

Delta Functions: An Introduction to Generalised Functions

R.F. HOSKINS

Research Professor

Department of Mathematical Sciences

De Montfort University

Leicester



**Horwood Publishing
Chichester**

Contents

1 Results from Elementary Analysis	
1.1 The real number system	1
1.2 Functions	5
1.3 Continuity	7
1.4 Differentiability	9
1.5 Taylor's theorem	11
1.6 Integration	15
1.7 Improper integrals	19
1.8 Note on uniform convergence	23
1.9 Differentiating an integral	25
2 The Dirac delta function	
2.1 Simple discontinuities and the unit step function	28
2.2 Derivative of the unit step function	33
2.3 The delta function as a limit	36
2.4 Delta functions and Stieltjes integrals	39
2.5 Historical note	44
3 Properties of the delta function and its derivatives	
3.1 The delta function as a generalised function	47
3.2 Addition and multiplication	50
3.3 Differentiation	55
3.4 Derivatives of the delta function	58
3.5 Pointwise description of $\delta'(t)$	62
3.6 Integration of delta functions	65
3.7 Change of variable	67
4 Time-invariant linear systems	
4.1 Systems and operators	75
4.2 Step response and impulse response	80
4.3 Convolution as an operation	84
4.4 Generalised impulse response functions	86
4.5 Transfer function and frequency response	89
4.6 Example: electrical network	94

5	The Laplace transform	
5.1	The classical Laplace transform	98
5.2	Laplace transforms of generalised functions	105
5.3	The computation of Laplace transforms	110
5.4	Note on inversion	119
6	Fourier series and Fourier transforms	
6.1	Fourier series	124
6.2	Generalised Fourier series	129
6.3	Fourier transforms	133
6.4	Generalised Fourier transforms	142
7	Other types of generalised function	
7.1	Fractional calculus	151
7.2	The finite part of a divergent integral	157
7.3	Pseudo-functions	162
8	Introduction to distributions	
8.1	Test functions	170
8.2	Functionals and distributions	176
8.3	The calculus of distributions	183
8.4	General Schwartz distributions	186
9	Integration theory	
9.1	The Riemann-Stieltjes integral	188
9.2	Extension of the basic integral	192
9.3	Lebesgue and Riemann integrals	197
10	NSA and generalised functions	
10.1	A nonstandard universe	204
10.2	Nonstandard extensions	212
10.3	Applications of NSA to elementary analysis	214
10.4	Internal objects	220
10.5	Nonstandard delta functions	229
10.6	Periodic delta functions	239
10.7	Nonstandard distributions	245
	Solutions to Exercises	251
	References	259
	Index	261