Demonstration of Monogamy Relations for Einstein-Podolsky-Rosen Steering in Gaussian Cluster States

Xiaowei Deng,[1,2] Yu Xiang,[2,3] Caixing Tian,[1,2] Gerardo Adesso,[4] Qiongyi He,[2,3,*] Qihuang Gong,[2,3] Xiaolong Su,[1,2,†] Changde Xie,[1,2] and Kunchi Peng[1,2]

[1]*State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan 030006, China*

[2]*Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan 030006, China*

[3]*State Key Laboratory of Mesoscopic Physics, School of Physics, Peking University, Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*

[4]*Centre for the Mathematics and Theoretical Physics of Quantum Non-Equilibrium Systems (CQNE), School of Mathematical Sciences, The University of Nottingham, Nottingham NG7 2RD, United Kingdom*

(Received 20 February 2017; published 7 June 2017)

Understanding how quantum resources can be quantified and distributed over many parties has profound applications in quantum communication. As one of the most intriguing features of quantum mechanics, Einstein-Podolsky-Rosen (EPR) steering is a useful resource for secure quantum networks. By reconstructing the covariance matrix of a continuous variable four-mode square Gaussian cluster state subject to asymmetric loss, we quantify the amount of bipartite steering with a variable number of modes per party, and verify recently introduced monogamy relations for Gaussian steerability, which establish quantitative constraints on the security of information shared among different parties. We observe a very rich structure for the steering distribution, and demonstrate one-way EPR steering of the cluster state under Gaussian measurements, as well as one-to-multimode steering. Our experiment paves the way for exploiting EPR steering in Gaussian cluster states as a valuable resource for multiparty quantum information tasks.

DOI: 10.1103/PhysRevLett.118.230501

Schrödinger [1] put forward the term "steering" to describe the "spooky action-at-a-distance" phenomenon pointed out by Einstein, Podolsky, and Rosen (EPR) in their famous paradox [2,3]. Wiseman, Jones, and Doherty [4] rigorously defined the concept of steering in terms of violations of a local hidden state model, and revealed that steering is an intermediate type of quantum correlation between entanglement [5,6] and Bell nonlocality [7,8], where local measurements on one subsystem can apparently adjust (steer) the state of another distant subsystem [9–12]. Such correlation is intrinsically asymmetric with respect to the two subsystems [13–19], and allows verification of shared entanglement even if the measurement devices of one subsystem are untrusted [11]. Because of this intriguing feature, steering has been identified as a physical resource for one-sided device-independent (1sDI) quantum cryptography [20–24], secure quantum teleportation [25–27], and subchannel discrimination [28].

Recently, experimental observation of multiparty EPR steering was reported in optical networks [29] and photonic qubits [30,31]. These experiments offer insights into understanding whether and how this special type of quantum correlation can be distributed over many different systems, a problem which has been recently studied theoretically by deriving so-called *monogamy relations* [32–38]. It has been shown that the residual Gaussian steering stemming from a monogamy inequality [36] can act as a quantifier of genuine multipartite steering [39] for pure three-mode Gaussian states, and acquires an operational interpretation in the context of a 1sDI quantum secret sharing protocol [40]. However, beyond [29], no systematic experimental exploration of monogamy constraints for EPR steering has been reported to date.

As generated via an Ising-type interaction, a cluster state features better persistence of entanglement than that of a Greenberger-Horne-Zeilinger (GHZ) state, and, hence, is considered as a valuable resource for one-way quantum computation [41–45] and quantum communication [46–49]. Continuous variable (CV) cluster states [50,51], which can be generated deterministically, have been successfully produced for eight [52], 60 [53], and up to 10 000 quantum modes [54]. Several quantum logical operations based on prepared CV cluster states have been experimentally demonstrated [55–58]. While the previous studies of multipartite steering mainly focus on the CV GHZ-like states [59], comparatively little is known about EPR steering and its distribution according to monogamy constraints in CV cluster states.

