Demand commitment bargaining

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DEMAND COMMITMENT BARGAINING:
- THE CASE OF APEX GAMES

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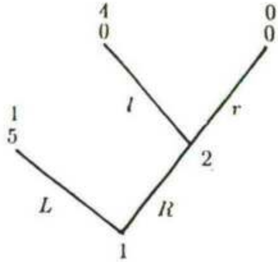
better name, we call *credible* subgame perfect equilibrium (CSPE). In the next section, we show that credibility leads to a drastic reduction in the number of equilibria.

To motivate our refinement, consider the following game.



When it is his turn to move, player 2 is indifferent between l and r. Player 1's optimal strategy therefore depends on his belief about how player will play in the face of this indifference. Harsanyi and Selten (1998) argue that a rational player should randomize equally among all alternatives over which he is indifferent. If player 1 believes that player 2 will choose l and r with equal probability, then player 1 should choose R, and player 2 will obtain a payoff of 0. However, if player 1 believes that player 2 will play r, then player 1 will choose L and player 2 will obtain a payoff of 5. The threat by player 2 to play r is completely rational, since player 2 is indifferent between l and r. Moreover, the threat, if believed, yields player 2 a higher payoff. In our view, therefore, player 2 will indeed threaten to play r and player 1 has every reason to believe that player 2 will carry out his threat, so player 1 should choose L.

The logic above is quite different from that underlying forward induction arguments. The usual forward induction argument (see, for example Van Damme (1989)) is that player 1, by his action, can indicate his desire to play a particular subgame perfect equilibrium in the subgame that follows this action. In demand commitment games, as well as the game above, the forward induction logic is not compelling: although player 1 may indicate his desire to play a particular subgame perfect strategy combination, he has no

DEMAND COMMITMENT BARGAINING:
- The Case of Apex Games*

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Abstract

An apex game is a bargaining situation in which there is one major (apex) player and n "minor" players. The only profitable coalitions contain either the apex player and any one of the minor players or else all of the minor players. The demand commitment model is a bargaining procedure, i.e. an extensive form game. This paper investigates the payoffs that result (as subgame perfect outcomes) for apex games when players use the demand commitment bargaining procedure. We show that whenever the apex player has the first move he forms a coalition with a minor player and obtains the fraction $(n-1)/n$ of the coalition's value while his (minor-player) partner obtains the remaining $1/n$. When a minor player has the first move he either forms a coalition with the apex player (and obtains $1/n$) or else forms a coalition with all of the remaining minor players. When this minor-player coalition forms there are many subgame perfect payoff distributions. A refinement of subgame perfection is proposed and is shown to select a unique payoff distribution ($1/n$ for each minor player) for the minor-player coalition.

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1 Introduction

In this paper we present a noncooperative model for bargaining in characteristic function games, and explore its implications for the class of apex games. An *apex game* is a bargaining situation in which there is one major (apex) player and n minor players; only the coalitions consisting of the apex player and any one of the minor players, and the coalition consisting of all of the minor players are profitable, and these coalitions have equal value. (It is convenient, and involves no loss of generality, to normalize payoffs so that the value of each of these coalitions is n, the number of minor players.) In our model, bargaining takes place by means of a procedure which we call *demand commitment*.[1] Each player in turn may set a price (a payoff demand, expressed in utility terms) for his participation in any coalition. Having set his price a player can form a coalition if his partners in the coalition have already named their prices and the coalition can afford these prices. The game terminates as soon as one coalition is formed; players not belonging to the "successful" coalition receive a payoff of zero.

Our purpose in this paper is to investigate the effects of the existence of alternative coalitions on bargaining outcomes. The demand commitment model is well-suited to this purpose: In this model, each player must make a trade-off between the higher payoff he might obtain by setting a higher price and the possibility of pricing himself out of the market entirely. We focus on apex games since they represent the simplest situations in which competition for partners is relevant.[2]

Considerable attention has been focused on providing extensive form models for bargaining in characteristic function games. The seminal work in this direction is that of Nash (1953) on the two-person simple bargaining problem. Nash argued for the importance of modeling the bargaining process explicitly by means of an extensive form

[1] The demand commitment procedure was first suggested by Reinhard Selten. We are grateful to him stimulating us to carry out the research described in this paper.

[2] An apex game can be viewed as a unanimity game played among the minor players augmented with a single outside option for each player (that of forming a coalition with the apex player).

game, and viewed cooperative and noncooperative approaches as complementary. The cooperative theory avoids any specification of the bargaining process, hence, it achieves great generality, is tractable and easy to apply. However, it is typically difficult to assess (the reasonableness of) the intuitions (axioms) underlying a cooperative solution, or the range of situations in which it should apply, without having a specific bargaining procedure in mind. A cooperative solution is in serious doubt if it is not compatible with some sensible noncooperative bargaining procedure. Conversely, a noncooperative bargaining procedure is not likely to be sensible if its outcomes are not supported by (the intuition of) some cooperative solution. Seminal papers on extensive form models for situations with many players and many potential coalitions (i.e., characteristic function games) are those of Harsanyi (1971) and Selten (1981). These papers exemplify the insight that can be gained by reinterpreting cooperative solution concepts as equilibria of noncooperative games. As is the case in many other models where time pressure plays no role, the models of Harsanyi and Selten are plagued with a mulitude of equilibria; Harsanyi and Selten use equilibrium selection to argue for "sensible" outcomes.

More recent papers in this area include those of Binmore (1985) and Chatterjee et al (1990). Both papers can be viewed as offspring of a mating between Rubinstein's noncooperative model of the two-person simple bargaining problem and Selten's noncooperative model of characteristic function games. Rubinstein (1982) describes a model in which two players alternate in making offers on the division of their total payoff, until agreement is reached. Pressure to reach agreement comes because players discount future payoffs. Selten (1981) describes a model in which many players bargain over which coalition to form and the division of payoffs within the coalitions. In Selten's model (the proposal-making model) players (sequentially) propose coalitions and feasible divisions of the payoffs obtainable in these coalitions. Binmore's "telephone bargaining" model and Chatterjee et al's model explore the effects of time pressure in the proposal-making model. In Section 7 we discuss the results obtained by Chatterjee et al, for now it suffices to remark that Binmore (1985) argues that an instability is built into this model since it imposes constraints that players would like to violate.

