

Delay-Compensation-Based State Estimation for Time-Varying Complex Networks with Incomplete Observations and Dynamical Bias

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Abstract—In this paper, a delay-compensation-based state estimation (DCBSE) method is given for a class of discrete time-varying complex networks (DTVCNs) subject to network-induced incomplete observations (NIOs) and dynamical bias. The NIOs include the communication delays and fading observations, where the fading observations are modelled by a set of mutually independent random variables. Moreover, the possible bias is taken into account, which is depicted by a dynamical equation. A predictive scheme is proposed to compensate the influences induced by the communication delays, where the predictive-based estimation mechanism is adopted to replace the delayed estimation transmissions. This paper focuses on the problems of estimation method design and performance discussions for addressed DTVCNs with NIOs and dynamical bias. In particular, a new distributed state estimation approach is presented, where a locally minimized upper bound is obtained for the estimation error covariance matrix and a recursive way is designed to determine the estimator gain matrix. Furthermore, the performance evaluation criteria regarding the monotonicity are proposed from the analytic perspective. Finally, some experimental comparisons are proposed to show the validity and advantages of new DCBSE approach.

Index Terms—Time-varying stochastic complex networks; Delay-compensation-based estimation; Communication delays; Incomplete Observations; Dynamical bias; Monotonicity analysis.

I. INTRODUCTION

Over the past years, the investigation of the complex networks (CNs) has become an emerging research topic due to its strong advantages on modelling/analyzing the practical systems, such as biological networks, power grid, traffic networks and technological networks [35], [40], [41]. Accordingly, a rich body of analysis strategies has been given to understand the structure and dynamical characteristic of CNs with a

large number of coupled nodes [10], [12], [37]. Among the exiting results, both the state estimation and synchronization problems have gained the primary interest and become two important research topics for CNs [6], [27], [32]. So far, a great number of sufficient criteria combined with certain performance requirements have been proposed to handle the synchronization problems for CNs with different coupling impacts [7], [13], [14]. In comparison with the synchronization problem, special effort has been devoted on the state estimation problems in order to better understand the intrinsic features of the networks by properly taking the coupling relationships and shared information into account [11], [34]. Recently, the dynamics behaviour analysis of CNs subject to time-varying characteristics has received increasing attention on the estimation of node's state [11], [17], [18]. For example, a new estimation scheme under the error variance constraint has been presented in [11] based on the recursive matrix inequality technique, where both the changeable typology structure and quantized measurements of time-varying CNs have been taken into consideration and examined accordingly. In [19], the recursive estimation method under the error covariance optimization has been developed, where both the time-varying characteristics of concerned CNs and stochastic impacts of the noises have been adequately reflected in the proposed estimation strategy for the first time. Subsequently, the estimation method in [19] has been extended to propose the recursive estimation algorithms for time-varying CNs with random/nonlinear coupling influences in [28], [29].

As it is well known, the practical systems are commonly influenced by different type disturbances/inputs, which should be properly handled during the system modelling and analysis [9], [15], [46]. Consequently, special attention has been made on dealing with the joint estimation problem of unknown inputs and system states [24], [38]. For example, some optimal estimation criteria in the minimum mean square error (MMSE) sense have been established in [25], [44] for linear stochastic systems, in which both unknown inputs and system states have been well estimated within a unified framework. In [21], as a special type of unknown inputs, the stochastic bias has been modeled by a dynamical equation. Subsequently, the state estimation problems have received ongoing research attention for dynamical systems with stochastic bias [22], [23], [42]. To be more specific, the two-step estimation schemes in the MMSE sense have been given in [20] for system with unknown dynamical bias and in [23] to deal with the incomplete information situation of dynamical bias via the adaptive mech-

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anism. Very recently, the state estimation problem under the minimization consideration of the estimation error covariance has been dealt with in [42] for two-dimensional stochastic systems, where a locally optimized estimation criterion has been developed against the dynamical bias and the random parameter perturbations simultaneously. So far, the design problem of state estimation algorithm has not been discussed for discrete *time-varying* complex networks (DTVCNs) with dynamical bias despite its practical importance, which deserves further investigation.

In the networked communication environment, the transmissions among different nodes/components may undergo the network-enhanced complexities due to the limited capability of communication channels [8], [33], [47], [48]. As such, accompanying with the increasing popularity of networked systems, the analysis problems with respect to missing measurements and communication delays have become active research topics [4], [26], [39], [45]. For example, the state estimation problems have been tackled in [31] for time-invariant neural networks subject to multiple missing measurements and in [3] for large and diverse road networks with missing data. To cater the time-varying feature, new optimized estimation scheme of the recursive manner has been proposed in [19] for discrete-time CNs subject to missing measurements. Nevertheless, it is noticed that most available state estimation algorithms against the missing measurements can be applicable for handling the completely lost or successfully transferred cases only, and the estimation methods corresponding to fading observations with partial missing measurement are very few [16]. Apart from the missing/fading measurements mentioned above, the existence of the communication delays is another reason, which causes the deterioration of the estimation accuracy [1], [43]. However, it is worthwhile to point out that most of the existing state estimation methods have relied on the delay information only and the potential impacts of the communication delays among different nodes have not been taken into account [2], [5]. For the purpose of performance improvements, the predictive-based idea has been preliminarily introduced to deal with the control problems subject to communication delays [30], [36]. To the best of our knowledge, few methods are available to handle the state estimation problem for DTVCNs with fading observations and communication delays. Consequently, there is an urgent need to address the state estimation problem for DTVCNs and develop a predictive compensation estimation scheme accordingly.

Inspired by the aforementioned discussions, this paper is concerned with the delay-compensation-based state estimation (DCBSE) problem for a class of DTVCNs, where the effects from the communication delays, fading observations and dynamical bias disturbances are handled. Compared with existing state estimation problems, the major challenges of conducted DCBSE problem lie in: i) how to better characterize the network-induced incomplete observations (NIOs) as well as dynamical bias disturbances and examine the corresponding influences on the estimation performance accordingly? ii) how to propose an efficient estimation scheme applicable for the real-time calculation? and iii) how to develop an analysis method with rigorous theoretical proof on the estimation

performance evaluation? To be more specific, the NIOs including communication delays and fading observations are taken into account within a same framework, in which a hybrid compensation method is proposed to attenuate the impacts caused by NIOs during the estimation algorithm design. The dynamical bias disturbances are described and estimated as a by-product when estimating the node states. In addition, the algorithm evaluation criteria are provided to further reveal the engineering insight via the theoretical analysis. The major contributions can be highlighted from the following three aspects. 1) The state estimation problem in the variance-constrained sense is, for the first time, discussed for DTVCNs with NIOs and dynamical bias. 2) Both the NIOs and dynamical bias are well discussed, and a distributed DCBSE algorithm is developed accordingly, which possesses a recursive manner applicable for online implementations. In particular, the active compensation way based on the predictive idea is proposed and the communication delays are well examined accordingly. 3) The monotonicity analyses are provided to discuss the impacts from the fading observations described by different modelling ways onto the estimation performance. Finally, some comparative experiments are made, and both the validity and advantages of newly proposed DCBSE approach are illustrated.

The rest of the paper is arranged as follows. In Section II, both the problem formulation and preliminaries are briefly introduced, where a new delay-compensation-based estimator is designed by taking the impacts of the communication delays and fading observations into account. In Section III, the recursions of estimation error covariance matrix and its related upper bound are provided, moreover, the optimized estimator gain matrix is presented by solving a recursive matrix equation (RME). Subsequently, in Section IV, the performance evaluation problems are conducted to discuss the monotonicity analyses of the presented state estimation algorithm and the corresponding theoretical proofs are proposed accordingly. The simulations with comparative discussions are given in Section V to demonstrate the effectiveness of new DCBSE algorithm and some conclusions are pointed out in Section VI.

Notations. Throughout this paper, \mathbb{R}^n stands for the n -dimensional Euclidean space. For a matrix A , A^T , A^{-1} and $A > 0$ denote the transpose of A , the inverse of A and A being a symmetric positive-definite matrix, respectively. $\text{sym}(A)$ denotes $A + A^T$. \circ represents the Hadamard product, where $[A \circ B]_{ij} = A_{ij} \cdot B_{ij}$. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation of ' \cdot '. Furthermore, we assume that the algebraic calculations are compatible when the matrix dimensions are not specified.

II. PROBLEM STATEMENT AND PRELIMINARY

In this paper, we consider the following class of DTVCNs with dynamical bias:

$$x_{i,k+1} = A_{i,k}x_{i,k} + \sum_{j=1}^N \omega_{ij,k}\Gamma x_{j,k} + B_{i,k}b_{i,k} + D_{i,k}\theta_{i,k}, \quad (1)$$

$$y_{i,k} = \Psi_{i,k} C_{i,k} x_{i,k} + \eta_{i,k}, \quad i = 1, 2, \dots, N \quad (2)$$

where $x_{i,k}$ refers to the n_x -dimensional state of the i -th node, $x_{i,0}$ represents the initial value with mean $\bar{x}_{i,0}$, $y_{i,k}$ denotes the n_y -dimensional measurement vector, $\Omega_k = [\omega_{ij,k}]_{N \times N}$ is the coupling strength matrix, and $\Gamma = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_n\}$ is the inner coupling matrix. $\Psi_{i,k} = \text{diag}\{\psi_{i1,k}, \psi_{i2,k}, \dots, \psi_{in_y,k}\}$ depicts the phenomena of the fading observations, where $\psi_{it,k}$ ($t = 1, 2, \dots, n_y$) are the random variables over the interval $[a_{it}, b_{it}]$ with $0 \leq a_{it} \leq b_{it} \leq 1$. Moreover, the mathematical means and variances of random variables $\psi_{it,k}$ are $\bar{\psi}_{it,k}$ and $\check{\psi}_{it,k}$, respectively. $\theta_{i,k}$ and $\eta_{i,k}$ represent the n_θ -dimensional process noise and n_y -dimensional measurement noise, where $\mathbb{E}\{\theta_{i,k}\} = 0$, $\mathbb{E}\{\eta_{i,k}\} = 0$, $\mathbb{E}\{\theta_{i,k}\theta_{i,k}^T\} = Q_{i,k}^\theta$ and $\mathbb{E}\{\eta_{i,k}\eta_{i,k}^T\} = Q_{i,k}^\eta$ with $Q_{i,k}^\eta > 0$. The dynamics model of the random bias is described as follows:

$$b_{i,k+1} = F_{i,k} b_{i,k} + \beta_{i,k}, \quad (3)$$

where $b_{i,k} \in \mathbb{R}^{n_b}$ depicts the bias of unknown magnitude. $\beta_{i,k}$ denotes the white noise, where $\mathbb{E}\{\beta_{i,k}\} = 0$ and $\mathbb{E}\{\beta_{i,k}\beta_{i,k}^T\} = Q_{i,k}^\beta$ with $Q_{i,k}^\beta > 0$. $A_{i,k}$, $B_{i,k}$, $C_{i,k}$, $D_{i,k}$ and $F_{i,k}$ are known matrices of proper dimensions.

