A Delayed Choice Quantum Eraser

Yoon-Ho Kim, R. Yu, S.P. Kulik*, and Y.H. Shih
Department of Physics, University of Maryland, Baltimore County,
Baltimore, MD 21250

Marlan O. Scully
Department of Physics, Texas A & M University, College Station, TX 77842
and Max-Planck Institut für Quantenoptik, München, Germany
(submitted to PRL)

This paper reports a "delayed choice quantum eraser" experiment proposed by Scully and Drühl in 1982. The experimental results demonstrated the possibility of simultaneously observing both particle-like and wave-like behavior of a quantum via quantum entanglement. The which-path or both-path information of a quantum can be erased or marked by its entangled twin even after the registration of the quantum.

PACS Number: 03.65.Bz, 42.50.Dv

Complementarity, perhaps the most basic principle of quantum mechanics, distinguishes the world of quantum phenomena from the realm of classical physics. Quantum mechanically, one can never expect to measure both precise position and momentum of a quantum at the same time. It is prohibited. We say that the quantum observables "position" and "momentum" are "complementary" because the precise knowledge of the position (momentum) implies that all possible outcomes of measuring the momentum (position) are equally probable. In 1927, Niels Bohr illustrated complementarity with "wave-like" and "particle-like" attributes of a quantum mechanical object [1]. Since then, complementarity is often superficially identified with "wave-particle duality of matter". Over the years the two-slit interference experiment has been emphasized as a good example of the enforcement of complementarity. Feynman, discussing the two-slit experiment, noted that this wave-particle dual behavior contains the basic mystery of quantum mechanics [2]. The actual mechanisms that enforce complementarity vary from one experimental situation to another. In the two-slit experiment, the common "wisdom" is that the position-momentum uncertainty relation $\delta x \delta p \geq \frac{\hbar}{2}$ makes it impossible to determine which slit the photon (or electron) passes through without at the same time disturbing the photon (or electron) enough to destroy the interference pattern. However, it has been proven [3] that under certain circumstances this common interpretation may not be true. In 1982, Scully and Drühl found a way around this position-momentum uncertainty obstacle and proposed a quantum eraser to obtain which-path or particle-like information without scattering or otherwise introducing large uncontrolled phase factors to disturb the interference. To be sure the interference pattern disappears when which-path information is obtained. But it reappears when we erase (quantum erasure) the which-path information [3,4]. Since 1982, quantum eraser behavior has been reported in several experiments [5]; however, the original scheme has not been fully demonstrated.

One proposed quantum eraser experiment very close to the 1982 proposal is illustrated in Fig.1. Two atoms labeled by A and B are excited by a laser pulse. A pair of entangled photons, photon 1 and photon 2, is then emitted from either atom A or atom B by atomic cascade decay. Photon 1, propagating to the right, is registered by a photon counting detector D_0, which can be scanned by a step motor along its x-axis for the observation of interference fringes. Photon 2, propagating to the left, is injected into a beamsplitter. If the pair is generated in atom A, photon 2 will follow the A path meeting BSA with 50% chance of being reflected or transmitted. If the pair is generated in atom B, photon 2 will follow the B path meeting BSB with 50% chance of being reflected or transmitted. Under the 50% chance of being transmitted by either BSA or BSB, photon 2 is detected by either detector D_3 or D_4. The registration of D_3 or D_4 provides which-path information (path A or path B) of photon 2 and in turn provides which-path information of photon 1 because of the entanglement nature of the two-photon state of atomic cascade decay. Given a reflection at either BSA or BSB photon 2 will continue to follow its A path or B path to meet another 50-50 beamsplitter BS and then be detected by either detector D_1 or D_2, which are placed at the output ports of the beamsplitter BS. The triggering of detectors D_1 or D_2 erases the which-path information. So that either the absence of the interference or the restoration of the interference can be arranged via an appropriately contrived photon correlation study. The

---

[*] Permanent Address: Department of Physics, Moscow State University, Moscow, Russia

experiment is designed in such a way that L_0, the optical distance between atoms A, B and detector D_0, is much shorter than L_i, which is the optical distance between atoms A, B and detectors D_1, D_2, D_3, and D_4, respectively. So that D_0 will be triggered much earlier by photon 1. After the registration of photon 1, we look at these "delayed" detection events of D_1, D_2, D_3, and D_4 which have constant time delays, $\tau_i \simeq (L_i - L_0)/c$, relative to the triggering time of D_0. It is easy to see these "joint detection" events must have resulted from the same photon pair. It was predicted that the "joint detection" counting rate R_{01} (joint detection rate between D_0 and D_1) and R_{02} will show interference pattern when detector D_0 is scanned along its x-axis. This reflects the wave property (both-path) of photon 1. However, no interference will be observed in the "joint detection" counting rate R_{03} and R_{04} when detector D_0 is scanned along its x-axis. This is clearly expected because we now have indicated the particle property (which-path) of photon 1. It is important to emphasize that all four "joint detection" rates R_{01}, R_{02}, R_{03}, and R_{04} are recorded at the same time during one scanning of D_0 along its y-axis. That is, in the present experiment we "see" both wave (interference) and which-path (particle-like) with the same apparatus.

