Mannheim Research Institute for the Economics of Aging

Demographic Change, Human Capital and Welfare

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Demographic Change, Human Capital and Welfare*

Alexander Ludwig[†] Thomas Schelkle[‡] Edgar Vogel[§]

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Abstract

This paper employs a large scale overlapping generations (OLG) model with endogenous human capital formation using a Ben-Porath (1967) technology to evaluate the quantitative role of human capital adjustments for the economic consequences of demographic change. We find that endogenous human capital formation is a quantitatively important adjustment mechanism which substantially mitigates the macroeconomic impact of population aging. On the aggregate level, the predicted decrease of the rate of return to physical capital is only one third of the predicted decrease in a standard model with a fixed human capital profile. In terms of welfare, while young agents with little assets gain up to 0.8% in consumption from increasing wages in both models, welfare losses from decreasing returns of older and asset rich households are substantial. But importantly, these losses are about $50 - 70\%$ higher in the model without endogenous human capital formation. Ignoring this adjustment channel thus leads to quantitatively important biases of the welfare assessment of demographic change. We also document that not reforming the social security system but letting contribution rates increase will largely offset any positive welfare effects for future generations.

JEL classification: C68, E17, E25, J11, J24
Keywords: population aging; human capital; rate of return; distribution of welfare

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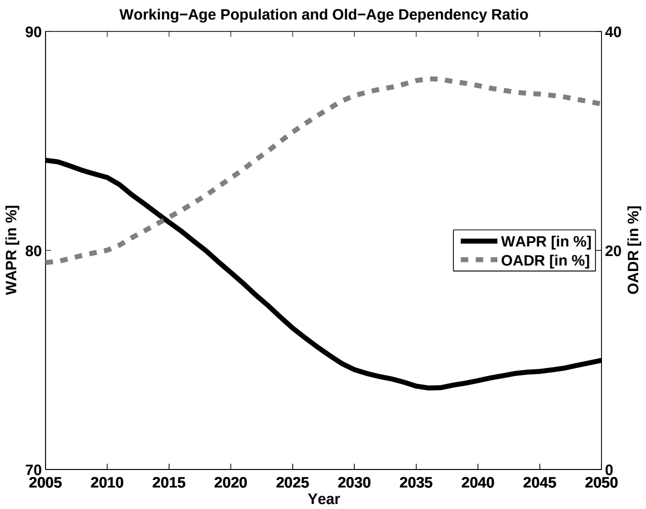
1 Introduction

As in all major industrialized countries the population of the United States is aging over time. This process is driven by increasing life-expectancy and a decline in birth rates from the peak levels of the baby boom. Consequently, the fraction of the population in working-age will decrease and the fraction of people in old-age will increase. Figure 1 presents two summary measures of these demographic changes: The working-age population ratio is predicted to decrease from 84% in 2005 to 75% in 2050 and the old-age dependency ratio increases from 19% in 2005 to 34% in 2050.

Figure 1: Working age and old-age dependency ratio



Notes: Working age population ratio (WAPR, left scale): ratio of population of age $16-64$ to total adult population of age $16-90$. Old-age dependency ratio (OADR, right scale): ratio of population of age $65-90$ to working age population.
Source: Own calculations based on Human Mortality Database (2008).

These projected changes in the population structure will have important macroeconomic effects on the balance between physical capital and labor. Specifically, labor is expected to be scarce, relative to physical capital, with an ensuing decline in real returns on physical capital and increases in gross wages. As shown in this paper, a strong incentive to invest in human capital emanates from the combined effects of increasing life expectancy and changes in relative prices particularly if social security systems are reformed so that contribution rates are held constant. In general equilibrium, such endogenous human capital adjustments substantially mitigate the effects of demographic change on macroeconomic aggregates and individual welfare. The key contribution of our paper is to show that the human capital adjustment mechanism is quantitatively important.

We add endogenous human capital accumulation to an otherwise standard large-scale OLG model in the spirit of Auerbach and Kotlikoff (1987). The central part of our analysis is then to work out the differences between our model with endogenous human capital adjustments and endogenous labor supply and the "standard" models in the literature with a fixed (exogenous) productivity profile.

We find that as a consequence of demographic change the decrease of the return to physical capital in our model with endogenous human capital is only one third of the decrease in the standard model. Welfare consequences from increasing wages and declines in rates of return can be substantial. Newborns in 2005 experience welfare gains in the order of up to 0.8% of lifetime consumption when contribution rates to the pension system are held constant and welfare losses worth −3% of lifetime consumption when the generosity of the pension system is maintained. In contrast, asset-rich households currently alive lose from the decline in rates of return and these losses can be large depending on the future evolution of the pension system.

But importantly, these losses are about $50 - 70\%$ higher when the human capital adjustment mechanism is shut down. Ignoring this adjustment channel thus leads to quantitatively important biases of the welfare assessment of demographic change.

Our work relates to a vast number of papers that have analyzed the economic consequences of population aging and possible adjustment mechanisms. Important examples in closed economies with a focus on social security adjustments include Huang et al. (1997), De Nardi et al. (1999) and, with respect to migration, Storesletten (2000). In open economies, Börsch-Supan et al. (2006), Attanasio et al. (2007) and Krüger and Ludwig (2007), among others, investigate the role of international capital flows during the demographic transition. We add to this literature by highlighting an additional mechanism through which households can respond to demographic change.

Our paper is closely related to the theoretical work on longevity, human capital, taxation and growth[1] and to Fougère and Mérette (1999) and Sadahiro and Shimasawa (2002) who also investigate demographic change in large-scale OLG models with individual human capital decisions. In contrast to their work, we focus our analysis on the relative price changes during the demographic transition and therefore consider an exogenous growth specification.[2] We also extend their analysis along various

[1]See for example de la Croix and Licandro (1999), Boucekkine et al. (2002), Kalemli-Ozcan et al. (2000) Echevarria and Iza (2006), Heijdra and Romp (2008) and Ludwig and Vogel (2009). Our paper is also related to a literature emphasizing the role of endogenous human capital accumulation for the analysis of changes to the tax or social security system as in Lord (1989), Trostel (1993), Perroni (1995), Dupor et al. (1996) and Lau and Poutvaara (2006), among others.

[2]Whether the trend growth rate endogenously fluctuates during the demographic transition or is held constant is of minor importance for the questions we are interested in. This is shown in our earlier unpublished working paper. Results are available upon request.

dimensions. We use realistic demographic projections instead of stylized scenarios. More importantly, our model contains a labor supply-human capital formation-leisure trade-off. It can thus capture effects from changes in individual labor supply, i.e., human capital utilization, on the return of human capital investments. As has already been stressed by Becker (1967) and Ben-Porath (1967) it is important to model human capital and labor supply decisions jointly in a life-cycle framework. Along this line, a key feature of our quantitative investigation, is to employ a Ben-Porath (1967) human capital model and calibrate it to replicate realistic life-cycle wage profiles.[3] Furthermore, we put particular emphasis on the welfare consequences of population aging for households living through the demographic transition.

The paper is organized as follows. In section 2 we present the formal structure of our quantitative model. Section 3 describes the calibration strategy and our computational solution method. Our results are presented in section 4. Finally, section 5 concludes the paper. A separate online appendix[4] contains additional results, a description of our demographic model and technical details.

2 The Model

We employ a large scale OLG model à la Auerbach and Kotlikoff (1987) with endogenous labor supply and endogenous human capital formation. The population

[3] The Ben-Porath (1967) model of human capital accumulation is one of the workhorses in labor economics used to understand such issues as educational attainment, on-the-job training, and wage growth over the life cycle, among others, see Browning, Hansen, and Heckman (1999) for a review. More recently, extended versions of the model have been applied to study the significant changes to the U.S. wage distribution and inequality observed since the early 1970s by Heckman, Lochner, and Taber (1998) and Guvenen and Kuruscu (2009).

[4] The online appendix is available at www.wiso.uni-koeln.de/aspsamp/cmr/alexludwig/downloads/HKApp.pdf.

structure is exogenously determined by time varying demographic processes for fertility and mortality, the main driving forces of our model.[5] Firms produce with a standard constant returns to scale production function in a perfectly competitive environment. We assume that the U.S. is a closed economy.[6] Agents contribute a share of their wage to the pension system and retirees receive a share of current net wages as pensions. Technological progress is exogenous.

2.1 Timing, Demographics and Notation

Time is discrete and one period corresponds to one calendar year t. Each year, a new generation is born. Birth in this paper refers to the first time households make own decisions and is set to real life age of 16 (model age $j = 0$). Agents retire at an exogenously given age of 65 (model age $jr = 49$). Agents live at most until age 90 (model age $j = J = 74$). At a given point in time t, individuals of age j survive to age $j+1$ with probability $\varphi_{t,j}$, where $\varphi_{t,J} = 0$. The number of agents of age j at time t is denoted by $N_{t,j}$ and $N_t = \sum_{j=0}^{J} N_{t,j}$ is total population in t.

2.2 Households

Each household comprises of one representative agent who decides about consumption and saving, labor supply and human capital investment. The household maxi-

[5]We model neither endogenous life-expectancy or fertility, nor endogenous migration and assume that all exogenous migration is completed before agents start making economically relevant decisions (cf. online appendix).

[6]For our question, the closed economy assumption is a valid approximation. As documented in Krüger and Ludwig (2007), demographically induced changes in the return to physical capital and wages from the U.S. perspective do not differ much between small and open economy scenarios. The reason is that demographic processes are correlated across countries and, in terms of speed of the aging processes, the U.S. is somewhere in the middle when looking at all OECD countries.

mizes lifetime utility at the beginning of economic life ($j = 0$) in period t,

$$\max \sum_{j=0}^{J} \beta^j \pi_{t,j} \frac{1}{1-\sigma} \{c_{t+j,j}^{\phi}(1 - \ell_{t+j,j} - e_{t+j,j})^{1-\phi}\}^{1-\sigma}, \quad \sigma > 0, \tag{1}$$

where the per period utility function is a function of individual consumption c, labor supply ℓ and time investment into formation of human capital, e. The agent is endowed with one unit of time, so $1-l-e$ is leisure time. β is the pure time discount factor, ϕ determines the weight of consumption in utility and σ is the inverse of the inter-temporal elasticity of substitution with respect to the Cobb-Douglas aggregate of consumption and leisure time. $\pi_{t,j}$ denotes the (unconditional) probability to survive until age j, $\pi_{t,j} = \prod_{i=0}^{j-1} \varphi_{t+i,i}$, for $j > 0$ and $\pi_{t,0} = 1$.

Agents earn labor income (pension income when retired) as well as interest payments on their savings and receive accidental bequests. When working they pay a fraction τ_t from their gross wages to the social security system. The net wage income in period t of an agent of age j is given by $w_{t,j}^n = \ell_{t,j} h_{t,j} w_t (1 - \tau_t)$, where w_t is the (gross) wage per unit of supplied human capital at time t. There are no annuity markets and households leave accidental bequests. These are collected by the government and redistributed in a lump-sum fashion to all households. Accordingly, the dynamic budget constraint is given by

$$a_{t+1,j+1} = \begin{cases} (a_{t,j} + tr_t)(1 + r_t) + w_{t,j}^n - c_{t,j} & \text{if } j < jr \\ (a_{t,j} + tr_t)(1 + r_t) + p_{t,j} - c_{t,j} & \text{if } j \geq jr, \end{cases} \tag{2}$$

where $a_{t,j}$ denotes assets, tr_t are transfers from accidental bequests, r_t is the real interest rate, the rate of return to physical capital, and $p_{t,j}$ is pension income. Initial household assets are zero ($a_{t,0} = 0$) and the transversality condition is $a_{t,J+1} = 0$.

2.3 Formation of Human Capital

The key element of our model is endogenous formation of human capital. Households enter economic life with a predetermined and cohort invariant level of human capital $h_{t,0} = h_0$. Afterwards, they can invest a fraction of their time into acquiring additional human capital. We adopt a version of the Ben-Porath (1967) human capital technology[7] given by

$$h_{t+1,j+1} = h_{t,j}(1 - \delta^h) + \xi(h_{t,j}e_{t,j})^{\psi} \quad \psi \in (0,1),\ \ \xi > 0,\ \ \delta^h \geq 0, \tag{3}$$

where ξ is a scaling factor, the average learning ability, ψ determines the curvature of the human capital technology, δ^h is the depreciation rate of human capital and $e_{t,j}$ is time investment into human capital formation.

The costs of investing into human capital in this model are only the opportunity costs of foregone wage income and leisure. We understand the process of accumulating human capital as a mixture of knowledge acquired by formal schooling and on the job training programs after schooling. Human capital can be accumulated until retirement age but agents' optimally chosen time investment converges to zero some time before retirement.

2.4 The Pension System

The pension system is a simple balanced budget pay-as-you-go system. Workers contribute a fraction τ_t of their gross wages and pensioners receive a fraction ρ_t of

[7] This functional form is widely used in the human capital literature, cf. Browning, Hansen, and Heckman (1999) for a review.

the current average net wages of workers.[8] The level of pensions in each period is then given by $p_{t,j} = \rho_t(1-\tau_t)w_t\bar{h}_t$, where $\bar{h}_t = \frac{\sum_{j=0}^{jr-1}\ell_{t,j}h_{t,j}N_{t,j}}{\sum_{j=0}^{jr-1}\ell_{t,j}N_{t,j}}$ denotes average human capital of workers. Using the formula for $p_{t,j}$, the budget constraint of the pension system[9] simplifies to

$$\tau_t \sum_{j=0}^{jr-1} \ell_{t,j}N_{t,j} = \rho_t(1-\tau_t)\sum_{j=jr}^{J} N_{t,j} \quad \forall t. \tag{4}$$

Below, we consider two polar scenarios of parametric adjustment of the pension system to demographic change. In our first scenario ("const. τ"), we hold the contribution rate constant, $\tau_t = \bar{\tau}$, and endogenously adjust the replacement rate to balance the budget of the pension system. In the other extreme scenario ("const. ρ"), we hold the replacement rate constant, $\rho_t = \bar{\rho}$, and endogenously adjust the contribution rate.

2.5 Firms and Equilibrium

Firms operate in a perfectly competitive environment and produce one homogenous good according to the Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(A_tL_t)^{1-\alpha}, \tag{5}$$

where α denotes the share of capital used in production. K_t, L_t and A_t are the stocks of physical capital, effective labor and the level of technology, respectively. Output

[8] In the U.S. system, pension benefits are linked to individual monthly earnings which are indexed and averaged over the life-cycle (Diamond and Gruber 1999). The replacement rate, however, is a decreasing function of monthly earnings so that the earnings related linkage is incomplete. By ignoring this earnings related linkage, we somewhat overstate the distortion of the labor-human capital formation-leisure decision induced by the pension system.