In this Letter, we experimentally investigate properties of bipartite steering within a CV four-mode square Gaussian cluster state (see Fig. 1), and quantitatively test its monogamy relations [33–37]. By reconstructing the covariance matrix of the cluster state, we measure the quantifier of EPR steering under Gaussian measurements introduced in [15], for various bipartite splits. We find that the two- and

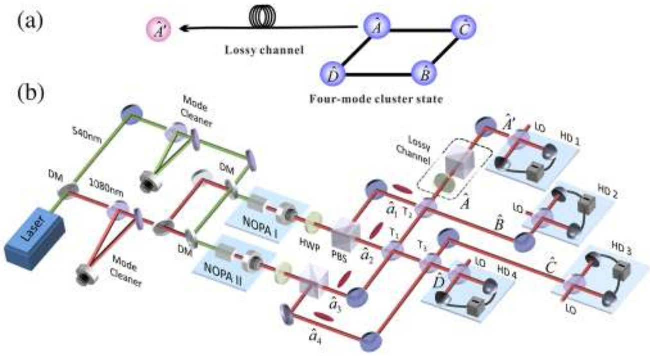

FIG. 1. Scheme of the experiment. (a) An optical mode ($\hat{A}$) of a four-mode square cluster state is distributed over a lossy quantum channel. (b) The experimental setup. The squeezed states with -3 dB squeezing at the sideband frequency of 3 MHz are generated from two nondegenerate optical parametric amplifiers (NOPAs). T_1, T_2, and T_3 are the beam splitters used to generate the cluster state. The lossy channel is composed by a half-wave plate (HWP) and a polarization beam splitter (PBS). $\mathrm{HD}_{1\text{–}4}$ denote homodyne detectors; LO denotes the local oscillator; and DM denotes dichroic mirror.

three-mode steering properties are determined by the geometric structure of the cluster state. Interestingly, a given mode of the state can be steered by its diagonal mode which is not directly coupled but cannot be steered even by collaboration of its two nearest neighbors, although they are coupled by direct interaction. These properties are different from those of a CV four-mode GHZ-like state. We further present for the first time an experimental observation of a "reverse" steerability, where the party being steered comprises more than one mode. With this ability, we precisely validate four types of monogamy relations recently proposed for Gaussian steering (see Table I) in the presence of loss [33–37]. Our study helps quantify how steering can be distributed among different parties in cluster states and link the amount of steering to the security of channels in a communication network.

The CV cluster quadrature correlations (so-called nullifiers) can be expressed by [45,50,51]

TABLE I. Classification of monogamy relations for the bipartite quantifier $\mathcal{G}^{j\to k}$ of EPR steerability of party k by party j under Gaussian measurements, in a tripartite $(n_A + n_B + n_C)$-mode system ABC. Note: I⊑II and III⊑IV, where "⊑" indicates being generalized by; the relations in types II and IVb can be violated for $n_C > 1$.

Type	Ref.	Inequality	Specifications
I	[33]	$\mathcal{G}^{A\to C} > 0 \Rightarrow \mathcal{G}^{B\to C} = 0$	$n_A = n_B = n_C = 1$
II	[34,35]	$\mathcal{G}^{A\to C} > 0 \Rightarrow \mathcal{G}^{B\to C} = 0$	$n_A,\, n_B \geq 1;\; n_C = 1$
IIIa	[36]	$\mathcal{G}^{C\to(AB)} - \mathcal{G}^{C\to A} - \mathcal{G}^{C\to B} \geq 0$	$n_A = n_B = n_C = 1$
IIIb	[36]	$\mathcal{G}^{(AB)\to C} - \mathcal{G}^{A\to C} - \mathcal{G}^{B\to C} \geq 0$	$n_A = n_B = n_C = 1$
IVa	[37]	$\mathcal{G}^{C\to(AB)} - \mathcal{G}^{C\to A} - \mathcal{G}^{C\to B} \geq 0$	$n_A,\, n_B,\, n_C \geq 1$
IVb	[37]	$\mathcal{G}^{(AB)\to C} - \mathcal{G}^{A\to C} - \mathcal{G}^{B\to C} \geq 0$	$n_A,\, n_B \geq 1;\; n_C = 1$

$$\left(\hat{p}_a - \sum_{b\in N_a} \hat{x}_b\right) \to 0, \quad \forall\ a \in G \tag{1}$$