In our model and Binmore's (1985) "market demand" model players make demands and not proposals. This distinction is important because commitment to a demand means that bargaining takes place in many different coalitions simultaneously (a player's price is a demand for payoff which is uniform across *all* of his potential coalitions while a proposal specifies payoffs for only one coalition). The importance of this difference can be seen by contrasting the outcomes of Binmore's "telephone bargaining" and his "market demand" model.

Our model is also different in a second important way: competition among coalitions for players and not time pressure is the driving force in determining the division of payoffs and consequently in determining which coalitions are going to form. See Section 7 for further discussion of this issue.

Demand commitment games are extensive form games with perfect information. As is well-known, finite extensive form games with perfect information that have different payoffs for each player at different terminal nodes, have unique subgame perfect equilibria. Although demand commitment games are games with perfect information and finite horizons, they are not finite since each player's payoff demand can be any nonnegative real number. Moreover, for each player there will be many terminal nodes that yield the player the same payoff. As a consequence, it will typically be the case that players are faced with choices among which *they* (but not the other players) are indifferent and in such cases multiplicity of subgame perfect equilibria may result.

We show that such multiplicity does not arise when the apex player moves first or when there are only two minor players: in the first instance the apex player forms a coalition with one of the minor players and, in the second instance, the first two players to move — whether or not one of them is the apex player — form a coalition. In either case successful[3] minor players each obtain a payoff of 1 and the apex player, when successful,

[3] "Successful" means that the player is a member of the coalition that forms.

obtains a payoff of $n-1$. In cases in which there are at least three minor players and a minor player moves first, however, one can generate a continuum of subgame perfect equilibrium outcomes by varying players' conjectures about the choices other players will make when faced with choices among which they are indifferent.

In our view, not all conjectures about how players will resolve their indifferences are equally convincing. We introduce, in Section 5, a refinement of subgame perfection, called *credible subgame perfect equilibrium* which is based on the idea that players strategically exploit their indifferences. We assume that a player i can credibly threaten player j to resolve his indifferences in a particular way, if carrying out the threat will not reduce i's payoff, will reduce j's payoff and if j's best response to the threat will increase i's payoff.

In Section 6 we show that allowing credible threats drastically reduces the number of equilibrium outcomes. The only outcomes resulting from credible subgame perfect strategies are: formation of the coalition of all minor players with each minor player obtaining a payoff of 1 and formation of a coalition consisting of the apex player and one of the minor players with the apex player obtaining a payoff of $n-1$ and the minor player obtaining a payoff of 1.

Section 7 concludes the paper with a discussion the relationship between our model and other cooperative and noncooperative models for apex games.

2 Apex Games and Demand Commitment

Let $N_0 = \{0, 1, \dots, n\}$ be the set of players. We call the players in $N = \{1, \dots, n\}$ the *minor* players and call player 0 the *apex* player. The characteristic function of the apex game is given by

$$v(S) = \begin{cases} n \text{ if } S = \{0, j\} \text{ with } j \in N \text{ or } S = N \\ 0 \text{ otherwise} \end{cases}$$

In demand commitment games play proceeds by each player, in turn, setting a price (a demand for payoff in utility terms) for his coalitional participation. Having set his price a player can form a coalition if his partners in the coalition have named their prices and the coalition can afford these prices. If the player doesn't form a coalition, he selects, as the player to have the next move, any player who hasn't announced his price. Before stating the rules more formally, we introduce the following notation.

Let p_j denote the price demanded by player j. The coalition S is *feasible* for i if $v(S) = n$ (i.e., S is a profitable coalition), S contains player i, every player in $j \in S$ has already announced his price, and $\sum_{j \in S} p_j \leq v(S)$.

The demand commitment game is played according to the following rules. Nature randomly selects a player from N_0 to move first. This player announces a demand (a nonnegative real number) which is then made known to all players and selects a player (any player who has not yet moved) to move next. When it is player i's turn to move, he announces a demand and either forms a feasible coalition or selects a player to move next. (If every other player has already moved and the last player to announce a demand does not form a coalition, the game ends and each player obtains a payoff of 0.) If a feasible coalition does form the game ends: each player in the coalition is paid his demand while players who are left out obtain nothing.

The demand commitment game is a finite length extensive form game with perfect information and continuous action spaces. (We prefer to work with the continuum rather than with discrete money units to avoid making additional case distinctions.) The solution concept we will employ is subgame perfect equilibrium in pure strategies, i.e. we will be looking for a pure strategy profile that induces a Nash equilibrium in every subgame. As this concept has by now become a standard element in the economist's toolkit there is no need to be more formal at this point. There is just one special feature of SPE in extensive form games with continuous action spaces of which the reader should be aware. This is the fact that in such a game not every SPE of a subgame can be extended to an SPE of the overall game because ties cannot always be broken in an arbitrary way. Requiring a strategy combination to be an SPE of the overall game may constrain a player to choose one action (consistent with one SPE of the subgame) rather than another action (consistent with a different SPE of the subgame) in situations where the player making the decision is indifferent between the two actions, this in order to ensure that players moving earlier in the game indeed have best responses. The simple analysis of the 3-person 3-cakes problem in Section 3 suffices to acquaint the reader with this peculiar property of SPEa in continuum games. For general results on SPEa in continuum games the reader is referred to Hellwig et al (1990).

3 The Three Player / Three Equal Cakes Problem

We begin with the simple case of an apex game with two minor player to familiarize the reader with the basic steps of the general argument. The apex game with two minor players is special because the apex player is not in a distinguished role — any two players can form a profitable coalition. In the literature, such bargaining situations have become known as 3-player/3-cakes problems (see for instance Binmore (1985)). Since apex games with two minor players are entirely symmetric, we deviate from our general notation and refer to the players simply as 1, 2 and 3. For this case the characteristic function is given by

$$v(S) = \begin{cases} 2 & \text{if } |S| = 2 \\ 0 & \text{otherwise.} \end{cases}$$

We will show that the game has a unique[4] SPE and that this SPE results in the outcome

[4]There is a recurring nonuniqueness which we ignore and will continue to ignore: the nonuniqueness which occurs when a player must choose among otherwise indistinguishable players. In this case the

where the two first moving players divide their cake equally.

