Delay-Dependent Control for Time-Delayed T-S Fuzzy Systems Using Descriptor Representation

Eun Tae Jeung, Do Chang Oh, and Hong Bae Park

Abstract: This paper presents a design method of delay-dependent control for T-S fuzzy systems with time delays. Based on parallel distributed compensation (PDC) and a descriptor model transformation of the system, a delay-dependent control is utilized. An appropriate Lyapunov-Krasovskii functional is chosen for delay-dependent stability analysis. A sufficient condition for delay-dependent control is represented in terms of linear matrix inequalities (LMIs).

Keywords: Delay-dependent control, descriptor representation, LMI, T-S fuzzy model.

1. INTRODUCTION

Since most physical systems and control systems are represented by nonlinear differential equations, the stability analysis and stabilizing controller design of nonlinear systems are very important. A typical approach for the analysis and synthesis of nonlinear control systems is to utilize local linearization. The possibility of analysis and synthesis of nonlinear control systems began to be realized with the publication of the T-S fuzzy model introduced by Takagi and Sugeno [1]. Recently, Wang *et al.* [2,3] proposed an efficient method for designing fuzzy controller for T-S fuzzy systems using the concept of PDC. The idea is to design linear feedback control gain for each local linear model. Therefore, the design method of the PDC-type fuzzy controller is similar to that of the linear controller.

Time-delay often occurs in the state, the control input, and the measurement output of numerous systems such as transportation systems, communication systems, chemical processing systems, environmental systems, and power systems [4]. Since time-delay is frequently a source of instability, the stability problems related to time-delay systems have received considerable attention over the last 3 decades (see [4-6] and the references therein). It is well known that the choice of a Lyapunov-Krasovskii functional is decisive for showing stability. There are two stability criteria. One is delay-independent, the other is (less conservative) delay-dependent. A few papers [7,8] for nonlinear systems with time delay using the T-S fuzzy model have been written, but these are only for delay-independent control. There are no publications yet for delay-dependent control for time-delayed T-S fuzzy systems.

In the current paper, we present a method of designing delay-dependent control for time-delayed T-S fuzzy control systems using the concept of PDC. We also use the equivalent descriptor model transformation proposed by Fridman [9]. An appropriate Lyapunov-Krasovskii functional is chosen for delay-dependent stability analysis. A sufficient condition for the existence of a PDC-type controller is represented in terms of LMIs.

2. A DESCRIPTOR MODEL TRANSFORMATION OF T-S FUZZY SYSTEMS

The T-S fuzzy model is an effective way to represent a nonlinear dynamic system. It uses a linear state-variable description as its consequence of individual plant rules. A time-delayed T-S fuzzy model is composed of r plant rules that can be represented as follows:

Plant Rule i:
IF $z_1(t)$ is M_{i1} and … and $z_p(t)$ is M_{ip}
THEN $\dot{x}(t) = A_i x(t) + D_i x(t-\tau) + B_i u(t)$ (1)

$$x(t) = \phi(t), \qquad t \in [-\tau, 0],$$

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where $i=1,2,\cdots,r$. M_{ij} is the fuzzy set and r is the number of rules. $x(t)\in \mathbf{R}^n$ is the state, $u(t)\in \mathbf{R}^m$ is the control input, $\tau>0$ is the time delay of the system, $\phi(\cdot)$ is a vector-valued initial continuous function, A_i, D_i, and B_i are constant matrices with appropriate dimensions, and $z(t)=[z_1(t)\, z_2(t)\, \cdots\, z_p(t)]$ are known premise variables, which may be functions of the states, external disturbances, and/or time. Given a pair of $[x(t),\ u(t),\ z(t)]$, by using the center of gravity for defuzzification, the final state of the T-S fuzzy system is inferred as follows:

$$\dot{x}(t)=\sum_{i=1}^{r} h_i(z(t))[A_i x(t)+D_i x(t-\tau)+B_i u(t)], \quad (2)$$

