

Received September 15, 2020, accepted September 21, 2020, date of publication September 24, 2020, date of current version October 6, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3026230

Delta-Modulator-Based Quantised Output Feedback Controller for Linear Networked Control Systems

CHATHURA WANIGASEKARA¹, (Graduate Student Member, IEEE),
DHAFAER ALMAKHLES², (Senior Member, IEEE), AKSHYA SWAIN¹, (Senior Member, IEEE),
AND SING KIONG NGUANG¹, (Senior Member, IEEE)

¹Department of Electrical, Computer, and Software Engineering, The University of Auckland, Auckland 1010, New Zealand

²Renewable Energy Laboratory (REL), Department of Communication and Networks, College of Engineering, Prince Sultan University (PSU), Riyadh 11586, Saudi Arabia

Corresponding author: Dhafer Almahles (dalmakhles@psu.edu.sa)

This work was supported by the Renewable Energy Laboratory (REL), College of Engineering, Prince Sultan University, Riyadh, Saudi Arabia.

ABSTRACT This article proposes a Δ -Modulator (Δ -M) based quantised output feedback controller for linear networked systems. The proposed Δ -M is essentially a 2-level quantiser, in contrast to some of the existing quantisers such as 2^p level ($p \geq 1$) uniform-interval-nearest-neighbour quantiser, and offers various advantages which include lower design complexity, less noisy and lower cost. The three key components of the control system: the *controller*, the *filter* and the *quantiser* are designed to achieve the desired performance. The stability conditions of the Δ -M are derived and conditions for the existence of *zig-zag* behaviour in steady-state are determined. The performance of the proposed controller is illustrated through simulations considering practical communication network based on ZigBee protocol. The results of the simulation demonstrate that the proposed controller could effectively achieve desired performance under various imperfections of the practical communication network.

INDEX TERMS Δ -modulation, quantised control, networked control systems (NCSs), output feedback control.

I. INTRODUCTION

During the past few decades, a new paradigm of controller design and implementation, based on a communication network, has emerged. This is called as networked control systems (NCSs) [1]. In NCS, various control system components exchange data through a communication network, and there exists a strong interaction between communication and control [2], [3]. The communication networks in NCS allow sharing of data packets among control components and therefore avoid the point-to-point wiring installation, associated with the traditional control system, which increases flexibility and maintainability of the system. Further, the well established and proven communication protocols often ensure the data packets to be successfully transmitted between the con-

trol components and thereby makes the NCS very reliable. This has widely been used in various applications which include remote control [4], telemanipulation [5], robotics [6], process control [7] and so on.

Although this control paradigm has been very successful [8], [9], there exist many challenges such as *quantisation errors* [10], *network-induced delay* [11], *packet losses*, which may result in system instability [12]. This is because the communication networks transmit data packets in the form of digital signals after sampling and quantisation of the continuous-time signal. Moreover, the limited bandwidth of the network often causes traffic congestion which leads to delays and packet dropouts.

In the past few decades, researchers have developed several effective control design methods and have proved their stability, both for linear and nonlinear systems [13]–[19]. Some of the existing control methods such as sliding mode

The associate editor coordinating the review of this manuscript and approving it for publication was Mohsin Jamil¹.

control [20], event-triggered control [21], H_∞ control [22], formation tracking control [23], model predictive control [24], distributed control [25] and so on have been tailored for networked control to mitigate the effects of various network imperfections and make the NCS more robust. These have been applied to many engineering applications such as fault detection, identifying cyberattacks, robot applications and many more.

One of the major issue with NCS is the bit-rate constraint (i.e. bandwidth utilisation) of the communication network. One of the possible methods to alleviate this problem is through quantisation. This is a process of mapping a large set of input values to a small set of output values where the continuous-time signals are represented by quantised signals. In the past, different types of quantisers have been proposed by researchers which include nearest neighbour quantisers [26], [27], logarithmic quantisers [28], [29], neural network quantisers [30] and so on. Note that as the number of quantisation levels decreases, the effectiveness of the quantisers in bandwidth utilisation increases. However, the accuracy of the generated control action decreases due to less number of quantisation levels (quantisation error increases). Although by using a higher number of bits could reduce the quantisation error, this increases the delay in the control action. Because all the bits need to be received in order to reconstruct the input to the controller [31].

The single-bit quantiser is a possible alternative to alleviate the problems associated with bit-rate constraints (or network bandwidth utilisation). During the past decade, various researchers have used the single-bit quantisers based on either Δ -Modulator (Δ -M), Delta-Sigma Modulator ($\Delta\Sigma$ -M), Hybrid-Delta Modulator (Δ_H -M) [32]–[34], to develop single-bit controllers. The output of these quantisers (modulators) are called bit-streams and the associated controllers are popularly known as bit-stream controllers [35]. The stability of bit-stream controllers have been established and the guidelines to tune the controllers' parameters have been reported in [8], [36]–[39].

In these methods, instead of using a microprocessor to implement the control functions, the controllers are implemented in hardware using bit-streams inside programmable logic devices such as field-programmable gate arrays (FPGAs). This technique differs from the traditional digital implementation where the continuous-time signal is represented by a single-bit signal. Moreover, since all control elements are implemented in parallel, the addition of extra functionality to a given design will consume extra silicon area with little impact on the timing of the system, unlike micro-controller based systems which execute control functions sequentially and may exceed the available execution time as more functionality is added. The success of single-bit quantisers has been demonstrated in areas such as control, mobile communication, and biomedical applications [35], [40]–[43].

However, the applications of single-bit quantisers to NCS have received comparatively less attention from researchers

except recently in [3], where quantised controllers are designed for NCS which consist of three major components such as the controller, the filter and the quantiser. In [3], a uniform-interval-nearest-neighbour quantiser, with 2^p levels, have been used and the performance of the system have been investigated by varying the number of bits p in the quantiser. This quantiser becomes a single-bit quantiser when $p = 1$, and it gives the minimum variance of the output.

Motivated by the success of the bit-stream controllers based on Δ -M, $\Delta\Sigma$ -M, Δ_H -M [8], [36]–[39], the present study proposes an alternate approach of designing a quantised controller for networked control systems using Δ -M. Amongst all these modulators, Δ -M offers many advantages such as lower noise, cost-effective operations, lower complexity and uses 2-levels in contrast to 2^p levels as used in [3]. The goal of this study is to investigate:

- (i.) Whether it is possible to use the Δ -M as a quantiser for networked control systems?
- (ii.) How to determine the step size (quantiser gain) of the Δ -M?
- (iii.) What are the conditions which will ensure the stability of Δ -M?
- (iv.) Whether the proposed control strategy can achieve better performance in a ZigBee protocol based real communication network?

For this purpose, a Δ -M based quantised output feedback controller is designed for linear networked systems. The bounds of the quantisation gain λ and the stability conditions are derived. The control architecture consisting of the *controller*, the *filter* and the *quantiser* is implemented in a real ZigBee protocol based communication network. The steady-state behaviour of the system is studied, and the conditions for the existence of the periodic behaviour are determined.

Note that, this study addresses the problem of designing quantised output-feedback controllers for networked control systems under disturbance. Theoretical results are derived for the design of the output-feedback controller which minimises a given cost function. The filter is designed to minimise the variance of the input signal to the Δ -M based quantiser. Further, the value of quantiser gain which ensures both the stability of the overall closed-loop system and periodic behaviour of the modulator output.

