

# Demand and Pricing Effects on the Radio Resource Allocation of Multimedia Communication Systems

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**Abstract**—Over the years, Radio Resource Management has been benchmarked mostly by its technical merits. For a service provider, however, also economics must be considered. When the financial needs of the provider and the satisfaction of the users are considered, common objectives in radio resource management like maximising throughput or meeting various quality constraints, are no longer sufficient. We analyse next generation communication systems by including models of economics, presented in the literature, and reasonable considerations to depict the users/provider relationship in a generalised multimedia environment. In particular, we develop a model of users' satisfaction, in which both requested Quality of Service and price paid are taken into account. The model enables us to investigate how resource allocation dynamics affect operator revenues and to derive some useful insights. The Radio Resource Management can be shown to highly depend on economic considerations. The provider's task to determine the best usage of the network capacity is heavily affected by the users' service demand and their reactions to the pricing policy. Thus, the economic scenario needs to be taken into account to efficiently exploit the constrained radio resource.

## I. INTRODUCTION

The use of multimedia services is expected to increase rapidly. With new devices and applications appearing every so often, the introduction of more flexible applications and forms of information exchange seems inevitable. Soft or adaptive applications adjust their information exchange according to their ability to present data to current link conditions so that radio resources can be conserved. However, the added flexibility also increases the complexity of determining the best usage of the radio resources.

In the recent literature, various researchers have studied the possibility to include micro-economic concepts in the Radio Resource Management (RRM) of wireless networks [1] [2] [3]. The objective of this approach is to maximise the user welfare, according to some model of the perceived Quality of Service (QoS). By means of game theory and utility functions the radio resource is divided among the users to find the greatest overall user satisfaction. With this approach, only the users have, until now, been considered in the game. The provider has merely been an arbitrator (or mediator).

For the provider to have a sustainable business model, the network operation must generate adequate revenue. The operator provides a data delivery service. Hence, the more data that can be delivered, the higher the potential revenue. The pricing and allocation strategies of the provider heavily influence the behaviour of the users. Users who are faced with inadequate QoS or with (unjustifiably) high QoS at a very high price are likely dissatisfied. Conversely, the provider is not

likely to want to sell too cheaply. The total revenue depends on both allocation and pricing policies. Nevertheless, most studies neglect the pricing strategy's effect on user satisfaction. We believe, however, that it is important to also incorporate the economic considerations of the provider in the analysis.

We develop an original framework, where we model the user satisfaction by means of a function describing the user's probability of entering an offered service. It includes the trade-off between paid price and perceived quality and allows a direct evaluation of the revenue. The soft degradation of the users perceived QoS is described by means of a simple utility function. The tariff for each user can be described as a continuous function in a similar way [4]. The price is an independent parameter set by the provider<sup>1</sup>.

In the following we study the rate allocation for a CDMA network. The framework is, however, applicable also to other forms of resource management. The achievable revenues for two pricing strategies and a class of allocation policies are evaluated and compared. We also study the pricing and the resource allocation by highlighting some inter-dependencies between these two aspects.

The main conclusions will justify the introduction of the model and show how the RRM techniques are impacted by the economic scenario and vice versa. In particular the revenue depends on the way in which the QoS is assigned to the users, and the system dimensioning is also affected, whereas price and demand determine different behaviours of the RRM.

The work is organised as follows: in Section II we present the model of users' behaviour, by involving the trade-off between perceived quality and paid price. In Section III we analyse an explanatory allocation scheme, in which the provider assigns the rate to users by exploiting knowledge of their utilities. In Section IV we present the results of this RRM policy, by highlighting points of trade-off and possible improvements. Finally, Section V concludes the paper.

## II. MODEL FOR THE BEHAVIOUR OF NETWORK USERS

The model presented here is developed from the concept of utility function that has been widely used in the recent literature [6] to depict the QoS perceived by the users of a wireless network. The idea is to employ this concept derived from micro-economics [7] to mathematically depict the QoS degree perceived by the users. There are several possibilities to define a QoS evaluation with a numerical value: an example

<sup>1</sup>Note that the pricing strategy in this work relates to the money exchange between user and service provider and, thus, should not be confused with strategies where virtual prices are used in conjunction with power control to improve the assignment of the radio resource [5].

of such kind could be the 5-level mean-opinion-score (MOS) [8], that directly considers the perception of the service and numerically grades the QoS via subjective testing. Different cases of resource assignment are considered and the grades can be easily transformed into a function by means of interpolation between the samples.