where

$$h_i(z(t))=\frac{w_i(z(t))}{\sum_{j=1}^{r} w_j(z(t))},$$

$$w_i(z(t))=\prod_{j=1}^{p} M_{ij}(z_j(t)),$$

$M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}, and it is assumed that

$$w_i(z(t))\geq 0, \qquad i=1,\ 2,\ \cdots,\ r$$

$$\sum_{i=1}^{r} w_i(z(t))>0$$

for all t. Therefore

$$h_i(z(t))\geq 0, \qquad i=1,\ 2,\ \cdots,\ r$$

$$\sum_{i=1}^{r} h_i(z(t))=1$$

for all t. From [9], we represent (3) in the equivalent descriptor form:

$$\dot{x}(t)=y(t)$$

$$0=-y(t)+\sum_{i=1}^{r} h_i(z(t))[A_i x(t)+D_i x(t-\tau)+B_i u(t)] \quad (3)$$

or

$$E\dot{\eta}(t)=\sum_{i=1}^{r} h_i(z(t))[\bar{A}_i\eta(t)+\bar{D}_i E_{10}\eta(t-\tau)+\bar{B}_i u(t)], \quad (4)$$

where

$$\eta(t)=\begin{bmatrix} x(t) \\ y(t)\end{bmatrix},\ E=\begin{bmatrix} I & 0 \\ 0 & 0\end{bmatrix},\ E_{10}=\begin{bmatrix} I & 0\end{bmatrix},$$

$$\bar{A}_i=\begin{bmatrix} 0 & I \\ A_i & -I\end{bmatrix},\ \bar{D}_i=\begin{bmatrix} 0 \\ D_i\end{bmatrix},\ \bar{B}_i=\begin{bmatrix} 0 \\ B_i\end{bmatrix}.$$

In the next section, we will present a sufficient condition of delay-dependent stability and an existence condition of stabilizing state feedback gain for T-S fuzzy time-delayed systems. Before closing this section, we recall Park's inequality, which will be used to prove the main results.

Lemma 1[10]**:** Assume that $a(\alpha)\in \mathbf{R}^n$ and $b(\alpha)\in \mathbf{R}^m$ are given for $\alpha\in\Omega$. Then, for any matrices $X>0$ and M, the following holds:

$$-2\int_{\Omega} b^T(\alpha)a(\alpha)d\alpha$$

$$\leq \int_{\Omega}\begin{bmatrix} a(\alpha) \\ b(\alpha)\end{bmatrix}^T\begin{bmatrix} X & XM \\ M^T X & (2,2)\end{bmatrix}\begin{bmatrix} a(\alpha) \\ b(\alpha)\end{bmatrix}d\alpha, \quad (5)$$

where (2,2) denotes $(M^T X+I)X^{-1}(XM+I)$.

3. MAIN RESULTS

In the sequel, we present our main results on delay-dependent stability analysis and stabilization for time-delayed T-S fuzzy systems based on the LMI method.

3.1. Stability analysis

First we derive the stability condition of unforced time-delayed systems as follows:

$$E\dot{\eta}(t)=\sum_{i=1}^{r} h_i(z(t))[\bar{A}_i\eta(t)+\bar{D}_i E_{10}\eta(t-\tau)]. \quad (6)$$

Since $\eta(t-\tau)=\eta(t)-\int_{t-\tau}^{t}\dot{\eta}(s)ds$, (6) can be rewritten as

$$E\dot{\eta}(t)=\sum_{i=1}^{r} h_i(z(t))[(\bar{A}_i+\bar{D}_i E_{10})\eta(t) - \bar{D}_i\int_{t-\tau}^{t} y(s)ds]. \quad (7)$$