[9] The budget constraint is given by $\tau_t w_t \sum_{j=0}^{jr-1}\ell_{t,j}h_{t,j}N_{t,j} = \sum_{j=jr}^{J} p_{t,j}N_{t,j} \quad \forall t$.

can be either consumed or used as an investment good. We assume that labor inputs and human capital of different agents are perfect substitutes and effective labor input L_t is accordingly given by $L_t = \sum_{j=0}^{jr-1} \ell_{t,j} h_{t,j} N_{t,j}$. Factors of production are paid their marginal products, i.e. $w_t = (1-\alpha)\frac{Y_t}{L_t}$ and $r_t = \alpha\frac{Y_t}{K_t} - \delta_t$, where w_t is the gross wage per unit of efficient labor, r_t is the interest rate and δ_t denotes the depreciation rate of physical capital. Total factor productivity, A_t, is growing at the exogenous rate of g_t^A: $A_{t+1} = A_t(1+g_t^A)$.

The definition of equilibrium is standard and relegated to our online appendix.

2.6 Thought Experiments

We take as exogenous driving process a time-varying demographic structure. Computations start in year 1850 ($t = 0$) assuming an artificial initial steady state. We then compute the model equilibrium from 1850 to 2400 ($t = T = 551$) – when the new steady state is assumed and verified to be reached[10] – and report simulation results for the main projection period of interest, from 2005 ($t = 156$) to 2050 ($t = 206$). We use data during our calibration period, $1960-2005$ (from $t_0 = 111$ to $t_1 = 156$), to determine several structural model parameters (cf. section 3).

Our main interest is to compare the time paths of aggregate variables and welfare across two model variants for two social security scenarios. Our first model variant is one in which households adjust their human capital and our second variant is one in which human capital is held constant across cohorts. Therefore, our strategy is to first

[10] In fact, changes in variables which are constant in steady state are numerically irrelevant already around the year 2300.

solve for the transitional dynamics using the model as described above. Next, we use the endogenously obtained profile of time investment into human capital formation to compute an average time investment and associated human capital profile which is then fed into our alternative model in which agents are restricted with respect to their time investment choice. We do so separately for the two polar social security scenarios described in subsection 2.4. The average time investment is computed as $\bar{e}_j = \frac{1}{t_1 - t_0 + 1} \sum_{t=t_0}^{t_1} e_{t,j}$ for our calibration period ($t_0 = 111$ and $t_1 = 156$). In the alternative model, we then add the constraint $e_{t,j} = \bar{e}_j$. The human capital profile is then obtained from (3) by iterating forward on age.[11]

3 Calibration and Computation

To calibrate the model, we choose model parameters such that simulated moments match selected moments in NIPA data and the endogenous wage profiles match the empirically observed wage profile in the U.S. during the calibration period 1960 − 2005.[12] The calibrated parameters are summarized in table 1 below.

[11]By imposing the restriction of identical time investment profiles for all cohorts (instead of, e.g., imposing the restriction only on cohorts born after 2005) we shut down direct effects from changing mortality on human capital and indirect anticipation effects of changing returns. This alternative model is a "standard" model of endogenous labor supply and an exogenously given age-specific productivity profile – as used in numerous studies on the consequences of demographic change – with the only exception being that the time endowment is age-specific. By setting time endowment to $1 - \bar{e}_j$ rather than 1 we avoid re-calibration across model variants, further see below.

[12]We perform this moment matching in the endogenous human capital model and the constant contribution rate scenario. We do not re-calibrate model parameters across social security scenarios or for the alternative human capital model because simulated moments do not differ much. Furthermore, we are interested in how our welfare conclusions are affected by imposing various constraints on the model – either through our social security scenarios or by restricting human capital formation – and any parametric change in this comparison would confound our welfare analysis.

3.1 Demographics

Actual population data from 1950 − 2005 are taken from the Human Mortality Database (2008). Our demographic projections beyond 2004 are based on a population model that is described in detail in the online appendix.[13]

3.2 Household Behavior

The parameter σ, the inverse of the inter-temporal elasticity of substitution, is set to 2. The time discount factor β is calibrated to match the empirically observed capital-output ratio of 2.8 which requires $\beta = 0.988$. The weight of consumption in the utility function, ϕ, is calibrated such that households spend one third of their time working on average which requires $\phi = 0.411$.

3.3 Individual Productivity Profiles

We choose values for the parameters of the human capital production function such that average simulated wage profiles resulting from endogenous human capital formation replicate empirically observed wage profiles. Data for age specific productivity are taken from Huggett et al. (2007)[14]. We first normalize $h_0 = 1$, and then determine the values of the structural parameters $\{\xi, \psi, \delta^h\}$ by indirect inference methods (Smith 1993; Gourieroux et al. 1993). To this end we run separate regressions of the

[13]The key assumptions of this model are as follows: First, the total fertility rate is constant at 2005 levels of 2.0185 until 2100 when fertility is adjusted slightly such as to keep the number of newborns constant for the remainder of the simulation period. Second, life expectancy monotonically increases from a current (2004) average life expectancy at birth of 77.06 years to 88.42 years in 2100 when it is held constant. Third, total migration is constant at the average migration for 1950 − 2005 for the remainder of the simulation period. These assumptions imply that a stationary population is reached in about 2200.

[14]We thank Mark Huggett for sending us the data.

data and simulated wage profiles on a 3rd-order polynomial in age given by

$$\log w_j = \eta_0 + \eta_1 j + \eta_2 j^2 + \eta_3 j^3 + \epsilon_j. \tag{6}$$

Here, w_j is the age specific productivity. Denote the coefficient vector determining the slope of the polynomial estimated from the actual wage data by $\overrightarrow{\eta} = [\eta_1, \eta_2, \eta_3]'$ and the one estimated from the simulated average human capital profile of cohorts born in $1960 - 2004$ by $\overrightarrow{\hat{\eta}} = [\hat{\eta}_1, \hat{\eta}_2, \hat{\eta}_3]'$. The latter coefficient vector is a function of the structural model parameters $\{\xi, \psi, \delta^h\}$. Finally, the values of our structural model parameters are determined by minimizing the distance $\|\overrightarrow{\eta} - \overrightarrow{\hat{\eta}}\|$, see subsection 3.6 for further details.

Figure 2 presents the empirically observed productivity profile and the estimated polynomials. Our coefficients[15] and the shape of the wage profile are in line with others reported in the literature, especially with those obtained by Hansen (1993) and Altig et al. (2001). The estimate of $\delta^h = 0.011$ is reasonable (Arrazola and de Hevia (2004), Browning, Hansen, and Heckman (1999)), and the estimate of $\psi = 0.67$ is just in the middle of the range reported in Browning, Hansen, and Heckman (1999).[16]

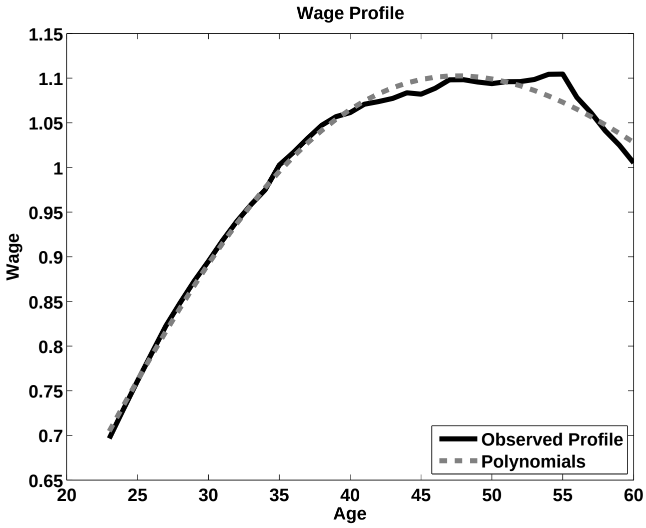
3.4 Production

We calibrate the capital share in production, α, to match the income share of labor in the data which requires $\alpha = 0.33$. The average growth rate of total factor

[15]The coefficient estimates from the data are η_0: -1.6262, η_1: 0.1054, η_2: -0.0017 and η_3: 7.83e-06. We do not display the calibrated profiles in figure 2 because they perfectly track the polynomial obtained from the data.

[16]In a sensitivity analysis we have shown that the estimate of the average time investment productivity, $\xi = 0.16$, depends on the predetermined value of h_0, whereas the other parameters are rather insensitive to this choice. We have also found that parameterizations with a different value for h_0 yield the same results for the effects of demographic change on aggregate variables and welfare.

Figure 2: Wage Profiles



Notes: Observed profile: average life-cycle wage profiles taken from Huggett, Ventura, and Yaron (2007). Polynomials: predicted wage profile based on estimated polynomial coefficients of (6). Both profiles were normalized by their respective means.

productivity, $\bar{g}^A$, is calibrated to match the growth rate of the Solow residual in the data. Accordingly, $\bar{g}^A = 0.018$. Finally, we calibrate $\bar{\delta}$ (and thereby scale the exogenous time path of depreciation, δ_t) such that our simulated data match an average investment output ratio of 20% which requires $\bar{\delta} = 0.039$.

3.5 The Pension System

In our first social security scenario ("const. τ") we fix contribution rates and adjust replacement rates of the pension system. We calculate contribution rates from NIPA data for $1960 - 2004$ and freeze the contribution rate at the year 2004 level for all following years. When simulating the alternative social security scenario with constant replacement rates ("const. ρ") we feed the equilibrium replacement rate obtained in the "const. τ" scenario into the model and hold it constant at the 2004 level for all the remaining years. Then, the contribution rate endogenously adjusts to balance the budget of the social security system.

3.6 Computational Method

For a given set of structural model parameters, solution of the model is by outer and inner loop iterations. On the aggregate level (outer loop), the model is solved by guessing initial time paths of four variables: the capital intensity, the ratio of bequests to wages, the replacement rate (or contribution rate) of the pension system and the average human capital stock for all periods from $t = 0$ until T. On the individual level (inner loop), we start each iteration by guessing the terminal values for consumption and human capital. Then we proceed by backward induction and iterate over these terminal values until convergence of the inner loop iterations.[17] In each outer loop, disaggregated variables are aggregated each period. We then update aggregate variables until convergence using the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2007).

To calibrate the model in the "const. τ" scenario, we consider additional "outer outer" loops to determine structural model parameters by minimizing the distance between the simulated average values and their respective calibration targets for the calibration period $1960 - 2004$. To summarize the description above, parameter values determined in this way are β, ϕ, δ, ξ, ψ and δ^h.

[17] As described in our online appendix, we implement a check for uniqueness of the solution at each age when computing optimal human capital decisions.

Table 1: Model Parameters

Preferences	σ	Inverse of Inter-Temporal Elasticity of Substitution	2.00
	β	Pure Time Discount Factor	0.988
	ϕ	Weight of Consumption	0.411
Human Capital	ξ	Scaling Factor	0.16
	ψ	Curvature Parameter	0.67
	δ^h	Depreciation Rate of Human Capital	1.1%
	h_0	Initial Human Capital Endowment	1.00
Production	α	Share of Physical Capital in Production	0.33
	$\bar{\delta}$	Depreciation Rate of Physical Capital	3.9%
	$\bar{g}^A$	Exogenous Growth Rate	1.8%

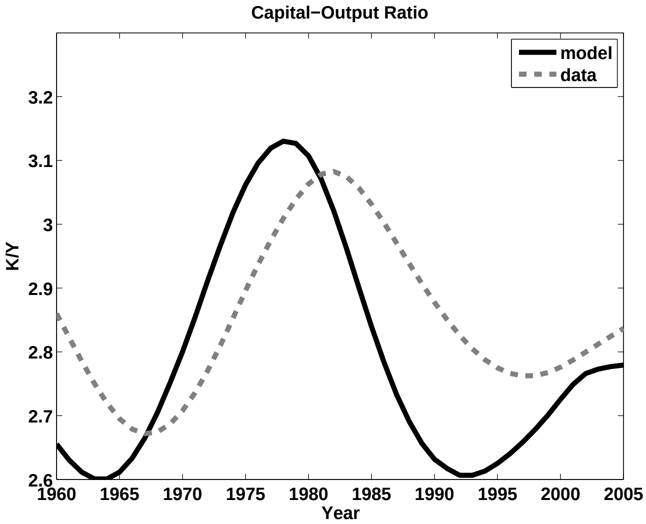
4 Results

Before using our model to investigate the effects of future demographic change, we show how well it can replicate observed aggregate variables and individual life-cycle profiles in the past. Next, we turn to the analysis of the transitional dynamics for the period 2005 to 2050 whereby we focus especially on the developments of major macroeconomic variables and the welfare effects of demographic change.

4.1 Backfitting

In order to backfit our model we do the following. First, we estimate series of TFP and actual depreciation using NIPA data. We HP filter these data series and then feed them into the model for the period 1950 to 2005. Thereafter, both parameters, g and δ are held constant at their respective means, see table 1. A key variable that determines paths of the rate of return to physical capital and wages is the capital output ratio. Figure 3 shows actual and fitted values for the period 1960-2005. Evidently, the fit of our model is quite remarkable along this key dimension of the data. Our model tracks the observed long-run swings in the data. The predicted amplitudes are slightly bigger in the model than they are in the data.

Figure 3: Capital Output Ratio

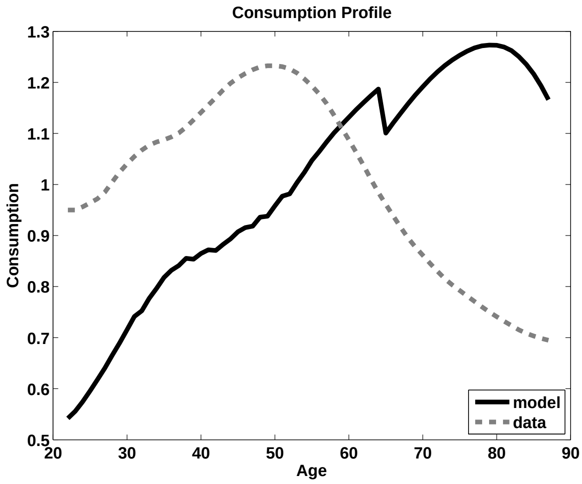


Notes: Capital-output ratio in the model and in aggregate data. The data is HP filtered.
Data Source: National income and product accounts (NIPA).