We do not investigate strategies to derive utility functions in detail. We simply assume that a utility function  $u(g)$  maps some quality-related parameter  $g$ ,  $0 \leq g < \infty$ , onto an interval of real numbers, discrete or continuous. Note that, in the case of RRM,  $g$  represents the resource of the network given to the users, and could be mono- or multi-dimensional.

Since utilities map the perceived quality, they are increasing functions of the  $g$ -parameters, i.e., it is assumed that the greater the resource allocated to a user, the higher its satisfaction. This implies the following requirements on the derivative of the function  $u(g)$ :

$$\frac{du(g)}{dg} \geq 0, \quad \lim_{g \rightarrow \infty} \frac{du(g)}{dg} = 0, \quad (1)$$

The right part of Equation (1) is known in economics as the law of diminishing marginal utilities. This reflects the phenomenon according to which the improvement of the QoS is vanishing when an already high grade of satisfaction has been reached. That is a realistic assumption for general cases.

In this work we focus only on rate assignment, even though the developed considerations can easily be translated to other kinds of RRM without loss of generality. For this reason  $g$  will be identified with the assigned rate. Note that in the case of bandwidth management, technological limits do not allow to exploit channel assignments larger than a given threshold, depending essentially on the kind of service and the type of terminal. For this reason in the following we will use:

$$\lim_{g \rightarrow \infty} u(g) = l \quad (2)$$

with constant  $l$ , that is a stronger condition than Eq. (1). Equation (2) reflects the fact that there is an upper bound to the perception of the QoS for every kind of service.

There is also a maximum value for  $g$ , called in the next  $g_{max}$ , due to technological constraints. Henceforth, we will consider assignment of the resource only in the range  $0 \leq g \leq g_{max}$ . It is reasonable to assume that in practical cases  $g_{max}$  supplies a utility close to  $l$ . This is equivalent to considering only users able to achieve satisfactory utilities for large  $g$ . Let the minimum achievable utility

$$u_0 \triangleq \min_{g \in [0, g_{max}]} u(g) \quad (3)$$

be the utility of not receiving service, i.e.,  $u_0 = u(0)$ . In the following we will assume  $u_0 = 0$ . Note that both these conditions can be considered due to a perfect Admission Control (AC)<sup>2</sup>.

Moreover, a utility function is often also supposed to have certain properties of regularity, which usually include

<sup>2</sup>In practice, we are assuming that the AC is actually blocking the users with low values of the utility even for  $g$  close to  $g_{max}$  and this decision is error-free, i.e., there are no admission errors that cause call dropping or degradation of the service of already connected users. In these cases the utility could go below  $u(0)$ , being an interruption of the service more annoying than a block in admission.

continuous differentiability, at least piece-wise. In particular, when this is verified for every value of  $g$ , we speak of *elastic traffic* [9]. Note that this property applied to (1) and (2) implies concavity of  $u(g)$  at least for  $g$  greater than a given value, i.e.:

$$\exists g_c : u''(g) < 0, \quad \forall g \geq g_c \quad (4)$$

The exact behaviour of the utility depends on the kind of multimedia traffic we are assigning to the users. For the simplest kind of service, e.g., GSM voice-like calls, it is commonly assumed that the quality degree of the service is on/off, i.e.,  $u(g)$  is bound to have only two values, which mean complete satisfaction or dissatisfaction for the user. This is not true when next-generation services like data transfer or audio/video streaming are taken into account. These services can be considered elastic traffic, since the services themselves allow different degrees of perceived quality according to the assigned rate, with a soft degradation from the best possible choice to the minimum acceptable quality. Therefore, we consider continuous functions to model the utility for the users.

One of the goals of the RRM is to achieve a good users' welfare, considered as an aggregate of their utilities, subject to feasibility constraints. In the case of rate allocation, the main constraint is the limited capacity of the network. However, it seems unrealistic to measure only the welfare without taking into account the role of pricing. The first reason is that the operator will not provide the service if the revenue coming from the users is insufficient. On the other hand, the perception of the service for the users is not always the same if the price is changed: in practice, users are satisfied with the service if both quality and price paid are considered as acceptable.

We propose to take this effect into account by defining an *acceptance probability* for every user that requests service. Note that this concept was not strictly necessary for the GSM-like services, in which the QoS can be assumed equal for each admitted users and the price fixed a priori (so that the QoS metrics are usually assumed to be the probability of not achieve the desired Signal-to-Interference Ratio or to have the connection refused by the Admission Controller).