The rest of the paper is organised as follows. Section-II describes the problem formulation and the architecture of the Δ -M based networked control system is described in Section-III. Design procedures of designing the controller, the filter and the quantisation gain are described in Section-IV. The effectiveness of the proposed control strategy is demonstrated using a practical communication network, considering two examples in Section-V followed by conclusions in Section-VI.

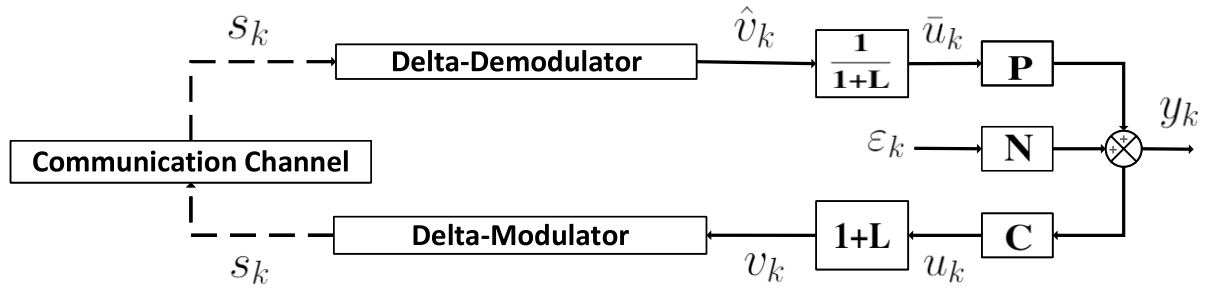


FIGURE 1. Closed-loop system with delta-modulator (Δ-M).

II. PROBLEM FORMULATION

Consider a single-input-single-output linear time invariant (LTI) continuous-time system given by:

$$\dot{x}(t) = A_{cont} x(t) + B_{cont} u(t) + K_{cont} \varepsilon(t), \quad (1a)$$

$$y(t) = C_{cont} x(t) + \varepsilon(t), \quad (1b)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^1$ and $y(t) \in \mathbb{R}^1$ are the system (plant) states, system input and system output, respectively. The objective of the study is to design a controller which would stabilise the plant under bit rate constraint in the communication channel between the controller and the plant. The bit rate of the communication channel is assumed to be b_r bits per second which give the smallest possible control update period δ_1 where,

$$\delta_1 = \frac{1}{b_r}. \quad (2)$$

The output from the plant is sampled at δ_1 seconds, and an anti-aliasing filter is deployed which operates at the same sampling period. Note that filtering of the signals at lower sampling periods is ideal for controllers.

The discrete equivalent of (1) at a sampling period of δ_1 seconds, in innovations form, [3], [44] is described as:

$$x_{k+1} = Ax_k + B\bar{u}_k + K\varepsilon_k, \quad (3a)$$

$$y_k = Cx_k + \varepsilon_k, \quad (3b)$$

where $x_k \in \mathbb{R}^n$, $\bar{u}_k \in \mathbb{R}^1$, $y_k \in \mathbb{R}^1$, $\varepsilon_k \in \mathbb{R}^1$ are the system (plant) states, system input, system output and innovations sequence having variance σ_ε^2 , respectively.

It is assumed that the system transfer function $P(z)$ ($P(z) = C(zI - A)^{-1}B$) has relative degree $d + 1 < n$. Hence, for $d = 0$, $CB \neq 0$ and for $d \geq 1$,

$$CA^i B = 0 \quad \forall i = 0, 1, \dots, d-1; CA^d B \neq 0. \quad (4)$$

For the rest of the paper, the following assumptions are used:

Assumption 1: The discrete-time transfer function from \bar{u}_k to y_k is stable and minimum phase.

Assumption 2: The communication between the controller and the plant are carried over a ZigBee protocol based wireless network supporting b_r bits per second.

Assumption 3: The communication channel is error-free.

Note that the assumption of error-free communication channel is only a working hypothesis. However, the results are shown by implementing the NCS on a real wireless network using ZigBee protocol where network imperfections are present.

III. ARCHITECTURE OF THE Δ-MODULATOR BASED NETWORKED CONTROL SYSTEM

The schematic of the proposed control scheme is shown in Figure-1 which consists of the plant (P), the controller (C), the filters (L), the communication channel and the discrete-time Δ-M. The Δ-M consists of a transmitter (encoder or modulator) and a receiver (decoder or demodulator). In this scheme, the input signal is oversampled at a higher rate than the Nyquist rate to achieve better resolution of the input signal at the end of the demodulator. The transmitter consists of a switch (relay component) which introduces non-linearity and add complexity to the overall system. Therefore, the stability of the overall system and Δ-M needs to be carefully addressed [32], [33].

The relations within various signals in this modulator can be described (see Figure-2) as:

$$\hat{v}_{k+1} = \hat{v}_k + \lambda \operatorname{sgn}(\bar{v}_k), \quad (5a)$$

$$\bar{v}_k = v_k - \hat{v}_k, \quad (5b)$$

where $\lambda > 0$, $\bar{v}_k \in \mathbb{R}$ and

$$\operatorname{sgn}(\bar{v}_k) = \begin{cases} +1, & \text{if } \bar{v}_k \geq 0, \\ -1, & \text{if } \bar{v}_k < 0. \end{cases}$$

As can be seen in Figure-2, the communication channel between the modulator and the demodulator carries 1-bit signal which is defined as $s_k = \frac{1}{2} [1 + \operatorname{sgn}(\bar{v}_k)]$. This implies $s_k \in \{0, 1\}^T$. For proper operation of Δ-M, the gain λ must be same in both the modulator and the demodulator [40].

IV. DESIGN PROCEDURE

The control law is computed by minimising the cost function:

$$J_A(p) = \frac{1}{p} \sum_{i=1}^p E \left\{ y^2(k+p+d+i) \right\}, \quad (6)$$

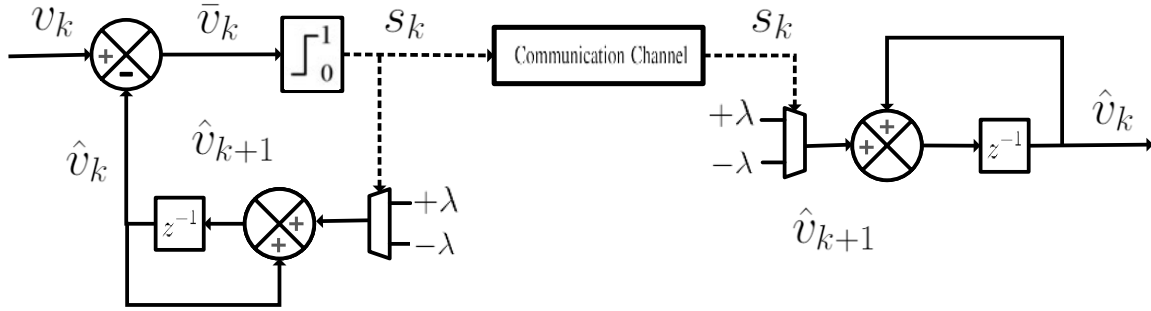


FIGURE 2. Discrete-time delta-modulator (Δ-M).

where p, d denote respectively the number of bits of the quantiser and the relative degree of the system. This essentially represents the variance of the output.

Remark 1: It is shown in [3] that for a p -bit representation, the output of the plant is cyclostationary with a period p . Therefore, the performance of the controller is investigated by minimising the cost function (6) for different values of p at each time step k .