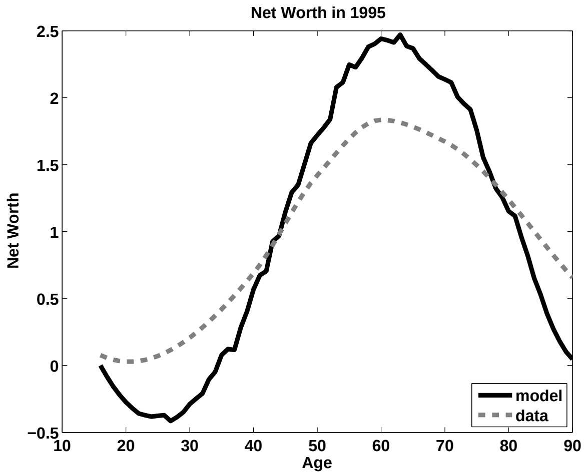
Turning to the individual level, we recognize that our model fails to replicate the empirically observed life-cycle consumption profile, cf. figure 4(a). The increase of consumption over the life cycle is too steep and the peak is too late compared to the data. Since in a model without idiosyncratic risk the decrease of consumption after the peak is solely caused by falling survival rates, we cannot expect to match this dimension of the data (cf. Hansen and Imrohoroglu (2008), Fernández-Villaverde and Krüger (2007)). As shown in Ludwig et al. (2007), omitting idiosyncratic risk has only a negligible effect on welfare calculations. This is because welfare calculations are based on differences in consumption profiles and the exact shape of the consumption profile is therefore less important.

Figure 4: Life-Cycle and Cross-Sectional Profiles

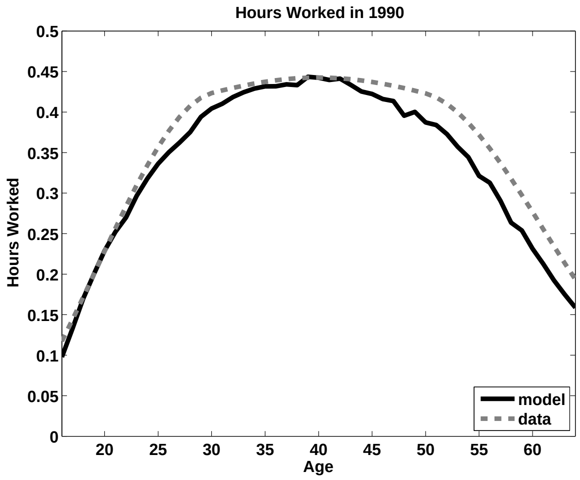
(a) Consumption



(b) Assets



(c) Labor Supply



(d) Wages

Notes: Model and data profiles for consumption, assets, labor supply and wages. The model consumption profile is the life-cycle consumption profile for the cohort born in 1960. The other profiles are cross-sectional profiles in 1990 and 1995. Consumption, asset and wage profiles are normalized by their respective mean. Hours data is normalized by 76 total hours per week.
Data Sources: Based on consumption profile estimated by Fernández-Villaverde and Krüger (2007), SCF net worth data obtained from Bucks et al. (2006), hours worked data from McGrattan and Rogerson (2004) and PSID wage data.

We next look at asset profiles. Figure 4(b) shows household net worth data from the Survey of Consumer Finances for a cross-section in 1995 obtained from Bucks et al. (2006) and the corresponding cross-sectional asset profile in the model. Our model matches the broad pattern in the data. Observed discrepancies are threefold: First, as borrowing constraints are absent from our model, initial assets are negative whereas they are positive in the data. Second, the run-up of wealth until retirement age is stronger in our model than it is in the data. Third, decumulation of assets is stronger as well. This last fact is due to the fact that our model neither has health risks as in De Nardi et al. (2009) nor explicit bequest motives.

Our model does a good job in matching the cross-sectional hours profile observed in 1990 Census data taken from McGrattan and Rogerson (2004), see figure 4(c).[18] We relegate a discussion of Frisch labor supply elasticities to the online appendix.

Figure 4(d) shows the cross-sectional wage profile observed in PSID data in 1990.[19] Although our model matches the broad pattern observed in the data, the fit is much better in 1970 and 1980, cf. the online appendix.[20]

4.2 Transitional Dynamics

We divide our analysis of the transitional dynamics into two parts. First, we analyze the behavior of several important aggregate variables from 2005 to 2050. Second, we investigate the welfare consequences of demographic change for generations already alive in 2005 and for households born in the future. Throughout, we demonstrate how the design of the social security system affects our results.

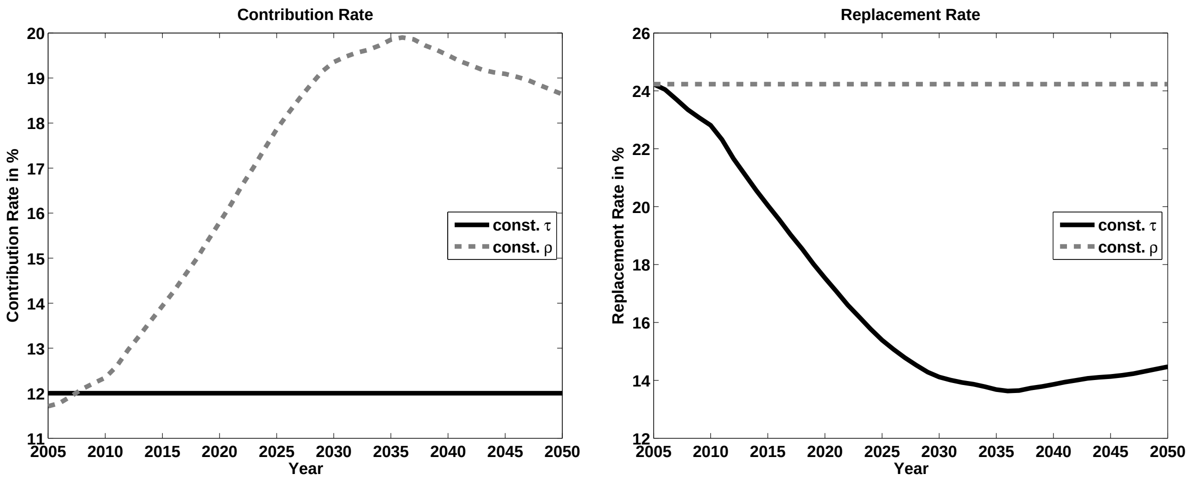
[18]The hours data is normalized with total hours per week equal to 76. This might appear to be a low number for total available hours. But such a magnitude is needed to make the McGrattan and Rogerson (2004) hours data broadly consistent with the common belief that agents spend about one third of their time working and standard practice of macroeconomists to calibrate their models (which we have followed).

[19]In order to smooth the data we show a centered average of five subsequent PSID samples.

[20]Part of this is due to the fortunate cohorts born after the war (see Ehrlich (2007) for a discussion).

Figure 5: Evolution of Policy Variables

(a) Contribution Rate (b) Replacement Rate



Notes: Pension system contribution and replacement rate for the two social security scenarios. "const. τ": constant contribution rate scenario. "const. ρ": constant replcement rate scenario.

4.2.1 Aggregate Variables

The evolution of the policy variables in the two social security scenarios are presented in figure 5.[21] In the "const. τ" scenario pensions become less generous over time represented by a decrease in the replacement rate from around 24% in 2005 to 14% in 2050. In contrast, in the "const. ρ" scenario the generosity of the pension system remains at the 2005 level implying that contribution rates have to increase from around 12% in 2005 to 19% in 2050.

Figure 6 reports the dynamics of four major macroeconomic variables for the two model variants – with endogenous and exogenous human capital – in the "const. τ" social security scenario and figure 7 does so in the "const. ρ" scenario.

[21]Figure drawn for the endogenous human capital model. The policy variables in the exogenous human capital model are similar.

In figures 6(a) and 7(a) we show the evolution of the rate of return to physical capital for the different models.[22] In the "standard" models with endogenous labor supply only, the rate of return decreases from an initial level of around 8% in 2005 to 7.1% in the "const. τ" scenario and to 7.5% in the "const. ρ" scenario in 2050.[23] This magnitude is in line with results reported elsewhere in the literature, cf., e.g., Börsch-Supan et al. (2006) and Krüger and Ludwig (2007) whereas Attanasio et al. (2007) find slightly bigger effects. On the contrary, in the two models with endogenous human capital adjustment, the rate of return is expected to fall by only 0.3 (0.1) percentage points in the "const. τ" ("const. ρ") scenario. This difference in the decrease of the rate of return between the exogenous and the endogenous human capital models is large, at a factor of about 3 (4.5).

In figure 6(b) and 7(b) we depict the evolution of average hours worked by all working age individuals. Average hours worked increase both for the endogenous and exogenous human capital models. Observe that there are level differences between the two model variants. This is mainly caused by differences in time investment into human capital formation.

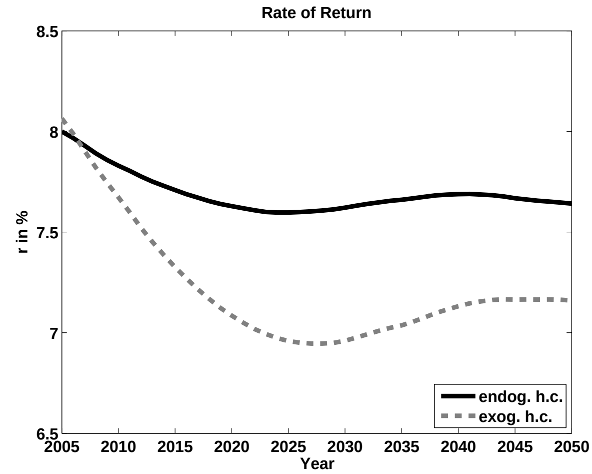
Figures 6(c) and 7(c) show that time investment into human capital formation increases when agents are allowed to adjust their human capital. Specifically, with endogenous human capital in the "const. τ" ("const. ρ") scenario average human capital per working hour increases by around 15% (10%) until 2050.

[22]There are two reasons for the small level differences in 2005 across the various scenarios. First, our calibration targets are averages for the period 1960 − 2004. Second, as already discussed in section 3, we do not recalibrate across scenarios. Such level differences in initial values can be observed in all of the following figures.

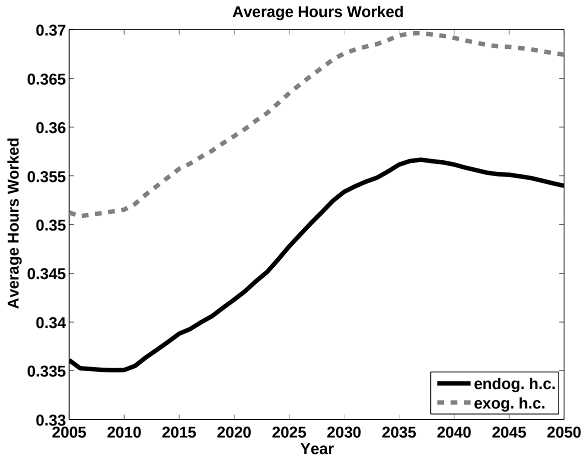
[23]The high initial level of the rate of return is caused by the baby boom in the past which increases the labor force and hence decreases capital intensity.

Figure 6: Aggregate Variables for Constant Contribution Rate Scenario

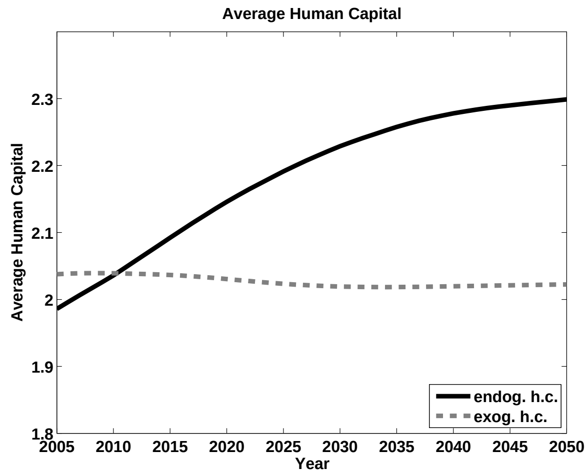
(a) Rate of Return to Physical Capital



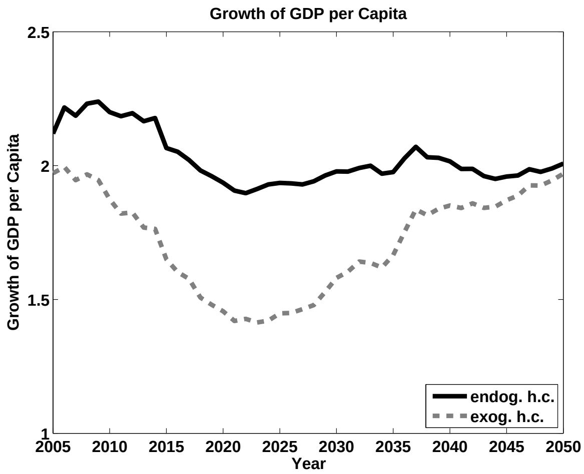
(b) Average Hours Worked



(c) Average Human Capital



(d) Growth of GDP per Capita in %

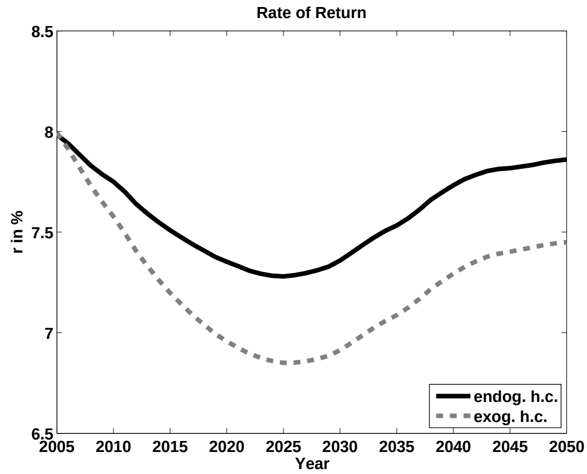


Notes: Rate of return to physical capital, average hours worked of the working age population, average human capital per working hour and growth of GDP per capita in the constant contribution rate social security scenario for two model variants. "endog. h.c.": endogenous human capital model. "exog. h.c.": exogenous human capital model.