The random inner coupling matrix Γ is described by:

$$\Gamma = \bar{\Gamma} + \Delta\bar{\Gamma}, \quad (4)$$

where $\bar{\Gamma} = \text{diag}\{\bar{\gamma}_1, \bar{\gamma}_2, \dots, \bar{\gamma}_N\}$ is the nominal inner coupling matrix, and $\Delta\bar{\Gamma}$ represents the random parameter perturbation satisfying

$$\mathbb{E}\{\Delta\bar{\Gamma}\} = 0, \quad \mathbb{E}\{\Delta\bar{\Gamma}\Delta\bar{\Gamma}^T\} \leq \bar{\gamma}I, \quad (5)$$

in which $\bar{\gamma}$ is a known positive constant.

Remark 1: It is worth pointing out that the coupling strengthens among the state elements are different if $\gamma_i \neq \gamma_j$. Hence, the random inner-coupling strengthens characterized by (4)-(5) are introduced to present more flexibility of the CNs modelling.

Subsequently, set

$$\begin{aligned} \bar{x}_{i,k} &= [x_{i,k}^T \quad b_{i,k}^T]^T, \quad C_{i,k} = [C_{i,k} \quad 0], \\ A_{i,k} &= \begin{bmatrix} A_{i,k} & B_{i,k} \\ 0 & F_{i,k} \end{bmatrix}, \quad \bar{\Gamma} = \begin{bmatrix} \bar{\Gamma} & 0 \\ 0 & 0 \end{bmatrix}, \\ \Delta\bar{\Gamma} &= \begin{bmatrix} \Delta\bar{\Gamma} & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{D}_{i,k} = \begin{bmatrix} D_{i,k} & 0 \\ 0 & I \end{bmatrix}, \\ \zeta_{i,k} &= [\theta_{i,k}^T \quad \beta_{i,k}^T]^T. \end{aligned} \quad (6)$$

Then, it follows from the above notations that the following compact form can be obtained:

$$\begin{aligned} \bar{x}_{i,k+1} &= A_{i,k} \bar{x}_{i,k} + \sum_{j=1}^N \omega_{ij,k} (\bar{\Gamma} + \Delta\bar{\Gamma}) \bar{x}_{j,k} + \mathcal{D}_{i,k} \zeta_{i,k}, \quad (7) \\ y_{i,k} &= \Psi_{i,k} C_{i,k} \bar{x}_{i,k} + \eta_{i,k}. \end{aligned} \quad (8)$$

In what follows, we assume that $\theta_{i,k}$, $\beta_{i,k}$, $\psi_{it,k}$, $\eta_{i,k}$, $x_{i,0}$ and the element of $\Delta\bar{\Gamma}$ are mutually independent in i , t and k .

Next, the time-varying state estimator in the distributed manner is designed for each node based on the related measurements and adjacent node's estimation. During the

information transmission, the communication delays between the node j and the node i are considered, which are denoted by τ_{ij} . As such, the estimation at the delayed instant can be available only during the estimator design. For the purpose of estimation accuracy improvement, the following predictive compensation rule is employed:

$$\begin{aligned} \hat{x}_{j,k-\tau_{ij}+1|k-\tau_{ij}} &= \mathcal{A}_{j,k-\tau_{ij}} \hat{x}_{j,k-\tau_{ij}}, \\ \hat{x}_{j,k-\tau_{ij}+2|k-\tau_{ij}} &= \mathcal{A}_{j,k-\tau_{ij}+1} \hat{x}_{j,k-\tau_{ij}+1|k-\tau_{ij}}, \\ &\vdots \\ \hat{x}_{j,k-1|k-\tau_{ij}} &= \mathcal{A}_{j,k-2} \hat{x}_{j,k-2|k-\tau_{ij}}, \\ \hat{x}_{j,k|k-\tau_{ij}} &= \mathcal{A}_{j,k-1} \hat{x}_{j,k-1|k-\tau_{ij}}. \end{aligned} \quad (9)$$

Define $\mathcal{A}_{j,\tau_{ij}} \triangleq \prod_{t=1}^{\tau_{ij}} \mathcal{A}_{j,k-t}$. According to (9), one has $\hat{x}_{j,k|k-\tau_{ij}} = \mathcal{A}_{j,\tau_{ij}} \hat{x}_{j,k-\tau_{ij}}$, then the predictive estimation $\hat{x}_{j,k|k-\tau_{ij}}$ at the current instant is derived based on the delayed estimation $\hat{x}_{j,k-\tau_{ij}}$.

For the i -th node, the following state estimator under the predictive compensation mechanism is constructed:

$$\begin{aligned} \hat{x}_{i,k+1} &= (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \hat{x}_{i,k} + \sum_{j=1, j \neq i}^N \omega_{ij,k} \bar{\Gamma} \hat{x}_{j,k|k-\tau_{ij}} \\ &\quad + \mathcal{K}_{i,k} (y_{i,k} - \bar{\Psi}_{i,k} C_{i,k} \hat{x}_{i,k}), \end{aligned} \quad (10)$$

where $\hat{x}_{i,k}$ represents the estimate of state $\bar{x}_{i,k}$ at the time step k , $\bar{\Psi}_{i,k} = \text{diag}\{\bar{\psi}_{i1,k}, \bar{\psi}_{i2,k}, \dots, \bar{\psi}_{in_y,k}\}$ and $\bar{\psi}_{it,k}$ is the mathematical expectation of the random variable $\psi_{it,k}$ ($t = 1, 2, \dots, n_y$), and $\mathcal{K}_{i,k}$ is the estimator parameter matrix to be determined later.

Remark 2: In order to improve the estimation accuracy, there is a need to examine the network-enhanced complexities. It should be pointed out that the hybrid compensation idea is presented to attenuate the effects from NIIOs when designing the new time-varying state estimator in (10). To be more specific, the communication delays among the different nodes are taken into account and those impacts are actively compensated in (9) by employing the delayed estimation information properly. In particular, it is worthwhile to mention that a new predictive scheme is presented based on $\hat{x}_{j,k-\tau_{ij}}$ at the time step $k-\tau_{ij}$ and the desirable estimation $\hat{x}_{j,k|k-\tau_{ij}}$ at the time step k can be obtained iteratively. Accordingly, this predictive-based scheme is given to compensate the influences caused by the communication delays during the transmissions among the adjacent nodes. Moreover, the mathematical expectation $\bar{\Psi}_{i,k}$ is also introduced in (10) to reflect the fading observations. In this paper, such a hybrid compensation estimation manner, which contains the predictive and probability-dependent mechanisms, is developed to improve the estimation algorithm performance and the corresponding estimation accuracy comparisons under different cases of NIIOs will be provided later.

Remark 3: The new state estimator in (10) under the predictive compensation mechanism has the following features: i) the communication delays among the different transmission channels and the fading observations are addressed within same framework, where a hybrid compensation estimation manner is provided; ii) a Kalman-type estimator is constructed based

on the predictive estimation and the locally optimal estimation scheme is expected, which performs an attractive advantage in the online implementations; and iii) the dynamical bias disturbances are discussed and estimated simultaneously.

Define $e_{i,k+1} \triangleq \bar{x}_{i,k+1} - \hat{x}_{i,k+1}$ as the estimation error and $P_{i,k+1} \triangleq \mathbb{E}\{e_{i,k+1}e_{i,k+1}^T\}$ as the estimation error covariance matrix. Next, we are in a position to state the major objectives of the DCBSE problem for DTVCNs with NIOs and dynamical bias, where the following three requirements are guaranteed simultaneously.

1) Design the state estimator (10) of a recursive manner and provide a sufficient criterion to look for an upper bound covariance matrix $\mathcal{P}_{i,k+1}$ with respect to $P_{i,k+1}$.

2) Present the expression form of the estimator parameter matrix $\mathcal{K}_{i,k+1}$ in order to minimize $\mathcal{P}_{i,k+1}$.

3) Provide the theoretical proofs of the monotonicity analysis between the fading observations and estimation accuracy.

III. MAIN RESULTS

In this section, according to the definitions, both the state covariance matrix and the estimation error covariance matrix are calculated. Subsequently, a locally minimized upper bound is given regarding the estimation error covariance and the recursive expression form of the estimator parameter matrix is provided accordingly.