The schematic of the proposed control system is shown in Figure-1. This consists of three major components; the controller (C), the filter (L) and the Δ-M. While designing the controller and the filter, it is assumed that the quantisation error $\bar{v}_k = 0$. The controller (C) is designed to minimise $J_A(p)$ and the filter (L) is designed to minimise the variance of \bar{v}_k (quantisation error). The quantisation gain (λ) is determined such that the Δ-M is stable.

A. DESIGN OF THE CONTROLLER (C)

Lemma 1: For a system with a transfer function $C(zI - A)^{-1}B$, the Δ-M based optimal controller, which minimises (6), is given by

$$x_{k+1}^c = A_c x_k^c + B_c u_{k-1-d} + K_c y_k, \quad (7a)$$

$$u_k = C_c x_k^c + D_c y_k + a u_{k-1}, \quad (7b)$$

where,

$$A_c = (A - K C), \quad B_c = B, \quad K_c = K, \quad (8a)$$

$$C_c = -\frac{g_2^T}{g_2^T g_2} C A^{d+1} (A - K C), \quad (8b)$$

$$D_c = -\frac{g_2^T}{g_2^T g_2} C A^{d+1} K, \quad (8c)$$

$$a = -\frac{g_2^T g_1}{g_2^T g_2}, \quad g_1 = C A^{d+1} B, \quad g_2 = C A^d B. \quad (8d)$$

Note that, for a system with relative degree d , $d + 1 < n$. For $d = 0$, $CB \neq 0$ and for $d \geq 1$,

$$C A^i B = 0 \quad \forall i = 0, 1, \dots, d - 1; \quad C A^d B \neq 0.$$

Proof: From (3) and (4), the predicted output at k^{th} instant can be expressed as:

$$\hat{y}_{(k+1)+(d+1)} = C A^{d+1} (A - K C) x_k + C A^{d+1} K y_k + C A^{d+1} B u_{k-1} + C A^d B u_k. \quad (9)$$

Since the norm of the vector in R.H.S of (9) is a square function of u_k , the optimal control law u_k is given by,

$$u_k = C_c x_k + D_c y_k + a u_{k-1}. \quad (10)$$

When the states of the plant x_k are not available, they are estimated by an observer which is given by,

$$x_{k+1}^c = A x_k^c + B u_{k-1-d} + K (y_k - C x_k^c). \quad (11)$$

Replacing x_k by x_k^c , in (10) gives the control law u_k in (7b). \square

B. DESIGN OF FILTERS (L)

Lemma 2: The optimum filter $(1+L)$, which minimises the variance of \bar{v} (input to the Δ-M), is given by:

$$x_{k+1}^l = A_l x_k^l + B_l u_{k-1} + K_l (u_k - a u_{k-1}), \quad (12a)$$

$$v_k = C_l x_k^l + (u_k - a u_{k-1}), \quad (12b)$$

where,

$$A_l = A + K_l C_l, \quad B_l = B, \quad C_l = -(C_c + D_c C). \quad (13)$$

The Kalman gain K_l is computed from

$$\tilde{x}_{k+1} = \tilde{A} \tilde{x}_k + \tilde{w}_k, \quad (14a)$$

$$\tilde{y}_k = \tilde{C} \tilde{x}_k + \tilde{v}_k, \quad (14b)$$

where,

$$\tilde{A} = A, \quad \tilde{C} = C_l, \quad (15)$$

and

$$E \left\{ \begin{pmatrix} \tilde{w}_k \\ \tilde{v}_k \end{pmatrix} \begin{pmatrix} \tilde{w}_k^T & \tilde{v}_k^T \end{pmatrix} \right\} = \begin{bmatrix} K K^T & K D_C^T \\ (K D_C^T)^T & (D_C)^2 \end{bmatrix} \sigma_\varepsilon^2. \quad (16)$$

The inverse filter, $(1+L)^{-1}$ is given by:

$$x_{k+1}^i = A x_k^i + B u_{k-1} + K_l \hat{v}_k, \quad (17a)$$

$$\bar{u}_k = -C_l x_l^i + a\bar{u}_{k-1} + \hat{v}_k. \quad (17b)$$

Proof: Consider the controller given in (7b). Since the controller state is (asymptotically) equal to the plant state, (7b) can be rewritten as:

$$u_k = C_c x_k + D_c y_k + a u_{k-1}. \quad (18)$$

Using the plant model (3a) and (3b):

$$u_k = C_c x_k + D_c (C x_k + \varepsilon_k) + a u_{k-1}. \quad (19)$$

Further, (19) can be seen as a system with process noise

$$K \varepsilon_k \sim \mathcal{N}(0, \sigma_\varepsilon^2 K K^T), \quad (20)$$

and measurement noise

$$D_c \varepsilon_k \sim \mathcal{N}(0, \sigma_\varepsilon^2 (D_c)^2). \quad (21)$$

The process noise and measurement noise have cross-covariance given by $\sigma_\varepsilon^2 K D_c^T$.

This system can be rewritten in innovations form as:

$$x_{k+1}^i = A x_k^i + B \bar{u}_{k-1} + K_l \hat{v}_k, \quad (22a)$$

$$\bar{u}_k = (C_c + D_c C) x_k^i + a \bar{u}_{k-1} + \hat{v}_k, \quad (22b)$$

where K_l is the Kalman gain and v_k is the new innovation sequence. Note that, (22a) and (22b) describe the dynamics of the inverse filter $\frac{1}{1+L}$ with input v_k and output \bar{u}_k . The filter $(1+L)$, is therefore given by:

$$x_{k+1}^l = A x_k^l + B u_{k-1} + K_l (u_k - (C_c + D_c C) x_k^l - a u_{k-1}), \quad (23a)$$

$$v_k = u_k - (C_c + D_c C) x_k^l - a u_{k-1}. \quad (23b)$$

This result proves (12a) and (12b). Further, since v_k is an innovations sequence, it is i.i.d with minimal variance. The inverse filter, as stated in (17a), (17b) is given by (22a) and (22b) respectively. This concludes the proof of of Lemma-2. \square

C. DESIGN OF QUANTIZATION GAIN (λ)

Following assumption is made while designing the quantization gain λ .

Assumption 4: Let $v(t)$ be the signal to be Δ -modulated and $V(f)$ denote its spectrum. It is assumed that the signal $v(t)$ is bounded and its spectrum $V(f)$ is band-limited i.e $|v(t)| \leq \mathcal{V}$ that $V(f) = 0, \forall f > \mathcal{B}_v$ where, \mathcal{V} and \mathcal{B}_v are known positive constants [45]. Further it is assumed that the sampling rate of the Δ -M, f_s satisfies the relation

$$\frac{f_s}{2\mathcal{B}_v} > 2^\alpha, \quad (24)$$

where $2\mathcal{B}_v$ is the Nyquist frequency of the signal $v(t)$, α is the over sampling ratio ($\alpha \in \mathbb{R}^1$).

Remark 2: At the demodulation side, the signal-to-noise-and-distortion ratio (SNDR) is significantly improved by doubling the sampling rate f_s .

Remark 3: In standard practice, over sampling rate f_s is chosen such that $\alpha > 5$.