We wish to report a realization of the above quantum eraser experiment. The schematic diagram of the experimental setup is shown in Fig.2. Instead of atomic cascade decay, spontaneous parametric down conversion (SPDC) is used to prepare the entangled two-photon state. SPDC is a spontaneous nonlinear optical process from which a pair of signal-idler photons is generated when a pump laser beam is incident onto a nonlinear optical crystal [6]. In this experiment, the $351.1nm$ Argon ion pump laser beam is divided by a double-slit and incident onto a type-II phase matching [7] nonlinear optical crystal BBO ($\beta - BaB_2O_4$) at two regions A and B. A pair of $702.2nm$ orthogonally polarized signal-idler photon is generated either from A or B region. The width of the SPDC region is about $0.3mm$ and the distance between the center of A and B is about $0.7mm$. A Glen-Thompson prism is used to split the orthogonally polarized signal and idler. The signal photon (photon 1, either from A or B) passes a lens LS to meet detector D_0, which is placed on the Fourier transform plane (focal plane for collimated light beam) of the lens. The use of lens LS is to achieve the "far field" condition, but still keep a short distance between the slit and the detector D_0. Detector D_0 can be scanned along its x-axis by a step motor. The idler photon (photon 2) is sent to an interferometer with equal-path optical arms. The interferometer includes a prism PS, two 50-50 beamsplitters BSA, BSB, two reflecting mirrors M_A, M_B, and a 50-50 beamsplitter BS. Detectors D_1 and D_2 are placed at the two output ports of the BS, respectively, for erasing the which-path information. The triggering of detectors D_3 and D_4 provide which-path information of the idler (photon 2) and in turn provide which-path information of the signal (photon 1). The electronic output pulses of detectors D_1, D_2, D_3, and D_4 are sent to coincidence circuits with the output pulse of detector D_0, respectively, for the counting of "joint detection" rates R_{01}, R_{02}, R_{03}, and R_{04}. In this experiment the optical delay $(L_i - L_0)$ is chosen to be $\simeq 2.5m$, where L_0 is the optical distance between the output surface of BBO and detector D_0, and L_i is the optical distance between the output surface of the BBO and detectors D_1, D_2, D_3, and D_4, respectively. This means that any information one can learn from photon 2 must be at least $8ns$ later than what one has learned from the registration of photon 1. Compared to the $1ns$ response time of the detectors, $2.5m$ delay is good enough for a "delayed erasure".

Figs.3, 4, and 5 report the experimental results, which are all consistent with prediction. Figs.3 and 4 show the "joint detection" rates R_{01} and R_{02} against the x coordinates of detector D_0. It is clear we have observed the standard Young's double-slit interference pattern. However, there is a π phase shift between the two interference fringes. The π phase shift is explained as follows. Fig.5 reports a typical R_{03} (R_{04}), "joint detection" counting rate between D_0 and "which-path" D_3 (D_4), against the x coordinates of detector D_0. An absence of interference is clearly demonstrated. There is no significant difference between the curves of R_{03} and R_{04} except the small shift of the center.

To explain the experimental results, a standard quantum mechanical calculation is presented in the following. The "joint detection" counting rate, R_{0i}, of detector D_0 and detector D_j, on the time interval T, is given by the Glauber formula [8]:

$$\begin{aligned} R_{0j} &\propto \frac{1}{T}\int_0^T \int_0^T dT_0 dT_j \langle \Psi | E_0^{(-)} E_j^{(-)} E_j^{(+)} E_0^{(+)} | \Psi \rangle \\ &= \frac{1}{T}\int_0^T \int_0^T dT_0 dT_j |\langle 0 | E_j^{(+)} E_0^{(+)} | \Psi \rangle|^2, \end{aligned} \quad (1)$$

where T_0 is the detection time of D_0, T_j is the detection time of D_j($j = 1, 2, 3, 4$) and $E_{0,j}^{(\pm)}$ are positive and negative-frequency components of the field at detectors D_0 and D_j, respectively. $|\Psi\rangle$ is the entangled state of SPDC,