To begin with, let $\tilde{\Psi}_{i,k} \triangleq \Psi_{i,k} - \bar{\Psi}_{i,k}$ with $\tilde{\Psi}_{i,k} = \text{diag}\{\tilde{\psi}_{i1,k}, \tilde{\psi}_{i2,k}, \dots, \tilde{\psi}_{in_y,k}\}$. Then, it is straight to testify that $\mathbb{E}\{\tilde{\psi}_{it,k}\} = 0$ ($t = 1, 2, \dots, n_y$). Next, it follows from (7) and (10) that $e_{i,k+1}$ can be given by

$$\begin{aligned} e_{i,k+1} &= (\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})e_{i,k} \\ &\quad - \mathcal{K}_{i,k}\tilde{\Psi}_{i,k}\mathcal{C}_{i,k}\bar{x}_{i,k} + \sum_{j=1}^N \omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k} \\ &\quad + \sum_{j=1, j \neq i}^N \omega_{ij,k}\bar{\Gamma}(\bar{x}_{j,k} - \hat{x}_{j,k|k-\tau_{ij}}) \\ &\quad + \mathcal{D}_{i,k}\zeta_{i,k} - \mathcal{K}_{i,k}\eta_{i,k}. \end{aligned} \quad (11)$$

Firstly, the recursion of the state covariance is computed. Secondly, the recursive expressions of estimation error covariance matrix and its upper bound are presented.

Theorem 1: Consider the DTVCNs (7) with the state estimator (10). For the positive scalars $\epsilon_{i,k}$, the state covariance matrix $\bar{X}_{i,k+1} = \mathbb{E}\{\bar{x}_{i,k+1}\bar{x}_{i,k+1}^T\}$ satisfies:

$$\begin{aligned} \bar{X}_{i,k+1} &\leq (1 + \epsilon_{i,k})\mathcal{A}_{i,k}\bar{X}_{i,k}\mathcal{A}_{i,k}^T \\ &\quad + N(1 + \epsilon_{i,k}^{-1}) \sum_{j=1}^N \omega_{ij,k}^2 \bar{\Gamma}\bar{X}_{j,k}\bar{\Gamma}^T \\ &\quad + N \sum_{j=1}^N \omega_{ij,k}^2 \bar{\gamma} \lambda_{\max}(\bar{X}_{j,k})I + \mathcal{D}_{i,k}Q_{i,k}^{\zeta}\mathcal{D}_{i,k}^T \\ &\triangleq \mathcal{X}_{i,k+1}, \end{aligned} \quad (12)$$

where $Q_{i,k}^{\zeta} = \text{diag}\{Q_{i,k}^{\theta}, Q_{i,k}^{\beta}\}$.

Proof: According to the definition of state covariance matrix, it follows that

$$\bar{X}_{i,k+1} = \mathbb{E}\{\bar{x}_{i,k+1}\bar{x}_{i,k+1}^T\}$$

$$\begin{aligned} &= \mathcal{A}_{i,k}\bar{X}_{i,k}\mathcal{A}_{i,k}^T + \text{sym}(\mathcal{E}_{i,k}) \\ &\quad + \mathbb{E}\left\{\left(\sum_{j=1}^N \omega_{ij,k}\bar{\Gamma}\bar{x}_{j,k}\right)\left(\sum_{j=1}^N \omega_{ij,k}\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\} \\ &\quad + \mathbb{E}\left\{\left(\sum_{j=1}^N \omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)\left(\sum_{j=1}^N \omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\} \\ &\quad + \sum_{r=1}^5 \text{sym}(\mathcal{F}_{ri,k}) + \mathcal{D}_{i,k}\mathbb{E}\{\zeta_{i,k}\zeta_{i,k}^T\}\mathcal{D}_{i,k}^T, \end{aligned} \quad (13)$$

where

$$\begin{aligned} \mathcal{E}_{i,k} &= \mathcal{A}_{i,k}\mathbb{E}\left\{\bar{x}_{i,k}\left(\sum_{j=1}^N \omega_{ij,k}\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\}, \\ \mathcal{F}_{1i,k} &= \mathcal{A}_{i,k}\mathbb{E}\left\{\bar{x}_{i,k}\left(\sum_{j=1}^N \omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\}, \\ \mathcal{F}_{2i,k} &= \mathcal{A}_{i,k}\mathbb{E}\{\bar{x}_{i,k}\zeta_{i,k}^T\}\mathcal{D}_{i,k}^T, \\ \mathcal{F}_{3i,k} &= \mathbb{E}\left\{\left(\sum_{j=1}^N \omega_{ij,k}\bar{\Gamma}\bar{x}_{j,k}\right)\left(\sum_{j=1}^N \omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\}, \\ \mathcal{F}_{4i,k} &= \mathbb{E}\left\{\left(\sum_{j=1}^N \omega_{ij,k}\bar{\Gamma}\bar{x}_{j,k}\right)\zeta_{i,k}^T\right\}\mathcal{D}_{i,k}^T, \\ \mathcal{F}_{5i,k} &= \mathbb{E}\left\{\left(\sum_{j=1}^N \omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)\zeta_{i,k}^T\right\}\mathcal{D}_{i,k}^T. \end{aligned} \quad (14)$$

Next, it can be observed that

$$\begin{aligned} \text{sym}(\mathcal{E}_{i,k}) &\leq \epsilon_{i,k}\mathcal{A}_{i,k}\bar{X}_{i,k}\mathcal{A}_{i,k}^T + \epsilon_{i,k}^{-1}\mathbb{E}\left\{\left(\sum_{j=1}^N \omega_{ij,k}\bar{\Gamma}\bar{x}_{j,k}\right)\right. \\ &\quad \left.\times \left(\sum_{j=1}^N \omega_{ij,k}\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\}, \end{aligned} \quad (15)$$

in which $\epsilon_{i,k} > 0$ are constant scalars. Moreover, we obtain

$$\begin{aligned} &\mathbb{E}\left\{\left(\sum_{j=1}^N \omega_{ij,k}\bar{\Gamma}\bar{x}_{j,k}\right)\left(\sum_{j=1}^N \omega_{ij,k}\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\} \\ &\leq N \sum_{j=1}^N \omega_{ij,k}^2 \bar{\Gamma}\bar{X}_{j,k}\bar{\Gamma}^T, \\ &\mathbb{E}\left\{\left(\sum_{j=1}^N \omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)\left(\sum_{j=1}^N \omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\} \\ &\leq N \sum_{j=1}^N \omega_{ij,k}^2 \bar{\gamma} \lambda_{\max}(\bar{X}_{j,k})I. \end{aligned} \quad (16)$$

Furthermore, we can show that $\mathcal{F}_{ri,k}$ ($r = 1, 2, \dots, 5$) in (14) are all zero terms. As such, it follows from (13) and (15)-(17) that the inequality in (12) can be obtained readily. ■

Theorem 2: Consider the DTVCNs (7) with the state estimator (10). The recursion equation of $P_{i,k+1}$ can be expressed by:

$$\begin{aligned} P_{i,k+1} &= (\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})P_{i,k}(\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} \end{aligned}$$

$$\begin{aligned}
& -\mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})^T + \mathcal{K}_{i,k}\mathbb{E}\left\{\bar{\Psi}_{i,k}\mathcal{C}_{i,k}\bar{x}_{i,k}\bar{x}_{i,k}^T\mathcal{C}_{i,k}^T\bar{\Psi}_{i,k}^T\right\}\mathcal{K}_{i,k}^T \\
& + \mathbb{E}\left\{\left(\sum_{j=1}^N\omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)\left(\sum_{j=1}^N\omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\} \\
& + \mathbb{E}\left\{\left[\sum_{j=1,j\neq i}^N\omega_{ij,k}\bar{\Gamma}(\bar{x}_{j,k}-\hat{x}_{j,k|k-\tau_{ij}})\right]\right. \\
& \quad \left.\times\left[\sum_{j=1,j\neq i}^N\omega_{ij,k}\bar{\Gamma}(\bar{x}_{j,k}-\hat{x}_{j,k|k-\tau_{ij}})\right]^T\right\} \\
& + \text{sym}(\mathcal{H}_{i,k}) + \mathcal{D}_{i,k}\mathcal{Q}_{i,k}^{\zeta}\mathcal{D}_{i,k}^T + \mathcal{K}_{i,k}\mathcal{Q}_{i,k}^{\eta}\mathcal{K}_{i,k}^T, \quad (18)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{H}_{i,k} = & \mathbb{E}\left\{(\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})e_{i,k}\right. \\
& \quad \left.\times\left[\sum_{j=1,j\neq i}^N\omega_{ij,k}\bar{\Gamma}(\bar{x}_{j,k}-\hat{x}_{j,k|k-\tau_{ij}})\right]^T\right\}. \quad (19)
\end{aligned}$$