The SPE is found by backwards induction, so w.l.o.g. let us assume that the players 1 and 2 have already moved and that they have demanded prices p_1, resp. p_2. The optimal choice for player 3 is to form a coalition with the player that asked the lowest price, at least when $\min(p_1, p_2) < 2$. If both players have demanded the same price, he may pick either one.

Next, consider the decision problem faced by player 2 after player 1 has demanded p_1. Assume $p_1 > 0$. If player 2 accepts[5] the demand of player 1 his payoff is $2 - p_1$. If player 2 rejects this demand, then he can (in SPE) count on the cooperation of player 3 only if he demands $p_2 \leq p_1$, hence accepting p_1 is strictly optimal if $2 - p_1 > p_1$, or $p_1 < 1$. If $p_1 > 1$, then player 2 is sure that player 3 will accept his demand if $p_2 < p_1$, hence player 2 can almost (but not quite) guarantee p_1. We see that, if it would be the case that in the subgame where both player 1 and player 2 have demanded p_1 with $p_1 > 1$, player 3 would form the coalition with player 1, then player 2 would not have a best response after player 1 has demanded $p_1 > 1$. Hence, to guarantee existence of an SPE, player 3 has to break ties in the favor of player 2 if player 1 demands more than 1. Consequently, in any SPE, if player 1 demands $p_1 > 1$, then player 2 puts $p_2 = p_1$ and player 3 forms the coalition with player 2. By the same argument one sees that the subgame with $p_1 = 1$ admits two SPEa: player 2 accepts the demand of player 1 or player 2 matches player 1's demand and 3 forms the coalition with 2.

Finally, consider player 1's decision problem. The above analysis has shown that player 1 will end up with zero if he demands $p_1 > 1$ and that any demand $p_1 < 1$ will be accepted by player 2. Again we have that player 1 does not have a best response unless player 2 breaks the tie in 1's favor if $p_1 = 1$. Hence, player 2 should break the ties in

first player must select one of two indistinguishable players, in subsequent sections the apex player must select one among several indistinguishable minor players.

[5]By "player i accepts the demand of player j" we mean that player i accepts the residual payoff in the coalition $\{i, j\}$ as his price and forms the coalition $\{i, j\}$.

this way, and we have shown

Theorem 1 . *For the 3-person apex game, the demand commitment model has a unique SPE. The outcome of this equilibrium is that the two first moving players agree on an equal split of their cake.*

4 The n-player Apex Game – Subgame Perfect Equilibria

In this section we derive two main results for apex games with at least 4 players

(i) if the apex player starts the game there is a unique SPE outcome: the apex player demands $n-1$ from some minor player and the latter accepts, (i.e. he demands 1 and forms the coalition with the apex player)

(ii) if a minor player starts the game there are infinitely many SPE outcomes and the first moving minor player may obtain any payoff between 1 and n (hence, he may actually obtain the entire cake).

We proceed by proving a series of lemmas about SPEa in the various continuation games that may arise. It will be notationally convenient (and without loss of generality) to assume that the minor players have to move in the order $1, 2, \ldots, n$. (Alternatively one simply doesn't fix the players' names in advance, player i is the ith moving minor player.) For a vector $p = (p_1, \ldots, p_{i-1}) \in \mathbf{R}_+^{i-1}$ with $i \leq n$ it is also convenient to write

$$P_i = \begin{cases} 0 & \text{if } i = 1 \\ \sum_{j=1}^{i-1} p_j & \text{if } i > 1 \end{cases} \tag{4.1}$$

$$p^i = \begin{cases} \infty & \text{if } i = 1 \\ \min_{j<i} p_j & \text{if } i > 1 \end{cases} \tag{4.2}$$

$$r_i(p) = (n - P_i)\,/(n - i + 1) \tag{4.3}$$

Note that $r_i(p)$ is the per capita remainder in the coalition N of all minor players when player i has to move. p^i is the minimal demand of all minor players that moved before minor player i: If i wants the cooperation of the apex player then he can demand at most p^i. Intuitively it is therefore clear that player i's optimal decision will depend on p^i and $r_i(p)$. We will analyse only those subgames with $0 \leq p_0 \leq n$, $0 \leq p_i \leq n$ (for all $i \in N$) and $r_n(p) \geq 0$, this to avoid having to make uninteresting case distinctions. The reader can easily verify that subgames that do not satisfy these restrictions will not be reached by any SPE.

Lemma 1 . *Assume that the apex player (player 0) has already moved and that he demanded p_0. Consider the subgame starting with player i after each player $j < i$ has demanded p_j. Then*

- *(i) If $r_i(p) < n - p_0$, then in the unique SPE of the subgame player i forms the coalition with the apex player.*
- *(ii) If $r_i(p) > n - p_0$, then in the unique SPE of the subgame player i asks for $n - P_i - (n - p_0)(n - i)$ and the coalition N forms with each remaining minor player $j > i$ asking for $n - p_0$.*
- *(iii) If $r_i(p) = n - p_0$, then there are exactly two SPEa in the subgame: Player i demands $p_i = n - p_0$ and either*
  - *(a) forms the coalition with the apex player or*
  - *(b) selects another minor player and every remaining minor player demands $p_j = n - p_0$ and the minor-player coalition, N, forms.*

Proof. The proof is by induction with respect to i in N. The result is obviously true for $i = n$, so assume the statements have all been proved for $j > i$ and consider player i's decision problem.

(i) Suppose $r_i(p) < n - p_0$. Then player i can guarantee $n - p_0$ by forming the coalition with player 0. Assume player i asks $p_i \geq n - p_0$ and gives the move to $i + 1$. Then $r_{i+1}(p) < n - p_0$ so that $i + 1$ will form the coalition with 0. Hence, the coalition N is suboptimal for player i, player i's unique optimal action is to form the coalition with player 0.

(ii) Suppose $r_i(p) > n - p_0$. If player i asks for p_i with $n - p_0 < p_i < n - P_i - (n - p_0)(n - i)$, then $r_{i+1}(p) > n - p_0$ and, by induction, the coalition N will form with player i getting the payoff p_i. To ensure that player i has a best response, the players $j > i$ also have to continue with the equilibrium from *(iii)b* if player i demands $p_i = n - P_i - (n - p_0)(n - i)$. Hence, the unique equilibrium is as described.

