \leq M_3\tilde{u}(0,t)^2 + \frac{\|\hat{w}(t)\|^2}{4} \\
& \left| 2\dot{D}(t) \int_0^1 (1+x)\hat{w}_x(x,t)s_x(x,t)dx \right| \\
& \leq M_4 (\|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2 + |X(t)|^2) \\
& \left| 2\hat{D}(t)^2 \int_0^1 (1+x)AKe^{A\hat{D}(t)x}B\tilde{u}(0,t)\hat{w}_x(x,t)dx \right| \\
& \leq M_5\tilde{u}(0,t)^2 + \frac{\|\hat{w}_x(t)\|^2}{4} \\
& \left| \dot{D}(t) \right| \\
& \leq 4M_6 (|X(t)|^2 + \|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2) \\
& 2\hat{w}_x(1,t)^2 \\
& \leq \dot{D}(t)^2 M_7 (|X(t)|^2 + \|\hat{w}(t)\|^2) + M_8\tilde{u}(0,t)^2.
\end{aligned}$$

Then

$$\begin{aligned}
\dot{V}(t) \leq & -\frac{\lambda_{\min}(Q)}{2}|X(t)|^2 - \frac{b_2}{2}\hat{w}(0,t)^2 - b_2\hat{w}_x(0,t)^2 \\
& - \left(b_2 - \frac{b_2}{4}\right)\|\hat{w}(t)\|^2 - \left(b_2 - \frac{b_2}{4}\right)\|\hat{w}_x(t)\|^2 \\
& - b_1\|\tilde{u}(t)\|^2 - \left(b_1 - b_2\left(\frac{1}{2} + M_3 + M_5 + M_8\right)\right) \\
& \times \tilde{u}(0,t)^2 + b_2M_7\dot{D}(t)^2 (|X(t)|^2 + \|\hat{w}(t)\|^2) \\
& + 2b_3\left|\dot{D}(t)\right|\left|\dot{D}(t)\right| + b_1M_1\left|\dot{D}(t)\right| \\
& \times (\|\tilde{u}(t)\|^2 + \|\hat{w}_x(t)\|^2 + \|\hat{w}(t)\|^2 + |X(t)|^2) \\
& + b_1M_1\left|\dot{D}(t)\right| \\
& \times (\|\tilde{u}(t)\|^2 + \|\hat{w}_x(t)\|^2\|\hat{w}(t)\|^2 + |X(t)|^2)
\end{aligned}$$

$$\begin{aligned}
s_x(x,t) = & \frac{1}{\hat{D}(t)} [(1-x)\hat{w}_{xx}(x,t) - \hat{w}_x(x,t)] + Ke^{A\hat{D}(t)x} \left(A\hat{D}(t)(1-x) - I \right) B(KX(t) + \hat{w}(0,t)) + \hat{D}(t) \\
& \times \int_0^x \hat{w}(y,t) \left[K \left(BK + A(2I + A\hat{D}(t)(x-y)) \right) \right] e^{A\hat{D}(t)(x-y)} B + \hat{D}(t) \\
& \quad \times \int_y^x K \left(BK + A \left(2I + A\hat{D}(t)(x-\xi) \right) \right) e^{A\hat{D}(t)(x-\xi)} BK e^{(A+BK)\hat{D}(t)(\xi-y)} B d\xi \Big] dy + KB\hat{w}(x,t) \\
& + \left[K(A^2\hat{D}(t) + BK)e^{A\hat{D}(t)x} + \hat{D}(t) \right. \\
& \quad \left. \times \int_0^x K \left(BK + A \left(2I + A\hat{D}(t)(x-y) \right) \right) \times e^{A\hat{D}(t)(x-y)} BK e^{(A+BK)\hat{D}(t)y} dy \right] X(t)
\end{aligned} \tag{43}$$

$$\begin{aligned}
 & + \left| \dot{D}(t) \right| \left(4b_2 M_2 (\|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2 + |X(t)|^2) \right. \\
 & \quad + 4b_2 M_4 (\|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2 + |X(t)|^2) \\
 & \quad \left. + 2b_2 (\|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2) \right)
 \end{aligned}$$

where we have chosen $b_2 \geq 8|PB|/\lambda_{\min}(Q)$. By choosing $b_1 > b_2(1/2 + M_3 + M_5 + M_8)$, and defining $\eta = \min\{\lambda_{\min}(Q)/2, b_2/2, b_1 - b_2(1/2 + M_3 + M_5 + M_8)\}$ and $V_0(t) = |X(t)|^2 + \|\hat{u}(t)\|^2 + \|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2$, where $\eta > 0$