Proof: Based on the definition of $P_{i,k+1}$, we have

$$\begin{aligned}
& P_{i,k+1} \\
= & (\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})P_{i,k}(\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} \\
& - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})^T + \mathcal{K}_{i,k}\mathbb{E}\left\{\bar{\Psi}_{i,k}\mathcal{C}_{i,k}\bar{x}_{i,k}\bar{x}_{i,k}^T\mathcal{C}_{i,k}^T\bar{\Psi}_{i,k}^T\right\}\mathcal{K}_{i,k}^T \\
& + \mathbb{E}\left\{\left(\sum_{j=1}^N\omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)\left(\sum_{j=1}^N\omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\} \\
& + \mathbb{E}\left\{\left[\sum_{j=1,j\neq i}^N\omega_{ij,k}\bar{\Gamma}(\bar{x}_{j,k}-\hat{x}_{j,k|k-\tau_{ij}})\right]\right. \\
& \quad \left.\times\left[\sum_{j=1,j\neq i}^N\omega_{ij,k}\bar{\Gamma}(\bar{x}_{j,k}-\hat{x}_{j,k|k-\tau_{ij}})\right]^T\right\} \\
& + \mathcal{D}_{i,k}\mathcal{Q}_{i,k}^{\zeta}\mathcal{D}_{i,k}^T + \mathcal{K}_{i,k}\mathcal{Q}_{i,k}^{\eta}\mathcal{K}_{i,k}^T + \text{sym}(\mathcal{H}_{i,k}) \\
& - \text{sym}(\mathcal{G}_{1i,k}) + \text{sym}(\mathcal{G}_{2i,k}) + \text{sym}(\mathcal{G}_{3i,k}) - \text{sym}(\mathcal{G}_{4i,k}) \\
& - \text{sym}(\mathcal{G}_{5i,k}) - \text{sym}(\mathcal{G}_{6i,k}) - \text{sym}(\mathcal{G}_{7i,k}) + \text{sym}(\mathcal{G}_{8i,k}) \\
& + \text{sym}(\mathcal{G}_{9i,k}) + \text{sym}(\mathcal{G}_{10i,k}) - \text{sym}(\mathcal{G}_{11i,k}) \\
& + \text{sym}(\mathcal{G}_{12i,k}) - \text{sym}(\mathcal{G}_{13i,k}) - \text{sym}(\mathcal{G}_{14i,k}), \quad (20)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{G}_{1i,k} & = \mathbb{E}\{(\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})e_{i,k} \\
& \quad \times \bar{x}_{i,k}^T\mathcal{C}_{i,k}^T\bar{\Psi}_{i,k}^T\mathcal{K}_{i,k}^T\}, \\
\mathcal{G}_{2i,k} & = \mathbb{E}\left\{(\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})e_{i,k}\right. \\
& \quad \left.\times\left(\sum_{j=1}^N\omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\}, \\
\mathcal{G}_{3i,k} & = \mathbb{E}\{(\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})e_{i,k}\zeta_{i,k}^T\mathcal{D}_{i,k}^T\}, \\
\mathcal{G}_{4i,k} & = \mathbb{E}\{(\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})e_{i,k}\eta_{i,k}^T\mathcal{K}_{i,k}^T\}, \\
\mathcal{G}_{5i,k} & = \mathbb{E}\left\{\mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k}\bar{x}_{i,k}\left(\sum_{j=1}^N\omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)^T\right\}, \\
\mathcal{G}_{6i,k} & = \mathbb{E}\left\{\mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k}\bar{x}_{i,k}\left[\sum_{j=1,j\neq i}^N\omega_{ij,k}\bar{\Gamma}\right.\right.
\end{aligned}$$

$$\left.\left.\times(\bar{x}_{j,k}-\hat{x}_{j,k|k-\tau_{ij}})\right]^T\right\},$$

$$\mathcal{G}_{7i,k} = \mathbb{E}\{\mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k}\bar{x}_{i,k}\zeta_{i,k}^T\mathcal{D}_{i,k}^T\},$$

$$\mathcal{G}_{8i,k} = \mathbb{E}\{\mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k}\bar{x}_{i,k}\eta_{i,k}^T\mathcal{K}_{i,k}^T\},$$

$$\begin{aligned}
\mathcal{G}_{9i,k} & = \mathbb{E}\left\{\left(\sum_{j=1}^N\omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)\left[\sum_{j=1,j\neq i}^N\omega_{ij,k}\bar{\Gamma}\right.\right. \\
& \quad \left.\left.\times(\bar{x}_{j,k}-\hat{x}_{j,k|k-\tau_{ij}})\right]^T\right\},
\end{aligned}$$

$$\mathcal{G}_{10i,k} = \mathbb{E}\left\{\left(\sum_{j=1}^N\omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)\zeta_{i,k}^T\mathcal{D}_{i,k}^T\right\},$$

$$\mathcal{G}_{11i,k} = \mathbb{E}\left\{\left(\sum_{j=1}^N\omega_{ij,k}\Delta\bar{\Gamma}\bar{x}_{j,k}\right)\eta_{i,k}^T\mathcal{K}_{i,k}^T\right\},$$

$$\mathcal{G}_{12i,k} = \mathbb{E}\left\{\left[\sum_{j=1,j\neq i}^N\omega_{ij,k}\bar{\Gamma}(\bar{x}_{j,k}-\hat{x}_{j,k|k-\tau_{ij}})\right]\zeta_{i,k}^T\mathcal{D}_{i,k}^T\right\},$$

$$\mathcal{G}_{13i,k} = \mathbb{E}\left\{\left[\sum_{j=1,j\neq i}^N\omega_{ij,k}\bar{\Gamma}(\bar{x}_{j,k}-\hat{x}_{j,k|k-\tau_{ij}})\right]\eta_{i,k}^T\mathcal{K}_{i,k}^T\right\},$$

$$\mathcal{G}_{14i,k} = \mathcal{D}_{i,k}\mathbb{E}\{\zeta_{i,k}\eta_{i,k}^T\}\mathcal{K}_{i,k}^T,$$

and $\mathcal{H}_{i,k}$ is denoted in (19). Subsequently, it is easy to derive the recursion in (18). ■

Now, we are in a position to look for an upper bound $\mathcal{Q}_{i,k+1}$ of $Q_{i,k+1}$. Accordingly, the locally minimized upper bound is obtained by resorting to the mathematical induction method and completing square technique, moreover, the estimator gain matrix $\mathcal{K}_{i,k}$ is parameterized via the solution to an RME.

Theorem 3: Consider the recursion of estimation error covariance matrix in (18) and let $\mu_{i,k} > 0$ be constant scalars. Under the initial condition $\mathcal{T}_{i,0} = \bar{X}_{i,0} > 0$, if the following RME

$$\begin{aligned}
\mathcal{T}_{i,k+1} & = (1 + \mu_{i,k})(\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})\mathcal{T}_{i,k} \\
& \quad \times (\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma} - \mathcal{K}_{i,k}\bar{\Psi}_{i,k}\mathcal{C}_{i,k})^T \\
& \quad + 2(1 + \mu_{i,k}^{-1})(N-1)\sum_{j=1,j\neq i}^N\omega_{ij,k}^2 \\
& \quad \times \bar{\Gamma}(\mathcal{X}_{j,k} + \hat{x}_{j,k|k-\tau_{ij}}\hat{x}_{j,k|k-\tau_{ij}}^T)\bar{\Gamma}^T \\
& \quad + \mathcal{K}_{i,k}[\bar{\Psi}_{i,k} \circ (\mathcal{C}_{i,k}\mathcal{X}_{i,k}\mathcal{C}_{i,k}^T)]\mathcal{K}_{i,k}^T \\
& \quad + N\bar{\gamma}\sum_{j=1}^N\omega_{ij,k}^2\lambda_{\max}(\mathcal{X}_{j,k})I \\
& \quad + \mathcal{D}_{i,k}\mathcal{Q}_{i,k}^{\zeta}\mathcal{D}_{i,k}^T + \mathcal{K}_{i,k}\mathcal{Q}_{i,k}^{\eta}\mathcal{K}_{i,k}^T \quad (21)
\end{aligned}$$

has a solution $\mathcal{T}_{i,k+1} > 0$, then we have

$$P_{i,k+1} \leq \mathcal{T}_{i,k+1}. \quad (22)$$

Furthermore, it is shown that $\mathcal{T}_{i,k+1}$ can be minimized if $\mathcal{K}_{i,k}$ is designed by

$$\mathcal{K}_{i,k} = [(1 + \mu_{i,k})(\mathcal{A}_{i,k} + \omega_{ii,k}\bar{\Gamma})\mathcal{T}_{i,k}\mathcal{C}_{i,k}^T\bar{\Psi}_{i,k}^T]\Xi_{i,k}^{-1} \quad (23)$$

with

$$\Xi_{i,k} = (1 + \mu_{i,k})\bar{\Psi}_{i,k}\mathcal{C}_{i,k}\mathcal{T}_{i,k}\mathcal{C}_{i,k}^T\bar{\Psi}_{i,k}^T + \mathcal{Q}_{i,k}^{\eta}$$

$$\begin{aligned} & + \check{\Psi}_{i,k} \circ (\mathcal{C}_{i,k} \mathcal{X}_{i,k} \mathcal{C}_{i,k}^T), \\ \check{\Psi}_{i,k} & = \text{diag}\{\check{\psi}_{i1,k}, \check{\psi}_{i2,k}, \dots, \check{\psi}_{in_y,k}\}. \end{aligned} \quad (24)$$

Proof: The proof of this theorem includes two steps. That is, an upper bound for the estimation error covariance is firstly derived and a locally minimized upper bound is given in terms of the completing square technique. In addition, the expression form of the estimator parameter matrix is characterized based on the solution to an RME.

Based on the property of the Hadamard product, the second term in (18) equals to

$$\begin{aligned} & \mathcal{K}_{i,k} \mathbb{E}\{\check{\Psi}_{i,k} \mathcal{C}_{i,k} \vec{x}_{i,k} \vec{x}_{i,k}^T \mathcal{C}_{i,k}^T \check{\Psi}_{i,k}^T\} \mathcal{K}_{i,k}^T \\ & = \mathcal{K}_{i,k} [\check{\Psi}_{i,k} \circ (\mathcal{C}_{i,k} \vec{X}_{i,k} \mathcal{C}_{i,k}^T)] \mathcal{K}_{i,k}^T, \end{aligned} \quad (25)$$

where $\check{\Psi}_{i,k} = [\mathbb{E}\{\check{\psi}_{is,k} \check{\psi}_{it,k}\}] n_y \times n_y$ and \circ is the Hadamard product. Notice that $\psi_{it,k}$ ($t = 1, 2, \dots, n_y$) are mutually independent in different s and t . Therefore, we have $\check{\Psi}_{i,k} = \text{diag}\{\psi_{i1,k}, \psi_{i2,k}, \dots, \psi_{in_y,k}\}$.