(iii) Suppose $r_i(p) = n - p_0$. Player i gets payoff $n - p_0$ by forming the coalition with player 0. Player i cannot get more from the coalition N since if $p_i > n - p_0$, then $r_{i+1}(p) < n - p_0$ and $i + 1$ forms the coalition with 0. Hence, there is an SPE in which i accepts the demand of player 0. On the other hand, the induction step and *(iii)b* guarantee that there exists an SPE in which N is formed if player i demands $p_i = n - p_0$ and gives the move to $i + 1$. It follows that there are two SPEa. □

Corollary 1 . *If the apex player starts the game there is a unique SPE outcome: the apex player demands $p_0 = n - 1$ from a minor player and the latter accepts.*

Proof. This follows immediately from Lemma 1 together with the fact that the first moving minor player should, in equilibrium, resolve ties in favor of player 0. □

Before moving to the most interesting case where player i has the choice between calling the apex player or calling player $i + 1$, let us consider the behavior of the apex player when he is called.

Lemma 2 . *Assume that player i gives the move to player 0. Then in the subgame starting with the move of player 0*

(i) *If $i = n$ or if $p^{i+1} < r_{i+1}(p)$, then in any SPE player 0 forms a coalition with some player $j \leq i$ with $p_j = p^{i+1}$.*

(ii) *If $i < n$ and $p^{i+1} > r_{i+1}(p)$, then in the unique SPE, player 0 rejects all previous demands, instead he asks for $p_0 = n - r_{i+1}(p)$ and gives the move to player $i+1$ who accepts.*

(iii) *If $i < n$ and $p^{i+1} = r_{i+1}(p)$, then there are multiple equilibria that correspond to those of the cases* (i) *and* (ii).

Proof. The assertion clearly holds if player 0 is the last one to move (that is, $i = n$), so assume $i < n$. Player 0 is guaranteed $n - p^{i+1}$ by forming a coalition with some player $j \leq i$. (Recall from (4.4) that p^i is the minimum price of any player *preceeding* player i.) If $p^{i+1} < r_{i+1}(p)$ and player 0 demands $p_0 \geq n - p^{i+1}$ from player $i+1$, then $r_{i+1}(p) > n - p_0$ so that (by Lemma 1) the coalition N will be formed. This proves (i). To prove (ii) one notices that, if $p^{i+1} > r_{i+1}(p)$, then player 0 is guaranteed of the cooperation of player $i+1$ as long as he asks $p_0 < n - r_{i+1}(p)$. (Of course, one also invokes the usual tie-breaking argument.) The proof of (iii) is a combination of the above arguments. □

Lemma 3 shows that if the players preceeding i have made modest demands (so that the per capita remainder exceeds the lowest previous price), the minor-player coalition will recessarily form. Lemma 4 on the other hand shows that, if one or more of the preceeding players has been "greedy" (so that the per capita remainder is less than the lowest previous price), then (in some SPE continuation) player i may form a coalition with the apex player.

Lemma 3 . *Consider a subgame where i has to move and where the apex player has not yet moved. Then, if $r_i(p) > p^i$, in the unique SPE player i demands $p_i = (n -$*

$i)(r_i(p) - p^i) + r_i(p)$, each player $j > i$ demands p^i and the minor-player coalition N is formed.

Proof. Induction w.r.t. i. The statement clearly holds if $i = n$ (with the obvious modification that n forms the coalition N). Assume $i < n$ and that the statement has already been proved for $j > i$. If player i demands p_i with $p^i < p_i < (n-i)(r_i(p) - p^i) + r_i(p)$, then $r_{i+1}(p) > p^{i+1}$ so that, by the induction hypothesis i will end up with p_i. Hence, player i can guarantee $(n-i)(r_i(p) - p^i) + r_i(p)$ by cooperating with N. On the other hand, player 0 will accept i's demand only if $p_i \leq p^i$. Clearly, i's unique optimal action is to cooperate with N and to demand the highest possible price that doesn't jeopardize the formation of that coalition. □

Lemma 4 . *Consider a subgame where player i has to move and where the apex player has not yet moved. Then if $r_i(p) \leq p^i$, there exists an SPE where player i calls the apex player who forms the coalition with i. In this SPE player i demands $p_i = p^i$ if $i = n$ and $p_i = r_i(p)$ if $i < n$.*

Proof. Induction w.r.t. i. The statement is obviously true for $i = n$. Consider player $i < n$ and suppose that the statement has already been proved for $j > i$. Also assume $r_i(p) \leq p^i$. If i demands p_i with $p_i < r_i(p)$, then $p_i = p^{i+1} < r_{i+1}(p)$ and Lemma 2 guarantees that in this case the apex player accepts the demand of player i. Hence, by the usual argument, player i can guarantee $r_i(p)$ from the apex player. Lemma 2 also shows that the apex player will reject i's demand if $p_i > r_i(p)$. Assume i demands $p_i > r_i(p)$ from $i+1$. Then $r_{i+1}(p) < p^{i+1}$, so that by the induction hypothesis there exists an SPE continuation where $i+1$ calls the apex player. If player $i+1$ chooses the latter continuation for each $p_i > r_i(p)$, then it is optimal for player i to put $p_i = r_i(p)$ and to call the apex player. □

The condition from Lemma 4 is obviously satisfied for the first moving minor player, hence

Corollary 2 . *If player 1 starts the game there exists an SPE where player 1 demands* $p_1 = 1$ *and calls the apex player who accepts player 1's demand.*

Note that Lemma 3 implies that player 1 can also enforce that the coalition N is formed by demanding p_1 slightly less than 1. Hence, player 1 has at least a payoff of 1 in any SPE. In fact, the previous Lemmas (together with the usual tie breaking arguments) imply that there exists an SPE in which each minor player i demands $p_i = 1$ and in which the coalition N of all minor players is formed. In this SPE, player 2 threatens to call the apex player (i.e. to play the SPE from Lemma 4) as soon as $p_1 > 1$. Note, however, that Lemma 4 does not imply that in a subgame with $p_1 > 1$, player 2 necessarily calls the apex player. Indeed, also in subgames with $r_i(p) \leq p^i$ there exist SPEa in which the coalition N is formed, hence, player 1 may obtain more than 1 in some SPE. In fact it is easy to see that player 1 can obtain the entire cake. Namely, suppose player 1 demands $p_1 = n$. Then player 2, as well as any other remaining minor player, knows that his payoff will be zero anyhow. (The payoff is zero in the coalition N, but the apex player will exploit this fact and demand the entire cake for himself as well when given the move.) Facing this fait accompli, one may as well accept it and agree to the formation of N. The following corollary, describing the worst and best payoffs player 1 can receive in the minor-player coalition, summarizes the above discussion.