where $\hat{x}_a = \hat{a} + \hat{a}^\dagger$ and $\hat{p}_a = (\hat{a} - \hat{a}^\dagger)/i$ stand for amplitude and phase quadratures of an optical mode $\hat{a}$, respectively. The modes of $a \in G$ denote the vertices of the graph G, while the modes of $b \in N_a$ are the nearest neighbors of mode $\hat{a}$. For an ideal cluster state the left-hand side of Eq. (1) tends to zero, so that the state is a simultaneous zero eigenstate of these quadrature combinations in the limit of infinite squeezing [45].

As a unit of a two-dimensional cluster state, a four-mode square cluster state as shown in Fig. 1(a) can be used to establish a quantum network [40,60]. The cluster state of the optical field is prepared by coupling two phase-squeezed and two amplitude-squeezed states of light on an optical beam-splitter network, which consists of three optical beam splitters with transmittance of $T_1 = 1/5$ and $T_2 = T_3 = 1/2$, respectively, as shown in Fig. 1(b) [61]. We distribute mode $\hat{A}$ of the state in a lossy channel [Fig. 1(a)]. The output mode is given by $\hat{A}' = \sqrt{\eta}\hat{A} + \sqrt{1-\eta}\hat{\upsilon}$, where η and $\hat{\upsilon}$ represent the transmission efficiency of the quantum channel and the vacuum mode induced by loss into the quantum channel, respectively.

The properties of a $(n_A + m_B)$-mode Gaussian state ρ_{AB} of a bipartite system can be determined by its covariance matrix

$$\sigma_{AB} = \begin{pmatrix} A & C \\ C^\top & B \end{pmatrix}, \tag{2}$$

with elements $\sigma_{ij} = \langle \hat{\xi}_i \hat{\xi}_j + \hat{\xi}_j \hat{\xi}_i \rangle / 2 - \langle \hat{\xi}_i \rangle \langle \hat{\xi}_j \rangle$, where $\hat{\xi} \equiv (\hat{x}_1^A, \hat{p}_1^A, \ldots, \hat{x}_n^A, \hat{p}_n^A, \hat{x}_1^B, \hat{p}_1^B, \ldots, \hat{x}_m^B, \hat{p}_m^B)$ is the vector of the amplitude and phase quadratures of optical modes. The submatrices A and B are corresponding to the reduced states of Alice's and Bob's subsystems, respectively. The partially reconstructed covariance matrix $\sigma_{A'BCD}$, which corresponds to the distributed mode $\hat{A}'$ and modes $\hat{B}$, $\hat{C}$ and $\hat{D}$, is measured by four homodyne detectors [61,66].

The steerability of Bob by Alice ($A \to B$) for a $(n_A + m_B)$-mode Gaussian state can be quantified by [15]

$$\mathcal{G}^{A\to B}(\sigma_{AB}) = \max\left\{0,\ -\sum_{j:\bar{\nu}_j^{AB\backslash A}<1} \ln(\bar{\nu}_j^{AB\backslash A})\right\}, x \tag{3}$$

where $\bar{\nu}_j^{AB\backslash A}$ $(j = 1, \ldots, m_B)$ are the symplectic eigenvalues of $\bar{\sigma}_{AB\backslash A} = B - C^\mathsf{T} A^{-1} C$, derived from the Schur complement of A in the covariance matrix σ_{AB}. The quantity $\mathcal{G}^{A\to B}$ is a monotone under Gaussian local operations and classical communication [37] and vanishes iff the state described by σ_{AB} is nonsteerable by Gaussian measurements [15]. The steerability of Alice by Bob $[\mathcal{G}^{B\to A}(\sigma_{AB})]$ can be obtained by swapping the roles of A and B.