Consider a Lyapunov-Krasovskii functional for the system (6) as

$$V(\eta(t))=V_1(\eta(t))+V_2(\eta(t))+V_3(\eta(t)),$$

$$V_1(\eta(t))=\eta^T(t)EP\eta(t),$$

$$V_2(\eta(t))=\tau^{-1}\int_{t-\tau}^{t} x^T(\theta)Rx(\theta)d\theta, \quad (8)$$

$$V_3(\eta(t))=\int_{-\tau}^{0}\int_{t+\theta}^{t} y^T(s)Ry(s)dsd\theta,$$

where

$$P=\begin{bmatrix} P_1 & 0 \\ P_2 & P_3\end{bmatrix},\ P_1>0,\ R>0. \quad (9)$$

Theorem 1: For a given $\tau>0$, the equilibrium of the system (6) is asymptotically stable in the large if there exist matrices $Q_1 > 0$, Q_2, Q_3, $U > 0$ that satisfy the following LMIs.

$$\begin{bmatrix} \Phi_i & \begin{bmatrix} Q_1^T \\ \tau D_i M^T \end{bmatrix} & \begin{bmatrix} \tau Q_2^T \\ \tau Q_3^T \end{bmatrix} & \begin{bmatrix} 0 \\ \tau D_i M^T \end{bmatrix} \\ \begin{bmatrix} Q_1 & \tau M D_i^T \end{bmatrix} & -\tau U & 0 & 0 \\ \begin{bmatrix} \tau Q_2 & \tau Q_3 \end{bmatrix} & 0 & -\tau U & 0 \\ \begin{bmatrix} 0 & \tau M D_i^T \end{bmatrix} & 0 & 0 & -\tau U \end{bmatrix} < 0,$$
$$i = 1, 2, \cdots, r \tag{10}$$

where

$$\Phi_i = \begin{bmatrix} Q_2 + Q_2^T & Q_1 (A_i + D_i)^T - Q_2^T + Q_3 \\ (A_i + D_i) Q_1 - Q_2 + Q_3^T & -Q_3 - Q_3^T + \tau D_i (M^T + M + U) D_i^T \end{bmatrix}. \tag{11}$$

Proof: The proof will be completed by showing $\dot{V}(\eta(t)) < 0$. The time derivative of $V_1(\eta(t))$ is

$$\begin{aligned} \dot{V}_1(\eta(t)) &= 2\eta^T(t) P^T E \dot{\eta}(t) \\ &= 2\eta^T(t) P^T \left\{ \sum_{i=1}^{r} h_i(z(t)) \times \left[(\bar{A}_i + \bar{D}_i E_{10}) \eta(t) - \bar{D}_i \int_{t-\tau}^{t} y(s) ds \right] \right\} \\ &= 2\eta^T(t) P^T \left\{ \sum_{i=1}^{r} h_i(z(t)) (\bar{A}_i + \bar{D}_i E_{10}) \eta(t) \right\} - 2\xi(t), \end{aligned} \tag{12}$$

where

$$\xi(t) = \sum_{i=1}^{r} h_i(z(t)) \eta^T(t) P^T \bar{D}_i \int_{t-\tau}^{t} y(s) ds. \tag{13}$$

From lemma 1, we have

$$\begin{aligned} -2\xi(t) \le & \int_{t-\tau}^{t} y^T(\theta) R y(\theta) d\theta \\ & + 2\sum_{i=1}^{r} h_i(z(t)) \eta^T(t) P^T \bar{D}_i M^T R \int_{t-\tau}^{t} y(\theta) d\theta \\ & + \tau \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \eta^T(t) P^T \\ & \times \bar{D}_i (M^T R + I) R^{-1} (RM + I) \bar{D}_j^T P \eta(t). \end{aligned} \tag{14}$$

Since the last term in the right-hand side of (14) is less than or equal to

$$\sum_{i=1}^{r} h_i(z(t)) \eta^T(t) \bar{D}_i [RM + I]^T R^{-1} [RM + I] \bar{D}_i^T P \eta(t),$$

the left-hand side of (14) has an upper bound as shown below:

$$\begin{aligned} -2\xi(t) \le & \int_{t-\tau}^{t} y^T(\theta) R y(\theta) d\theta \\ & + 2\sum_{i=1}^{r} h_i(z(t)) \eta^T(t) P^T \bar{D}_i M^T R \int_{t-\tau}^{t} y(\theta) d\theta \\ & + \tau \sum_{i=1}^{r} h_i(z(t)) \eta^T(t) P^T \\ & \times \bar{D}_i (M^T R + I) R^{-1} (RM + I) \bar{D}_i^T P \eta(t). \end{aligned} \tag{15}$$