Corollary 3 .

(a) If player 1 starts the game there is an SPE where player 1 demands $p_1 = 1$*, every other minor player demands 1 and the minor-player coalition forms.*

(b) If player 1 starts the game there is an SPE in which player 1 demands $p_1 = n$ *every other minor player demands 0 and the minor-player coalition forms.*

The previous lemmas also allow us to describe the set of all SPE payoffs that can result in the demand commitment game if the coalition of all minor players is formed.

Theorem 2 . *If player 1 starts the game, then there exists an SPE in which the minor-player coalition N forms and agrees on the payoff vector $p \in \mathbb{R}^n_+$ if and only if p satisfies*

$$\sum_{i=1}^{n} p_i = n \tag{4.4}$$

$$0 \leq r_i(p) \leq p_i \qquad \textit{(all i)} \tag{4.5}$$

$$p_{n-1} = r_{n-1}(p) \tag{4.6}$$

Proof. (Necessity.) Condition (4.4) is obvious. Since each player i can guarantee a payoff zero (by putting $p_i = 0$) we must have $r_i(p) \geq 0$. Lemma 3 implies that if there exists an SPE with payoffs p, then $r_i(p) \leq p^i$ for all $i = 2, \ldots, n$. Namely, if $r_i(p) > p^i$, then player $i-1$ can increase his demand without jeopardizing the formation of the coalition N. Therefore, for $i = 1, \ldots, n-1$, we must have

$$p_i \geq p^{i+1} \geq r_{i+1}(p) = r_i(p) - \frac{p_i - r_i(p)}{n-i} \tag{4.7}$$

which shows that $p_i \geq r_i(p)$ for $i = 1, \ldots, n-1$. Condition (4.4) already implies that $p_n \geq r_n(p)$. We finally must have $p_{n-1} = r_{n-1}(p)$ since otherwise (by (4.7)) $r_n(p) < p^n$ and player n prefers to form the coalition with the apex player.
(Sufficiency.) Assume $\bar{p}$ satisfies the conditions (4.4) – (4.6) and consider the following strategy combination:

For the apex player: Play in accordance with the strategies from Lemma 2, breaking ties in favor of that minor player who moved last.

For a minor player $i \in N$:

(i) If the apex player has already moved, play in accordance with Lemma 1, breaking ties in favor of the apex player.

(ii) If the apex player has not yet moved

a) if each $j < i$ demanded $\bar{p}_j$, demand $\bar{p}_i$ and call player $i + 1$ (if you are $i = n$, form the coalition N in this case);

b) if you are in a subgame covered by Lemma 3, continue with the SPE described in that Lemma.

c) In all cases not covered by a) or b), play in accordance with the SPE from Lemma 4.

By construction, this strategy profile constitutes an SPE for each subgame that starts with a move of the apex player or that is covered by the cases (ii) a) or (ii) c). Hence, it remains to be verified that along the equilibrium path no profitable deviations are possible i.e. it suffices to check that the strategies form a Nash equilibrium. Now note that since $\bar{p}$ satisfies (4.4) – (4.6) we have that

$$r_i(\bar{p}) \leq \bar{p} \qquad (\text{all } i) \qquad \text{and} \qquad r_n(\bar{p}) = \bar{p}^n \tag{4.8}$$

so that definitely player n cannot profitably deviate if all other players conform. If player $i < n$ is the first minor player to deviate to a demand $p_i > \bar{p}_i$, then Lemma 2 and (ii) c) guarantee that the coalition $\{0, i+1\}$ will be formed after this deviation. Hence, no player can profit by deviating unilaterally and we have an SPE. □

5 Credible Subgame Perfect Equilibria

In the previous section we found a plethora of equilibria with a corresponding continuum of payoff divisions in the minor-player coalition. However the strategies that support many of these equilibria rest on logic that seems uncompelling. In this section, we formalize this intuition as a refinement of subgame perfect equilibrium which, for lack of a better name, we call *credible* subgame perfect equilibrium (CSPE). In the next section, we show that credibility leads to a drastic reduction in the number of equilibria.

To motivate our refinement, consider the following game.

When it is his turn to move, player 2 is indifferent between l and r. Player 1's optimal strategy therefore depends on his belief about how player will play in the face of this indifference. Harsanyi and Selten (1998) argue that a rational player should randomize equally among all alternatives over which he is indifferent. If player 1 believes that player 2 will choose l and r with equal probability, then player 1 should choose R, and player 2 will obtain a payoff of 0. However, if player 1 believes that player 2 will play r, then player 1 will choose L and player 2 will obtain a payoff of 5. The threat by player 2 to play r is completely rational, since player 2 is indifferent between l and r. Moreover, the threat, if believed, yields player 2 a higher payoff. In our view, therefore, player 2 will indeed threaten to play r and player 1 has every reason to believe that player 2 will carry out his threat, so player 1 should choose L.

The logic above is quite different from that underlying forward induction arguments. The usual forward induction argument (see, for example Van Damme (1989)) is that player 1, by his action, can indicate his desire to play a particular subgame perfect equilibrium in the subgame that follows this action. In demand commitment games, as well as the game above, the forward induction logic is not compelling: although player 1 may indicate his desire to play a particular subgame perfect strategy combination, he has no means of enforcing this strategy combination since he has no further moves in the game.

An intuition similar to ours led Leininger (1986) to define a refinement of subgame perfection called *strategic equilibrium*. However his formalization of this intuition is different from ours. One manifestation of this difference is that every credible subgame perfect equilibrium is self-consistent while strategic equilibria need not be. That is, a credible subgame perfect equilibrium necessarily induces a credible subgame perfect equilibrium in every subgame; a strategic equilibrium need not induce a strategic equilibrium in every subgame.[6]

We now turn to the formal description. Some terminology first. We restrict attention to the class of demand commitment games but the definition of credible subgame perfect equilibria can be readily extended to the class of all extensive form games with perfect information. Let Γ^* be a demand commitment game based on an $(n+1)$-person apex game. Let Γ be a subgame of Γ^* and denote its length, i.e. the maximum number of moves on a path in Γ, by $l(\Gamma)$. Let σ be a strategy combination in Γ. If τ_i is an alternative strategy of player i in Γ then $\sigma\backslash\tau_i$ denotes the strategy profile in which all players play in accordance with σ except player i who plays τ_i. The subgame γ is said to be *consistent* with σ if γ is reached if σ is played. We say that τ_i' is an *ε-best reply* of player i to σ in Γ if $h_i(\sigma\backslash\tau_i') \geq \sup_{\tau_i} h_i(\sigma\backslash\tau_i)-\varepsilon$, where h_i is player i's payoff in Γ.