Lemma 3 [45]: Consider the signal $v(t)$ which satisfies assumption-4. Let $\dot{v}(t)$ be its derivative. Then,

$$|\dot{v}(t)| \leq 2\pi \mathcal{B}_v \mathcal{V}, \quad \forall t \in \mathbb{R}^+. \quad (25)$$

Further, let v_k denote the discrete samples of $v(t)$ and f_s is the over-sampling frequency of Δ -M which satisfies (24). Then,

$$|\Delta v_k| \leq \frac{\pi}{2^\alpha} \mathcal{V}, \quad (26)$$

where $\Delta v_k = v_{k+1} - v_k$.

Stability of the proposed control scheme is dependent on the type of the quantiser and the associated gains. The range of the stability margin of the quantiser gain is derived in the following.

Lemma 4: If the quantiser input \bar{v}_k (shown in Figure-2) satisfies the condition, $|\bar{v}_{k+1}| \leq |\bar{v}_k|$, and

$$\sup_{k \geq 0} |\Delta v_k| < \lambda < \infty, \quad (27)$$

then,

$$|\bar{v}_\infty| \leq 2\lambda < \beta, \quad (28)$$

where β is a positive constant and λ is the quantisation gain.

Proof: From Figure-2, the dynamics \bar{v}_k can be expressed as:

$$\begin{aligned} \bar{v}_{k+1} &= v_{k+1} - \hat{v}_{k+1}, \\ &= v_{k+1} - \hat{v}_k - \lambda \operatorname{sgn}(\bar{v}_k), \\ &= v_{k+1} - v_k + \bar{v}_k - \lambda \operatorname{sgn}(\bar{v}_k), \\ &= \bar{v}_k + \Delta v_k - \lambda \operatorname{sgn}(\bar{v}_k). \end{aligned} \quad (29)$$

For a Δ -M, it has been shown that the system is stable if $|\bar{v}_{k+1}| \leq |\bar{v}_k|$ [32]. Let us investigate using (29), the behaviour of the trajectory for two possible cases: $\bar{v}_k > 0$ and $\bar{v}_k \leq 0$.

When $\bar{v}_k > 0$:

$$\begin{aligned} 1\bar{v}_{k+1} &= \Delta v_k + \bar{v}_k - \lambda, \\ &< \bar{v}_k < 2\lambda. \end{aligned}$$

Similarly, when $\bar{v}_k \leq 0$, then:

$$\begin{aligned} 1\bar{v}_{k+1} &= \Delta v_k + \bar{v}_k + \lambda, \\ &> \bar{v}_k > -2\lambda. \end{aligned}$$

From Lemma-3, it can be seen that Δv_k is function of over-sampling frequency. From the above two cases $|\bar{v}_k|$ is bounded such that $|\bar{v}_k| < 2\lambda$. Further, as $k \rightarrow \infty$, \bar{v}_k will be bounded as:

$$|\bar{v}_\infty| \leq 2\lambda < \beta.$$

This completes the proof of Lemma-4. \square

In order to find the bound of \bar{v}_k , the following lemma is presented [45].

Lemma 5 [45]: The closed loop system (5) exhibits quasi-sliding motion from any arbitrary initial value if,

$$\lambda > \frac{\pi \mathcal{V}}{2^n}, \quad (30)$$

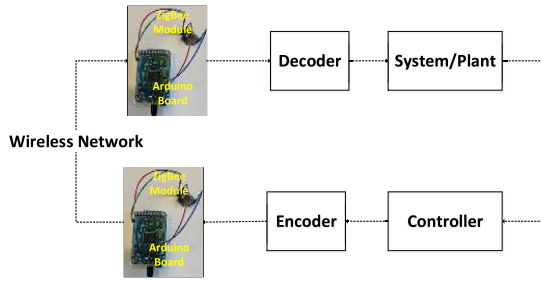


FIGURE 3. Networked control using ZigBee protocol based communication network.

and the switching function \bar{v}_k converges to a boundary layer Ω ($|\bar{v}_k| \leq \Omega$) in finite time.

Remark 4: When \bar{v}_k is bounded such that $|\bar{v}_k| \leq \Omega$, the trajectory of \bar{v}_k will exhibit periodic (zig-zag) behaviour [45]. The period can change depending on the choice of quantisation gain λ and sampling time T_s .

Remark 5: When \bar{v}_k exhibits a zig-zag behaviour, then v_k and \hat{v}_k also exhibit similar behaviour.

Remark 6: If v_k is a constant signal, $\Delta v_k = 0$. Then, from (29),

$$\bar{v}_{k+1} = \bar{v}_k - \lambda \operatorname{sgn}(\bar{v}_{k-1}).$$

By iterating this $k - 1$ times gives

$$\bar{v}_k = \bar{v}_0 - \lambda \sum_{i=0}^{k-1} \operatorname{sgn}(\bar{v}_i).$$

This implies that \bar{v}_k has a period of $k - 1$.

V. RESULTS

The effectiveness of the proposed controller design and implementation framework in a networked environment is demonstrated, considering two examples under the effects of three types of noise (i.e. process noise, measurement noise, and quantisation noise).

A. ZigBee BASED NETWORKED CONTROL SYSTEM

The block diagram of the networked control system is shown in Figure-3 where the network is implemented using the ZigBee protocol. Note that the plant, the controller, the filters and the modulators run in MATLAB/Simulink environment. The modulated signal is transmitted to the Zigbee module in Arduino board 2 from the Zigbee module in Arduino board 1. This board is connected to the computer and with the plant, the controller, the filter and the quantiser. The Zigbee module in Arduino board 2 acts as a hop device in another computer which transmits the signal back to the Zigbee module in Arduino board 1 which then transmits the signal into the demodulator.

This study follows the common practice, reported in the literature, where the wireless communication channel is implemented only on one side of the networked control system to transmit the control signal from the controller node to the system. During the simulations, it is found

TABLE 1. Data packet format.

Name	Length(bytes)
Header	2
Source Port	1
Destination Port	1
Receiver Address	2
Data	μ
Packet End	1

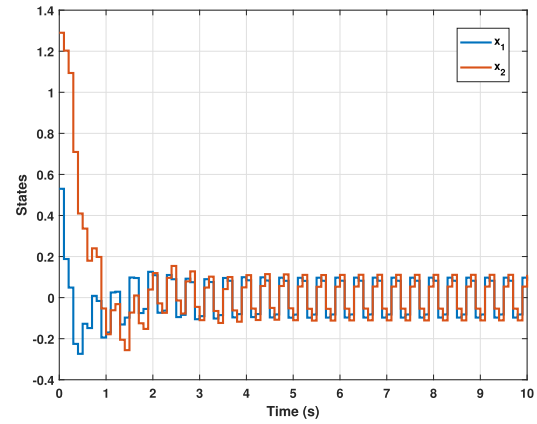


FIGURE 4. Behaviour of states x_1 and x_2 with time for $\lambda = 0.2$ and $\delta_1 = 0.1$.

out that the transmission delay τ of this network equals to 0.02 seconds.

The format of the transmitted data packet using the ZigBee protocol is given in Table-1. Note that the length of the information in the transmitted data packet is assumed to be μ bytes. For each control action, the total length of each data packet (Υ):

$$\Upsilon = \mu + 7.$$

The average transmission time T is given by:

$$T = \frac{8 \times \Upsilon}{\eta}, \quad (31)$$

where η is the average rate of radio transmission of the ZigBee module. The average energy E consumed by each transmission is calculated as:

$$E = v \times i \times T, \quad (32)$$

where v and i denote respectively the operating voltage and the current. The values of the parameters of the Δ -M and ZigBee module used in this study are: $\Upsilon = 8$; $\eta = 250$ K bits/s; $T = 256$ ms; $v = 3.3$ V; $i = 0.3$ A; $E = 25.3$ mJ.