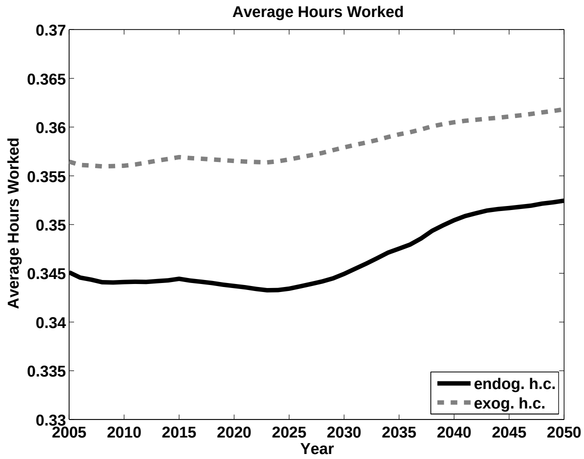
Finally, we focus on the evolution of the growth rate of GDP per capita as shown in figures 6(d) and 7(d). When the U.S. aging process peaks in 2025 (cf. figure 1), the growth rate of per capita GDP falls in all scenarios to its lowest level. The drop is least pronounced for the endogenous human capital model with a fixed contribution

Figure 7: Aggregate Variables for Constant Replacement Rate Scenario

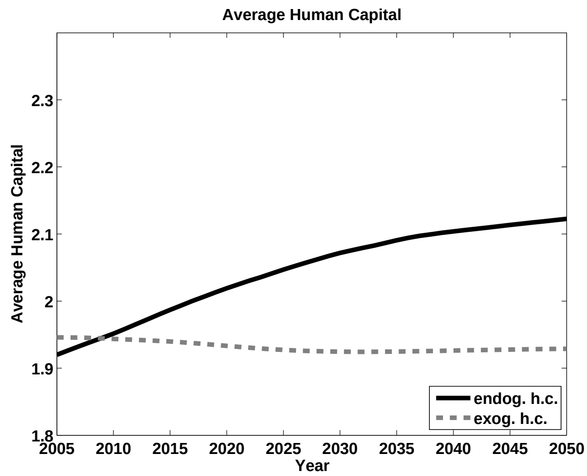
(a) Rate of Return to Physical Capital



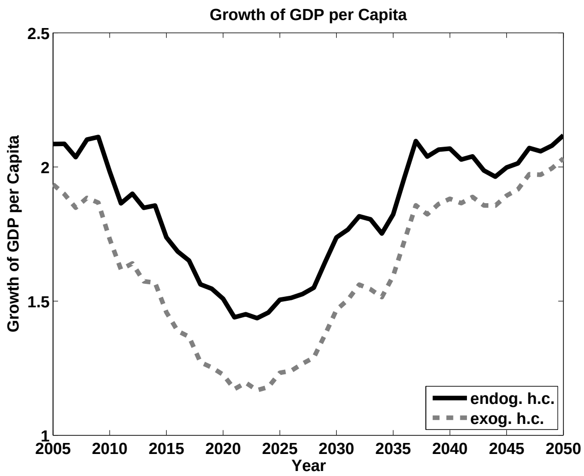
(b) Average Hours Worked



(c) Average Human Capital



(d) Growth of GDP per Capita in %



Notes: Rate of return to physical capital, average hours worked of the working age population, average human capital per working hour and growth of GDP per capita in the constant replacement rate social security scenario for two model variants. "endog. h.c.": endogenous human capital model. "exog. h.c.": exogenous human capital model.

rate. There, the growth rate gradually declines from 2.2% in 2005 to 1.9% in 2025.[24] Comparing the two "const. τ" scenarios, it can be seen that not adjusting the human capital profile entails a big drop in the growth rate. The maximum difference in about 2025 is 0.5 percentage points. Although the difference across human capital models

[24]The high initial growth rate is a consequence of the past baby boom, cf. footnote 23.

is only 0.3 percentage points in case the replacement rate is held constant ("const. ρ" scenarios), the same conclusion applies. The ageing process induces relative price changes so that agents increase their time investment into human capital formation and thereby cushion the negative effects of demographic change on growth.[25]

4.2.2 Welfare Effects

In our model, a household's welfare is affected by two consequences of demographic change. First, her lifetime utility changes because her own survival probabilities increase. Second, households face a path of declining interest rates, increasing gross wages and decreasing replacement rates (increasing contribution rates), relative to the situation without a demographic transition.

We want to isolate the welfare consequences of the second effect. To this end, we compare for an agent born at time t and of current age j her lifetime utility when she faces equilibrium factor prices, transfers and contribution (replacement) rates as documented in the previous section, with her lifetime utility when she instead faces prices, transfers and contribution (replacement) rates that are held constant at their 2005 value. For both of these scenarios we fix the households' individual survival probabilities at their 2005 values.[26] Following Attanasio et al. (2007) and Krüger and Ludwig (2007), we then compute the consumption equivalent variation $g_{t,j}$, i.e.

[25] In our online appendix we show that the cumulative effect of these growth rate differences between the endogenous and exogenous human capital model on the level of GDP per capita are large. With human capital adjustments the detrended level of GDP per capita will increase by around 15% (10%) more until 2050 in the "const. τ" ("const. ρ") scenario than without these adjustments.

[26] Of course, they fully retain their age-dependency. We show in the online appendix that varying the survival probabilities according to the underlying demographic projections leaves our conclusions on welfare in the comparison across the two models essentially unchanged.

the percentage increase in consumption that needs to be given to an agent with characteristics t, j at each date in her remaining lifetime at fixed prices to make her as well off as under the situation with changing prices.[27] Positive numbers of $g_{t,j}$ thus indicate that households obtain welfare gains from the general equilibrium effects of demographic change, negative numbers are welfare losses.

Welfare of Generations Alive in 2005

Of particular interest is how the welfare of all generations already alive in 2005 will be affected by demographic change. This analysis allows for an inter-generational welfare comparison of the consequences of demographic change in terms of wellbeing that is not possible using aggregated figures such as per capita GDP. Newborns and young generations benefit from increasing wages as well as decreasing returns if they borrow to finance their human capital formation. However, older – and thus asset-rich – generations are expected to lose lifetime utility: First, they benefit less from increasing wages because they do not significantly adjust their human capital and because their remaining working period is short, second, falling returns diminish their capital income and, third, retirement income decreases in our scenario with constant contribution rates.

Results, shown in figure 8, can be summarized as follows: First, newborn agents experience welfare gains in the "const. τ" scenarios of roughly 1% of life-time consumption and welfare losses of roughly 3% in the "const. ρ" scenarios. As explained

[27]With our assumptions on preferences, $g_{t,j}$ can be calculated as $g_{t,j} = \left(\frac{\bar{V}_{t,j}}{\bar{V}_j^{2005}}\right)^{\frac{1}{\phi(1-\sigma)}} - 1$, where $\bar{V}_{t,j}$ denotes lifetime utility at changing prices and $\bar{V}_j^{2005}$ at fixed 2005 prices.

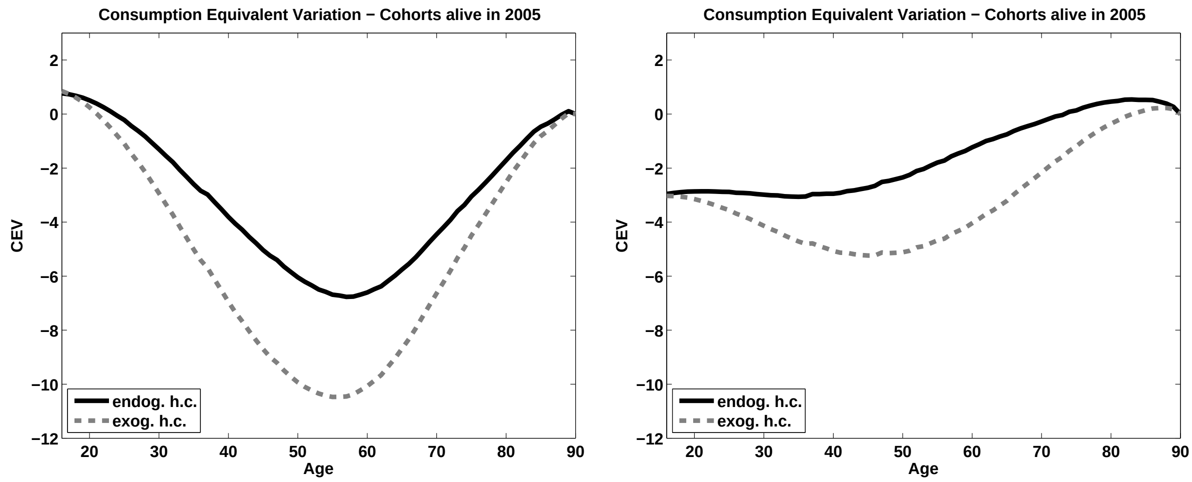
in our online appendix, the fact that these gains (and losses) are almost identical in the two human capital models is due to a complex interaction between the value of human capital adjustments which is positive and differential general equilibrium effects which partially offset this. Second, middle-aged agents incur the highest losses in the "const. τ" scenarios: the maximum loss of agents is much larger compared to a scenario with fixed replacement rates. Clearly, constant replacement rates decrease net wages of the young but keep pensions more generous. This decreases lifetime utility of the young but narrows the loss of utility of the old (compared to a situation with falling replacement rates). The redistribution through the pension system shifts the balance somewhat in favor of the old. This also explains why the maximum of the losses occurs at a much higher age in the "const. τ" scenario in which agents close to retirement lose interest income *and* receive lower pensions. Third, independent of future pension policy, agents lose relatively less in the endogenous human capital model. Younger agents can adjust their human capital in response to higher wages whereas older (asset-rich) households benefit from a smaller drop in the interest rate (cf. figures 6(a) and 7(a)) and higher pension payments.[28]

Table 2 finally provides numbers on the maximum welfare loss displayed in figure 8 as a summary statistic. It is important to emphasize that, in the exogenous human capital model, the maximum loss is about 3.7 (2.1) percentage points or 55% (71%) higher in the "const. τ" ("const. ρ") scenario than in the endogenous human capital model. This exemplifies that ignoring the adjustment channel through human capi-

[28] In the online appendix, we decompose the welfare differences between the two models into effects stemming from differential changes in factor prices and the relative rise in social security benefits which is caused by additional human capital formation.

Figure 8: Consumption Equivalent Variation of Agents alive in 2005

(a) Constant Contribution Rate Scenario (b) Constant Replacement Rate Scenario



Notes: Consumption equivalent variation (CEV) in the two social security scenarios.

tal formation leads to quantitatively important biases of the welfare assessments of demographic change.

Table 2: Maximum Utility Loss for Generations alive in 2005

	Human Capital	
	Endogenous	Exogenous
Const. τ ($\tau_t = \bar{\tau}$)	-6.8%	-10.5%
Const. ρ ($\rho_t = \bar{\rho}$)	-3.1%	-5.2%

Welfare of Future Generations

We next look at the welfare consequences for all future newborns. Due to increasing wages, agents born into a "const τ"-world with endogenous human capital experience gains of lifetime utility throughout the entire projection window. Agents with exogenous human capital born after 2035 incur utility losses of up to 1% of lifetime consumption. However, welfare losses for future generations can be quite large despite the human capital channel if the social security system is not reformed ("const ρ"). Despite of human capital adjustments, they are at about -7% of lifetime consumption for cohorts born around and after 2030 (-8% for exogenous human capital).[29]

5 Conclusion

This paper finds that increased investments in human capital may substantially mitigate the macroeconomic impact of demographic change with profound implications for individual welfare. As labor will be relatively scarce and capital will be relatively abundant in an aging society, interest rates will fall. As we emphasize, these effects will be much smaller once we account for changes in human capital formation. For the U.S., our simulations predict that if contribution rates (replacement rates) are kept constant then the rate of return will fall by only 0.4 (0.7) percentage points until 2025 with endogenous human capital, compared to 1.1 (1.1) percentage points in the standard model with a fixed human capital profile.

We also document that the welfare consequences from the increase in wages and declines in rates of return can be substantial, in the order of up to 0.8% (-3%) with constant contribution (replacement) rates in lifetime consumption for newborns in 2005. Thus, welfare gains for newborns only come along if social security contribution rates are held constant at current levels. Households that have already accumulated assets, on the other hand, lose from the decline in rates of return. Im-

[29]See graphs in the online appendix for more details.

portantly, we find that our model with exogenous human capital overstates these losses by $50 - 70\%$.

However we have operated in a frictionless environment where all endogenous human capital adjustments are driven by relative price changes. If instead human capital formation is characterized by substantial market failures then these automatic adjustments will be inhibited. In this case appropriate education and training policies in aging societies are an important topic for future research and the policy agenda.

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Overview

This is an online appendix for the paper "Demographic Change, Human Capital and Welfare" by Alexander Ludwig, Thomas Schelkle and Edgar Vogel. It contains further material that could not be included in the paper due to space limitations. The appendix is organized as follows. Section A contains the formal equilibrium definition. Section B provides more results on the fit of our model to observed life-cycle profiles of hours and wages, the implied labor supply elasticities of our model and additional results on predicted aggregate variables during the demographic transition as well as the associated welfare effects. Our population model is explained in section C. Details on our computational procedures can be found in section D.

A Equilibrium

Denoting current period/age variables by x and next period/age variables by x', a household of age j solves, at the beginning of period t, the maximization problem

$$V(a,h,t,j) = \max_{c,\ell,e,a',h'} \{u(c, 1-\ell-e) + \varphi\beta V(a',h',t+1,j+1)\} \tag{7}$$

subject to $w^n_{t,j} = \ell_{t,j} h_{t,j} w_t (1-\tau_t)$, (2), (3) and the constraints $\ell \in [0,1)$, $e \in [0,1)$.