Together with (5), one has

$$\begin{aligned} & \mathbb{E}\left\{\left(\sum_{j=1}^N \omega_{ij,k} \Delta \vec{x}_{j,k}\right) \left(\sum_{j=1}^N \omega_{ij,k} \Delta \vec{x}_{j,k}\right)^T\right\} \\ & \leq N \mathbb{E}\left\{\sum_{j=1}^N \omega_{ij,k}^2 \Delta \vec{x}_{j,k} \Delta \vec{x}_{j,k}^T \Delta \vec{\Gamma}^T\right\} \\ & \leq N \bar{\gamma} \sum_{j=1}^N \omega_{ij,k}^2 \lambda_{\max}(\vec{X}_{j,k}) I. \end{aligned} \quad (26)$$

Moreover, the fifth term in (18) is tackled as follows:

$$\begin{aligned} & \text{sym}(\mathcal{H}_{i,k}) \\ & \leq \mu_{i,k} (A_{i,k} + \omega_{ii,k} \vec{\Gamma} - \bar{\lambda}_{i,k} \mathcal{K}_{i,k} \mathcal{C}_{i,k}) P_{i,k} \\ & \quad \times (A_{i,k} + \omega_{ii,k} \vec{\Gamma} - \bar{\lambda}_{i,k} \mathcal{K}_{i,k} \mathcal{C}_{i,k})^T \\ & \quad + \mu_{i,k}^{-1} \mathbb{E}\left\{\left[\sum_{j=1, j \neq i}^N \omega_{ij,k} \vec{\Gamma}(\vec{x}_{j,k} - \hat{x}_{j,k|k-d_{ij}})\right]\right. \\ & \quad \left. \times \left[\sum_{j=1, j \neq i}^N \omega_{ij,k} \vec{\Gamma}(\vec{x}_{j,k} - \hat{x}_{j,k|k-d_{ij}})\right]^T\right\} \end{aligned} \quad (27)$$

with $\mu_{i,k} > 0$ being constant scalars. By taking (18) and (25)-(27) into account, we have the following inequality

$$\begin{aligned} & P_{i,k+1} \\ & \leq (1 + \mu_{i,k}) (\mathcal{A}_{i,k} + \omega_{ii,k} \vec{\Gamma} - \mathcal{K}_{i,k} \bar{\Psi}_{i,k} \mathcal{C}_{i,k}) P_{i,k} \\ & \quad \times (A_{i,k} + \omega_{ii,k} \vec{\Gamma} - \mathcal{K}_{i,k} \bar{\Psi}_{i,k} \mathcal{C}_{i,k})^T \\ & \quad + \mathcal{K}_{i,k} [\check{\Psi}_{i,k} \circ (\mathcal{C}_{i,k} \vec{X}_{i,k} \mathcal{C}_{i,k}^T)] \mathcal{K}_{i,k}^T \\ & \quad + N \bar{\gamma} \sum_{j=1}^N \omega_{ij,k}^2 \lambda_{\max}(\vec{X}_{j,k}) I \\ & \quad + (1 + \mu_{i,k}^{-1}) \mathbb{E}\left\{\left[\sum_{j=1, j \neq i}^N \omega_{ij,k} \vec{\Gamma}(\vec{x}_{j,k} - \hat{x}_{j,k|k-\tau_{ij}})\right]\right. \\ & \quad \left. \times \left[\sum_{j=1, j \neq i}^N \omega_{ij,k} \vec{\Gamma}(\vec{x}_{j,k} - \hat{x}_{j,k|k-\tau_{ij}})\right]^T\right\} \\ & \quad + \mathcal{D}_{i,k} \mathcal{Q}_{i,k}^{\zeta} \mathcal{D}_{i,k}^T + \mathcal{K}_{i,k} \mathcal{Q}_{i,k}^{\eta} \mathcal{K}_{i,k}^T. \end{aligned} \quad (28)$$

Subsequently, the following inequality can be readily obtained:

$$\begin{aligned} & \mathbb{E}\left\{\left[\sum_{j=1, j \neq i}^N \omega_{ij,k} \vec{\Gamma}(\vec{x}_{j,k} - \hat{x}_{j,k|k-\tau_{ij}})\right]\right. \\ & \quad \left. \times \left[\sum_{j=1, j \neq i}^N \omega_{ij,k} \vec{\Gamma}(\vec{x}_{j,k} - \hat{x}_{j,k|k-\tau_{ij}})\right]^T\right\} \\ & \leq 2(N-1) \mathbb{E}\left\{\sum_{j=1, j \neq i}^N \omega_{ij,k}^2 \vec{\Gamma}(\vec{x}_{j,k} \vec{x}_{j,k}^T\right. \\ & \quad \left. + \hat{x}_{j,k|k-\tau_{ij}} \hat{x}_{j,k|k-\tau_{ij}}^T) \vec{\Gamma}^T\right\} \\ & = 2(N-1) \sum_{j=1, j \neq i}^N \omega_{ij,k}^2 \vec{\Gamma} (X_{j,k} + \hat{x}_{j,k|k-\tau_{ij}} \hat{x}_{j,k|k-\tau_{ij}}^T) \vec{\Gamma}^T. \end{aligned} \quad (29)$$

Together with (28) and (29) leads to

$$\begin{aligned} & P_{i,k+1} \\ & \leq (1 + \mu_{i,k}) (\mathcal{A}_{i,k} + \omega_{ii,k} \vec{\Gamma} - \mathcal{K}_{i,k} \bar{\Psi}_{i,k} \mathcal{C}_{i,k}) P_{i,k} \\ & \quad \times (A_{i,k} + \omega_{ii,k} \vec{\Gamma} - \mathcal{K}_{i,k} \bar{\Psi}_{i,k} \mathcal{C}_{i,k})^T \\ & \quad + \mathcal{K}_{i,k} [\check{\Psi}_{i,k} \circ (\mathcal{C}_{i,k} \vec{X}_{i,k} \mathcal{C}_{i,k}^T)] \mathcal{K}_{i,k}^T \\ & \quad + N \bar{\gamma} \sum_{j=1}^N \omega_{ij,k}^2 \lambda_{\max}(\vec{X}_{j,k}) I + 2(1 + \mu_{i,k}^{-1})(N-1) \\ & \quad \times \sum_{j=1, j \neq i}^N \omega_{ij,k}^2 \vec{\Gamma} (X_{j,k} + \hat{x}_{j,k|k-\tau_{ij}} \hat{x}_{j,k|k-\tau_{ij}}^T) \\ & \quad + \mathcal{D}_{i,k} \mathcal{Q}_{i,k}^{\zeta} \mathcal{D}_{i,k}^T + \mathcal{K}_{i,k} \mathcal{Q}_{i,k}^{\eta} \mathcal{K}_{i,k}^T. \end{aligned} \quad (30)$$

After the straightforward calculations, we arrive at

$$P_{i,k+1} \leq \mathcal{T}_{i,k+1}, \quad (31)$$

where $\mathcal{T}_{i,k+1}$ is denoted in (21).

Finally, $\mathcal{T}_{i,k+1}$ in (21) can be reorganized as:

$$\begin{aligned} & \mathcal{T}_{i,k+1} \\ & = (1 + \mu_{i,k}) (\mathcal{A}_{i,k} + \omega_{ii,k} \vec{\Gamma}) \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \vec{\Gamma})^T \\ & \quad + 2(1 + \mu_{i,k}^{-1})(N-1) \sum_{j=1, j \neq i}^N \omega_{ij,k}^2 \\ & \quad \times \vec{\Gamma} (\mathcal{X}_{j,k} + \hat{x}_{j,k|k-\tau_{ij}} \hat{x}_{j,k|k-\tau_{ij}}^T) \vec{\Gamma}^T \\ & \quad + N \bar{\gamma} \sum_{j=1}^N \omega_{ij,k}^2 \lambda_{\max}(\mathcal{X}_{j,k}) I + \mathcal{D}_{i,k} \mathcal{Q}_{i,k}^{\zeta} \mathcal{D}_{i,k}^T \\ & \quad - (1 + \mu_{i,k}) (\mathcal{A}_{i,k} + \omega_{ii,k} \vec{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \bar{\Psi}_{i,k}^T \mathcal{K}_{i,k}^T \\ & \quad - (1 + \mu_{i,k}) \mathcal{K}_{i,k} \bar{\Psi}_{i,k} \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \vec{\Gamma})^T \\ & \quad + \mathcal{K}_{i,k} [(1 + \mu_{i,k}) \bar{\Psi}_{i,k} \mathcal{C}_{i,k} \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \bar{\Psi}_{i,k}^T \\ & \quad + \check{\Psi}_{i,k} \circ (\mathcal{C}_{i,k} \mathcal{X}_{i,k} \mathcal{C}_{i,k}^T) + \mathcal{Q}_{i,k}^{\eta}] \mathcal{K}_{i,k}^T \\ & = (1 + \mu_{i,k}) (\mathcal{A}_{i,k} + \omega_{ii,k} \vec{\Gamma}) \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \vec{\Gamma})^T \\ & \quad + 2(1 + \mu_{i,k}^{-1})(N-1) \sum_{j=1, j \neq i}^N \omega_{ij,k}^2 \end{aligned}$$

$$\begin{aligned}
& \times \bar{\Gamma}(\mathcal{X}_{j,k} + \hat{x}_{j,k|k-\tau_{ij}} \hat{x}_{j,k|k-\tau_{ij}}^T) \\
& + N\bar{\gamma} \sum_{j=1}^N \omega_{ij,k}^2 \lambda_{\max}(\mathcal{X}_{j,k}) I + \mathcal{D}_{i,k} \mathcal{Q}_{i,k}^\zeta \mathcal{D}_{i,k}^T \\
& + \left(\mathcal{K}_{i,k} - (1 + \mu_{i,k})(\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \right) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \bar{\Psi}_{i,k}^T \\
& \times \Xi_{i,k}^{-1} \Xi_{i,k} \left(\mathcal{K}_{i,k} - (1 + \mu_{i,k})(\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \right) \\
& \times \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \bar{\Psi}_{i,k}^T \Xi_{i,k}^{-1} \Xi_{i,k} \Xi_{i,k}^{-1} \Xi_{i,k} \Xi_{i,k}^{-1} \\
& \times \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \bar{\Psi}_{i,k}^T \Xi_{i,k}^{-1} \bar{\Psi}_{i,k} \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T,
\end{aligned}$$

where

$$\begin{aligned}
\Xi_{i,k} &= (1 + \mu_{i,k}) \bar{\Psi}_{i,k} \mathcal{C}_{i,k} \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \bar{\Psi}_{i,k}^T + \mathcal{Q}_{i,k}^\eta \\
& + \check{\Psi}_{i,k} \circ (\mathcal{C}_{i,k} \mathcal{X}_{i,k} \mathcal{C}_{i,k}^T).
\end{aligned}$$