B. NUMERICAL EXAMPLE

The dynamics of the first simulated system is given by [3]:

$$x_{k+1} = A x_k + B_c \bar{u}_k + K \varepsilon_k, \quad (33a)$$

$$y_k = C x_k + \varepsilon_k, \quad (33b)$$

where,

$$A = \begin{bmatrix} 0.9 & -0.1 \\ 0.7 & 0.8 \end{bmatrix}, \quad B = \begin{bmatrix} 0.8 \\ 1.0 \end{bmatrix},$$

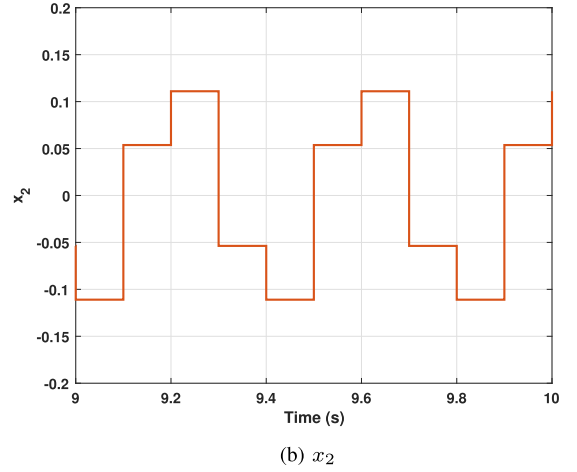
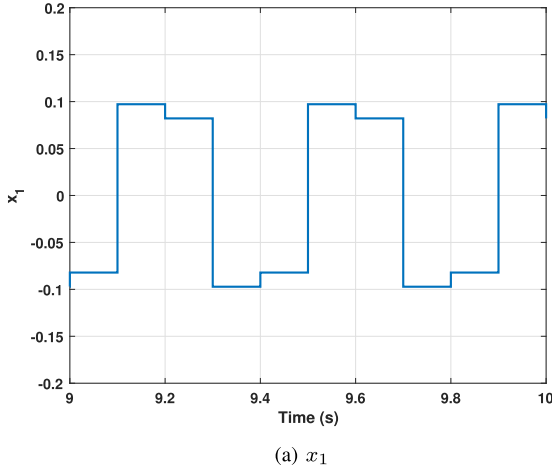


FIGURE 5. Zig-zag behaviour of states.

TABLE 2. Signal-to-noise-ratio of Example 1.

Case	SNR (dB)
Case 1	21.0690 dB
Case 2	21.1904 dB
Case 3	21.4342 dB

$$C = \begin{bmatrix} 1.0 & 0 \end{bmatrix}, \quad K = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}.$$

This linear time-invariant discrete-time system is stable, minimum phase and has relative degree $d = 1$. In this article, results are shown/compared considering three cases.

- 1) Case1: With the Δ -Modulator and Zigbee communication network.
- 2) Case2: With a 2^p -level quantizer.
- 3) Case3: With ideal communication channel.

For case 1, the design of the controller, the filter and the quantisation gain is carried out following the procedures described in the previous sections. Note that the initial conditions x_0 of the the plant is $x_0 = [0.7, 1]^T$. For this example, innovation variance is taken as 0.1, and the response of both the states are shown in Figure-4. Note that quantisation gain and sampling time are chosen to fulfil Lemma-3 and Lemma-4.

Results of case 2 have been shown in [3]. Results of the case 3 is shown in Figure-6 and Figure-7.

From case 1, it can be seen from the Figure-4 that all the states converge into a region Ω within finite time and stay within that region indefinitely. This is further obvious from Figure-5, where the response of the states is of zig-zag nature as described in Lemma-5. This is expected due to the presence of the quantiser. Also the period of the zig-zag motion for both states are 4 (see Figure-5). As discussed in Remark-4, this period is dependent on the value of the quantisation gain and the sampling frequency, and it is, therefore, possible to get different period with other values of quantisation gain and sampling frequency. From, Table-2, it can be observed that signal-to-noise-ratio (SNR) values are similar.

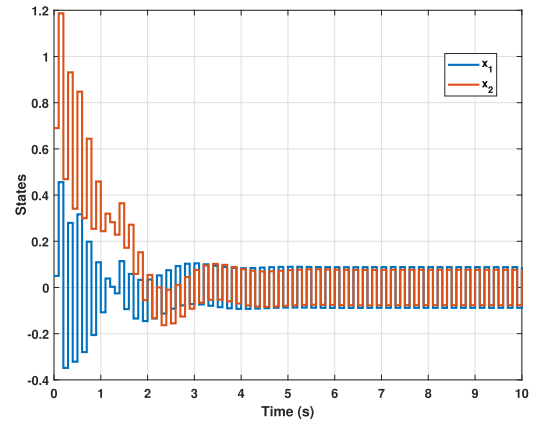


FIGURE 6. Behaviour of states for case 3.

C. FREQUENCY CONTROL OF A SINGLE AREA POWER SYSTEM

The second example considers the load frequency control problem of a single area power system. The block diagram of this system is shown in Fig-8.

The dynamics of this system in state-space form is expressed as [46]:

$$\dot{x}(t) = Fx(t) + Gu(t) + H \Delta P_d, \quad (34a)$$

$$y(t) = Lx(t), \quad (34b)$$

where,

$$F = \begin{bmatrix} -\frac{1}{T_p} & \frac{K_p}{T_p} & 0 & 0 \\ 0 & -\frac{1}{T_t} & \frac{1}{T_t} & 0 \\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} & -\frac{1}{T_g} \\ K_i & 0 & 0 & 0 \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \frac{1}{T_g} \\ 0 \end{bmatrix}$$

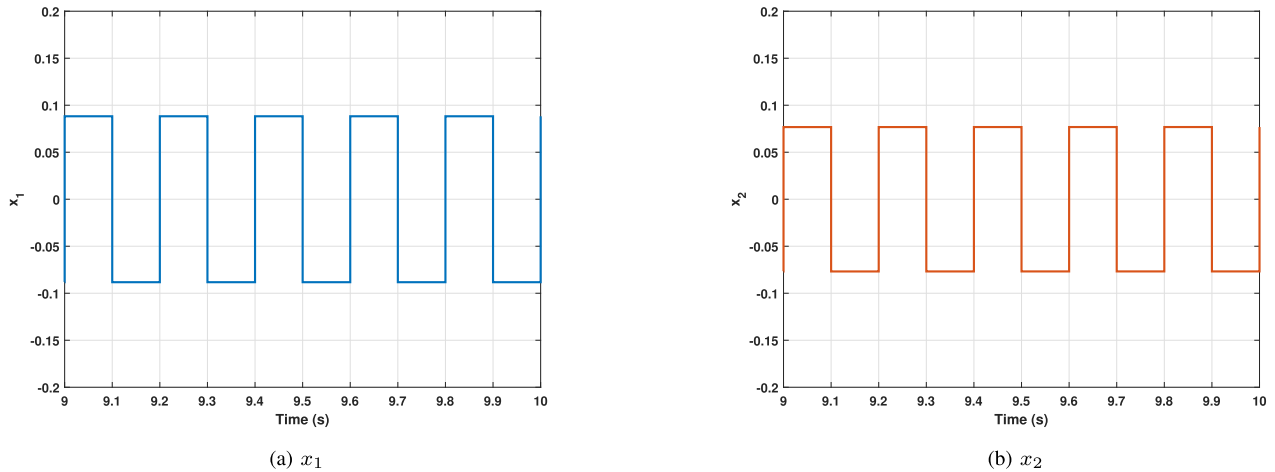


FIGURE 7. Zig-zag behaviour of states for case 3.