We define credible threats and credible subgame perfect equilibria (CSPE) by induction on the length of the game. If $l(\Gamma)=1$, then there are no credible threats so every SPE is a CSPE. If CSPEa have been defined in all subgames of length $l(\Gamma)-1$, then we define credible threats in Γ as follows:

Definition 1. Credible Threats.
Let σ be an SPE of a subgame Γ with $l(\Gamma)>1$. Let j be a player who has to move when σ is played. The strategy τ_j of player j is a *credible threat* of player j against i at σ if

[6] An example is available from the authors upon request.

the following three conditions are satisfied

(a) Ex post indifference: $h_j(\sigma\backslash\tau_j) = h_j(\sigma)$

(b) Ex ante improvement: There exists $\varepsilon > 0$ such that for every ε-best reply σ_i' of player i against $\sigma\backslash\tau_j$ we have $h_j(\sigma\backslash\tau_j\backslash\sigma_i') > h_j(\sigma)$.

(c) Credibility: For each choice σ_i' of player i, the strategy combination $\sigma\backslash\tau_j\backslash\sigma_i'$ induces a CSPE in the subgame that follows σ_i'.

Condition (a) requires that if j is called upon to carry out his threat, his is no worse off than by following the equilibrium strategy. Condition (b) captures the idea that i's best response must improve j's payoff. Since, however, players' demands are continuous variables, player i may not have an exact best response against j's threat. Hence, condition (b) requires that every 'almost best' response of player i to j's threat improves j's payoff. Definition 1 assumes that a player k ($k \notin \{i,j\}$) will not deviate from σ. Condition (c) from the definition guarantees that this assumption is justified: No matter what player i will do, player j's threat results in the players continuing with a credible equilibrium. Hence, condition (c) formalizes the idea that credible threats are accompanied by credible promises.

A credible subgame perfect equilibrium (CSPE) is an SPE in which there are no credible threats. Formally:

Definition 2. Credible subgame perfect equilibrium (CSPE).

(i) Initialization: Every SPE of Γ is a CSPE if $l(\Gamma) = 1$.

(ii) Induction: let Γ be a subgame with $l(\Gamma) > 1$ in which player i has the first turn to move. An SPE σ of Γ is a CSPE if

  (a) The strategy combination σ_γ that σ induces in γ is a CSPE of γ for each proper subgame γ of Γ, and

  (b) no player j who has a move on the path of σ has a credible threat against σ.

6 Credible SPEa of Demand Commitment Games

We next investigate the extent to which the credibility requirement reduces the set of SPEa in apex games. We show that only two types of outcomes survive: coalitions consisting of the apex player and a minor player, with payoffs of $n-1$ and 1 respectively, and the coalition of all minor players, with a payoff of 1 for each minor player.

In the demand commitment model a threat of player i against player j takes the form: "If you don't reduce your demand — thereby allowing me to obtain a higher payoff — I will not form a coalition with you." Such a threat is credible exactly when player i can in fact obtain the same payoff in a coalition without player j, while player j cannot obtain the same payoff in a coalition without player i.

We first consider subgames that have a unique SPE. The next lemma shows that for such a subgame the SPE is credible.

Lemma 5 . *If Γ admits a unique SPE σ, then σ is a CSPE of Γ.*

Proof. The proof is by induction with respect to the length of Γ and closely follows the arguments given in Section 4. In particular it uses the fact that the player moving first in such a subgame Γ can, by lowering his demand slightly force the others to accept. Hence, there can be no credible threats. For example, the minor player 1 has no credible threat against the apex player if the latter demands $p_0 < 1$. Consequently, the apex player can counter the threat that player 1 will form the coalition N if $p_0 = 1$ by demanding slightly less and giving the move to another minor player. Hence, player 1 does not have a credible threat if $p_0 = 1$. We leave further details to the reader. □

Hence, we may concentrate on the interesting subgames with multiple SPEa. We first consider subgames starting with player $i = n$ and in which the apex player still has to move.

Lemma 6 . *Assume that the apex player has not yet moved and consider a subgame* Γ *starting with player* $i = n$. *Then any SPE of* Γ *is a CSPE.*

Proof. The results from the previous section imply that if $r_n(p) \neq p^n$ there is a unique SPE of the subgame Γ, hence, this is a CSPE by Lemma 5. Assume $r_n(p) = p^n$ so that there are two SPE. The one in which N is formed is a CSPE since no player is moving after n. The SPE in which player n demands $r_n(p)$ from player 0 is a CSPE since the apex player's threat to form a coalition with $j \neq n$, can be countered by forming the coalition N. □

The next lemma describes two CSPEa for subgames starting with player $i < n$ in which the apex player has not yet moved.

Lemma 7 . *Assume the apex player has not yet moved and consider a subgame* Γ *starting with a move of a player* $i \in N$ *with* $i \neq n$ *and* $r_i(p) \leq p^i$. *Then the following two outcomes can be sustained by CSPEa of* Γ.

(i) *Player* i *demands* $p_i = r_i(p)$ *and gives the move to the apex player who accepts this demand.*

(ii) *Each player* $j \geq i$ *demands* $p_j = r_i(p)$ *and the coalition* N *is formed.*

Proof. Statement (i) follows immediately from Lemmas 2, 3 and 5: By demanding p_i slightly less than $r_i(p)$, player i forces any player to whom he gives the move to accept. Statement (ii) follows in a similar way by using an induction argument: If $j \in N \backslash \{i\}$ threatens not to accept player i's demand of $r_i(p)$, player i can counter by forming the coalition $\{0, i\}$. □

The next Lemma shows that the subgames described in Lemma 7 have no other CSPE outcomes.