Next, let $\mathcal{K}_{i,k}$

$$\mathcal{K}_{i,k} \triangleq [(1 + \mu_{i,k})(\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \bar{\Psi}_{i,k}^T] \Xi_{i,k}^{-1}.$$

Then, we have the locally minimized upper bound $\mathcal{T}_{i,k+1}$ denoted by

$$\begin{aligned}
& \mathcal{T}_{i,k+1} \\
& = (1 + \mu_{i,k})(\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \\
& + 2(1 + \mu_{i,k}^{-1})(N - 1) \sum_{j=1, j \neq i}^N \omega_{ij,k}^2 \\
& \times \bar{\Gamma}(\mathcal{X}_{j,k} + \hat{x}_{j,k|k-\tau_{ij}} \hat{x}_{j,k|k-\tau_{ij}}^T) \\
& + N\bar{\gamma} \sum_{j=1}^N \omega_{ij,k}^2 \lambda_{\max}(\mathcal{X}_{j,k}) I + \mathcal{D}_{i,k} \mathcal{Q}_{i,k}^\zeta \mathcal{D}_{i,k}^T \\
& - (1 + \mu_{i,k})^2 (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \bar{\Psi}_{i,k}^T \Xi_{i,k}^{-1} \\
& \times \bar{\Psi}_{i,k} \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T, \tag{32}
\end{aligned}$$

which completes the proof of this theorem. \blacksquare

Remark 4: So far, a new optimized estimation scheme has been given for DTVCNs subject to NIOs and dynamical bias. To be more specific, both the communication delays and fading observations have been adequately taken into account, where a predictive-based mechanism has been proposed to compensate the potential influences caused by the communication delays. Subsequently, both the predictive-based estimation information and available probability of fading observations have been actively introduced to design a new recursive state estimator. It should be noticed that some positive scalars $\mu_{i,k}$ have been introduced in Theorem 3. In the subsequent estimation algorithm implementation, the parameters $\mu_{i,k}$ can be fixed and then adjusted to enhance the solvability of the proposed DCBSE algorithm. Compared with the conventional Kalman estimation algorithm, the newly proposed DCBSE algorithm is a locally optimal one, where a locally minimized upper bound with regards to the estimation error covariance matrix has been found and the desirable estimator gain matrix has been parameterized in terms of the solution to an RME. The major reason lies in that it is impossible to obtain the exact value of the estimation error covariance matrix due to the existence of NIOs. As such, an alternative estimation approach has been proposed to achieve admissible estimation requirements.

For the ease of the implementation, the following DCBSE algorithm based on the predictive scheme is outlined, where the parameter selection and updating way are summarized.

Algorithm 1: Delay-Compensation-Based State Estimation Algorithm

Step 1. Give the initial conditions and parameters, and set $k = 0$.

Step 2. Obtain the predictive term $\hat{x}_{j,k|k-\tau_{ij}}$ by (9) based on delayed estimation $\hat{x}_{j,k-\tau_{ij}}$.

Step 3. Design the estimator parameter $\mathcal{K}_{i,k}$ by (23).

Step 4. Obtain the upper bound matrices $\mathcal{X}_{i,k+1}$ and $\mathcal{T}_{i,k+1}$ according to the parameter update formulas (12) and (21), respectively.

Step 5. Compute the estimation $\hat{x}_{i,k+1}$ according to (10).

Step 6. Setting $k = k + 1$, and go to *Step 2*.

Remark 5: As illustrated in Algorithm 1, based on (9) and the RME in (21), the predictive estimation term $\hat{x}_{j,k|k-\tau_{ij}}$ and the estimator parameter $\mathcal{K}_{i,k}$ can be obtained easily. Then, the addressed DCBSE problem for DTVCNs with NIOs and dynamical bias is solvable. Note that the desirable parameters are available via testifying the feasibility of some forward matrix difference equations. Consequently, the developed DCBSE scheme within time-varying framework is satisfactory for the requirement of online applications.

IV. PERFORMANCE ANALYSIS OF ESTIMATION METHOD

In this section, the algorithm performance analysis is conducted, where the monotonicity of the new DCBSE scheme is shown.

For ease of illustration, let the random variables $\psi_{it,k}$ ($t = 1, 2, \dots, n_y$) be $\psi_{i,k}$ with $\mathbb{E}\{\psi_{i,k}\} = \bar{\psi}_{i,k}$ and $\mathbb{E}\{(\psi_{i,k} - \bar{\psi}_{i,k})(\psi_{i,k} - \bar{\psi}_{i,k})^T\} = \check{\psi}_{i,k}$. Moreover, the relationship between $\psi_{i,k}$ and $\text{tr}(\mathcal{T}_{i,k+1})$ is discussed as $\psi_{i,k}$ is fixed. To further provide the monotonicity analyses, the following lemma is helpful.

Lemma 1: For the matrices A, B, X as well as Y with proper dimensions and the variable $t \in \mathbb{R}$, one has

$$\begin{aligned}
& \frac{d}{dt} \text{tr}(X + tY) = \text{tr}(Y), \\
& \frac{d}{dt} \text{tr}[B^T(X + tY)^{-1}A] \\
& = -\text{tr}[B^T(X + tY)^{-1}Y(X + tY)^{-1}A].
\end{aligned}$$

Theorem 4: If the occurrence probability $\bar{\psi}_{i,k}$ increases, it is observed that $\text{tr}(\mathcal{T}_{i,k+1})$ is non-increasing.

Proof: Notice that $\Xi_{i,k}$ in (24) and $\mathcal{T}_{i,k+1}$ in (32) contain the scalar $\bar{\psi}_{i,k}$. Then, it follows from 1 that

$$\begin{aligned}
& \frac{d \text{tr}\{\mathcal{T}_{i,k+1}\}}{d \bar{\psi}_{i,k}} \\
& = \frac{d \text{tr}}{d \bar{\psi}_{i,k}} \left\{ (1 + \mu_{i,k})(\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \right. \\
& \quad + 2(1 + \mu_{i,k}^{-1})(N - 1) \sum_{j=1, j \neq i}^N \omega_{ij,k}^2 \\
& \quad \times \bar{\Gamma}(\mathcal{X}_{j,k} + \hat{x}_{j,k|k-\tau_{ij}} \hat{x}_{j,k|k-\tau_{ij}}^T) \\
& \quad + N\bar{\gamma} \sum_{j=1}^N \omega_{ij,k}^2 \lambda_{\max}(\mathcal{X}_{j,k}) I + \mathcal{D}_{i,k} \mathcal{Q}_{i,k}^\zeta \mathcal{D}_{i,k}^T \\
& \quad \left. - (1 + \mu_{i,k})^2 \bar{\psi}_{i,k}^2 (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \right\}
\end{aligned}$$

$$\begin{aligned}
& \times \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \} \\
= & \text{tr} \left\{ -2(1 + \mu_{i,k})^2 \bar{\psi}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \right. \\
& \times \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \\
& + 2(1 + \mu_{i,k})^2 \bar{\psi}_{i,k}^2 (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \\
& \times [(1 + \mu_{i,k}) \bar{\psi}_{i,k} \mathcal{C}_{i,k} \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T] \\
& \left. \times \Xi_{i,k}^{-1} \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \right\}.
\end{aligned}$$

Adding some zero terms to the above equation, one has

$$\begin{aligned}
& \frac{\text{dtr}\{\mathcal{T}_{i,k+1}\}}{\text{d}\bar{\psi}_{i,k}} \\
= & \text{tr} \left\{ -2(1 + \mu_{i,k})^2 \bar{\psi}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \right. \\
& \times \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \\
& + 2(1 + \mu_{i,k})^2 \bar{\psi}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \\
& \times [(1 + \mu_{i,k}) \bar{\psi}_{i,k}^2 \mathcal{C}_{i,k} \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T + \mathcal{Q}_{i,k}^\eta \\
& + \check{\Psi}_{i,k} \circ (\mathcal{C}_{i,k} \mathcal{X}_{i,k} \mathcal{C}_{i,k}^T)] \Xi_{i,k}^{-1} \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \\
& - 2(1 + \mu_{i,k})^2 \bar{\psi}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \\
& \times [\mathcal{Q}_{i,k}^\eta + \check{\Psi}_{i,k} \circ (\mathcal{C}_{i,k} \mathcal{X}_{i,k} \mathcal{C}_{i,k}^T)] \\
& \left. \times \Xi_{i,k}^{-1} \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \right\}.
\end{aligned}$$

After the simple algebraic calculation, we arrive at

$$\begin{aligned}
& \frac{\text{dtr}\{\mathcal{T}_{i,k+1}\}}{\text{d}\bar{\psi}_{i,k}} \\
= & \text{tr} \left\{ -2(1 + \mu_{i,k})^2 \bar{\psi}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \right. \\
& \times [\mathcal{Q}_{i,k}^\eta + \check{\Psi}_{i,k} \circ (\mathcal{C}_{i,k} \mathcal{X}_{i,k} \mathcal{C}_{i,k}^T)] \Xi_{i,k}^{-1} \mathcal{C}_{i,k} \mathcal{T}_{i,k} \\
& \left. \times (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \right\}, \tag{33}
\end{aligned}$$

which leads to

$$\frac{\text{dtr}\{\mathcal{T}_{i,k+1}\}}{\text{d}\bar{\psi}_{i,k}} \leq 0.$$

Consequently, it is concluded that $\text{tr}\{\mathcal{T}_{i,k+1}\}$ is non-increasing when $\bar{\psi}_{i,k}$ increases. \blacksquare

As a special case, the same assertion can be obtained when $\bar{\psi}_{i,k}$ is governed by the Bernoulli distribution, which is shown in the following corollary.