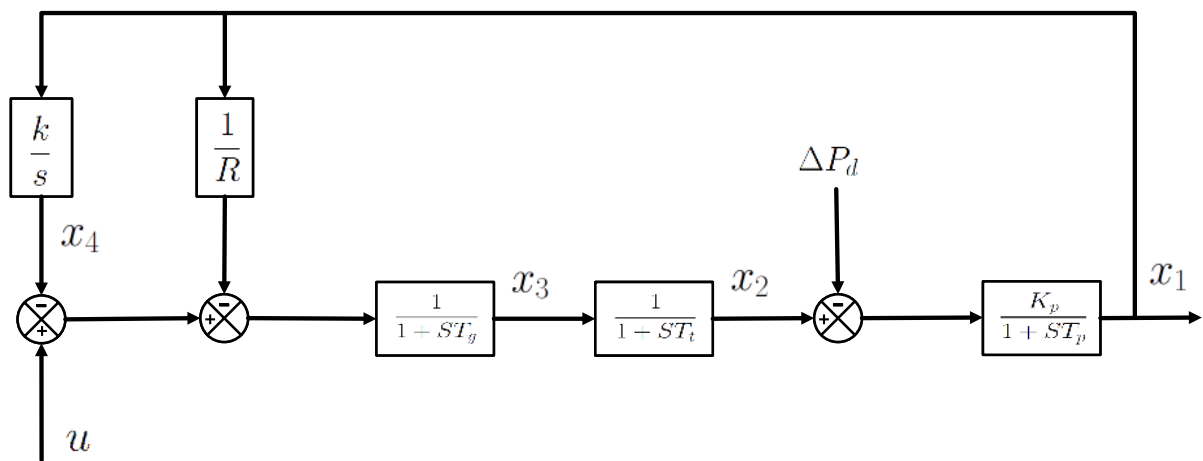


FIGURE 8. The block diagram of a single area power system.

$$H = \begin{bmatrix} -\frac{K_p}{T_p} & 0 & 0 & 0 \end{bmatrix}^T L = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

and ΔP_d is the load disturbance. This is equivalent to the innovations sequence ε_k used in (3).

The discrete equivalent of (34) at sampling time of $\delta_1 = 0.02$ seconds (using Euler discretisation and Lemma-5), is given by:

$$A = \begin{bmatrix} 1 - \frac{\delta_1}{T_p} & \frac{\delta_1 K_p}{T_p} & 0 & 0 \\ 0 & 1 - \frac{\delta_1}{T_t} & \frac{\delta_1}{T_t} & 0 \\ -\frac{\delta_1}{RT_g} & 0 & 1 - \frac{\delta_1}{T_g} & -\frac{\delta_1}{T_g} \\ \delta_1 K_i & 0 & 0 & 1 \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ \frac{\delta_1}{T_g} \\ 0 \end{bmatrix}$$

$$K = \begin{bmatrix} -\frac{\delta_1 K_p}{T_p} & 0 & 0 & 0 \end{bmatrix}^T C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

TABLE 3. Parameters for single area power system.

Parameter	Value	Unit
T_p	20	seconds
T_t	0.3	seconds
T_g	0.08	seconds
K_p	120	Hz/Mw p.u.
K_i	0.6	Hz/Mw p.u.
R	2.4	Hz/Mw p.u.

The values of various parameters of the power system, used in this study, are shown in Table-3. The main objective of the load frequency controller is to ensure zero steady-state error (i.e. the deviation of the frequency from the nominal value should be zero) when the system is subjected to load disturbance. Note that the frequency of the power system is governed by the well known $P-f$ mechanism [47] which states that a mismatch in the real power demand and generation would affect the frequency of the system. For example, when the load in the system increases/decreases, the frequency of the system decreases/increases from its nominal value.

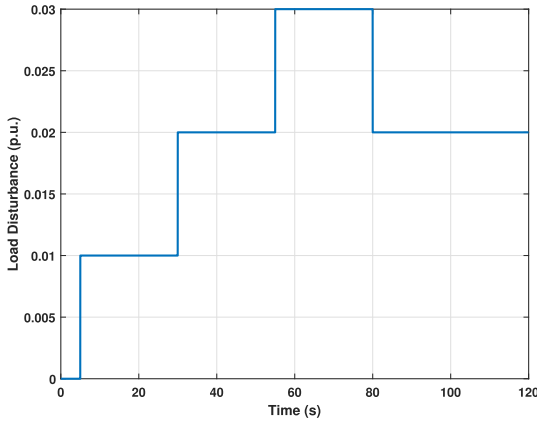


FIGURE 9. Change of load disturbance with time.

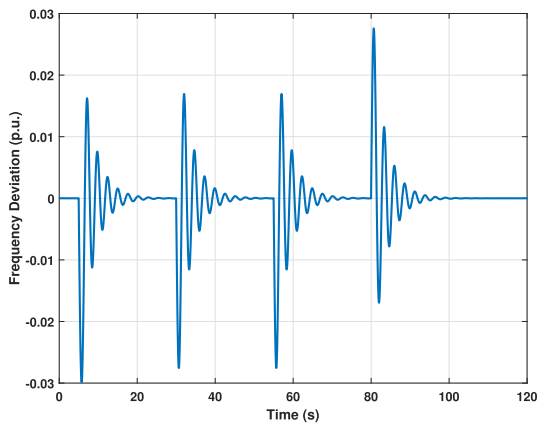


FIGURE 10. Frequency change with load disturbance.

To control these variations in frequency, sophisticated controllers are required, which determine the amount of mechanical input (e.g steam) to reduce the frequency deviations (from nominal value) to zero.

The power system is often subjected to a time-varying load disturbance shown in Figure-9. For case 1, the quantized controller is designed following the procedures discussed in earlier sections. The frequency deviation under load disturbances (Figure-9) is shown in Figure-10 for quantisation gain $\lambda = 0.01$ which is chosen according to Lemma-4.

For case 2, frequency deviation under load disturbances is shown in Figure-11. Case 3 results are shown in [46], hence not included in this manuscript.

It can be seen from the Figure-10 that the output (frequency deviation) converges towards zero within finite time (settling time) under time-varying load disturbances. Also, it is evident that overshoot of the networked system is within industry-accepted limits (i.e. less than 5% p.u.). The output variable, i.e. frequency deviation converge into a region Ω within finite time. From, Table-2, it can be observed that SNR values are similar for all three cases.

From both the examples, it can be seen that proposed architecture works well with the ZigBee protocol based wireless communication channel under various network constraints

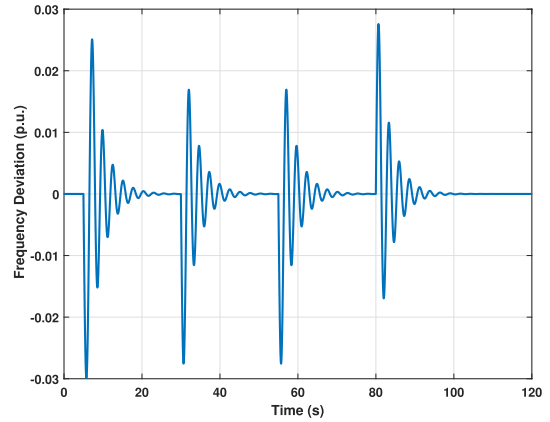


FIGURE 11. Frequency change with nearest neighbour quantiser ($p = 3$).