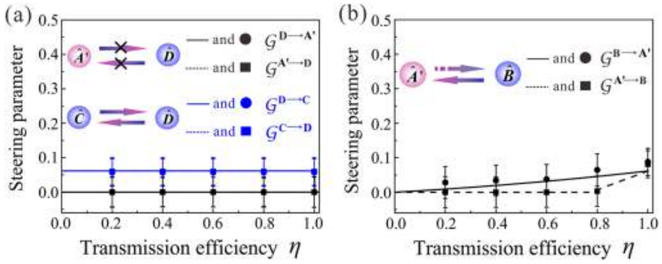

FIG. 2. Gaussian EPR steering between two modes of the cluster state. (a) There is no EPR steering between neighboring modes $\hat{A}'$ and $\hat{D}$ under Gaussian measurements, while diagonal modes $\hat{C}$ and $\hat{D}$ can steer each other with equal power. (b) One-way EPR steering between modes $\hat{A}'$ and $\hat{B}$ under Gaussian measurements. Additional $(1+1)$-mode partitions are shown in Fig. S2 in [61]. In all the panels, the quantities plotted are dimensionless. The lines and curves represent theoretical predictions based on the theoretical covariance matrix as calculated in [61]. The dots and squares represent the experimental data measured at different transmission efficiencies. Error bars represent ± one standard deviation and are obtained based on the statistics of the measured noise variances.

Figure 2 shows a selection of results for the steerability between any two modes [i.e., $(1+1)$-mode partitions] of the cluster state under Gaussian measurements. Surprisingly, as shown in Fig. 2(a) and Fig. S2 in [61], we find that steering does not exist between any two neighboring modes, as one might have expected due to the direct coupling as shown in the definition of a cluster state in Eq. (1). Instead, two-mode steering is present between diagonal modes which are not directly coupled, as shown in Fig. 2. This observation can be understood as a consequence of the monogamy relation (type-I) derived from the two-observable ($\hat{x}$ and $\hat{p}$) EPR criterion [33]: two distinct modes cannot steer a third mode simultaneously by Gaussian measurements. In fact, as shown in Fig. 1, mode $\hat{C}$ and mode $\hat{D}$ are completely symmetric in the cluster state. Thus, if $\hat{A}'$ could be steered by $\hat{C}$, it should be equally steered by $\hat{D}$ too, which, on the contrary, is forbidden by the type-I monogamy relation. However, there is no such constraint for mode $\hat{B}$. As a comparison, in a CV GHZ-like state, pairwise steering is strictly forbidden between any two modes based upon the same argument as the state is fully symmetric under mode permutations [32,67]. Thus, we conclude that a cluster state features richer steerability properties, due to the inherent asymmetry induced by its geometric configuration.

We further investigate quantitatively the robustness of the two-mode steering when transmission loss is imposed on one of the two parties. In Fig. 2(b), we show the steering parameter defined in Eq. (3) by varying the transmission efficiency η of the lossy channel. When the lossy mode $\hat{A}'$ is the steered party, we find that the nonlossy steering party $\hat{B}$ can always steer $\hat{A}'$, although the steerability is reduced with increasing loss. However, the presence of loss plays a vital role if $\hat{A}'$ is the steering party. In fact, if the transmission efficiency η is lower than a critical value of ~ 0.772, the Gaussian steering of $\hat{A}'$ upon $\hat{B}$ is completely destroyed. This leads to a manifestation of "one-way" steering within the region of $\eta \in (0, 0.772)$, as previously noted in other types of entangled states [17–19,29]. However, we remark that in our experiment we are limited to Gaussian measurements for the steering party, which leaves open the possibility that $A' \to B$ steering could still be demonstrated for smaller values of η by resorting to suitable non-Gaussian measurements [18,68].