Definition 1. *Given the exogenous population distribution and survival rates in all periods $\{\{N_{t,j}, \varphi_{t,j}\}_{j=0}^{J}\}_{t=0}^{T}$, an initial physical capital stock and an initial level of average human capital $\{K_0, \bar{h}_0\}$, and an initial distribution of assets and human capital $\{a_{t,0}, h_{t,0}\}_{j=0}^{J}$, a competitive equilibrium are sequences of individual variables $\{\{c_{t,j}, \ell_{t,j}, e_{t,j}, a_{t+1,j+1}, h_{t+1,j+1}\}_{j=0}^{J}\}_{t=0}^{T}$, sequences of aggregate variables $\{L_t, K_{t+1}, Y_t\}_{t=0}^{T}$, government policies $\{\rho_t, \tau_t\}_{t=0}^{T}$, prices $\{w_t, r_t\}_{t=0}^{T}$, and transfers $\{tr_t\}_{t=0}^{T}$ such that*

1. *given prices, bequests and initial conditions, households solve their maximization problem as described above,*
2. *interest rates and wages are paid their marginal products, i.e. $w_t = (1-\alpha)\frac{Y_t}{L_t}$ and $r_t = \alpha\frac{Y_t}{K_t} - \delta$,*
3. *per capita transfers are determined by*

$$tr_t = \frac{\sum_{j=0}^{J} a_{t,j}(1-\varphi_{t-1,j-1})N_{t-1,j-1}}{\sum_{j=0}^{J} N_{t,j}}, \tag{8}$$

4. *government policies are such that the budget of the social security system is balanced every period, i.e. equation (4) holds $\forall t$, and household pension income is given by $p_{t,j} = \rho_t(1-\tau_t)w_t\bar{h}_t$,*

5. *markets clear every period:*

$$L_t = \sum_{j=0}^{jr-1} \ell_{t,j} h_{t,j} N_{t,j} \tag{9a}$$

$$K_{t+1} = \sum_{j=0}^{J} a_{t+1,j+1} N_{t,j} \tag{9b}$$

$$Y_t = \sum_{j=0}^{J} c_{t,j} N_{t,j} + K_{t+1} - (1-\delta) K_t. \tag{9c}$$

Definition 2. *A stationary equilibrium is a competitive equilibrium in which per capita variables grow at the constant rate $1 + \bar{g}^A$ and aggregate variables grow at the constant rate $(1 + \bar{g}^A)(1 + n)$.*

B Further Results

B.1 Backfitting

Figure 9 presents the fit of our model to cross-sectional hours data from McGrattan and Rogerson (2004) for the years 1970, 1980, 1990 and 2000. We observe that our model does a very good job in matching the data along this dimension from 1980 onwards.

A comparison between wage profiles observed in PSID data and the model is shown in Figure 10. The fit of our model is very good in 1970 and 1980 and still broadly consistent with the data in 1990 and 2000.

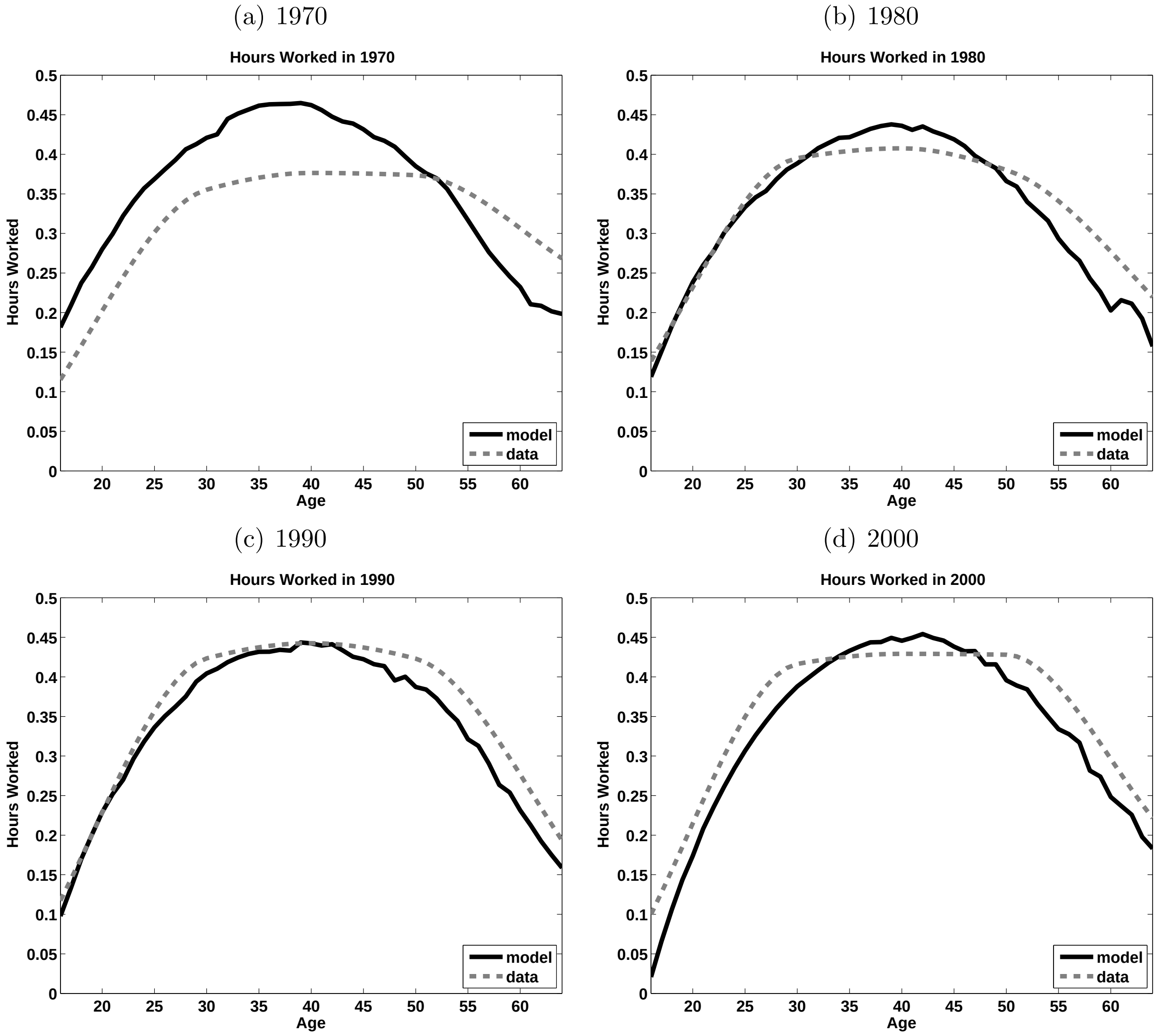
B.2 Labor Supply Elasticities

Since agents' human capital investments do not only depend on changes in relative returns but also on the extent of labor supply adjustments, realistic labor supply elasticities are key for our analysis. First, we compute the Frisch (or λ-constant) elasticity of labor supply that holds the marginal utility of wealth constant. We do so using the standard formula. In the context of our model this means holding time investment into human capital formation constant. It is then given by

$$\epsilon_{\ell,w}^{j} = \frac{1 - \phi(1-\sigma)}{\sigma} \frac{1 - \ell_j - e_j}{\ell_j}, \tag{10}$$

see Browning, Hansen, and Heckman (1999) for a derivation. In our model the Frisch elasticity depends on the amount of leisure and labor supply and therefore is age-dependent. As a consequence of the hump-shaped labor supply, the Frisch labor supply elasticity is u-shaped over the life-cycle. During the years 1960-2000 we find

Figure 9: Labor Supply

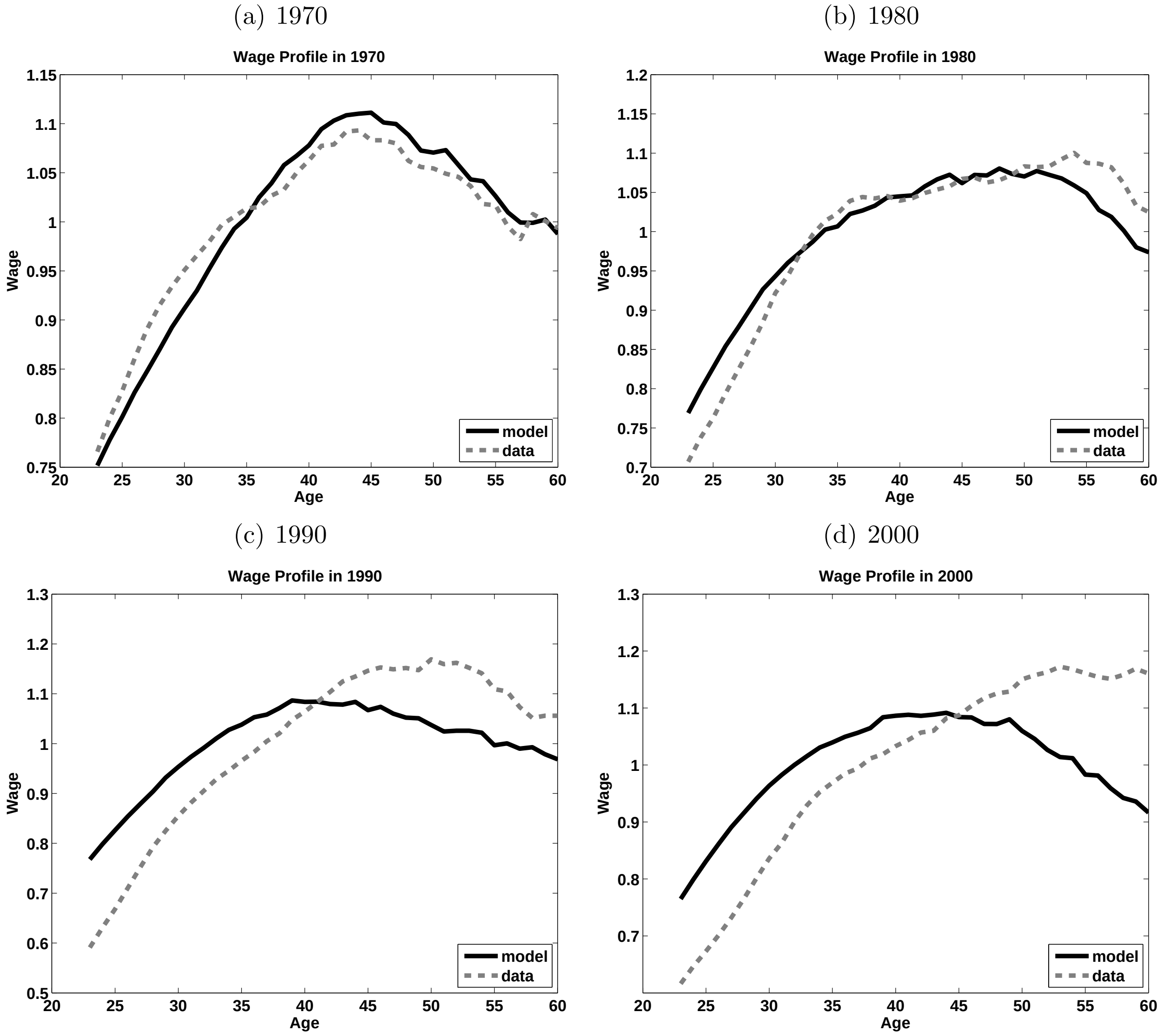


Notes: Model and data profiles for labor supply. Hours data is normalized by 76 total hours.
Data Sources: Based on hours worked data from the Decennial Censuses obtained from McGrattan and Rogerson (2004).

that agents between age 25 and 55 have a labor supply elasticity between 0.7 and 1.0, while it is higher for younger and older agents. For agents of age 30-50 (20-60) the average Frisch elasticity is around 0.86 (1.10), while across all agents the average is around 1.36. If we aggregate the u-shaped micro Frisch elasticities to a macro Frisch elasticity that takes the differing initial labor supply at different ages of life into account then this yields a number around 1.17 for the macro elasticity.

We also report a Frisch labor supply elasticity that allows time investment into human capital formation to vary. In the spirit of the Frisch elasticity concept we hold the marginal utility of human capital constant in addition to the marginal utility of

Figure 10: Wages



Notes: Model and data profiles for wages. The data is a centered average of five subsequent PSID samples.
Data Sources: Based on PSID wage data.

wealth. This Frisch elasticity is then given by

$$\widetilde{\epsilon}^j_{\ell,w} = \frac{1-\phi(1-\sigma)}{\sigma}\frac{1-\ell_j-e_j}{\ell_j} + \frac{1}{1-\psi}\frac{e_j}{\ell_j}. \tag{11}$$

As usual, an interior solution is assumed here. If we use this concept then the labor supply elasticity is higher because the second term is positive, i.e. agents invest less into human capital formation when they face a higher wage today and the marginal utility of human capital remains unchanged. Due to decreasing time investment into human capital formation, the second term decreases over the life-cycle. The resulting labor supply elasticity is still u-shaped over the life-cycle. Accordingly, during 1960-2000 for agents of age 30-50 (20-60) the resulting average Frisch elasticity with varying time investment is around 1.26 (1.79), while across all agents the average is around 2.47. Here, the macro Frisch elasticity is around 1.97 when accounting for the differing initial labor supply across agents of different age.

The numbers we find in our model are slightly higher than the standard estimates reported in the literature which are about 0.5, see e.g. Domeij and Flodén (2006), or even lower, see table 3.3 in Browning, Hansen, and Heckman (1999). However, the data used by the empirical literature usually refers to prime-age, full-time employed, male workers and therefore captures mainly the intensive margin. As reported above, if we restrict attention to a subset of agents in the model, e.g. those of age 30-50, that is most comparable to the data set of a typical empirical study then the estimates are quite close. Furthermore, the fact that the empirical literature focusses mostly on the intensive margin and neglects much of the extensive margin suggests that the empirical estimates are a lower bound on the true labor supply elasticity. Browning, Hansen, and Heckman (1999) also report that empirical estimates for females can be much higher than for males. The u-shape of labor supply elasticities in our model can be regarded as a good property because the extensive margin is probably most relevant towards the beginning and the end of the life-cycle.

Another potential source of downward bias of the empirical literature results from not considering endogenous human capital accumulation explicitly and thereby not correctly accounting for the true opportunity cost of time. This was shown by Imai and Keane (2004) in the context of a learning-by-doing model, so it is not directly applicable to our model. But similar biases might also be present here. With regard to the Frisch elasticity with varying time investment reported above we are unaware of any attempt to estimate the Frisch elasticity empirically in this model framework, which would mean to include the marginal utility of human capital in the set of conditioning variables.

Lastly, as shown in figures 9 and 10 our model does a good job in replicating observed life-cycle profiles of hours and wages. This is probably a more meaningful test of the ability of our model to explain the relation between hours worked and wages than comparing a single number that is very hard to identify empirically.

B.3 Transitional Dynamics

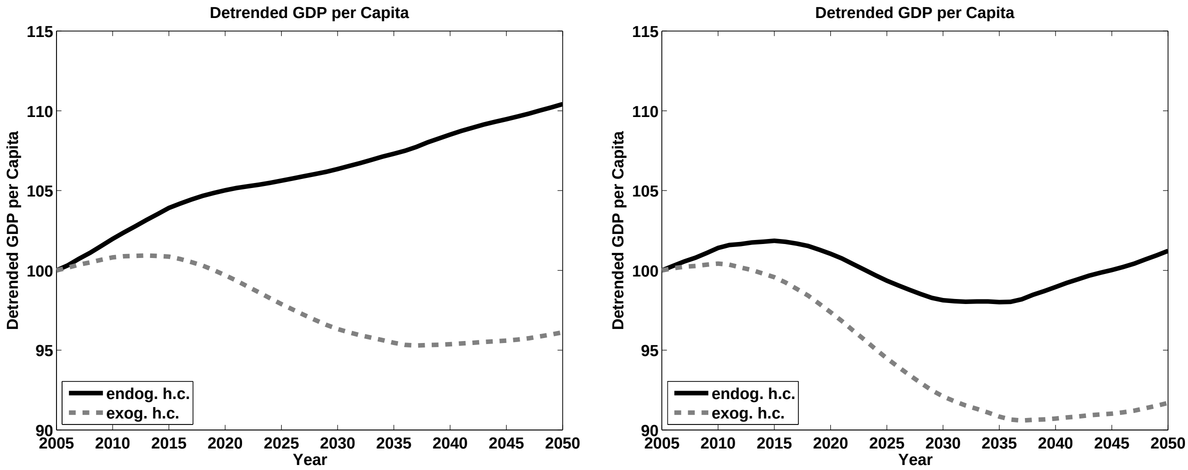
B.3.1 Aggregate Variables

The cumulative effect of the differences in growth rates on GDP per capita are displayed in figure 11. In the endogenous human capital model with constant contribution (replacement) rates, GDP per capita will increase by about 15% (10%) more until the year 2050 than without human capital adjustments.