Lemma 8 . *Assume the apex player has not yet moved and consider a subgame* Γ *starting with a move of player* $i \in N$ *with* $i \neq n$ *and* $r_i(p) \leq p^i$. *Then in each CSPE of* Γ *player* i *demands* $p_i = r_i(p)$.

Proof. Induction with respect to i. The results from the previous section imply that for $i = n-1$ or in case $r_i(p) = p^i$ the statement holds even for any SPE. Therefore, let $i \leq n-2$, assume $r_i(p) < p^i$ and consider an SPE in which player i has a pay-off more than $r_i(p)$. Hence, i demands $p_i > r_i(p)$ and gives the move to $i+1$. Let $0 < \eta < p_i - r_i(p)$. Player $i+1$ has the following credible threat against this SPE: If you demand $p_i' \leq r_i(p) + \eta$ then we continue with the CSPE from Lemma 7 (ii), if you demand $p_i' > r_i(p) + \eta$, then we continue with the CSPE from Lemma 7 (i). For an appropriate value of ε this threat indeed satisfies the conditions from Definition 1 (if ε is small enough, then in any ε-best response, player i still gives the move to player $i+1$ and the latter ex ante gains at least $(p_i - r_i(p) - \eta)/n$), hence the SPE is not credible. □

The following theorem summarizes the results obtained in this section.

Theorem 3 . *In the demand commitment game the following and only the following outcomes can be sustained by credible subgame perfect equilibria:*

(i) the apex player starts the game, demands $p_0 = n-1$ *and forms a coalition with a minor player.*

(ii) a minor player starts the game, demands 1 and forms the coalition with the apex player.

(iii) a minor player starts the game, demands 1 and calls on a minor player, who demands 1 and calls on another minor player ... and forms the minor-player coalition N.

A natural question is whether one can give additional arguments in favor of, or to dismiss, either of the outcomes described in Theorem 3 (ii), (iii). At first it seems that the minor player might prefer the coalition with the apex player since the apex player will accept for certain while attempting to form the coalition of all minor players seems more risky. (The apex player accepts $n-1$ for certain, because if he rejects he can demand at most $n-1$ from any other minor player, and risks the possibility that this player chooses the 'wrong' continuation.) Upon closer inspection this argument is not valid. Consider the decision situation faced by player i in N when each minor player $j < i$ has demanded $p_j = 1$ and the apex player has not yet moved. Clearly, if $i = n$, then i will prefer to form N: By forming N i has 1 for sure, if he gives the move to the apex player, he has to compete for this player's favor with the other minor players. Continuing inductively, we see that each player $i > 1$ will prefer to form the coalition N to avoid the competition with the minor player that already moved. Hence, player 1 not only knows that the apex player would accept his demand $p_1 = 1$, he also knows that, if he demands $p_1 = 1$ and gives the move to player 2, then the coalition N is formed for sure. Hence, player 1 is indifferent and both equilibria are viable.

7 Discussion

Next we relate the results of our model to those of other cooperative and noncooperative models.

In the early 1960's Davis and Maschler polled a number of prominent game theorists about the 4-minor player version of the apex game asking them what they thought should be the division of payoff between the apex player and a minor player. The replies are reported in Davis and Maschler (1965). Not surprisingly, different people made different suggestions but the majority of responses favored the payoff division (3,1). The discussion is quite interesting and reveals several intuitions that one might have about the game. Interesting is also the fact that two of the experts stress the importance of the extensive form. Martin Shubik demanded more information about the rules of the game, he did not want to commit himself using the argument that "one cannot predict anything on a game given solely in terms of the characteristic function form". Lloyd

Shapley noted that "A good deal depends here on the extensive form of the game; i.e. on whether the game is actually presented to the players as a pure coalition-forming exercise, or whether there is a structure of moves and strategies which just happen to yield the indicated characteristic function. The passage from extensive (or normal) form to characteristic function form is not without pitfalls; its validity depends to some extent on the nature of the solution-concept that is applied to the characteristic function."

Two concepts discussed in Davis and Maschler that do not yield the (3,1)-division are the bargaining set and the kernel. The bargaining set of Aumann and Maschler (1964) admits any division of payoff from (2,2) to (3,1); the kernel of Davis and Maschler (1965) admits only the payoff vector (2,2).[7].

The cooperative solution closest in spirit to our noncooperative model is that of bargaining aspirations. Bargaining aspirations were introduced independently by several authors — under a variety of names — the first was Albers (1974). Later is was recognized that bargaining aspirations are any extension of the bargaining set of Aumann and Maschler to the aspiration solution space (see Bennett and Zame (1988)). Formally, a vector $p = (p_0, p_1, \dots, p_n)$ is an aspiration if: (a) there is no payoff leftover in any coalition after its members are paid their prices ($\sum_{i \in S} p_i = v(S)$), and (b) no player is priced out of the market (for player i there is a coalition $S(i)$ containing i with $\sum_{i \in S(i)} p_i = v(S(i))$).

In the context of apex games, an *objection* of player i against player j takes the form: "If you don't lower your price — and thereby allow me to raise mine — I will not form a coalition with you." This objection is *justified* when player i can in fact obtain the same price in a coalition without player j, while player j cannot obtain his price in a coalition without player i. A vector p is a bargaining aspiration if it is an aspiration and there are no justified objections against it. Clearly similar intuitions lie behind bargaining aspira-

[7]The bargaining set and kernel were designed to analyze payoff divisions for situations with a fixed coalition structure, and not for situations with endogenous coalition formation. See Section 13 of

tions and credible threats; so it may not be surprising that CSPE prices are related to bargaining aspirations. For apex games with n minor players $(n-1, 1, \ldots, 1)$, the only price vector consistent with CSPE strategies, is also the unique bargaining aspiration.[8]

Chatterjee et al (1990) consider an alternative noncooperative bargaining model for the class of TU games. Their model extends those of Rubinstein (1982) and Selten (1981). In contrast to our model, in which each player announces a price, in their model, players make, accept, or reject proposals. (A proposal $< S, x >$ consists of a feasible coalition S together with a payoff division x for that coalition.) For the class of apex games, the rules are as follows. If there are no proposals on the table, the player who has to move makes a proposal $< S, x >$, where S is a coalition containing the given player. If there is a proposal on the table, the player who has to move may accept or reject it. If he accepts it, the proposal remains on the table and the move passes to the next player (the order of play within coalitions is given exogenously). If he rejects it, the proposal vanishes from the table and he must make a new proposal. The game terminates as soon as a proposal $< S, x >$ has been accepted by all players in S. Players discount payoffs by a factor of $(1-\varepsilon)$ for each rejection that has occurred, so if r is the number of rejections, then each player $j \in S$ receives $(1-\varepsilon)^r x_j$; players not in S receive 0.[9]

Chatterjee et al look for SPE of this game in stationary strategies. Let $\bar{x}_p = n/(2-\varepsilon)$, let $\bar{x}_r = n - \bar{x}_p$ and write $\bar{x} = (\bar{x}_p, \bar{x}_r)$. Then it is easily seen that the following strategies constitute such an SPE.