Corollary 1: If $\bar{\psi}_{i,k}$ is a Bernoulli distributed random variable and its mean $\bar{\psi}_{i,k}$ increases, we can also obtain the fact that $\text{tr}(\mathcal{T}_{i,k+1})$ is non-increasing.

Proof: In this case, we have $\bar{\Psi}_{i,k} = \bar{\psi}_{i,k} I$ and $\check{\Psi}_{i,k} = \bar{\psi}_{i,k} (1 - \bar{\psi}_{i,k}) I$. Along the same line of Theorem 4, one has

$$\begin{aligned}
& \frac{\text{dtr}\{\mathcal{T}_{i,k+1}\}}{\text{d}\bar{\psi}_{i,k}} \\
= & \text{tr} \left\{ -2(1 + \mu_{i,k})^2 \bar{\psi}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \right. \\
& \times \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \\
& + (1 + \mu_{i,k})^2 \bar{\psi}_{i,k}^2 (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \\
& \times [2(1 + \mu_{i,k}) \bar{\psi}_{i,k} \mathcal{C}_{i,k} \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T + (1 - 2\bar{\psi}_{i,k}) I \\
& \left. \circ (\mathcal{C}_{i,k} \mathcal{X}_{i,k} \mathcal{C}_{i,k}^T)] \Xi_{i,k}^{-1} \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \right\}
\end{aligned}$$

$$\begin{aligned}
& \leq \text{tr} \left\{ -2(1 + \mu_{i,k})^2 \bar{\psi}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \right. \\
& \times \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \\
& + (1 + \mu_{i,k})^2 \bar{\psi}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \\
& \times [2(1 + \mu_{i,k}) \bar{\psi}_{i,k}^2 \mathcal{C}_{i,k} \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T + 2\bar{\psi}_{i,k} \\
& \times (1 - \bar{\psi}_{i,k}) I \circ (\mathcal{C}_{i,k} \mathcal{X}_{i,k} \mathcal{C}_{i,k}^T) + 2\mathcal{Q}_{i,k}^\eta \\
& \left. - \bar{\psi}_{i,k} I \circ (\mathcal{C}_{i,k} \mathcal{X}_{i,k} \mathcal{C}_{i,k}^T)] \Xi_{i,k}^{-1} \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \right\} \\
= & \text{tr} \left\{ -(1 + \mu_{i,k})^2 \bar{\psi}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma}) \mathcal{T}_{i,k} \mathcal{C}_{i,k}^T \Xi_{i,k}^{-1} \right. \\
& \times [\bar{\psi}_{i,k} \circ (\mathcal{C}_{i,k} \mathcal{X}_{i,k} \mathcal{C}_{i,k}^T)] \Xi_{i,k}^{-1} \mathcal{C}_{i,k} \mathcal{T}_{i,k} (\mathcal{A}_{i,k} + \omega_{ii,k} \bar{\Gamma})^T \left. \right\} \\
& \leq 0. \tag{34}
\end{aligned}$$

As such, it is concluded that $\text{tr}\{\mathcal{T}_{i,k+1}\}$ is non-increasing when $\bar{\psi}_{i,k}$ increases. \blacksquare

Remark 6: Compared with some existing estimation results, this paper proposes a mathematical analysis method to discuss the monotonicity performance of the developed DCBSE algorithm with regard to the fading observations. Particularly, in Theorem 4 and Corollary 1, the theoretical proofs have been provided to reveal the monotonicity relationship between the fading probability and the estimation accuracy. As shown in the above analyses, it follows that the estimation accuracy becomes worse when the fading observations are severe. That is to say, the estimation error is larger if less measurement information can be utilized in the estimator side, which is indeed consistent with the essential intuition in the practical engineering.

V. AN ILLUSTRATIVE SIMULATION

In this section, the following simulation experiment is conducted and the usefulness of newly DCBSE scheme is discussed.

For the DTVCNs (1)-(2), consider the following related matrix parameters:

$$\begin{aligned}
A_{1,k} &= \begin{bmatrix} 0.35 + 0.35 \cos(k) & 0.52 \\ & -1.38 & 1.55 \end{bmatrix}, \\
A_{2,k} &= \begin{bmatrix} 0.65 & 0.5 \\ -0.05 & -0.75 \end{bmatrix}, \\
A_{3,k} &= \begin{bmatrix} 0.86 & 0.72 + 0.3 \sin(k) \\ -0.25 & 0.84 \end{bmatrix}, \\
D_{1,k} &= \begin{bmatrix} 0.31 \\ 0.32 \end{bmatrix}, D_{2,k} = \begin{bmatrix} 0.26 \\ 0.22 \end{bmatrix}, D_{3,k} = \begin{bmatrix} 0.38 \\ 0.40 \end{bmatrix}, \\
B_{1,k} &= \begin{bmatrix} 1.1 & 0.9 \\ 0.43 & -0.91 \end{bmatrix}, B_{2,k} = \begin{bmatrix} 0.65 & 0.98 \\ 0.37 & 0.56 \end{bmatrix}, \\
B_{3,k} &= \begin{bmatrix} 0.52 & -0.19 \\ 1.31 & 0.38 \end{bmatrix}, C_{1,k} = [1.6 \quad 1.5], \\
C_{2,k} &= [1.7 \quad 1.9], C_{3,k} = [1.8 \quad 1.5], \\
F_{1,k} &= \begin{bmatrix} 0.51 & 0.78 \\ -0.3 & 0.6 \end{bmatrix}, F_{2,k} = \begin{bmatrix} 0.46 & 0.48 \\ 0.4 & 0.36 \end{bmatrix}, \\
F_{3,k} &= \begin{bmatrix} 0.49 & 0.58 \\ 0.63 & -0.16 \end{bmatrix}.
\end{aligned}$$

Moreover, the other parameters are chose by $\bar{\Gamma} = \text{diag}\{0.25, 0.25\}$, $\bar{\gamma} = 0.01$, $w_{ij,k} = 0.5$ ($i \neq j$) and $w_{ii,k} = 1$ ($i = 1, 2, 3$).

During the simulation experiment, choose the initial conditions as $\bar{x}_{1,0} = [2 \ 1]^T$, $\bar{x}_{2,0} = [3 \ 1]^T$, $\bar{x}_{3,0} = [1 \ -4]^T$, $\bar{b}_{1,0} = [1 \ 1]^T$, $\bar{b}_{2,0} = [1 \ 1]^T$, $\bar{b}_{3,0} = [-1 \ -1]^T$, $\mathcal{T}_{i,0} = 5I$, $\mathcal{X}_{i,0} = \bar{x}_{i,0} \bar{x}_{i,0}^T$ with $\bar{x}_{i,0} = \begin{bmatrix} \bar{x}_{i,0} \\ \bar{b}_{i,0} \end{bmatrix}$, $\epsilon_{i,k} = 0.5$, $\mu_{i,k} = 1.5$, $Q_{i,k}^\beta = 0.15I$, $Q_{i,k}^\theta = 0.25$, $Q_{i,k}^\eta = 0.2$ ($i = 1, 2, 3$), $\hat{x}_{i,0} = [1 \ 1 \ 1 \ 1]^T$, and $\hat{x}_{i,k} = [0.2 \ 0.2 \ 0.2 \ 0.2]^T$ ($k < 0$) ($i = 1, 2, 3$). In order to depict the estimation performance, MSE_i denotes the mean square error (MSE) for the estimation of $x_{i,k}$ and is calculated by $\frac{1}{M} \sum_{s=1}^M (x_{i,k}^{l(s)} - \hat{x}_{i,k}^{l(s)})^2$, where $M = 1000$ is the run number of the simulation test and $x_{i,k}^l$ ($l = 1, 2; i = 1, 2, 3$) denotes the l -th element of $x_{i,k}$. Moreover, the sum of the MSE_i ($i = 1, 2, 3$) is denoted by $SMSE$.

To validate the efficiency and advantages of the proposed DCBSE algorithm, we compare the cases with/without prediction compensation, i.e., Case I: the state estimation under the DCBSE algorithm; Case II: the state estimation without the updating rule (9). Accordingly, the simulation results under $a_{it} = 0.5$, $b_{it} = 1$ and $\tau_{ij} = 5$ are presented in Figs. 1-5, where Figs. 1-3 plot the state trajectories of 3 nodes and their estimations with/without prediction compensation. Fig. 4 depicts the $\log(MSE_i)$ of 3 nodes and their upper bounds under Case I. The comparisons of $SMSE$ with/without delay compensation under 1000 iterations are shown in Fig. 5. It is observed from the simulations that the new DCBSE algorithm in Case I performs well than the one as in Case II. The major reason lies in that the predictive compensation mechanism has been introduced to attenuate the influences induced by the communication delays, thereby improving the estimation accuracy.

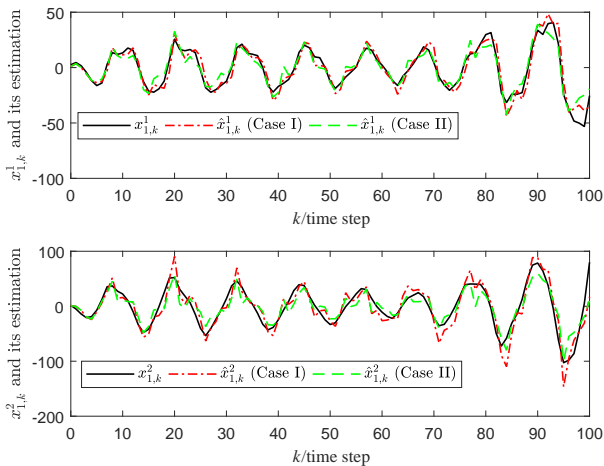


Fig. 1. The curves of $x_{1,k}$ and $\hat{x}_{1,k}$.