Figure 11: Detrended GDP per Capita [Index, 2005=100]

(a) Constant Contribution Rate Scenario

(b) Constant Replacement Rate Scenario



B.3.2 Welfare Effects

Welfare of Future Generations

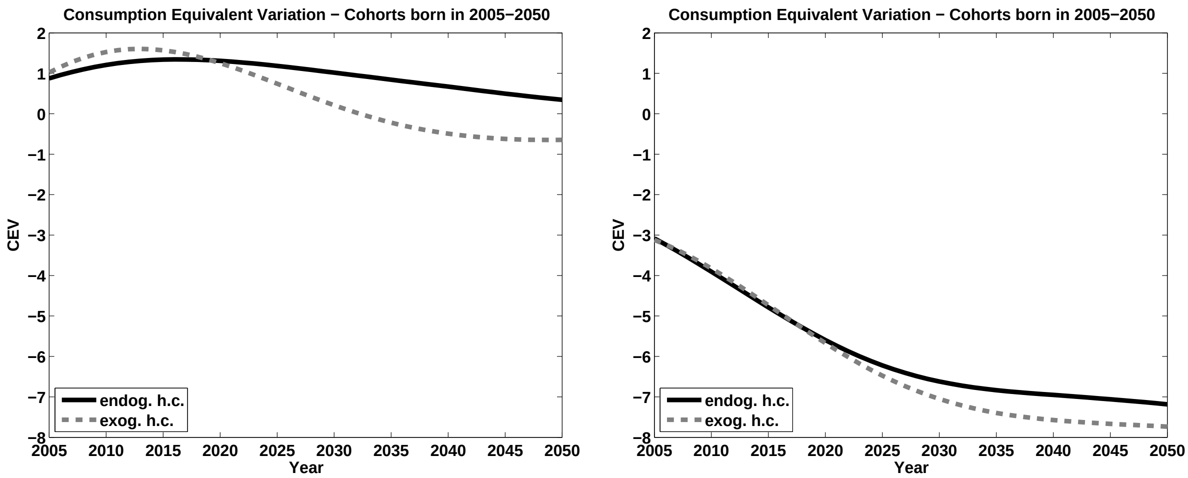
In our main text, we mostly analyze the welfare consequences for agents alive in 2005 and only briefly glance at the consequences for future generations. We here look at those. Figure 12 shows the consumption equivalent variation for the two models and the two social security scenarios. Agents born into a world with endogenous human capital and constant contribution rates experience gains of lifetime utility throughout the entire projection window. Even if agents are allowed to invest into human capital, welfare losses of future generation can be quite large if the contribution rates rise ("const ρ"). Notice again that, in our comparison across models, differences are not large because the positive value of human capital adjustments is offset by the more beneficial general equilibrium effects in the exogenous human capital model. For this reason welfare gains for some cohorts may even be slightly higher in the exogenous human capital model when the contribution rate is held constant.

The Value of Human Capital Adjustments

From figure 8 of our main text, we observe that welfare gains (and losses) for newborns are almost identical in the endogenous and exogenous human capital models. Detailed numbers are provided in table 3. The explanation for these similar welfare consequences is as follows: While the value of human capital adjustments is positive (see below), the increase of wages and the associated decrease of interest rates is much stronger in the exogenous human capital model. As newborn households generally benefit from the combined effects of increasing wages and decreasing returns,

Figure 12: Consumption Equivalent Variation of Agents born in 2005-2050

(a) Constant Contribution Rate Scenario (b) Constant Replacement Rate Scenario



Notes: Consumption equivalent variation (CEV) in the two social security scenarios.

welfare gains from these general equilibrium effects are higher in the exogenous human capital model. This explains why the overall welfare consequences for newborns across models do not differ much despite the fact that the value of human capital adjustments is positive.

Table 3: CEV for Generation Born in 2005 [in %]

	Human Capital	
	Endogenous	Exogenous
Const. τ ($\tau_t = \bar{\tau}$)	0.8%	0.9%
Const. ρ ($\rho_t = \bar{\rho}$)	-3.0%	-3.0%

Notes: CEV: consumption equivalent variation.

Our comparison across models does not tell us anything about the value of a flexible adjustment of human capital investments from the individual perspective, that is, about the value of human capital adjustments within the endogenous human capital model. To accomplish this, we store from our computation of $\bar{V}_j^{2005}$ (see above) the associated endogenous time investment profile, $\{e_j^{2005}\}_{j=0}^J$. Next, we compute $\bar{V}_{t,j}^{CE}$ as the lifetime utility of agents born at time t, age j facing constant 2005 survival rates, a sequence of equilibrium prices, transfers and contribution (replacement) rates as documented for the endogenous human capital model in the previous section, but keep the time investment profile fixed at $\{e_j^{2005}\}_{j=0}^J$. In correspondence to what we did before, we then compute

$$g_{t,j}^{CE} = \left(\frac{\bar{V}_{t,j}^{CE}}{\bar{V}_j^{2005}} \right)^{\frac{1}{\phi(1-\sigma)}} - 1, \tag{12}$$

as the consumption equivalent variation with constant time investment decisions. The difference $g_{t,j} - g_{t,j}^{CE}$ is then our measure of the value of endogenous human capital (where $g_{t,j}$ is the consumption equivalent variation with flexible time investments as computed above).[30]

The value of human capital adjustments is obviously positive and more or less monotonically decreasing with age (because of decreasing time investments over the life-cycle). Furthermore, for all future generations, the value of human capital adjustments can be expected to increase slightly because of the increasing rate of return to human capital formation. For sake of brevity, we do not report these results and confine ourselves to a comparison of the value of human capital adjustments of newborns in 2005, that is $g_{156,0} - g_{156,0}^{CE}$ across social security scenarios. As reported in table 4, the value of human capital adjustments in the "const. τ" scenario is 0.35% compared to only 0.07% in the "const. ρ" scenario and thereby around 5 times higher.

Table 4: The Value of Human Capital Adjustments in 2005

Const. τ ($\tau_t = \bar{\tau}$)	0.35%
Const. ρ ($\rho_t = \bar{\rho}$)	0.07%

Notes: The value of human capital adjustments is computed as $g_{t,j} - g_{t,j}^{CE}$.

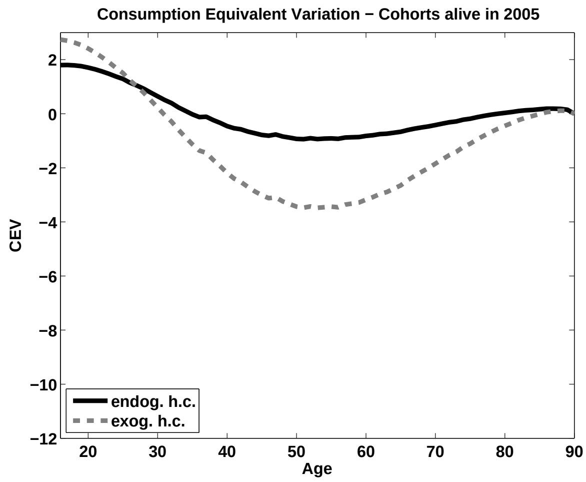
Role of the Pension System: Agents alive in 2005

We here provide a decomposition of our welfare results into the effects stemming from changes in relative factor prices and transfers and those of changing pension payments. To this end, figure 13 shows the welfare consequences of demographic change for agents alive in 2005 from changing factor prices alone, keeping pension payments constant. We here look only at our scenario with constant contribution rates. Table 5 presents the maximum utility loss for agents alive in 2005 with constant pension payments. In the exogenous human capital model, the maximum loss is about 2.6 percentage points (or 270%) higher than in the endogenous human capital model. Observe from table 2 of our main text that, in terms of the percentage point difference, this gain relative to the exogenous human capital model is roughly 3.7

[30] To see this more clearly, rewrite the welfare difference as $g_{t,j} - g_{t,j}^{CE} = (\bar{V}_j^{2005})^{-\frac{1}{\phi(1-\sigma)}} \left(\bar{V}_{t,j}^{\frac{1}{\phi(1-\sigma)}} - \bar{V}_{t,j}^{CE\,\frac{1}{\phi(1-\sigma)}} \right)$. The difference between the terms in the brackets is only due to the fact that agents are (or are not) allowed to adjust their human capital.

percentage points when pension payments adjust. From comparing these numbers we can therefore conclude that roughly two thirds of the overall gain of 3.7 percentage points can be attributed to differential changes in interest rates, wages and accidental bequests and one third to the relative rise in social security benefits which is caused by the additional human capital formation and the accompanying increase of average wages.

Figure 13: CEV of Agents alive in 2005 with constant pensions: Constant Contribution Rates



Notes: Consumption equivalent variation (CEV) in the constant contribution rate scenario with constant pension payments. "endog. h.c.": endogenous human capital model with constant pensions. "exog. h.c.": exogenous human capital model with constant pensions.

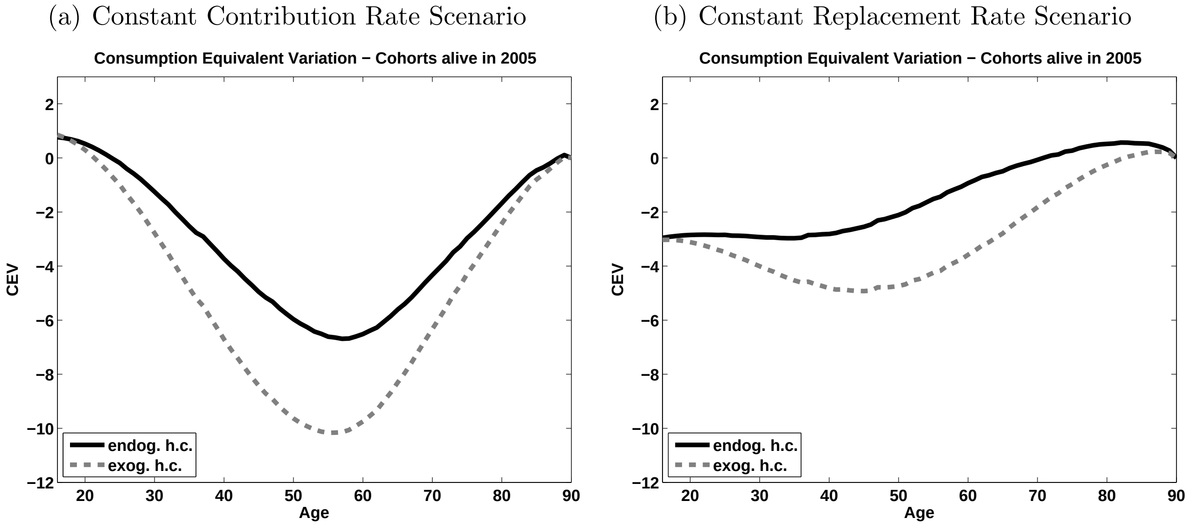
Table 5: Maximum Utility Loss for Generations alive in 2005 with Constant Pensions

	Human Capital	
	Endogenous	Exogenous
Const. τ ($\tau_t = \bar{\tau}$)	-0.94%	-3.47%

Role of Survival Rates for Welfare Calculations

So far, we computed the welfare effects of demographic change by holding survival rates constant. We here present welfare results for varying survival rates. Figures 14 and 15 present the results of these calculations. Table 6 presents the maximum utility loss for agents alive in 2005 with changing survival rates. Comparing these results to those of figures 8 and table 2 of our main text and those of figure 12, we can conclude that holding survival rates constant or varying them according to the underlying demographic projections does not affect our conclusions about the welfare consequences of demographic change in our comparisons across various scenarios.

Figure 14: CEV of Agents alive in 2005 with changing Survival Rates



Notes: Consumption equivalent variation (CEV) calculated with changing survival rates in the two social security scenarios.

Table 6: Maximum Utility Loss for Generations alive in 2005 with changing Survival Rates

	Human Capital	
	Endogenous	Exogenous
Const. τ ($\tau_t = \bar{\tau}$)	-6.7%	-10.2%
Const. ρ ($\rho_t = \bar{\rho}$)	-3.0%	-4.9%

C Demographic Data

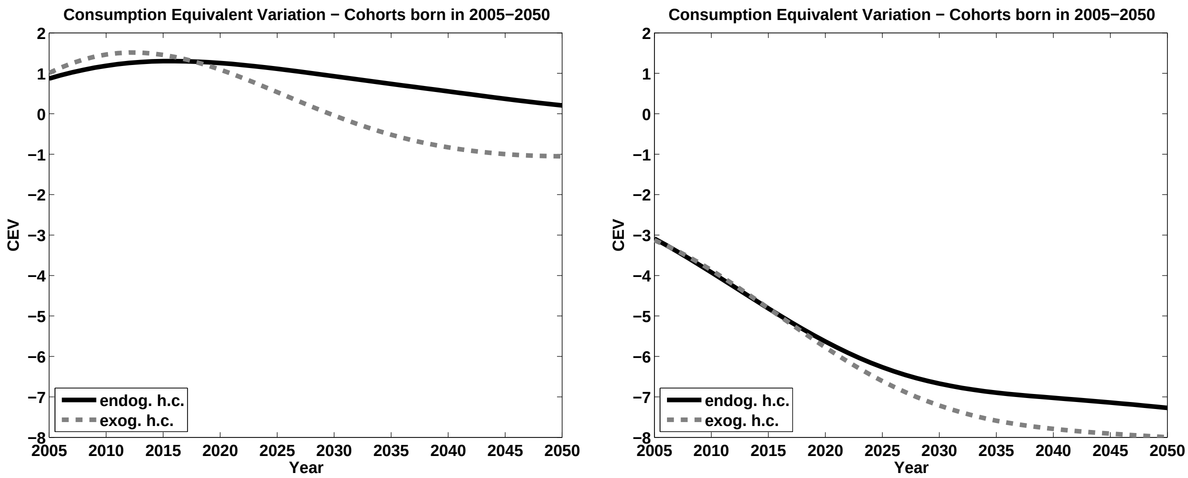
Our demographic data are based on the Human Mortality Database (2008). Population of age j in year t is determined by four factors: (i) an initial population distribution in year 0, (ii) age and time specific mortality rates, (iii) age and time specific fertility rates and (iv) age and time specific migration rates. We describe here how we model all of these elements and then briefly compare results of our demographic predictions with those of United Nations (2007).