$$|\Psi\rangle = \sum_{s,i} C(\mathbf{k}_s, \mathbf{k}_i)\ a_s^\dagger(\omega(\mathbf{k}_s))\ a_i^\dagger(\omega(\mathbf{k}_i))|0\rangle, \quad (2)$$

where $C(\mathbf{k}_s, \mathbf{k}_i) = \delta(\omega_s + \omega_i - \omega_p)\delta(\mathbf{k}_s + \mathbf{k}_i - \mathbf{k}_p)$, for the SPDC in which ω_j and $\mathbf{k_j}$ ($j = s, i, p$) are the frequency and wavevectors of the signal (s), idler (i), and pump (p), respectively, ω_p and $\mathbf{k}_p$ can be considered as constants, a single mode laser line is used for pump and $a_s^\dagger$ and $a_i^\dagger$ are creation operators for signal and idler photons, respectively. For the case of two scattering atoms, see ref. [3], and in the case of cascade radiation, see ref. [9], $C(\mathbf{k}_s, \mathbf{k}_i)$ has a similar structure but without the momentum delta function. The δ functions in eq.(2) are the results of approximations for an infinite size SPDC crystal and for infinite interaction time. We introduce the two-dimensional function $\Psi(t_0, t_j)$ as in eq.(1),

$$\Psi(t_0, t_j) \equiv \langle 0 | E_j^{(+)} E_0^{(+)} | \Psi \rangle. \tag{3}$$

$\Psi(t_0, t_j)$ is the joint count probability amplitude ("wavefunction" for short), where $t_0 \equiv T_0 - L_0/c$, $t_j \equiv T_j - L_j/c, j = 1, 2, 3, 4$, L_0 (L_j) is the optical distance between the output point on the BBO crystal and D_0 (D_j). It is straightforward to see that the four "wavefunctions" $\Psi(t_0, t_j)$, correspond to four different "joint detection" measurements, having the following different forms:

$$\begin{aligned} \Psi(t_0, t_1) &= A(t_0, t_1^A) + A(t_0, t_1^B), \\ \Psi(t_0, t_2) &= A(t_0, t_2^A) - A(t_0, t_2^B), \end{aligned} \tag{4}$$

$$\Psi(t_0, t_3) = A(t_0, t_3^A), \quad \Psi(t_0, t_4) = A(t_0, t_4^B), \tag{5}$$

where as in Fig.1 the upper index of t (A or B) labels the scattering crystal (A or B region) and the lower index of t indicates different detectors. The different sign between the two amplitudes $\Psi(t_0, t_1)$ and $\Psi(t_0, t_2)$ is caused by the transmission-reflection unitary transformation of the beamsplitter BS, see Fig.1 and Fig.2. It is also straightforward to calculate each of the $A(t_i, t_j)$ [10]. To simplify the calculations, we consider the longitudinal integral only and write the two-photon state in terms of the integral of k_e and k_o:

$$|\Psi\rangle = A_0' \int dk_e \int dk_o \ \delta(\omega_e + \omega_o - \omega_p) \times \Phi(\Delta_k L) a_{k_e}^\dagger a_{k_o}^\dagger |0\rangle, \tag{6}$$

where a type-II phase matching crystal with finite length of L is assumed. $\Phi(\Delta_k L)$ is a sinc-like function, $\Phi(\Delta_k L) = (e^{i(\Delta_k L)} - 1)/i(\Delta_k L)$. Using eqs. (3) and (6) we find,

$$A(t_i, t_j) = A_0 \int dk_e \int dk_o \delta(\omega_e + \omega_o - \omega_p) \times \Phi(\Delta_k L) f_i(\omega_e) f_j(\omega_o) e^{-i(\omega_e t_1^e + \omega_o t_2^o)}, \tag{7}$$

where $f_{i,j}(\omega)$, is the spectral transmission function of an assumed filter placed in front of the k_{th} detector and is assumed Gaussian to simplify the calculation. To complete the integral, we define $\omega_e = \Omega_e + \nu$ and $\omega_o = \Omega_o - \nu$, where Ω_e and Ω_o are the center frequencies of the SPDC, $\Omega_e + \Omega_o = \Omega_p$ and ν is a small tuning frequency, so that $\omega_e + \omega_o = \Omega_p$ still holds. Consequently, we can expand k_e and k_o around $K_e(\Omega_e)$ and $K_o(\Omega_o)$ to first order in ν:

$$\begin{aligned} k_e &= K_e + \nu \left. \frac{d\omega_e}{dk_e} \right|_{\Omega_e} = K_e + \frac{\nu}{u_e}, \\ k_o &= K_o - \nu \left. \frac{d\omega_o}{dk_o} \right|_{\Omega_o} = K_o - \frac{\nu}{u_o}, \end{aligned} \tag{8}$$

where u_e and u_o are recognized as the group velocities of the e-ray and o-ray at frequencies Ω_e and Ω_o, respectively. Completing the integral, the biphoton wavepacket of type-II SPDC is thus:

$$A(t_i, t_j) = A_0 \Pi(t_i - t_j) e^{-i\Omega_i t_i} e^{-i\Omega_j t_j}, \tag{9}$$

where we have dropped the e, o indices. The shape of $\Pi(t_1 - t_2)$ is determined by the bandwidth of the spectral filters and the parameter DL of the SPDC crystal, where $D \equiv 1/u_o - 1/u_e$. If the filters are removed or have large enough bandwidth, we have a rectangular pulse function $\Pi(t_1 - t_2)$.

$$\Pi(t_0 - t_j) = \begin{cases} 1 & \text{if } 0 \leq t_0 - t_j \leq DL, \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to find that the two amplitudes in $\Psi(t_0, t_1)$ and $\Psi(t_0, t_2)$ are indistinguishable (overlap in both $t_0 - t_j$ and $t_0 + t_j$), respectively, so that interference is expected in both the coincidence counting rates, R_{01} and R_{02}; however, with a π phase shift due to the different sign,

$$R_{01} \propto \cos^2(x\pi d/\lambda f), \quad \text{and} \quad R_{02} \propto \sin^2(x\pi d/\lambda f).$$

If we consider "slit" A and B both have finite width (not infinitely narrow), an integral is necessary to sum all possible amplitudes along slit A and slit B. We will have a standard interference-diffraction pattern for R_{01} and R_{02},

$$\begin{aligned} R_{01} &\propto \text{sinc}^2(x\pi a/\lambda f) \cos^2(x\pi d/\lambda f), \\ R_{02} &\propto \text{sinc}^2(x\pi a/\lambda f) \sin^2(x\pi d/\lambda f), \end{aligned} \tag{10}$$

where a is the width of the slit A and B (equal width), d is the distance between the center of slit A and B, $\lambda = \lambda_s = \lambda_i$ is the wavelength of the signal and idler, and f is the focal length of lens LS. We have also applied the "far field approximation" for the signal and equal optical distance of the interferometer for the idler. After considering the finite size of the detectors and the divergence of the pump beam for further integrals, the interference visibility is reduced to the level close to the observation.

For the "joint detection" R_{03} and R_{04}, it is seen that the "wavefunction" in eq.(5) (which clearly provides "which-path" information) has only one amplitude and no interference is expected.

In conclusion, we have realized a quantum eraser experiment of the type proposed in ref. [3]. The experimental results demonstrate the possibility of observing both particle-like and wave-like behavior of a light quantum via quantum mechanical entanglement. The which-path or both-path information of a quantum can be erased or marked by its entangled twin even after the registration of the quantum.

This work was supported, in part, by the U.S. Office of Naval Research, the Army Research Office - the National Security Agency, the National Science Foundation, and the Welch Foundation. MOS wishes to thank Roland Hagen for helpful and stimulating discussions.

---

[1] N. Bohr, Naturwissenschaften, **16**, 245 (1928).

[2] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Vol. III, Addison Wesley, Reading (1965).

[3] M.O. Scully and K. Drühl, Phys. Rev. A **25**, 2208 (1982).

[4] See Wheeler's "delayed choice", in *Quantum Theory and Measurement*, edited by J.A. Wheeler and W.H. Zurek, Princeton Univ. Press (1983).

[5] A.G. Zajonc *et al.*, Nature, **353**, 507 (1991); P.G. Kwiat *et al.*, Phys. Rev. A **49**, 61 (1994); T.J. Herzog *et al.*, Phys. Rev. Lett., **75**, 3034 (1995); T.B. Pittman *et al.*, Phys. Rev. Lett., **77**, 1917 (1996).

[6] D.N. Klyshko, *Photon and Nonlinear Optics*, Gordon and Breach Science, New York (1988); A. Yariv, *Quantum Electronics*, John Wiley and Sons, New York (1989).

[7] In type-I SPDC, signal and idler are both ordinary rays of the crystal; however, in type-II SPDC the signal and idler are orthogonal polarized, i.e., one is ordinary ray and the other is extraordinary ray of the crystal.