$$\begin{aligned}
 \dot{V}(t) & \leq -\eta (|X(t)|^2 + \|\hat{u}(t)\|^2 + \|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2 \\
 & \quad + \hat{u}(0, t)^2 + \hat{w}(0, t)^2 + \hat{w}_x(0, t)) \\
 & \quad + 16b_2 M_7 M_6^2 V_0(t)^3 + 4b_1 M_1 M_6 \bar{D} V_0(t)^2 \\
 & \quad + (8b_3 M_6 + b_1 M_1) \left| \dot{D}(t) \right| V_0(t) \\
 & \quad + 4M_6 (4b_2 M_2 + 4b_2 M_4 + 2b_2) V_0(t)^2 \\
 & \leq -\eta V_0(t) + (8b_3 M_6 + b_1 M_1) \left| \dot{D}(t) \right| V_0(t) \\
 & \quad + 4M_6 (b_1 M_1 \bar{D} + 2b_2 (2M_2 + 2M_4 + 1)) V_0(t)^2 \\
 & \quad + 16b_2 M_7 M_6^2 V_0(t)^3.
 \end{aligned}$$

To eliminate the parameter error term, we employ the bound

$$\begin{aligned}
 \left| \dot{D}(t) \right| & \leq \frac{\varepsilon}{2} + \frac{\check{D}(t)^2}{2\varepsilon} \\
 & \leq \frac{\varepsilon}{2} + \frac{1}{2b_3\varepsilon} (V(t) - \lambda_{\min}(P) |X(t)|^2 - b_1 \underline{D} \|\hat{u}(t)\|^2 \\
 & \quad - b_2 \underline{D} \|\hat{w}(t)\|^2 - b_2 \underline{D} \|\hat{w}_x(t)\|^2) \\
 & \leq \frac{\varepsilon}{2} + \frac{1}{2b_3\varepsilon} (V(t) - \min\{\lambda_{\min}(P), b_1 \underline{D}, b_2 \underline{D}\} V_0(t))
 \end{aligned}$$

which yields

$$\begin{aligned}
 \dot{V}(t) & \leq - \left(\eta - (8b_3 M_6 + b_1 M_1) \left(\frac{\varepsilon}{2} + \frac{1}{2b_3\varepsilon} V(t) \right) \right) V_0(t) \\
 & \quad - \left(\frac{(8b_3 M_6 + b_1 M_1) \min\{\lambda_{\min}(P), b_1 \underline{D}, b_2 \underline{D}\}}{2b_3\varepsilon} \right. \\
 & \quad \left. - 4M_6 (b_1 M_1 \bar{D} + 2b_2 (2M_2 + 2M_4 + 1)) \right. \\
 & \quad \left. - 16b_2 M_7 M_6^2 V_0(t) \right) V_0(t)^2. \quad (44)
 \end{aligned}$$

If we choose the analysis parameter ε as

$$\varepsilon \leq \min \left\{ \frac{2\eta}{8b_3 M_6 + b_1 M_1}, \frac{(8b_3 M_6 + b_1 M_1) \min\{\lambda_{\min}(P), b_1 \underline{D}, b_2 \underline{D}\}}{8b_3 M_6 (b_1 M_1 \bar{D} + 2b_2 (2M_2 + 2M_4 + 1))} \right\} \quad (45)$$

and restrict the initial conditions so that $V(0) \leq \rho_1$

$$\begin{aligned}
 \rho_1 & \triangleq \min \left\{ 2b_3\varepsilon \left(\frac{\eta}{8b_3 M_6 + b_1 M_1} - \frac{\varepsilon}{2} \right), \right. \\
 & \quad \times \frac{\min\{\lambda_{\min}(P), b_1 \underline{D}, b_2 \underline{D}\}}{16b_2 M_7 M_6^2} \\
 & \quad \left. \times \left(\frac{(8b_3 M_6 + b_1 M_1) \min\{\lambda_{\min}(P), b_1 \underline{D}, b_2 \underline{D}\}}{2b_3\varepsilon} \right. \right. \\
 & \quad \left. \left. - 4M_6 (b_1 M_1 \bar{D} + 2b_2 (2M_2 + 2M_4 + 1)) \right) \right\} \quad (46)
 \end{aligned}$$

we obtain $\dot{V}(t) \leq -\mu_1(t)V_0(t) - \mu_2(t)V_0(t)^2$, where

$$\begin{aligned}
 \mu_1(t) & = \eta - (8b_3 M_6 + b_1 M_1) \left(\frac{\varepsilon}{2} + \frac{1}{2b_3\varepsilon} V(t) \right) \\
 \mu_2(t) & = \frac{(8b_3 M_6 + b_1 M_1) \min\{\lambda_{\min}(P), b_1 \underline{D}, b_2 \underline{D}\}}{2b_3\varepsilon} \\
 & \quad - 4M_6 (b_1 M_1 \bar{D} + 2b_2 (2M_2 + 2M_4 + 1)) \\
 & \quad - \frac{6b_2 M_7 M_6^2}{\min\{\lambda_{\min}(P), b_1 \underline{D}, b_2 \underline{D}\}} V(t)
 \end{aligned}$$