Initial Population Distribution

We take as data the age and time specific population for the periods $1950 - 2004$.

Figure 15: CEV of Agents born in 2005-2050 with changing Survival Rates

(a) Constant Contribution Rate Scenario (b) Constant Replacement Rate Scenario



Notes: Consumption equivalent variation (CEV) calculated with changing survival rates in the two social security scenarios.

Mortality Rates

Our mortality model is based on sex, age and time specific mortality rates. To simplify notation, we suppress a separate index for sex. Using data from $1950-2004$, we apply a Lee-Carter procedure (Lee and Carter 1992) to decompose mortality rates as

$$\ln(1-\varphi_{t,j}) = a_j + b_j d_t, \qquad (13)$$

where a_j and b_j are vectors of age-specific constants and d_t is a time-specific index that equally affects all age groups. We assume that the time-specific index, d_t, evolves according to a unit-root process with drift,

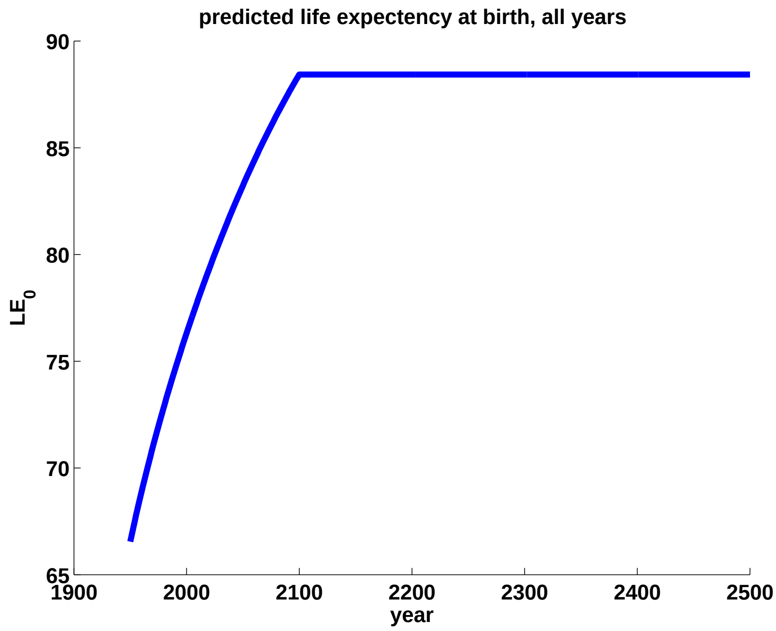
$$d_t = \chi + d_{t-1} + \epsilon_t. \qquad (14)$$

The estimate of the drift term is $\hat{\chi} = -1.2891$. We then predict mortality rates into the future (until 2100) by holding $\hat{a}_j$, $\hat{b}_j$ and $\hat{\chi}$ constant and setting $\epsilon_t = 0$ for all t. For all years beyond 2100 we hold survival rates constant at their respective year 2100 values. Figure 16 shows the corresponding path of life expectancy at birth.

Fertility Rates

Fertility in our model is age and time specific. For our predictions, we assume that age-specific fertility rates are constant at their respective year 2004 values for all periods $2005, \ldots, 2100$. For periods after 2100 we assume that the number of newborns is constant. Since the U.S. reproduction rate is slightly above replacement levels this

Figure 16: Life Expectancy at Birth



Notes: Own predictions of life-expectancy at birth based on Human Mortality Database (2008).

implies that the total fertility rate is slightly decreasing each year from 2100 onwards until about year 2200 when the population converges to a stationary distribution.

Population Dynamics

We use the estimated fertility and mortality data to forecast the future population dynamics. The transition of the population is accordingly given by

$$N_{t,j} = \begin{cases} N_{t-1,j-1}\varphi_{t-1,j-1} & \text{for } j > 0 \\ \sum_{i=0}^{J} f_{t-1,i} N_{t-1,i} & \text{for } j = 0, \end{cases} \tag{15}$$

where $f_{t,j}$ denotes age and time specific fertility rates. Population growth is then given by $n_t = \frac{N_{t+1}}{N_t} - 1$, where and $N_t = \sum_{j=0}^{J} N_{t,j}$ is total population in t.

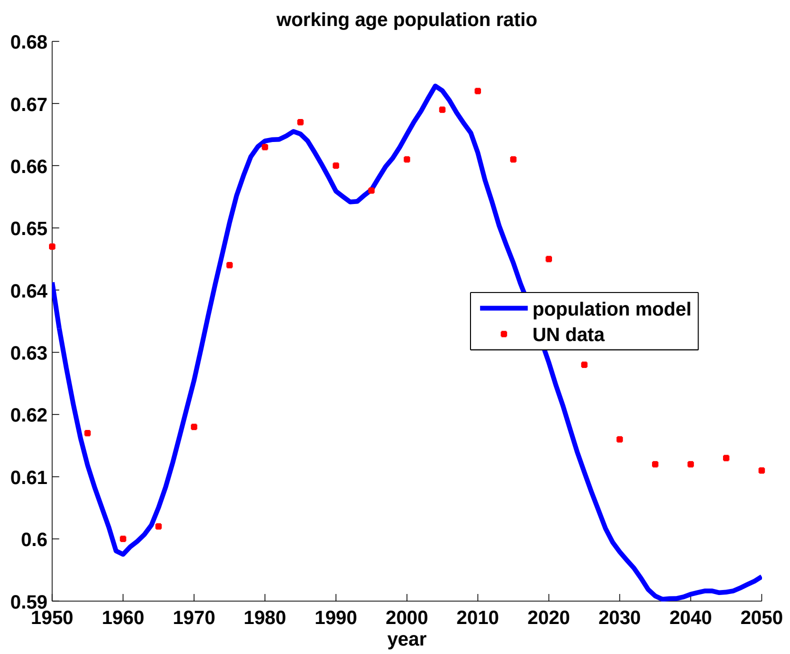
Migration

Migration is exogenous in our economic model. Setting migration to zero would lead us to overestimate future decreases in the working age population ratio and to overstate the increases in old-age dependency. We therefore restrict migration to ages $j \leq 15$ so that migration plays a similar role as fertility in our economic model. This simplifying assumption allows us to treat newborns and immigrants alike. We compute aggregate migration from United Nations (2007) and distribute age-specific migrants in each year equally across all ages $0, \ldots, 15$.

Evaluation

Figures 17-18 display the predicted working age population and old-age dependency ratios, according to our population model and according to United Nations (2007). Compared to this benchmark, our population model is close to the UN but predicts a slightly stronger decrease of the working age population ratio and a correspondingly stronger increase of the old-age dependency ratio until 2050.

Figure 17: Working Age Population Ratio



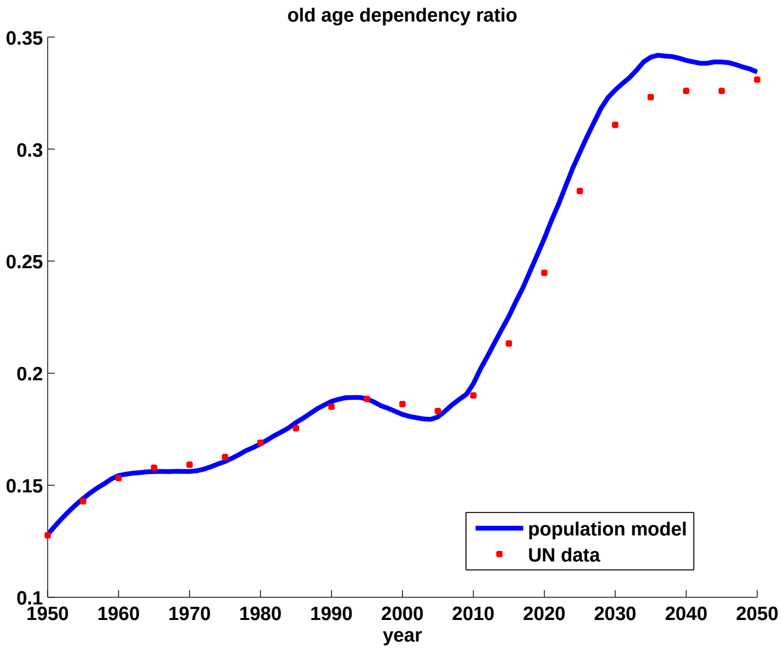
Notes: Population model: own predictions of the working age population ratio based on Human Mortality Database (2008). UN data: working age population ratio according to United Nations (2007).

D Computational Appendix

D.1 Household Problem

To simplify the description of the solution of the household model for given prices (wage and interest rate), transfers and social security payments, we focus on steady states and therefore drop the time index t. Furthermore, we focus on a de-trended version of the household problem in which all variables x are transformed to $\tilde{x} = \frac{x}{A}$ where A is the technology level growing at the exogenous rate g. To simplify notation, we do not denote variables by the symbol $\tilde{}$ but assume that the transformation is

Figure 18: Old Age Dependency Ratio



Notes: Population model: own predictions of the old-age dependency ratio based on Human Mortality Database (2008). UN data: old-age dependency ratio according to United Nations (2007).

understood. The de-trended version of the household problem is then given by

$$\begin{aligned} V(a,h,j) &= \max_{c,\ell,e,a',h'} \left\{ u(c, 1-\ell-e) + \beta s(1+g)^{\phi(1-\sigma)} V(a',h',j+1) \right\} \\ &\text{s.t.} \\ a' &= \frac{1}{1+g}\left((a+tr)(1+r)+y-c\right) \\ y &= \begin{cases} \ell h w(1-\tau) & \text{if } j < jr \\ p & \text{if } j \geq jr \end{cases} \\ h' &= g(h,e) \\ \ell &\in [0,1], \quad e \in [0,1]. \end{aligned} \tag{16}$$

Here, $g(h,e)$ is the human capital technology.

Let $\tilde{\beta} = \beta s(1+g)^{\phi(1-\sigma)}$ be the transformation of the discount factor. Using the budget constraints, now rewrite the above as

$$\begin{aligned} V(a,h,j) = \max_{c,\ell,e,a',h'} & \left\{ u(c,1-\ell-e) + \tilde{\beta} V\left(\frac{1}{1+g}\left((a+tr)(1+r)+y-c\right), g(h,e), j+1 \right) \right\} \\ & \text{s.t.} \\ & \ell \geq 0. \end{aligned}$$

where we have also replaced the bounded support of time investment and leisure with a one-side constraint on ℓ because the upper constraints, $\ell = 1$, respectively $e = 1$, and the lower constraint, $e = 0$, are never binding due to Inada conditions on the utility function and the functional form of the human capital technology (see below). Denoting by μ_ℓ the Lagrange multiplier on the inequality constraint for ℓ, we can write the first-order conditions as

$$c: \quad u_c - \tilde{\beta}\frac{1}{1+g}V_{a'}(a', h'; j+1) = 0 \tag{17a}$$

$$\ell: \quad -u_{1-\ell-e} + \tilde{\beta}hw(1-\tau)\frac{1}{1+g}V_{a'}(a', h', j+1) + \mu_\ell = 0 \tag{17b}$$

$$e: \quad -u_{1-\ell-e} + \tilde{\beta}g_e V_{h'}(a', h', j+1) = 0 \tag{17c}$$

and the envelope conditions read as

$$a: \quad V_a(a, h, j) = \tilde{\beta}\frac{1+r}{1+g}V_{a'}(a', h', j+1) \tag{18a}$$

$$h: \quad V_h(a, h, j) = \tilde{\beta}\left(\ell w(1-\tau)\frac{1}{1+g}V_{a'}(a', h', j+1) + g_h V_{h'}(a', h', j+1)\right). \tag{18b}$$

Note that for the retirement period, i.e. for $j \geq jr$, equations (17b) and (17c) are irrelevant and equation (18b) has to be replaced by

$$V_h(a, h, j) = \tilde{\beta}g_h V_{h'}(a', h', j+1).$$

From (17a) and (18a) we get

$$V_a = (1+r)u_c \tag{19}$$

and, using the above in (17a), the familiar inter-temporal Euler equation for consumption follows as

$$u_c = \tilde{\beta}\frac{1+r}{1+g}u_{c'}. \tag{20}$$

From (17a) and (17b) we get the familiar intra-temporal Euler equation for leisure,

$$u_{1-\ell-e} = hw(1-\tau)u_c + \mu_\ell. \tag{21}$$

From the human capital technology (3) we further have

$$g_e = \xi\psi(eh)^{\psi-1}h \tag{22a}$$

$$g_h = (1-\delta^h) + \xi\psi(eh)^{\psi-1}e. \tag{22b}$$

We loop backwards in j from $j = J-1, \ldots, 0$ by taking an initial guess of $[c_J, h_J]$ as given and by initializing $V_{a'}(\cdot, J) = V_{h'}(\cdot, J) = 0$. During retirement, that is for all ages $j \geq jr$, our solution procedure is by standard backward shooting using the first-order conditions. However, during the period of human capital formation, that is for all ages $j < jr$, the first order conditions would not be sufficient if the problem is not a convex-programming problem. And thus, our backward shooting algorithm will not necessarily find the true solution. In fact this may be the case in human capital models such as ours because the effective wage rate is endogenous (it depends on the human capital investment decision). For a given initial guess $[c_J, h_J]$ we therefore first compute a solution via first-order conditions and then, for each age $j < jr$, we check whether this is the unique solution. As an additional check, we consider variations of initial guesses of $[c_J, h_J]$ on a large grid. In all of our scenarios we never found any multiplicities.