[8] R.J. Glauber, Phys. Rev. **130**, 2529 (1963); **131**, 2766 (1963).

[9] M.O. Scully and M.S. Zubairy, *Quantum Optics*, Cambridge Univ. Press, Cambridge, UK (1997).

[10] M.H. Rubin, D.N. Klyshko, and Y.H. Shih, Phys. Rev. A **50**, 5122 (1994).

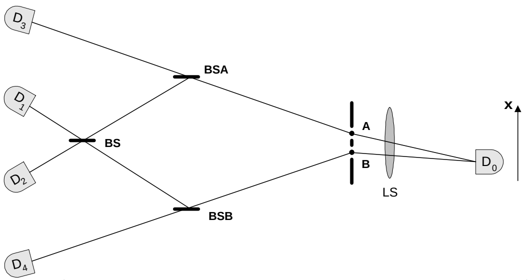

FIG. 1. A proposed quantum eraser experiment. A pair of entangled photons is emitted from either atom A or atom B by atomic cascade decay. "Clicks" at D_3 or D_4 provide which-path information and "clicks" at D_1 or D_2 erase the which-path information.

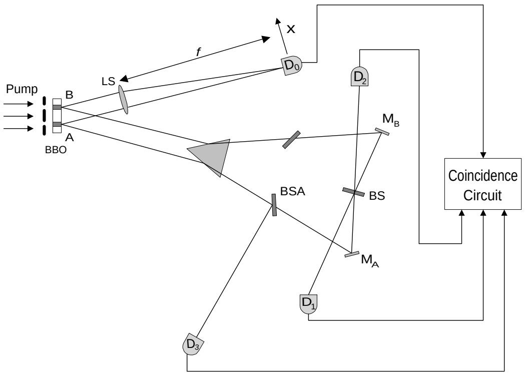

FIG. 2. Schematic of the experimental setup. The pump laser beam of SPDC is divided by a double-slit and incident onto a BBO crystal at two regions A and B. A pair of signal-idler photons is generated either from A or B region. The detection time of the signal photon is $8ns$ earlier than that of the idler.

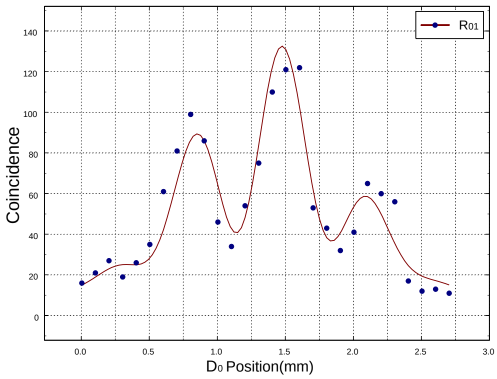

FIG. 3. R_{01} ("joint detection" rate between detectors D_0 and D_1) against the x coordinates of detector D_0. A standard Young's double-slit interference pattern is observed.

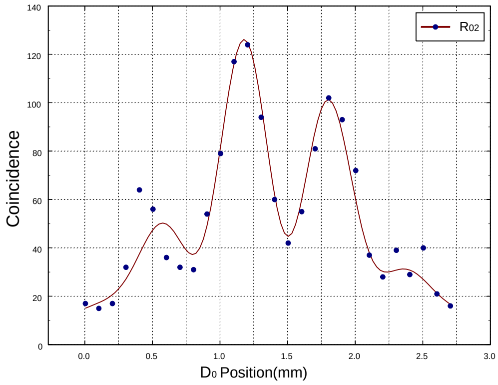

FIG. 4. R_{02} ("joint detection" rate between detectors D_0 and D_2) Note, there is a π phase shift compare to R_{01} shown in Fig.3

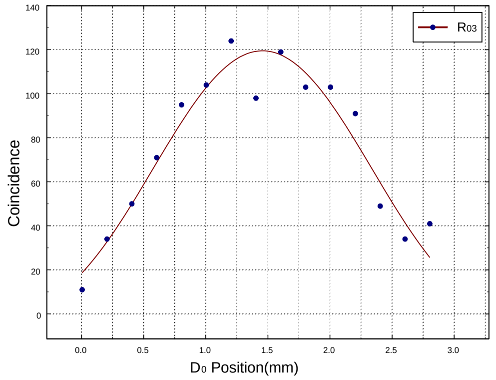

FIG. 5. R_{03} ("joint detection" rate between detectors D_0 and D_3). An absence of interference is clearly demonstrated.