are nonnegative functions if the initial conditions are as in (46). Hence, $V(t) \leq V(0), \forall t \geq 0$. From this result for $V(t)$, we the result for $\Upsilon(t)$. Using Lemma 5 (which holds both when \hat{D} is constant and with a time-varying $\hat{D}(t)$), we obtain

$$\begin{aligned}
 \Upsilon(t) & = |X(t)|^2 + \|u(t)\|^2 + \|\hat{u}(t)\|^2 + \|\hat{u}_x(t)\|^2 + \check{D}(t)^2 \\
 & \leq |X(t)|^2 + 2(\|\hat{u}(t)\|^2 + \|\hat{u}_x(t)\|^2) \\
 & \quad + \|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2 + \check{D}(t)^2 \\
 & \leq (1 + 3p_2 + p_4) |X(t)|^2 + 2\|\hat{u}(t)\|^2 \\
 & \quad + (3p_1 + p_3) \|\hat{w}(t)\|^2 + 4\|\hat{w}_x(t)\|^2 + \check{D}(t)^2 \\
 & \leq \max\{1 + 3p_2 + p_4, 3p_1 + p_3, 4\} \\
 & \quad \times (|X(t)|^2 + \|\hat{u}(t)\|^2 + \|\hat{w}(t)\|^2 \\
 & \quad + \|\hat{w}_x(t)\|^2 + \check{D}(t)^2) \\
 & \leq \frac{\max\{1 + 3p_2 + p_4, 3p_1 + p_3\}}{\min\{\lambda_{\min}(P), b_1 \underline{D}, b_2 \underline{D}, b_3\}} V(t) \\
 & \triangleq \rho_2 V(t) \leq \rho_2 V(0). \quad (47)
 \end{aligned}$$

Hence, from (46) and (47), we obtain $\rho = \rho_1 \rho_2$. Similarly, using Lemma 5, we show that

$$\begin{aligned}
 V(0) & \leq \max\{\lambda_{\max}(P), 2b_1 \bar{D}, 2b_2 \bar{D}, b_3\} \\
 & \quad \times (|X(t)|^2 + \|\hat{u}(t)\|^2 + \|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2) \quad (48) \\
 & \leq \max\{\lambda_{\max}(P), 2b_1 \bar{D}, 2b_2 \bar{D}, b_3\} \\
 & \quad \times \max\{1 + q_2 + q_4, 2, 2 + q_1 + q_3, 4\} \Upsilon(0). \quad (49)
 \end{aligned}$$

Then, using (47), (48), we complete the local stability proof with $R = (\max\{1 + 3p_2 + p_4, 3p_1 + p_3\} / \min\{\lambda_{\min}(P), b_1 \underline{D}, b_2 \underline{D}, b_3\}) \max\{\lambda_{\max}(P), 2b_1 \bar{D}, 2b_2 \bar{D}, b_3\} \max\{1 + q_2 + q_4, 2 + q_1 + q_3\}$. To prove regulation, from $V(t) \leq V(0)$, it follows that $X(t), \|\hat{u}\|, \|\hat{w}\|, \|\hat{w}_x\|, \check{D}(t)$ are uniformly bounded. Then, from (10), using Cauchy-Schwartz inequality, we obtain the uniform boundedness of $\|\hat{u}(t)\|$ and consequently also of $U(t)$ for $t \geq 0$ from (8). Thus, $u(0, t) = U(t - D)$ is uniformly bounded for $t \geq D$. Using (2), we get that $d|X(t)|^2/dt$ is uniformly bounded for $t \geq D$. From (44), it follows that $|X(t)|$ is square integrable. Finally, by Barbalat's lemma, we get that $X(t) \rightarrow 0$. To also prove the regulation of $U(t)$, we start by deducing from (44) the square integrability of $\|\hat{w}(t)\|$. Then, from Lemma 5, we have the square integrability of $\|\hat{u}\|$ and, from (8), using Cauchy-Schwartz inequality, the square integrability of $U(t)$. To establish the boundedness of $dU(t)^2/dt$, we compute it as

$$\begin{aligned}
 \frac{d}{dt} U(t)^2 & = 2U(t)K \left(e^{A\hat{D}(t)} \dot{X}(t) + \dot{D}(t)G_1(t) + \hat{D}(t)G_2(t) \right) \\
 G_1(t) & = Ae^{A\hat{D}(t)} X(t) + (x-1) \int_0^1 e^{A\hat{D}(t)(1-y)} B \hat{u}_x(y, t) dy \\
 & \quad + \int_0^1 \left(I + A\hat{D}(t)(1-y) \right) e^{A\hat{D}(t)(1-y)} B \hat{u}(y, t) dy \\
 G_2(t) & = \int_0^1 e^{A\hat{D}(t)(1-y)} B \hat{u}_x(y, t) dy.
 \end{aligned}$$