The details of our steps are as follows:

1. In each j, $h_{j+1}, V_{a'}(\cdot, j+1), V_{h'}(\cdot, j+1)$ are known.
2. Compute u_c from (17a).
3. For $j \geq jr$, compute h_j from (3) by setting $e_j = \ell_j = 0$ and by taking h_{j+1} as given and compute c_j directly from equation (26) below.
4. For $j < jr$:
   (a) Assume $\ell \in [0, 1)$ so that $\mu_\ell = 0$.
   (b) Combine (3), (17b), (17c) and (22a) to compute h_j as

$$h_j = \frac{1}{1-\delta^h}\left(h_{j+1} - \xi\left(\frac{\xi\psi\frac{1}{1+g}V_{h'}(\cdot, j+1)}{\omega(1-\tau)V_{a'}(\cdot, j+1)}\right)^{\frac{1}{1-\psi}}\right) \tag{23}$$

   (c) Compute e from (3) as

$$e_j = \frac{1}{h_j}\left(\frac{h_{j+1} - h_j(1-\delta^h)}{\xi}\right)^{\frac{1}{\psi}}. \tag{24}$$

   (d) Calculate $lcr_j = \frac{1-e_j-\ell_j}{c_j}$, the leisure to consumption ratio from (21) as follows: From our functional form assumption on utility marginal utilities are given by

$$\begin{aligned} u_c &= \left(c^\phi(1-\ell-e)^{1-\phi}\right)^{-\sigma}\phi c^{\phi-1}(1-\ell-e)^{1-\phi} \\ u_{1-\ell-e} &= \left(c^\phi(1-\ell-e)^{1-\phi}\right)^{-\sigma}(1-\phi)c^\phi(1-\ell-e)^{-\phi} \end{aligned}$$

   hence we get from (21) the familiar equation:

$$\frac{u_{1-\ell-e}}{u_c} = hw(1-\tau) = \frac{1-\phi}{\phi}\frac{c}{1-\ell-e},$$

   and therefore:

$$lcr_j = \frac{1-e_j-\ell_j}{c_j} = \frac{1-\phi}{\phi}\frac{1}{hw(1-\tau)}. \tag{25}$$

(e) Next compute c_j as follows. Notice first that one may also write marginal utility from consumption as

$$u_c = \phi c^{\phi(1-\sigma)-1}(1-\ell-e)^{(1-\sigma)(1-\phi)}. \tag{26}$$

Using (25) in (26) we then get

$$\begin{aligned} u_c &= \phi c^{\phi(1-\sigma)-1}(lcr \cdot c)^{(1-\sigma)(1-\phi)} \\ &= \phi c^{-\sigma} \cdot lcr^{(1-\sigma)(1-\phi)}. \end{aligned} \tag{27}$$

Since u_c is given from (17a), we can now compute c as

$$c_j = \left(\frac{u_{c_j}}{\phi \cdot lcr_j^{(1-\sigma)(1-\phi)}} \right)^{-\frac{1}{\sigma}}. \tag{28}$$

(f) Given c_j, e_j compute labor, ℓ_j, as

$$\ell_j = 1 - lcr_j \cdot c_j - e_j.$$

(g) If $\ell_j < 0$, set $\ell_j = 0$ and iterate on h_j as follows:

i. Guess h_j

ii. Compute e as in step 4c.

iii. Noticing that $\ell_j = 0$, update c_j from (26) as

$$c = \left(\frac{u_c}{\phi(1-e)^{(1-\sigma)(1-\phi)}} \right)^{\frac{1}{\phi(1-\sigma)-1}}.$$

iv. Compute μ_ℓ from (17b) as

$$\mu_\ell = u_{1-\ell-e} - \tilde{\beta} h w(1-\tau) V_{a'}(\cdot, j+1)$$

v. Finally, combining equations (17b), (17c) and (22a) gives the following distance function f

$$f = e - \left(\frac{\tilde{\beta}\xi\psi h^{\psi} \frac{1}{1+g} V_{h'}(\cdot)}{\tilde{\beta}\omega h(1-\tau)V_{a'}(\cdot) + \mu_\ell} \right)^{\frac{1}{1-\psi}}, \tag{29}$$

where e is given from step 4(g)ii. We solve for the root of f to get h_j by a non-linear solver iterating on steps 4(g)ii through 4(g)v until convergence.

(h) *Check for uniqueness I*: What is computed above is a candidate solution under the assumption that the first-order conditions are necessary and sufficient. As a consequence of potential non-convexities of our programming problem first-order conditions may however not be sufficient and our procedure may therefore not give the unique global optimum. To address this, we next compute solutions on a grid and check if the previously computed candidate solution is indeed the only solution to our system of equations. We do so as follows: For a grid of $e_j \in [\underline{e} = 0.0001, \overline{e} = 0.9999]$, denote the equally spaced grid points by $e_{j,i}, i = 1, \ldots, ne$ and:

i. For each $e_{j,i}$, compute the corresponding $h_{j,i}$ from (3).
ii. Compute the corresponding $c_{j,i}, \ell_{j,i}$ by the analogous steps as described above, again taking the case distinction for binding labor into account.
iii. Compute the corresponding value of the distance function in (29), $f_{j,i}$.[31]

If for all $e_{j,i}, i = 1, \ldots, ne$ the value of the distance function, $f_{j,i}$, changes signs only once, then our previously computed candidate solution is indeed the unique optimum. If it would change signs more than once, then there would be multiplicities and our first-order conditions would accordingly not be sufficient. Setting $ne = 200$ we never found this to be the case in any of our scenarios.

5. Update as follows:
   (a) Update V_a using either (18a) or (19).
   (b) Update V_h using (18b).

Next, loop forward on the human capital technology (3) for given h_0 and $\{e_j\}_{j=0}^J$ to compute an update of h_J denoted by h_J^n. Compute the present discounted value of consumption, PVC, and, using the already computed values $\{h_j^n\}_{j=0}^J$, compute the present discounted value of income, PVI. Use the relationship

$$c_0^n = c_0 \cdot \frac{PVI}{PVC} \tag{30}$$

to form an update of initial consumption, c_0^n, and next use the Euler equations for consumption to form an update of c_J, denoted as c_J^n. Define the distance functions

$$g_1(c_J, h_J) = c_J - c_J^n \tag{31a}$$
$$g_2(c_J, h_J) = h_J - h_J^n. \tag{31b}$$

In our search for general equilibrium prices, constraints of the household model are occasionally binding. Therefore, solution of the system of equations in (31) using Newton based methods, e.g., Broyden's method, is instable. We solve this problem by a nested Brent algorithm, that is, we solve two nested univariate problems, an outer one for c_J and an inner one for h_J.

Check for uniqueness II: Observe that our nested Brent algorithm assumes that the functions in (31) exhibit a unique root. As we adjust starting values $[c_J, h_J]$ with each outer loop iteration we thereby consider different points in a variable box of $[c_J, h_J]$ as starting values. For all of these combinations our procedure always converged. To systematically check whether we also always converge to the unique optimum, we fix, after convergence of the household problem, a large box around the previously computed $[c_J, h_J]$. Precisely, we choose as boundaries for this box ±50% of the solutions in the respective dimensions. For these alternative starting values we then check whether there is an additional solution to the system of equations (31). We never detected any such multiplicities.

[31] Notice that if $\ell_{j,i} > 0$, then we know from equation (23) that $x_{j,i} = \{c_{j,i}, \ell_{j,i}, e_{j,i}, h_{j,i}\}$ cannot a solution but we still proceed by computing $f_{j,i}$.

D.2 The Aggregate Model

For a given $r \times 1$ vector $\vec{\Psi}$ of structural model parameters, we first solve for an "artificial" initial steady state in period $t = 0$ which gives initial distributions of assets and human capital. We thereby presume that households assume prices to remain constant for all periods $t \in \{0, \ldots, T\}$ and are then surprised by the actual price changes induced by the transitional dynamics. Next, we solve for the final steady state of our model which is reached in period T and supported by our demographic projections, see appendix C. For both steady states, we solve for the equilibrium of the aggregate model by iterating on the $m \times 1$ steady state vector $\vec{P}_{ss} = [p_1, \ldots, p_m]'$. p_1 is the capital intensity, p_2 are transfers (as a fraction of wages), p_3 are social security contribution (or replacement) rates and p_4 is the average human capital stock. Notice that all elements of $\vec{P}_{ss}$ are constant in the steady state.

Solution for the steady states of the model involves the following steps:

1. In iteration q for a guess of $\vec{P}^q_{ss}$ solve the household problem.
2. Update variables in $\vec{P}_{ss}$ as follows:
   (a) Aggregate across households to get aggregate assets and aggregate labor supply to form an update of the capital intensity, p^n_1.
   (b) Calculate an update of bequests to get p^n_2.
   (c) Using the update of labor supply, update social security contribution (or replacement) rates to get p^n_3.
   (d) Use labor supply and human capital decisions to form an update of the average human capital stock, p^n_4.
3. Collect the updated variables in $\vec{P}^n_{ss}$ and notice that $\vec{P}^n_{ss} = H(\vec{P}_{ss})$ where H is a vector-valued non-linear function.
4. Define the root-finding problem $G(\vec{P}_{ss}) = \vec{P}_{ss} - H(\vec{P}_{ss})$ and iterate on $\vec{P}_{ss}$ until convergence. We use Broyden's method to solve the problem and denote the final approximate Jacobi matrix by B_{ss}.

Next, we solve for the transitional dynamics by the following steps:

1. Use the steady state solutions to form a linear interpolation to get the starting values for the $m(T-2) \times 1$ vector of equilibrium prices, $\vec{P} = [\vec{p}'_1, \ldots, \vec{p}'_m]'$, where $p_i, i = 1, \ldots, m$ are vectors of length $(T-2) \times 1$.
2. In iteration q for guess $\vec{P}^q$ solve the household problem. We do so by iterating backwards in time for $t = T-1, \ldots, 2$ to get the decision rules and forward for $t = 2, \ldots, T-1$ for aggregation.
3. Update variables as in the steady state solutions and denote by $\tilde{\vec{P}} = H(\vec{P})$ the $m(T-2) \times 1$ vector of updated variables.

4. Define the root-finding problem as $G(\vec{P}) = \vec{P} - H(\vec{P})$. Since T is large, this problem is substantially larger than the steady state root-finding problem and we use the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2007) to form and update guesses of an approximate Jacobi matrix of the system of $m(T-2)$ non-linear equations. We initialize these loops by using a scaled up version of B_{ss}.

D.3 Calibration of Structural Model Parameters

We split the $r \times 1$ vector of structural model parameters, $\vec{\Psi}$, as $\vec{\Psi} = \left[(\vec{\Psi}^e)', (\vec{\Psi}^f)'\right]'$. $\vec{\Psi}^f$ is a vector of predetermined (fixed) parameters, whereas the $e \times 1$ vector $\vec{\Psi}^e$ is estimated by minimum distance (unconditional matching of moments using e moment conditions). Denote by

$$u_t(\vec{\Psi}^e) = y_t - f(\vec{\Psi}^e) \text{ for } t = 0, \dots, T_0 \tag{32}$$

the GMM error as the distance between data, y_t, and model simulated (predicted) values, $f(\vec{\Psi}^e)$.

Under the assumption that the model is correctly specified, the restrictions on the GMM error can be written as

$$E[u_t(\vec{\Psi}^e_0)] = 0, \tag{33}$$

where $\tilde{\Psi}^e_0$ denotes the vector of true values. Denote sample averages of u_t as

$$g_{T_0}(\vec{\Psi}^e) \equiv \frac{1}{T_0 + 1} \sum_{t=0}^{T_0} u_t(\vec{\Psi}^e). \tag{34}$$

We estimate the elements of $\vec{\Psi}^e$ by setting these sample averages to zero (up to some tolerance level).

In our economic model, only two parameters are pre-determined and we therefore have that

$$\vec{\Psi}^f = [\sigma, h_0]'. \tag{35}$$

The vector $\vec{\Psi}^e$ is given by

$$\vec{\Psi}^e = \left[g, \alpha, \delta, \beta, \phi, \psi, \xi, \delta^h\right]'. \tag{36}$$

We estimate the structural model parameters using data from various sources for the period $1960, ..., 2004$, hence $T_0 = 44$. The parameters $\vec{\Psi}^e_1 = [g, \alpha]'$ are directly determined using NIPA data on GDP, fixed assets, wages and labor supply. The remaining structural model parameters, $\vec{\Psi}^e_2 = [\delta, \beta, \phi, \psi, \xi, \delta^h]'$ are estimated by simulation. Our calibration targets are summarized in table 7.

Table 7: Calibration Targets

Parameter	Target	Moment
$\vec{\Psi}^f$		
σ	predetermined parameter	
h_0	predetermined parameter	
$\vec{\Psi}_1^e$		
g^A	growth rate of Solow residual	0.018
α	share of wage income	0.33
$\vec{\Psi}_2^e$		
δ	investment output ratio	0.2
β	capital output ratio	2.8
ϕ	average hours worked	0.33
ψ, ξ, δ^h	coefficients of wage polynomial (from PSID)	

Determining the subset of parameters $\vec{\Psi}_2^e$ along the transition is a computationally complex problem that we translate into an equivalent simple problem. Point of departure of our procedure is the insight that calibrating the model for a steady state is easy and fast. However, simulated steady state moments may differ quite substantially from simulated averages along the transition even when the steady state is chosen to lie in the middle of the calibration period, in our case year 1980. We therefore proceed as follows:

1. Initialization: Choose a vector of scaling factors, $\vec{sf}$, of length e_2 that appropriately scales the steady state calibration targets (see below).
2. Calibrate the model in some steady state year, e.g., 1980, by solving the system of equations
$$\frac{\bar{y}_{2,i}^e}{sf_i} - f_{2,i}^{e,ss}(\vec{\Psi}) \tag{37}$$
for all $i = 1, \ldots, e_2$ to get $\hat{\vec{\Psi}}_2^e$. Here, $\bar{y}_{2,i}^e$ is the average of moment i in the data for the calibration period (1960-2004), e.g., the investment-output ratio for $i = 1$.
3. For the estimated parameter vector, $\hat{\vec{\Psi}}_2^e$, solve the model along the transition.
4. Compute the relevant simulated moments for the transition, $f_2^e(\vec{\Psi})$.
5. Update the vector of scaling vectors as
$$sf_i = \frac{f_{2,i}^e(\vec{\Psi})}{f_{2,i}^{e,ss}(\vec{\Psi})} \tag{38}$$
for all $i = 1, \ldots, e_2$.
6. Continue with step 2 until convergence on scaling factors (fixed point problem).

We thereby translate a complex root-finding problem into a combination of a simple root-finding problem (steady state calibration) and a fixed point iteration on scaling factors. Since scaling factors are relatively insensitive to Ψ_2^e, convergence is fast and robust. The resulting scaling factors range from 0.94 to 1.29 which means that differences between simulated moments in the artificial steady state year (1980) and averages during the transition are large (up to 30%). This also implies that calibrating the model in some artificial steady year only would lead to significantly biased estimates of structural model parameters.

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