We can mathematically model it by considering a utility function  $u(g)$ , as previously defined, to represent the QoS. The price could also be represented by a function  $p(g)$  (in general, dependent of the rate) for which no particular assumptions are made, even though it seems reasonable to require that  $p'(g) \geq 0$ . Thus, from the above discussion we can suppose to assign to each user an acceptance probability  $A(u, p)$ , for which we emphasise the dependence on the QoS (through the utility  $u$ ) and the paid price  $p$ . In fact, this probability has to increase for increasing utility and decreasing price. In more detail  $A(u, p)$  should satisfy:

$$\frac{\partial A}{\partial u} \geq 0, \quad \frac{\partial A}{\partial p} \leq 0 \quad (5)$$

$$\forall p > 0, \quad \lim_{u \rightarrow 0} A(u, p) = 0, \quad \lim_{u \rightarrow \infty} A(u, p) = 1 \quad (6)$$

$$\forall u > 0, \quad \lim_{p \rightarrow 0} A(u, p) = 1, \quad \lim_{p \rightarrow \infty} A(u, p) = 0 \quad (7)$$

where the second part of relationship (6) should be intended as more due to the duality between utility and price, than as in a real operative sense, because an infinite utility is not reachable, see Eq. (2). The values of  $A(0, 0)$  and  $A(\infty, \infty)$  are arbitrary, as the former is the acceptance probability of a blocked user

(that is not admitted, regardless of its value of  $A$ ), whereas the latter represents a case that never occurs in practical systems, due to limited utility. A choice that can assure the validity of conditions (6) and (7) is:

$$A(u, p) \triangleq 1 - e^{-C \cdot u^\mu \cdot p^{-\epsilon}} \quad (8)$$

with  $C, \mu, \epsilon$ , being appropriate positive constants.

The choice of this particular function is related to the Cobb-Douglas demand curves [7], that are widely used in economics. If we consider a high number of users in the system, each of them with a very low probability to have access to the system ( $C$  close to 0), it is then true that  $A$  tends to the demand for the access, i.e.,

$$\begin{aligned} A(p) &\sim d(p) \propto p^{-\epsilon} \quad \text{for given } u, \\ A(u) &\sim d(u) \propto u^\mu \quad \text{for given } p. \end{aligned}$$

However, the conclusions we obtain are quite general and do not depend on this particular choice: they are valid for every function that satisfies Equations (5)-(7).

With the probability  $A$  we can model the behaviour of users in a *centralised* resource assignment scheme in which the only choice left to the users is whether they want to accept the service or not. In this case the revenue is determined as:

$$R = \sum_{i=1}^N p_i A(u_i, p_i), \quad (9)$$

where the users are considered to be numbered from 1 to  $N$  and their relative utility and price to be  $u_i$  and  $p_i$  respectively.

### III. STRATEGIES OF RATE ALLOCATION AND PRICING

In this Section we present a rate allocation algorithm and a pricing scheme for the purpose of evaluating the above model. We refer in particular to a CDMA cellular system, with an interference-limited soft capacity.

We assume that the provider adopts a centralised and greedy rate assignment strategy. Further, we assume that the provider and, consequently, the resource manager know the relation  $g \rightarrow u_i(g)$  for every user  $i$ . Based on this information, the provider tries to choose a value for the rate  $g$  that might satisfy the user, being at the same time respectful of the limited amount of bandwidth that can be allocated. After the decision, the user can decide whether or not to accept the assigned value, according to the acceptance probability previously defined.

In more detail, the utilities are modelled as sigmoid curves, since they are well-known functions often used to describe QoS perception [2] [9]. We consider the following analytic expression for these curves:

$$u(g) \triangleq \frac{(g/K)^\zeta}{1 + (g/K)^\zeta}, \quad (10)$$

where  $\zeta \geq 2$  and  $K > 0$  are tunable parameter, according to which different users' utilities are differentiated. It is also assumed that the utilities are normalised to their upper limit, i.e., the asymptotic value of  $u(g)$  for large  $g$  (indicated in Eq. (2) as  $l$ ) is taken to be equal to 1.