Since mode $\hat{A}'$ is coupled to its two nearest neighbors $\hat{C}$ and $\hat{D}$ on each side, one may wonder whether the two neighboring modes can jointly steer $\hat{A}'$. Figures 3 and S3 in [61] show the steerability between one mode and any two other modes of the cluster state [i.e., $(1+2)$-mode and $(2+1)$-mode partitions] under Gaussian measurements. Interestingly, we find that mode $\hat{A}'$ still cannot be steered even by the collaboration of modes $\hat{C}$ and $\hat{D}$ ($\mathcal{G}^{CD\to A'} = 0$) [Fig. 3(a)] but can be steered so long as the diagonal mode $\hat{B}$ is involved ($\mathcal{G}^{BC\to A'} = \mathcal{G}^{BD\to A'} > 0$) [Fig. 3(b)]. This phenomenon is determined unambiguously from a generalized monogamy relation applicable to the case of the steering party consisting of an arbitrary number of modes (type-II) [34,35]. As mode $\hat{B}$ can always steer $\hat{A}'$ [shown in Fig. 2(b)], the other group $\{\hat{C}, \hat{D}\}$ is forbidden to steer the same mode simultaneously. We stress that this property is again in stark contrast to the case of a CV four-mode GHZ-like state, where any two modes $\{\hat{\imath}, \hat{\jmath}\}$ can collectively steer another mode $\hat{k}$ [67] as there is no two-mode steering to rule out this possibility. Similarly, mode $\hat{C}$ can only be steered by a group comprising the diagonal mode $\hat{D}$ [$\mathcal{G}^{BD\to C} > 0$ shown in Fig. 3(a), and $\mathcal{G}^{A'D\to C} > 0$ shown in Fig. 3(c)]. We also show that the collective steerability $\mathcal{G}^{BC(D)\to A'}$ [solid curve in Fig. 3(b)] is significantly higher than the steerability by $\hat{B}$ mode alone $\mathcal{G}^{B\to A'}$ [solid curve in Fig. 2(b)], suggesting that although the neighboring modes $\hat{C}$ and $\hat{D}$ cannot steer $\hat{A}$ by themselves, their roles in assisting collective steering with mode $\hat{B}$ are nontrivial.

We further measure, for the first time, the steerability when the steered party comprises more than one mode, i.e., steering parameters of $(1+2)$-mode configurations, which are shown in Fig. 3 and in Fig. S3 in [61]. The loss imposed on $\hat{A}$ also leads to asymmetric steerability $\mathcal{G}^{BC\to A'} \neq \mathcal{G}^{A'\to BC}$, and a parameter window for one-way steering (under the restriction of Gaussian measurements) with $\eta \in (0, 0.5]$, as shown in Fig. 3(b). In addition, our results $\mathcal{G}^{D\to BC} > 0$ [$\mathcal{G}^{D\to BC} = \mathcal{G}^{C\to BD}$, Fig. 3(a)] and $\mathcal{G}^{A'\to BC} > 0$ when $\eta > 0.5$ [Fig. 3(b)] also confirm experimentally that, when the steered system is composed of at least two modes, it can be steered by more than one party simultaneously; i.e., the type-II monogamy relation is lifted [35].

Using the results of $(1+2)$-mode steerability, we also present the first experimental examination of the type-III