TABLE 4. Signal-to-noise Ratio of Example 2.

Case	SNR (dB)
Case 1	17.5320 dB
Case 2	17.6352 dB
Case 3	18.1397 dB

like packet loss, transmission delay, random packet delay etc. It is evident that single-bit control is sufficient to keep the system stable and required control is achieved within acceptable time limits.

VI. CONCLUSION

The single-bit output feedback controller is designed for linear networked systems where Δ -M is used as 2-level quantiser. The step size (quantiser gain λ) and oversampling frequency (f_s) are determined which ensures the stability of the Δ -M. The effectiveness of the control strategy is shown considering a real ZigBee protocol based communication network with inherent imperfections such as bit-rate constraints, packet losses, transmission delays and so on using two simulated examples.

FUTURE RECOMMENDATIONS

Due to the current advancements of communication technology, most of the current control schemes need to be reexamined for their effectiveness; if they are to be used for the networked environment. In this study, we focus on designing output-feedback controller for networked systems assuming that the system is linear and single-input-single-output. The extension of the proposed controller to multi-input-multi-output (MIMO) systems and nonlinear systems is immediate future research work. Further, the design of such controllers for event-trigger networked control systems may be a better method from the perspective of bandwidth utilisation and will be investigated in future and be reported separately.

ACKNOWLEDGMENT

The authors would like to acknowledge the support given by The University of Auckland and Prince Sultan University to carry out this research.

REFERENCES

- [1] P. Antsaklis and J. Baillieul, "Special issue on networked control systems," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1421–1423, Sep. 2004.
- [2] U. Premaratne, S. K. Halgamuge, and I. M. Y. Mareels, "Event triggered adaptive differential modulation: A new method for traffic reduction in networked control systems," *IEEE Trans. Autom. Control*, vol. 58, no. 7, pp. 1696–1706, Jul. 2013.
- [3] G. C. Goodwin, M. E. Cea Garrido, A. Feuer, and D. Q. Mayne, "On the use of one bit quantizers in networked control," *Automatica*, vol. 50, no. 4, pp. 1122–1127, Apr. 2014.
- [4] K. Chang Lee and S. Lee, "Remote controller design of networked control system using genetic algorithm," in *Proc. IEEE Int. Symp. Ind. Electron.*, Jun. 2001, pp. 1845–1850.
- [5] T. T. Ho and H. Zhang, "Internet-based tele-manipulation," in *Proc. IEEE Can. Conf. Electr. Comput. Eng.*, May 1999, pp. 1425–1430.
- [6] K. Han, S. Kim, Y. Kim, and J. Kim, "Internet control architecture for Internet-based personal robot," *Auto. Robots*, vol. 10, no. 2, pp. 135–147, 2001.
- [7] S. H. Yang, X. Chen, and J. L. Alty, "Design issues and implementation of Internet-based process control systems," *Control Eng. Pract.*, vol. 11, no. 6, pp. 709–720, Jun. 2003.
- [8] D. Al-Makhles, A. K. Swain, and N. Patel, "Adaptive quantizer for networked control system," in *Proc. Eur. Control Conf. (ECC)*, Jun. 2014, pp. 1404–1409.
- [9] J. H. Braslavsky, R. H. Middleton, and J. S. Freudenberg, "Feedback stabilization over Signal-to-Noise ratio constrained channels," *IEEE Trans. Autom. Control*, vol. 52, no. 8, pp. 1391–1403, Aug. 2007.
- [10] W. P. M. H. Heemels, A. R. Teel, N. van de Wouw, and D. Nešić, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance," *IEEE Trans. Autom. Control*, vol. 55, no. 8, pp. 1781–1796, Aug. 2010.
- [11] W. Zhang, M. S. Branicky, and S. M. Phillips, "Stability of networked control systems," *IEEE Control Syst. Mag.*, vol. 21, no. 1, pp. 84–99, Feb. 2001.
- [12] J. Wu and T. Chen, "Design of networked control systems with packet dropouts," *IEEE Trans. Autom. Control*, vol. 52, no. 7, pp. 1314–1319, Jul. 2007.
- [13] S. Mobayen and F. Tchier, "Synchronization of a class of uncertain chaotic systems with Lipschitz nonlinearities using state-feedback control design: A matrix inequality approach," *Asian J. Control*, vol. 20, no. 1, pp. 71–85, Jan. 2018.
- [14] S. Mobayen, M. J. Yazdanpanah, and V. J. Majd, "A finite-time tracker for nonholonomic systems using recursive singularity-free FTSM," in *Proc. Amer. Control Conf.*, Jun. 2011, pp. 1720–1725.
- [15] S. Mobayen and F. Tchier, "Nonsingular fast terminal sliding-mode stabilizer for a class of uncertain nonlinear systems based on disturbance observer," *Scientia Iranica*, vol. 24, no. 3, pp. 1410–1418, Jun. 2017.
- [16] S. Kiamini, A. Jalilvand, and S. Mobayen, "LMI-based robust control of floating tension-leg platforms with uncertainties and time-delays in offshore wind turbines via T-S fuzzy approach," *Ocean Eng.*, vol. 154, pp. 367–374, Apr. 2018.
- [17] S. Mobayen and G. Pujol-Vázquez, "A robust LMI approach on nonlinear feedback stabilization of continuous state-delay systems with Lipschitzian nonlinearities: Experimental validation," *Iranian J. Sci. Technol., Trans. Mech. Eng.*, vol. 43, no. 3, pp. 549–558, Sep. 2019.
- [18] M. Jafari and S. Mobayen, "Second-order sliding set design for a class of uncertain nonlinear systems with disturbances: An LMI approach," *Math. Comput. Simul.*, vol. 156, pp. 110–125, Feb. 2019.
- [19] M. Afshari, S. Mobayen, R. Hajmohammadi, and D. Baleanu, "Global sliding mode control via linear matrix inequality approach for uncertain chaotic systems with input nonlinearities and multiple delays," *J. Comput. Nonlinear Dyn.*, vol. 13, no. 3, Mar. 2018, Art. no. 031008.
- [20] J. Wang, F. Fang, X. Yi, and Y. Liu, "Dynamic event-triggered fault estimation and sliding mode fault-tolerant control for networked control systems with sensor faults," *Appl. Math. Comput.*, vol. 389, Jan. 2021, Art. no. 125558.
- [21] L. Zhang and G. Guo, "Observer-based adaptive event-triggered sliding mode control of saturated nonlinear networked systems with cyber-attacks," *Inf. Sci.*, vol. 543, pp. 180–201, Jan. 2021.
- [22] L. Hou, D. Chen, and C. He, "Finite-time H_∞ bounded control of networked control systems with mixed delays and stochastic nonlinearities," *Adv. Difference Equ.*, vol. 2020, no. 1, pp. 1–23, Dec. 2020.
- [23] Y. Xu, C. Wang, X. Cai, Y. Li, and L. Xu, "Output-feedback formation tracking control of networked nonholonomic multi-robots with connectivity preservation and collision avoidance," *Neurocomputing*, vol. 414, pp. 267–277, Nov. 2020.
- [24] G. Xiao and F. Liu, "Observer-based cooperative distributed fault-tolerant model predictive control with imperfect network communication and asynchronous measurements," *Int. J. Robust Nonlinear Control*, vol. 30, no. 12, pp. 4531–4549, Aug. 2020.
- [25] W. Sun and Z. Ning, "Quantised output-feedback design for networked control systems using semi-Markov model approach," *Int. J. Syst. Sci.*, vol. 51, no. 9, pp. 1637–1652, Jul. 2020.
- [26] Z. Pan, L. Wang, Y. Wang, and Y. Liu, "Product quantization with dual codebooks for approximate nearest neighbor search," *Neurocomputing*, vol. 401, pp. 59–68, Aug. 2020.
- [27] J. Yang, B. Chen, and S.-T. Xia, "Mean-removed product quantization for large-scale image retrieval," *Neurocomputing*, vol. 406, pp. 77–88, Sep. 2020.
- [28] X. Shao and Y. Shi, "Neural adaptive control for MEMS gyroscope with full-state constraints and quantized input," *IEEE Trans. Ind. Informat.*, vol. 16, no. 10, pp. 6444–6454, Oct. 2020.
- [29] L. Gao, B. Chen, and L. Yu, "Fusion-based FDI attack detection in cyber-physical systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 67, no. 8, pp. 1487–1491, Aug. 2020.
- [30] J. Rodríguez Ramírez and Y. Minami, "Design of neural network quantizers for networked control systems," *Electronics*, vol. 8, no. 3, p. 318, Mar. 2019.
- [31] D. Liberzon, "On stabilization of linear systems with limited information," *IEEE Trans. Autom. Control*, vol. 48, no. 2, pp. 304–307, Feb. 2003.
- [32] D. Almahkles, A. K. Swain, A. Nasiri, and N. Patel, "An adaptive two-level quantizer for networked control systems," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 3, pp. 1084–1091, May 2017.
- [33] D. Almahkles, A. K. Swain, and A. Nasiri, "The dynamic behaviour of data-driven Δ -M and $\Delta\Sigma$ -M in sliding mode control," *Int. J. Control*, vol. 90, no. 11, pp. 2406–2414, 2016.
- [34] S. Haykin, *Communication Systems*. Hoboken, NJ, USA: Wiley, 2000.
- [35] D. J. Almahkles, A. K. Swain, and N. D. Patel, "Stability and performance analysis of bit-stream-based feedback control systems," *IEEE Trans. Ind. Electron.*, vol. 62, no. 7, pp. 4319–4327, Jul. 2015.
- [36] N. Patel, S. Nguang, G. Coghill, and A. Swain, "Online implementation of servo controllers using bit-streams," in *Proc. IEEE Region Conf. (TENCON)*, Nov. 2005, pp. 1–6.
- [37] D. J. Almahkles, N. D. Patel, and A. K. Swain, "Conventional and hybrid bit-stream in real-time system," in *Proc. 11th Workshop Intell. Solutions Embedded Syst. (WISES)*, 2013, pp. 1–6.
- [38] D. Almahkles, N. Pyle, H. Mehrabi, A. Swain, and A. P. Hu, "Single-bit modulator based controller for capacitive power transfer system," in *Proc. IEEE 2nd Annu. Southern Power Electron. Conf. (SPEC)*, Dec. 2016, pp. 1–6.
- [39] D. J. Almahkles, N. D. Patel, and A. K. Swain, "Bit-stream control system: Stability and experimental application," in *Proc. Int. Conf. Appl. Electron. (AE)*, 2013, pp. 1–6.
- [40] C. Canudas-de-Wit, F. Gomez-Estern, and F. R. Rubio, "Delta-modulation coding redesign for feedback-controlled systems," *IEEE Trans. Ind. Electron.*, vol. 56, no. 7, pp. 2684–2696, Jul. 2009.
- [41] F. Gomez-Estern, C. Canudas-de-Wit, and F. R. Rubio, "Adaptive delta modulation in networked controlled systems with bounded disturbances," *IEEE Trans. Autom. Control*, vol. 56, no. 1, pp. 129–134, Jan. 2011.
- [42] M. G. Cea and G. C. Goodwin, "An MPC-based nonlinear quantizer for bit rate constrained networked control problems with application to inner loop power control in WCDMA," in *Proc. 9th IEEE Int. Conf. Control Autom. (ICCA)*, Dec. 2011, pp. 153–158.
- [43] E. Dahlman, S. Parkvall, J. Skold, and P. Beming, *3G Evolution: HSPA and LTE for Mobile Broadband*. New York, NY, USA: Academic, 2010.
- [44] G. C. Goodwin and K. S. Sin, *Adaptive Filtering Prediction and Control*. Chelmsford, MA, USA: Courier Corporation, 2014.
- [45] A. Pilloni, M. Franceschelli, A. Pisano, and E. Usai, "Delta modulation (Δ -M) via second-order sliding-mode control technique," *Control Eng. Pract.*, vol. 92, Nov. 2019, Art. no. 104129.
- [46] A. Y. Sivaramakrishnan, M. V. Hariharan, and M. C. Srisailam, "Design of variable-structure load-frequency controller using pole assignment technique," *Int. J. Control*, vol. 40, no. 3, pp. 487–498, Sep. 1984.
- [47] P. Kundur, *Power System Stability and Control*, vol. 102. New York, NY, USA: McGraw-Hill, 2009.