The time derivatives of $V_2(\eta(t))$ and $V_3(\eta(t))$ are

$$\dot{V}_2(\eta(t)) = \tau^{-1} [x^T(t) R x(t) - x^T(t-\tau) R x(t-\tau)], \tag{16}$$

$$\begin{aligned} \dot{V}_3(\eta(t)) &= \int_{-\tau}^{0} y^T(t) R y(t) - y^T(t+\theta) R y(t+\theta) d\theta \\ &= \tau y^T(t) R y(t) - \int_{-\tau}^{0} y^T(t+\theta) R y(t+\theta) d\theta. \end{aligned} \tag{17}$$

From (12), (16), and (17),

$$\begin{aligned} \dot{V}(\eta(t)) \le & \sum_{i=1}^{r} h_i(z(t)) \eta^T(t) \left\{ P^T (\bar{A}_i + \bar{D}_i E_{10}) + (\bar{A}_i + \bar{D}_i E_{10})^T P \right\} \eta(t) \\ & + 2\sum_{i=1}^{r} h_i(z(t)) \eta^T(t) \\ & \times P^T \bar{D}_i M^T R \left[E_{10} \eta(t) - x(t-\tau) \right] \\ & + \tau^{-1} \eta^T(t) E_{10}^T R E_{10} \eta(t) \\ & - \tau^{-1} x^T(t-\tau) R x(t-\tau) \\ & + \tau\, y^T(t) R y(t) \\ & + \tau \sum_{i=1}^{r} h_i(z(t)) \eta^T(t) P^T \bar{D}_i [RM + I]^T \\ & \times R^{-1} [RM + I] \bar{D}_i^T P \eta(t) \end{aligned} \tag{18}$$

or

$$\begin{aligned} \dot{V}(\eta(t)) \le & \sum_{i=1}^{r} h_i(z(t)) \begin{bmatrix} \eta(t) \\ x(t-\tau) \end{bmatrix}^T \\ & \times \begin{bmatrix} \Psi_i & -P^T \bar{D}_i M^T R \\ -RM\bar{D}_i^T P & -\tau^{-1} R \end{bmatrix} \begin{bmatrix} \eta(t) \\ x(t-\tau) \end{bmatrix}, \end{aligned} \tag{19}$$

where

$$\Psi_i = P^T(\bar{A}_i + \bar{D}_i E_{10}) + (\bar{A}_i + \bar{D}_i E_{10})^T P + P^T \bar{D}_i M^T R E_{10} + E_{10}^T R M \bar{D}_i^T P + \tau^{-1} E_{10}^T R E_{10} + \tau E_{01}^T R E_{01} + \tau P^T \bar{D}_i (RM + I)^T R^{-1} (RM + I) \bar{D}_i^T P,$$

$$E_{01} = \begin{bmatrix} 0 & I \end{bmatrix}.$$

For simplicity, define

$$Q = \begin{bmatrix} Q_1 & 0 \\ Q_2 & Q_3 \end{bmatrix} = P^{-1}, \quad U = R^{-1},$$

then (10) and (11) are represented by

$$\begin{bmatrix} \Phi_i & * & * & * \\ \tau M \bar{D}_i^T + E_{10} Q & -\tau R^{-1} & 0 & 0 \\ \tau E_{10} Q & 0 & -\tau R^{-1} & 0 \\ \tau M \bar{D}_i^T & 0 & 0 & -\tau R^{-1} \end{bmatrix} < 0, \tag{20}$$