In order to further reveal the estimation algorithm performance, we conduct more experiments to discuss the relationship between $SMSE$ and different situations of the fading observations. For the comparison purpose, another three cases are taken into account, i.e., Case a: $a_{it} = 0$, $b_{it} = 1/3$, Case b: $a_{it} = 1/3$, $b_{it} = 2/3$, Case c: $a_{it} = 2/3$, $b_{it} = 1$. For the above mentioned cases, it can be observed that the fading

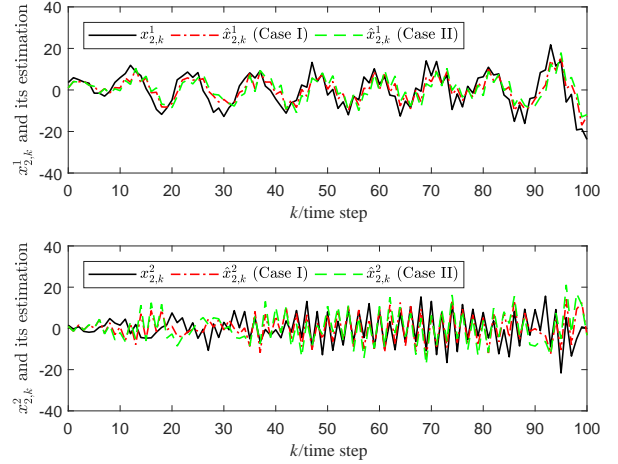


Fig. 2. The curves of $x_{2,k}$ and $\hat{x}_{2,k}$.

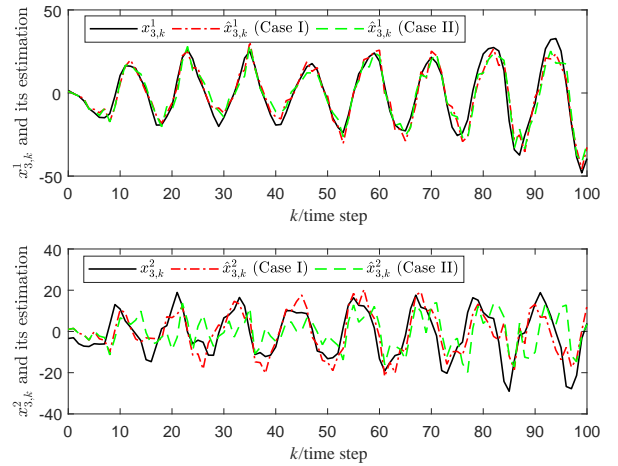


Fig. 3. The curves of $x_{3,k}$ and $\hat{x}_{3,k}$.

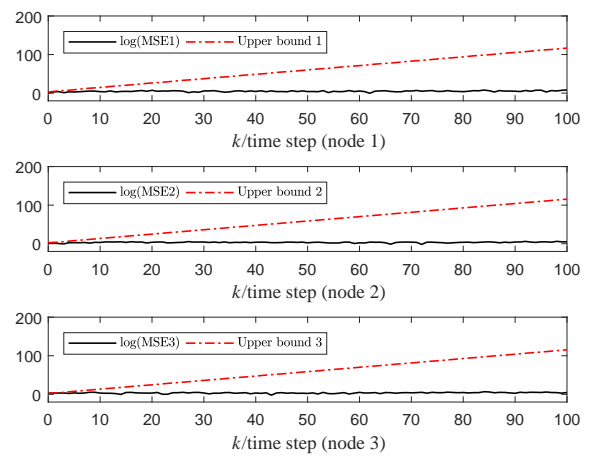


Fig. 4. $\log(MSE_i)$ and upper bounds.

observations are most severe in Case a. Based on the main

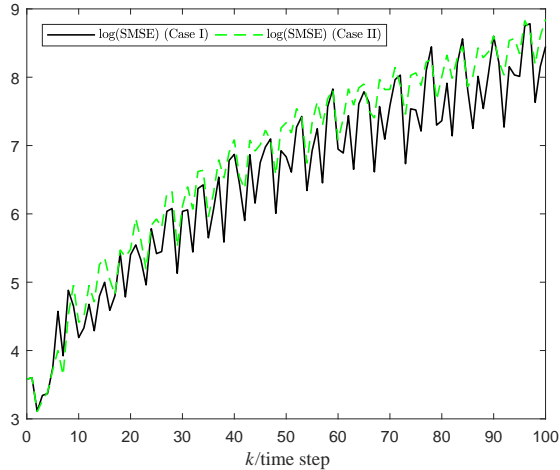


Fig. 5. $\log(\text{SMSE})$ with/without compensation (1000 iterations).

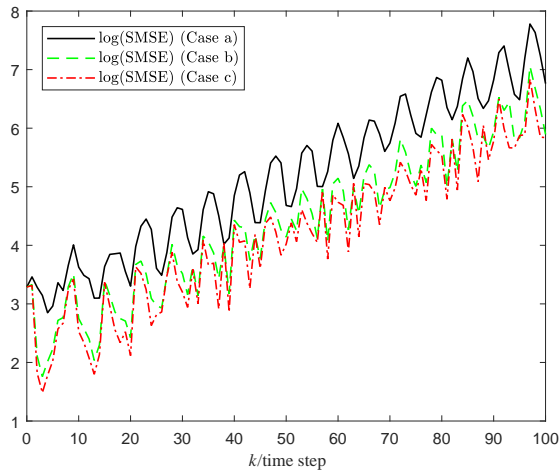


Fig. 6. $\log(\text{SMSE})$ with different fading observations (1000 iterations).

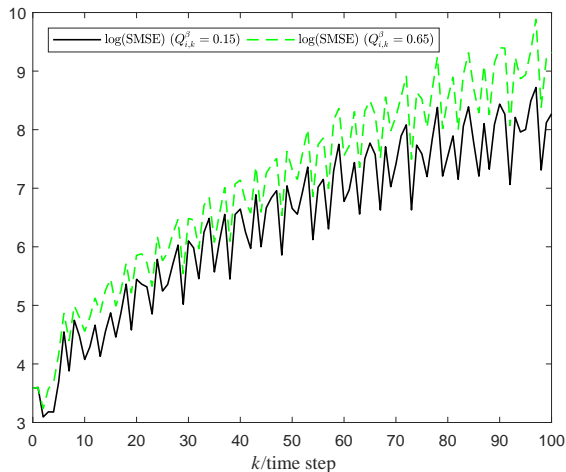


Fig. 7. $\log(\text{SMSE})$ with different $Q_{i,k}^\beta$ (1000 iterations).

results with related parameters, the developed DCBSE strategy

in Algorithm 1 can be implemented. Accordingly, the relationship between $\log(\text{SMSE})$ and different fading observations is plotted in Fig. 6 and the $\log(\text{tr}\{\mathcal{T}_{1,k+1}\})$ under three cases are listed in Tables I-III, which further show that the estimation error becomes larger when the fading observations are severe as in Case a. This is consistent with the practical viewpoint because less information is available in the remote estimator side in Case a, hence the estimation performance becomes worse compared with those in Case b and Case c.

TABLE I
 $\log(\text{tr}\{\mathcal{T}_{1,k+1}\})$ (node 1)

time(k)	...	65	66	67	68	69	...
Case a	...	76.358	77.267	78.354	79.627	80.994	...
Case b	...	76.038	77.017	78.17	79.448	80.763	...
Case c	...	75.984	76.977	78.14	79.416	80.717	...

TABLE II
 $\log(\text{tr}\{\mathcal{T}_{2,k+1}\})$ (node 2)

time(k)	...	65	66	67	68	69	...
Case a	...	74.944	76.172	77.349	78.461	79.542	...
Case b	...	74.767	76.018	77.189	78.281	79.347	...
Case c	...	74.748	76.002	77.172	78.262	79.326	...

TABLE III
 $\log(\text{tr}\{\mathcal{T}_{3,k+1}\})$ (node 3)

time(k)	...	65	66	67	68	69	...
Case a	...	74.876	76.118	77.307	78.401	79.44	...
Case b	...	74.656	75.929	77.125	78.213	79.243	...
Case c	...	74.624	75.901	77.098	78.187	79.217	...

For the comparison purpose, the covariance matrices of random bias with $Q_{i,k}^\beta = 0.15$ and $Q_{i,k}^\beta = 0.65$ are considered, where the effect caused by the dynamical bias is discussed. Subsequently, Fig. 7 depicts the $\log(\text{SMSE})$ in different bias cases under 1000 iterations, which shows that the estimation error is relatively larger when $Q_{i,k}^\beta = 0.65$. That is to say, the estimation accuracy becomes worse when the random bias is severe. Overall, the above comparisons reflect the estimation performance of proposed DCBSE method and further illustrate the usefulness of main results.

VI. CONCLUSIONS

In this paper, we have made one of the first attempts to handle the DCBSE problem for DTVCNs subject to NIOs and dynamical bias. By fully taking the available information of communication delays and fading observations, a simultaneous bias and state estimation scheme has been proposed in terms of the delay-prediction-compensation approach. The major features of the proposed DCBSE strategy include: 1) the NIOs have been actively compensated during the estimation algorithm design by properly employing the available information of NIOs; 2) a new estimation method of the recursive manner has been presented, where the expression form of the estimator gain matrix and the illustration of the proposed DCBSE algorithm have been given; and 3) the monotonicity proofs with regard to the fading observations characterized by

different probability distribution have been provided. Finally, some simulation experiments have been proposed, where the estimation performance comparisons with/without the predictive compensation mechanism have been demonstrated and the advantages of main results have been illustrated. Further research extensions include the design of DCBSE method for DTVCNs subject to communication protocols in order to save the limited network resources. Moreover, it is interesting to deal with the boundedness analysis issue of the proposed estimation strategy, which can be expected in the near future.

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