We consider a rate allocation strategy based on the derivative of the utility. The role of  $u'(g)$  is to describe the subjective perception of changes in the rate assignment. If  $u'_i(g)$  is close to 0 for  $g \geq g_0$ , there is no point in giving more resource

than  $g_0$  to user  $i$ . The improvements due to increasing of the assignment beyond  $g_0$  can be considered negligible.

The evaluation of the point  $g_0$  after which the incremental utility can be considered close to zero is still a degree of freedom for the provider, and there is a trade-off in its choice. For this reason we model the rate assignment performed by the greedy provider in the following way: a threshold value  $\vartheta > 0$  is determined a priori by the provider, and the rate assignment proposed to each user  $i$ ,  $g_i$ , starts from:

$$g_{i0} \triangleq \max(\{0\} \cup \{g \in ]0, g_{max}] : u'_i(g) \geq \vartheta\}). \quad (11)$$

Note that the threshold  $\vartheta$  numerically translates the general bandwidth management strategy into a single parameter.

According to the sigmoid-shape of the utilities, the greater the value of  $\vartheta$ , the lower the initial rate  $g_{i0}$  proposed to user  $i$ . This implies a trade-off for the provider in choosing  $\vartheta$ . With  $\vartheta \rightarrow 0$  the provider tries to supply users with very high utility. However, due to limitation in the total resource, such an assignment may prevent other users from entering the system, as there is no bandwidth left. On the other hand, too low rates, obtained with high threshold, save capacity for other users but decrease the acceptance probability.

The soft capacity of a CDMA system is taken into account by considering the feasibility of the rate assignment in an interference-limited system. We translate the rate to signal-to-interference ratio (SIR) by means of the well-known Shannon's capacity formula:

$$\gamma_{t,i} = 2^{g_i/W} - 1, \quad (12)$$

where  $\gamma_{t,i}$  is the target SIR for user  $i$ . The rate values  $g_i$  are determined for one user at time, by assuming that the allocation for user  $i$  happens when every user  $j$ ,  $1 \leq j < i$  has been considered. For each user  $i$ , the rate is initialised to the value  $g_{i0}$  determined by Eq. (11). If the set of the target SIRs for all users is feasible, this rate assignment is kept. Else, the new user's SIR is decreased in steps of 1dB, until the system is feasible. Note that the assignments for already allocated users are not changed. Finally, the rate  $g_i$  corresponding to the SIR according to Eq. (12) is assigned to the user  $i$ . Then, both utility  $u(g_i)$  and price  $p(g_i)$  are determined and the user acceptance of the service is evaluated by means of the probability  $A(u, p)$ .

For what concerns the pricing, one should observe that pricing strategies include a lot of different proposals [10] [11] and it is not clear whether all of them can be considered realistic or not. For the sake of simplicity, in this work we compare two different policies: a *flat price* strategy and a simple usage-based pricing where the price  $p(g)$  is linearly related to  $g$ , i.e.,  $p(g) = kg$ , with a given constant  $k$ . In particular, in the case of flat price, Eq. (9) can be rewritten by replacing  $p_i$  with a constant  $p$ . The effect of pricing is not neglected, as the value of  $p$ , defined a priori, can be subject to change. For linear pricing instead,  $p_i$  can be seen as  $p(g_i)$ , so that different prices are experienced by differently served users. These two strategies can be reasonably assumed to be present in next generation networks, due to their conceptual simplicity that might be appreciated by the users, even though a more complicated pricing may turn out to be better for both the users and the provider. However, the model can be applied to every fixed pricing relationship known by the users a priori.

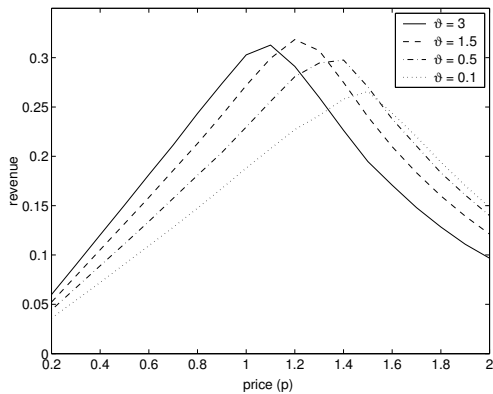


Fig. 1. Provider revenue for flat price, 160 users, as a function of the price

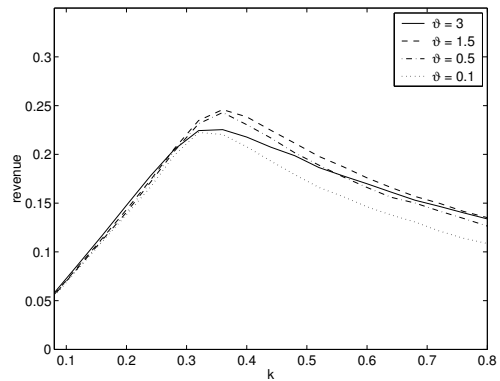


Fig. 2. Provider revenue for price  $p(g) = kg$ , 160 users, as a function of  $k$