For player 0: Once 'matched' with minor player i, never leave this player; propose $< \{0, i\}, \bar{x} >$, and accept any proposal x that gives you at least $\bar{x}_r$.

[8] Binmore (1985) presents a multilateral Nash bargaining model as the cooperative solution concept which supports his noncooperative model. A multilateral Nash bargaining model (similar in spirit but different in details) can also be shown to support this noncooperative model. See Bennett (1990) for an overview of multilateral bargaining models.

[9] Equivalently, we could assume that players do not discount payoffs, but interpret ε as the probability the game ends following a rejection.

For player $i \in N$: Propose $< \{0,i\}, \bar{x} >$, accept any proposal of player 0 that gives you at least $\bar{x}_r$; reject any proposal to form the coalition N unless you are the last, or next to last one to move in this coalition and your acceptance guarantees a payoff of at least $\bar{x}_r$.

For small ε, these strategies lead to the formation of a two person coalition (i.e., a coalition of the apex player and one minor player); as ε tends to 0, the payoff within this coalition tends to equal division. Thus, even when there are 1,000 minor players, the apex player does not fully exploit his bargaining power: the apex player and his minor partner each obtain 500.

This odd outcome seems to result from the requirement that players use stationary strategies. The apex player, ignoring the presence of other minor players to whom he might switch, remains with the minor player to whom he is initially matched. This strategy is sensible for the apex player only because the minor players also ignore the presence of other minor players, so the apex player cannot gain from switching. Put another way, the minor players "refuse to learn" during the game, and there is nothing the apex player can do to teach them; this severely limits the bargaining power of the apex player.

Chatterjee et al do not provide a convincing motivation for the assumption of stationarity. (They just note that without it, not much can be said: in strictly superadditive games with at least 3 players, any individually rational, efficient allocation can be generated by a SPE, for ε small enough.) In our opinion, stationarity is a strong behavioral assumption, and is not justified. Furthermore, there appears to be no convincing alternative way (yet) to select among the infinity of nonstationary equilibria. In the demand commitment model, there are also an infinity of SPE, but there is a relatively straightforward (and in our opinion convincing) way to reduce to multiplicity. We find the way in which the outcomes depend on the number of minor players in the demand commitment model to be considerably more satisfying.

We expect that the demand commitment game, or natural variations of it, can give interesting insights for other classes of cooperative games as well. Of course, the analysis in the present paper depends on the assumption that demands cannot be renegotiated. In general it may be more natural to allow for such renegotiation, that is to allow multiple bargaining rounds with players in each round having the opportunity to quote a new demand. Also in each new round one may want to select the player moving first in that round at random, this to avoid monopoly power of the player moving first. We plan to study such multiple-round demand commitment games in the future.

References

Albers, W. (1974). "Zwei Lösungskonzepte für Kooperative Mehrpersonspiele, die auf Anspruchsnivaus der Spieler Basieren", *OR-Verfahren*, **21**, 1-13.

Aumann, R. and J. Drèze (1974). "Cooperative Games with Coalition Structures", *International Journal of Game Theory*, **3**, 217-237.

Aumann, R. and M. Maschler (1964). "The Bargaining Set for Cooperative Games", in *Advances in Game Theory (Annals of Math. Studies 52*, Eds. M. Dresher,L. Shapley and A.W. Tucker, Princeton University Press, Princeton.

Bennett, E. (1986, revised 1990). "Multilateral Bargaining Problems", Economic Department, University of California at Los Angeles WP594.

Bennett, E. (1990a). "Three Approaches to Bargaining in NTU Games", *Strategic Bargaining*, Ed. R. Selten, Springer-Verlag, Berlin.

Bennett, E. and W. Zame (1988). "Bargaining in Cooperative Games" *International Journal of Game Theory*, **17**, 279-300.

Binmore, K. (1985). "Bargaining and Coalitions", *Game Theoretic Models of Bargaining*, Ed. Alvin E. Roth, Cambridge University Press.

Chatterjee, K., B. Dutta, D. Ray and K. Sengupta (1990). "A Non-

Cooperative Theory of Cooperative Bargaining", *Mimeo*, Penn State University.

Davis, M. and M. Maschler (1965). "The Kernel of a Cooperative Game", *Naval Research Logistic Quarterly*, **12**, 223-259.

Harsanyi, J. (1971). "An Equilibrium-Point Interpretation of Stable Sets and a Proposed Alternative Definition", *Management Science*, **20**, 1422-1495.

Harsanyi, J. and R. Selten (1988). *A Theory of Equilibrium Selection in Games*, MIT Press, Cambridge.

Hellwig, M., W. Leininger, P. Reny and A. Robson (1990). "Subgame Perfect Equilibrium in Continuous Games of Perfect Information: An Elementary Approach to Existence and to Approximation by Discrete Games", *Journal of Economic Theory*, forthcoming.

Leininger, W. (1986). "Strategic Equilibrium in Games with Perfect Recall", *Technical Memorandum*, Bell Laboratories.

Nash, J. (1950). "The Bargaining Problem", *Econometrica*, **18**, 155-162.

Nash, J. (1953). "Two-Person Cooperative Games", *Econometrica*, **21**, 128-140.

Rubinstein, A. (1982). "Perfect Equilibrium in a Bargaining Model", *Econometrica*, **50**, 97-108.

Selten, R. (1981). "A Noncooperative Model of Characteristic Function Bargaining", *Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern*, Eds. V. Böhm and H.H. Nachtkamp, 131-151.

Van Damme, E. (1989). "Stable Equilibria and Forward Induction", *Journal of Economic Theory*, **48**, 476-496.

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