$i = 1, 2, \ldots, r$

$$\Phi_i = (\bar{A}_i + \bar{D}_i E_{10}) Q + Q^T (\bar{A}_i + \bar{D}_i E_{10})^T + \tau \bar{D}_i (M^T + M + R^{-1}) \bar{D}_i^T,$$

respectively, where * denotes the transposed terms for symmetric positions. From Schur complements [11], (20) is equivalent to

$$\begin{aligned} &(\bar{A}_i + \bar{D}_i E_{10}) Q + Q^T (\bar{A}_i + \bar{D}_i E_{10})^T \\ &+ (\tau M \bar{D}_i^T + E_{10} Q)^T \tau^{-1} R (\tau M \bar{D}_i^T + E_{10} Q) \\ &+ \tau \bar{D}_i (M^T + M + R^{-1}) \bar{D}_i^T \\ &+ \tau Q^T E_{01}^T R E_{01} Q + \tau \bar{D}_i M^T R M \bar{D}_i^T < 0 \end{aligned} \tag{21}$$

or

$$\begin{aligned} &(\bar{A}_i + \bar{D}_i E_{10}) Q + Q^T (\bar{A}_i + \bar{D}_i E_{10})^T \\ &+ \bar{D}_i M^T R E_{10} Q + Q^T E_{10}^T R M \bar{D}_i^T \\ &+ \tau^{-1} Q^T E_{10}^T R E_{10} Q \\ &+ \tau \bar{D}_i (RM + I)^T R^{-1} (RM + I) \bar{D}_i^T \\ &+ \tau Q^T E_{01}^T R E_{01} Q + \tau \bar{D}_i M^T R M \bar{D}_i^T < 0, \end{aligned} \tag{22}$$

because the sum of the third and forth terms in the left-hand side of (21) are equal to

$$\bar{D}_i M^T R E_{10} Q + Q^T E_{10}^T R M \bar{D}_i^T + \tau^{-1} Q^T E_{10}^T R E_{10} Q + \tau \bar{D}_i (RM + I)^T R^{-1} (RM + I) \bar{D}_i^T.$$

By the pre- and post-multiplying of P^T and P, respectively, to (22), we get

$$\Psi_i + \tau P^T \bar{D}_i M^T R M \bar{D}_i^T P < 0. \tag{23}$$

From (19), (23) implies $\dot{V}(\eta(t)) < 0$. □

3.2. Delay-dependent stabilization of time-delayed T-S fuzzy systems

Our goal in this subsection is to stabilize time delayed T-S fuzzy systems using the PDC approach [2, 3]. That is, we present a sufficient condition of existence of the PDC-type delay-dependent controller. The PDC-type controller shares the same fuzzy sets with the time-delayed T-S fuzzy system (1). The PDC-type controller after defuzzification is

$$u(t) = \sum_{i=1}^{r} h_i(z(t)) K_i x(t) = \sum_{i=1}^{r} h_i(z(t)) K_i E_{10} \eta(t). \tag{24}$$

Substituting this controller into (4), we obtain the closed-loop system

$$E\dot{\eta}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \times \left\{ \tilde{A}_{ij} \eta(t) + \bar{D}_i E_{10} \eta(t - d) \right\}, \tag{25}$$

where

$$\tilde{A}_{ij} = \bar{A}_i + \bar{B}_i K_j E_{10}.$$

Theorem 2: Assume that the number of rules that fire for all t is less than or equal to s where $1 < s \le r$. For a given $\tau > 0$, the equilibrium of the closed-loop system (25) is asymptotically stable in the large if there exist matrices $Q_1 > 0$, Q_2, Q_3, $U > 0$, $Z \ge 0$, M, N_i that satisfy the following LMIs.