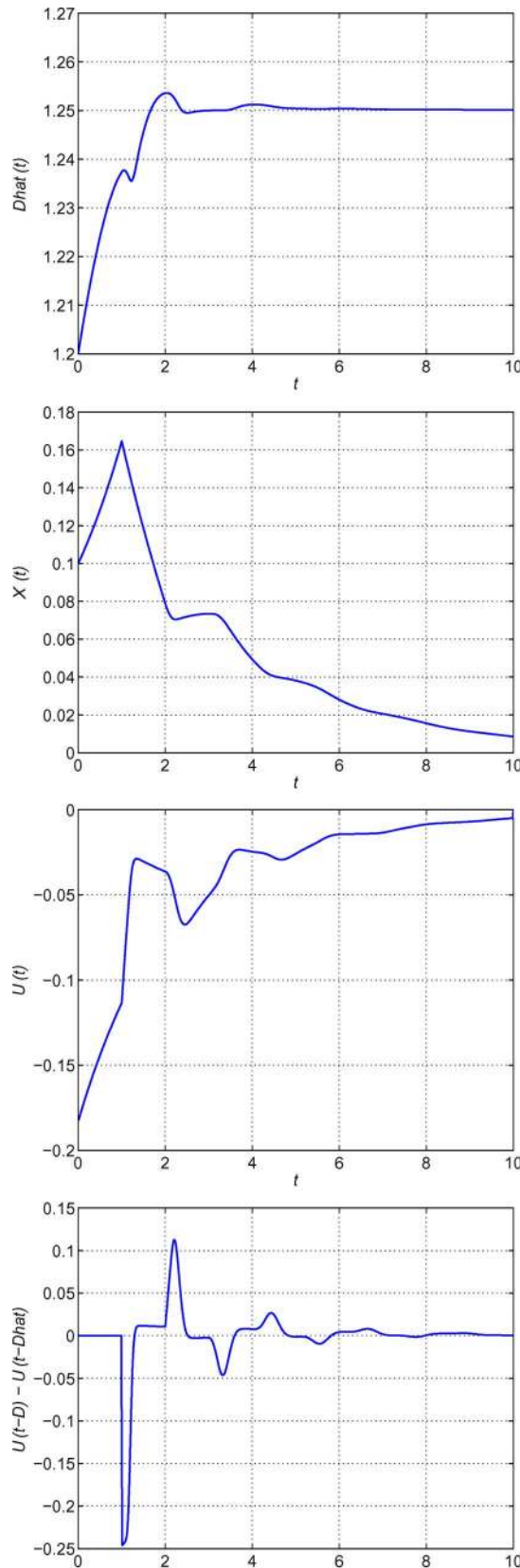


Fig. 2. System response of the system (2)–(4), (6)–(9) for $D = 1$. Bottom: the estimation error of the actuator state, $\hat{u}(0, t) = u(0, t) - \hat{u}(0, t) = U(t - D) - U(t - \hat{D}(t))$.

The signal $\hat{D}(t)$ is uniformly bounded for $t \geq D$ according to (32). By using the boundedness of $\dot{X}(t)$, $X(t)$, $\|\hat{u}(t)\|$ and $\|\hat{u}_x(t)\|$ over $t \geq D$, we get boundedness of $dU(t)^2/dt$ for $t \geq D$. Then, by Barbalat's lemma, $U(t) \rightarrow 0$ as $t \rightarrow \infty$. ■

IV. SIMULATIONS

Consider the system $X(s)/U(s) = e^{-Ds}B/(s - A)$, which is a model of the dynamics of an X-29 aircraft in an unstable regime [2], with the input being the control surface deflection, and with the output being a linear combination of the pitch rate (measured with a gyroscope) and the rate of change of the flight path (measured with a gyroscope). We take the plant parameters as $A = 0.75$, $B = 1$, and the nominal control gain as $K = -A - 1 = -1.75$ (which means that $P = 1$ for $Q = 2$). Fig. 2 shows a simulation example with $D = 1$, $A = 0.5$, $B = 1$, $K = -1.5$, $P = 1$, $Q = 2$, and $\gamma = 5$.

V. RELATED RESULTS

In this note, the only parametric uncertainty considered is the unknown delay. In a companion paper [1] we present an extension with unknown plant parameters and where the control objective is not regulation to zero but trajectory tracking. The design technique in this note is inspired by the techniques for parabolic PDEs in [7] and the non-adaptive techniques for hyperbolic PDEs [8]. Nonlinear extensions of predictor feedback are introduced in [5].

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