learning, and networked control systems. He was the recipient of the Fowlds Memorial Prize for the Most Distinguished Student in M.Eng., for the year 2016.

CHATHURA WANIGASEKARA (Graduate Student Member, IEEE) received the B.Sc. (Eng.) degree in electrical and electronic engineering from the University of Peradeniya, Sri Lanka, in 2013, and the M.Eng. degree in electrical and electronic engineering from The University of Auckland, New Zealand, in 2016, where he is currently pursuing the Ph.D. degree in control systems. His current research interests include nonlinear system identification and control, machine



identification and control, machine learning, and big data. He is a member of the Editorial Board of *International Journal of Automation and Control* and *International Journal of Sensors, Wireless Communications and Control*. He is an Associate Editor of IEEE SENSORS JOURNAL.

AKSHYA SWAIN (Senior Member, IEEE) received the B.Sc. (Eng.) and M.E. degrees, in 1985 and 1988, respectively, and the Ph.D. degree in control engineering from The University of Sheffield, in 1996. He has published over 200 papers in various international journals and conferences in a broad range of topics, including control system identification, wireless power transfer, machine learning, and power systems. His current research interests include nonlinear system



Director of the Science and Technology Unit. He is also the Leader of the Renewable Energy Research Team and the laboratory at Prince Sultan University. His research interests include the hardware implementation of control theory, signal processing, networked control systems, and sliding mode.

DHAFAER ALMAKHLES (Senior Member, IEEE) received the B.E. degree in electrical engineering from the King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia, in 2006, and the master's (Hons.) and Ph.D. degrees from The University of Auckland, New Zealand, in 2011 and 2016, respectively. Since 2016, he has been with Prince Sultan University, Saudi Arabia, where he is currently the Chairman of the Communications and Networks Engineering Department and the



filtering. He was the Chief-Editor of the *International Journal of Sensors, Wireless Communications and Control*.

SING KIONG NGUANG (Senior Member, IEEE) received the B.E. and Ph.D. degrees from The University of Newcastle, Australia, in 1992 and 1995, respectively. He has published over 400 papers in various journals and conferences. His research interests include robust nonlinear control and filtering. He was the Chief-Editor of the *International Journal of Sensors, Wireless Communications and Control*.

...