TABLE I  
LIST OF PARAMETERS OF SIMULATION SCENARIO

Parameter (symbol)	value
number of cells	19
bandwidth ( $W$ )	20 rate units
max assignable rate ( $g_{max}$ )	8 rate units
cell radius ( $d$ )	500 m
gain at 1 m ( $A$ )	-28dB
Hata path loss exponent ( $\alpha$ )	3.5
shadowing parameter ( $\sigma$ )	8dB
log-normal correlation downlink	0.5
log-normal correlation distance	75m
mean SNR at cell border	20dB
utility parameter $\zeta$	$2 \div 20$
utility parameter $K$	$0.2 \div 4.2$
acceptance prob. parameter $C$	0.05
acceptance prob. parameter $\mu$	2
acceptance prob. parameter $\epsilon$	4

#### IV. RESULTS

In the following, we consider rate assignment in a CDMA system. Table I shows the parameters of the simulation scenario. In particular, note that the users are uniformly distributed among the hexagonal cells, that are “wrapped around” so that no border effect is introduced.

The first results presented investigate how the price affects the revenue. Figure 1 shows the behaviour of the flat price strategy, whereas in Figure 2 the revenue for the usage-based pricing is plotted. Here 160 users have been considered. It is emphasised that there is a pricing which maximises the revenue. The existence of such a value comes from the hypotheses made in Section II. Thus, the price variations adjust the revenue by means of the acceptance probability.

There is also a dependence on the provider choices in assigning the bandwidth to the users. In fact, it should be noted that, besides the price, also the threshold value affects the revenue: both the maximising price and the maximum achievable revenue change if the operator adopts a different threshold  $\vartheta$ . The value of the threshold represents a measure of the QoS given to the users: in general  $u(g_i)$  increases for decreasing  $\vartheta$ , even though different users experience different qualities. Hence, the price and the rate allocation strategy have

to be carefully planned, possibly with a joint analysis.

Figures 3 and 4 show, for flat and usage-based price respectively, the fraction of users admitted into the system. In general, the higher the price, the lower the number of the users that accept the service. The trade-off in the choice of the utility that the provider assigns to each user (captured in the RRM with the threshold value  $\vartheta$ ) implies, however, different behaviours for different thresholds, even for a flat price strategy. For a low price the number of admitted users is constant and corresponds to a saturation of the bandwidth, due to which some users can not be admitted. In this case, the lower the threshold, the fewer the users. A low  $\vartheta$  generally means a high assigned rate. Therefore, few users are admitted in this case, whereas higher values of  $\vartheta$  allow the admission of more users, though with lower quality. In Figure 3, this phenomenon is reversed at high price, i.e., there are more users for low values of  $\vartheta$ . This happens because the decrease in the number of users is more consistent for threshold values that assign a poorer quality to the users. This does not occur in the usage-based price strategy since the price for low quality users is still lower. Finally, note that the maximum revenue is obtained approximately on the edge of saturation of the capacity, i.e., where the number of users starts to decrease. In this point, in fact, the effect of the decrease in admitted users overcomes the revenue increase due to higher price.

The curves presented in Figures 1–4 can be shown to be highly sensitive to the number of users. Thus, the demand effect on the revenue and pricing has to be underlined. In fact, the revenue that can be earned, and also its maximising price, heavily depends on the number of users that are requesting the service. Hence, we introduced also Figure 5, that shows how system performance depends on the load. Here is highlighted that the revenue saturates for high demand, due to the constrained capacity. In this case, the resource is fully allocated and the fraction of users admitted decreases linearly with the load from this point on (there is an almost constant number of users admitted, whereas the demand is increasing). On the other hand, if the resource is sufficient for the users demanding the service, the admission rate is approximately constant, being the fraction of users that consider the price acceptable for the perceived QoS. Note that in this range the revenue is approximately directly proportional to the demand, as the fraction of the used capacity increases (until it saturates). This difference implies that two different states for the system can be identified, i.e., high and low demand, that correspond