$$\begin{bmatrix} \frac{1}{2}(G_{ii} + G_{ii}^T) + \tau S_{ii} + (\nu - 1) Z & * & * & * \\ \tau M \bar{D}_i^T + E_{10} Q & -\tau U & 0 & 0 \\ \tau E_{01} Q & 0 & -\tau U & 0 \\ \tau M \bar{D}_i^T & 0 & 0 & -\tau U \end{bmatrix} < 0, \quad i = 1, 2, \cdots, r \tag{26}$$

$$\begin{bmatrix} G_{ij} + G_{ij}^T + \tau(S_{ij} + S_{ji}) - 2Z & * & * & * & * \\ \tau M \bar{D}_i^T + E_{10} Q & -\tau U & 0 & 0 & 0 \\ \tau M \bar{D}_j^T + E_{10} Q & 0 & -\tau U & 0 & 0 \\ \tau M (\bar{D}_i + \bar{D}_j)^T & 0 & 0 & -2\tau U & 0 \\ \tau E_{01} Q & 0 & 0 & 0 & -\frac{1}{2}\tau U \end{bmatrix} < 0, \quad i < j \le r \tag{27}$$

where

$$G_{ij} = (\bar{A}_i + \bar{A}_j) Q + \bar{B}_i N_j E_{10} + \bar{B}_j N_i E_{10} + (\bar{D}_i + \bar{D}_j) Q_1 E_{10} = \begin{bmatrix} 2Q_2 & 2Q_3 \\ G_{ij21} & -2Q_3 \end{bmatrix},$$

$$G_{ij21} = (A_i + A_j)Q_1 - 2Q_2 + B_i N_j + B_j N_i + (D_i + D_j)Q_1,$$

$$S_{ij} = \bar{D}_i (M^T + M + U)\bar{D}_j^T .$$

Furthermore, state feedback gains for each rule of the time-delayed T-S fuzzy system (4) are

$$K_i = N_i Q_1^{-1}, \quad i = 1, 2, \cdots, r . \tag{28}$$

Proof: For the closed-loop system (25), the time derivative of Lyapunov-Krasovskii functional (8) satisfies the following inequality:

$$\dot{V}(\eta(t)) \le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \begin{bmatrix} \eta(t) \\ x(t-\tau) \end{bmatrix}^T \times \begin{bmatrix} \Psi_{ij} & -P^T \bar{D}_i M^T R \\ -RM\bar{D}_i^T P & -\tau^{-1} R \end{bmatrix} \begin{bmatrix} \eta(t) \\ x(t-\tau) \end{bmatrix}, \tag{29}$$

where

$$\Psi_{ij} = P^T(\tilde{A}_{ij} + \bar{D}_i E_{10}) + (\tilde{A}_{ij} + \bar{D}_i E_{10})^T P + (\tau M\bar{D}_i^T P + E_{10})^T \tau^{-1} R (\tau M \bar{D}_i^T P + E_{10}) + \tau E_{01}^T R E_{01} + \tau P^T \bar{D}_i (M^T + M + R^{-1}) \bar{D}_i^T P.$$

From corollary 4 and theorem 5 in [12], the right-hand side of (29) is less than zero if there exist $P > 0$ and $\bar{Z} \ge 0$ such that

$$\begin{bmatrix} \Psi_{ii} + (v-1)\bar{Z} & -P^T \bar{D}_i M^T R \\ -RM\bar{D}_i^T P & -\tau^{-1} R \end{bmatrix} < 0, \quad i = 1, 2, \cdots, r \tag{30}$$

$$\begin{bmatrix} \Psi_{ij} + \Psi_{ji} - 2\bar{Z} & -P^T(\bar{D}_i + \bar{D}_j) M^T R \\ -RM(\bar{D}_i^T + \bar{D}_i^T) P & -2\tau^{-1} R \end{bmatrix} \le 0, \quad i < j \le r . \tag{31}$$

Now the proof will be completed if (30) and (31) are equivalent, respectively, to (26) and (27). By the pre- and post-multiplying of $\begin{bmatrix} -Q & 0 \\ 0 & I \end{bmatrix}^T$ and $\begin{bmatrix} -Q & 0 \\ 0 & I \end{bmatrix}$, respectively, to (30) and (31), manipulating their matrix inequalities using Schur complements, and letting $N_i = K_i Q_1$, (30) and (31) can be rewritten as (26) and (27), respectively. □

4. AN EXAMPLE

We will design a PDC-type controller for the following time-delayed nonlinear system:

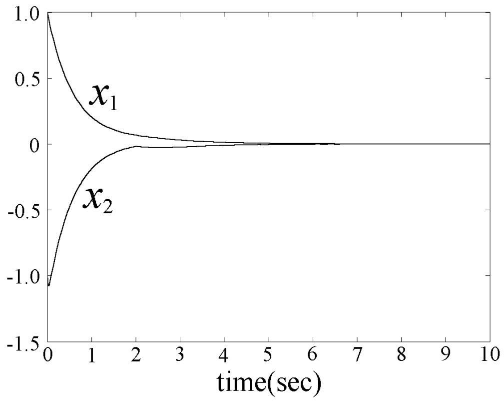

Fig. 1. The simulation results for $\tau = 2$.

$$\dot{x}_1(t) = x_2(t) - 0.2 x_1(t) x_1(t-\tau),$$
$$\dot{x}_2(t) = x_1^2(t) - x_2(t) + 4 x_1(t-\tau) + u(t). \tag{32}$$

It is assumed that $x_1(t)$ is measurable and $x_1(t) \in [-2,\ 2]$.

An equivalent T-S fuzzy model to (32) is represented by

Rule 1:

IF $x_1(t)$ is M_1

THEN

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -0.4 & 0 \\ 4 & 0 \end{bmatrix} x(t-\tau) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

Rule 2:

IF $x_1(t)$ is M_2

THEN

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0.4 & 0 \\ 4 & 0 \end{bmatrix} x(t-\tau) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

where $x(t) = [x_1(t) \quad x_2(t)]^T$ and $M_1 = \frac{2 - x_1}{4}$, $M_2 = 1 - M_1$. For $\tau = 2$, all parameters Q, U, Z, M, N_1, N_2 satisfying (26) and (27) are

$$Q = \begin{bmatrix} Q_1 & 0 \\ Q_2 & Q_3 \end{bmatrix} = \left[\begin{array}{cc|cc} 2.0953 & -3.9822 & 0 & 0 \\ -3.9822 & 9.8517 & 0 & 0 \\ \hline -1.8090 & 2.9352 & 3.5188 & 2.6533 \\ 1.4688 & -83.9144 & -6.3201 & 218.8701 \end{array}\right],$$

$$Z = \begin{bmatrix} 0.4415 & -0.9955 & 0.3761 & 1.9218 \\ -0.9955 & 32.9423 & -3.0847 & -15.7938 \\ 0.3761 & -3.0847 & 1.6691 & 8.5740 \\ 1.9218 & -15.7938 & 8.5740 & 77.0615 \end{bmatrix},$$

$$U = \begin{bmatrix} 7.1768 & -3.2729 \\ -3.2729 & 455.4994 \end{bmatrix}, \quad M = \begin{bmatrix} -3.2185 & 0 \\ 0.2910 & 0 \end{bmatrix},$$

$N_1 = [\text{-16.3316} \quad \text{-176.0110}]$,

$N_2 = [\text{-10.2798} \quad \text{-282.3263}]$.

The control gains of each rule are

$K_1 = [\text{-180.1349} \quad \text{-90.6800}]$,

$K_2 = [\text{-256.1661} \quad \text{-132.2049}]$.

Fig. 1 shows the simulation results for $\tau = 2$ with the initial value conditions of

$x_1(t) = 1(t \le 0)$ and $x_2(0) = -1$.

We obtain a maximum time delay τ of 2.705 with corresponding state-feedback gains of $K_1 = 10^6 \times [\text{-3.2505} \quad \text{-0.7307}]$ and $K_2 = 10^6 \times [\text{-3.4545} \quad \text{-0.7765}]$ for each rule.

5. CONCLUSIONS

By choosing an appropriate Lyapunov-Krasovskii functional, we have designed a delay-dependent PDC-type controller for time-delayed T-S fuzzy control systems. We have also used equivalent descriptor model transformation of the system. A sufficient condition for existence of the PDC-type controller has been represented in terms of LMIs and state feedback gains for each rule have been obtained from solutions of the LMIs. An illustrated example has been given to demonstrate our results.

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