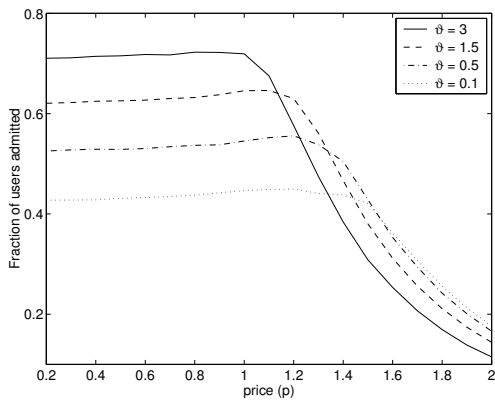


Fig. 3. Admission rate for flat price, 160 users, as a function of the price

to complete usage or not of the resource. The behaviour of the RRM and the values of the economic quantities are very different in these two cases, and also the load value, in which the separation occurs, is floating with respect to  $k$  and  $\vartheta$ ; therefore, an operator has to carefully plan the network by taking into account these aspects.

## V. CONCLUSIONS

The provider's task of determining the best usage of the network resources, so as to maximise its profit, does not have a trivial solution. The revenue depends on users' response to both radio resource management and pricing. To better take this into account, we introduced the *Acceptance-probability* model, which considers the joint effect of user utility and price. The model framework enables us to include economic considerations in the study of communications systems.

In this work the model was applied to a simple CDMA-like system. It was shown that similar RRM strategies behave differently when economic parameters like pricing strategies and user demand are taken into account. Thus, to efficiently control the performance of the system, the selection and tuning of RRM and pricing policies should be addressed jointly.

There is a trade-off between quality and price: users will not accept a high quality if they think it is too expensive. In fact, over-assignment can be considered wasteful: it hardly improves the revenue, but markedly deteriorates the admission rate. Appropriately setting the pricing strategy is crucial for the provider to have a satisfactory revenue. Too high prices drive customers away (in the long run, likely to competitors), with low or no revenue as result. Too low prices can easily be afforded by the users, but also yield very little revenue. Price variations also affect the expected number of users in the system; hence, they have to be considered in system dimensioning. Future research on self-tunable prices, obtained through negotiations [12] between the users and the provider, can give further insight on this matter. Finally, results indicated that the system performance setup can be addressed better if good estimates of the load, i.e. the demand, are available.

To sum up, the proposed model allows useful insights about the RRM strategy to be gained. The economic aspects of RRM should not be neglected, for they not only affect performance, but also require several strategic choices to be made. It is imperative for the provider to take into account these aspects; thus, our model can be useful to gain understanding of them and improve the RRM in real systems.

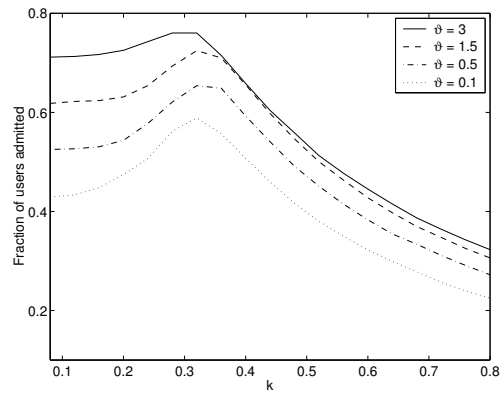


Fig. 4. Admission rate for price  $p(g) = kg$ , 160 users, as a function of  $k$

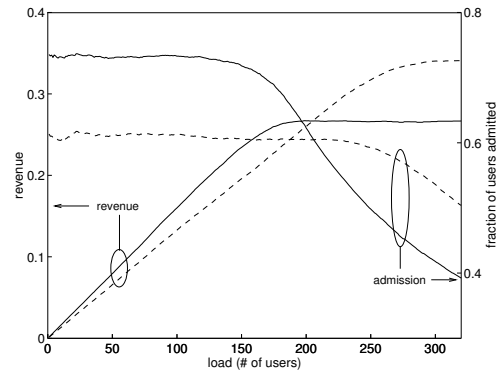


Fig. 5. Provider revenue and admission rate for price  $p(g) = kg$ , with  $k = 0.36$ ,  $\vartheta = 1.5$  (solid) and  $k = 0.44$ ,  $\vartheta = 3$  (dashed), as a function of load.

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