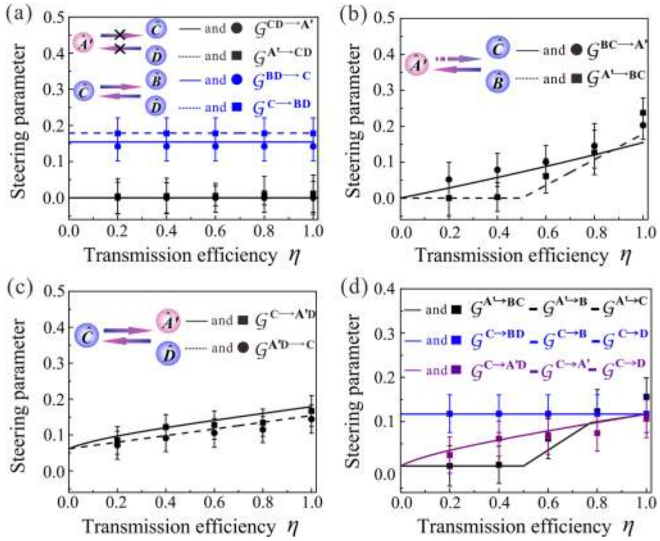

FIG. 3. Gaussian EPR steering between one and two modes of the cluster state. (a) Mode $\hat{A}'$ cannot be steered by the collaboration of two nearest neighboring modes $\{\hat{C}, \hat{D}\}$ even though they are directly coupled; while $\hat{C}$ and $\{\hat{B}, \hat{D}\}$ can steer each other. (b) One-way EPR steering between modes $\hat{A}'$ and $\{\hat{B}, \hat{C}\}$ under Gaussian measurements. (c) $\hat{C}$ and $\{\hat{A}', \hat{D}\}$ can steer each other asymmetrically and the steerability grows with increasing transmission efficiency, reflecting the different effect when loss happens on steering or a steered channel. (d) Validation of CKW-type monogamy for steering (type-III). Additional partitions are shown in Fig. S3 in [61]. In all the panels, the quantities plotted are dimensionless. The lines and curves represent theoretical predictions based on the theoretical covariance matrix as calculated in [61]. The dots and squares represent the experimental data measured at different transmission efficiencies. Error bars represent $\pm$ one standard deviation and are obtained based on the statistics of the measured noise variances.

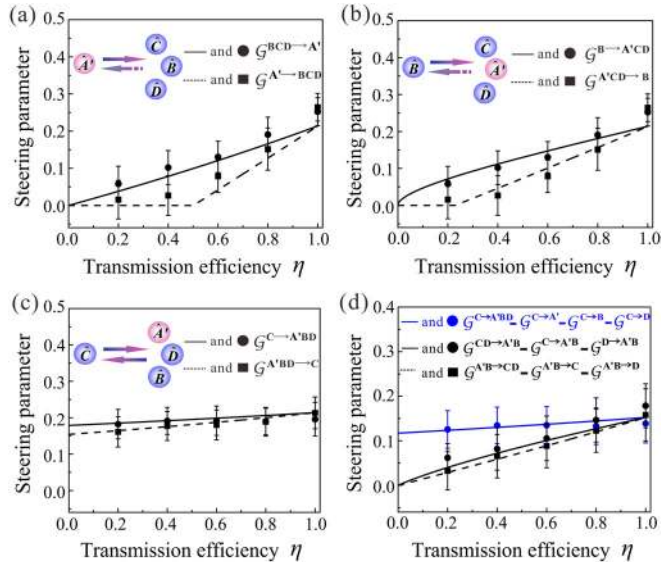

FIG. 4. Gaussian EPR steering between one and three modes in the cluster state. (a) One-way EPR steering under Gaussian measurements between modes $\hat{A}'$ and $\{\hat{B}, \hat{C}, \hat{D}\}$ with directional property. (b) One-way EPR steering under Gaussian measurements between modes $\hat{B}$ and $\{\hat{A}', \hat{C}, \hat{D}\}$. (c) Asymmetric steering between modes $\hat{C}$ and $\{\hat{A}', \hat{B}, \hat{D}\}$. (d) Monogamy of steering quantifier for $(1+3)$- and $(2+2)$-mode partitions. In all the panels, the quantities plotted are dimensionless. The lines and curves represent theoretical predictions based on the theoretical covariance matrix as calculated in [61]. The dots and squares represent the experimental data measured at different transmission efficiencies. Error bars represent $\pm$ one standard deviation and are obtained based on the statistics of the measured noise variances.

monogamy relation, called Coffman-Kundu-Wootters (CKW)-type monogamy in reference to the seminal study on monogamy of entanglement [32], which quantifies how the steering is distributed among different subsystems [36]. For a three-mode scenario, the CKW-type monogamy relation reads

$$\mathcal{G}^{k\to(i,j)}(\sigma_{ijk}) - \mathcal{G}^{k\to i}(\sigma_{ijk}) - \mathcal{G}^{k\to j}(\sigma_{ijk}) \geq 0, \tag{4}$$

where $i, j, k \in \{\hat{A}', \hat{B}, \hat{C}, \hat{D}\}$ in our case. We have experimentally verified that this monogamy relation is valid for all possible types of $(1+2)$-mode steering configurations; some of them are shown in Fig. 3(d).

Next, we study the steerability between one and the remaining three modes within the cluster state, i.e., $(1+3)$- and $(3+1)$-mode partitions. As shown in Figs. 4(a) and 4(b), one-way EPR steering (under Gaussian measurements) is observed for bipartitions $(\hat{A}' + \hat{B}\hat{C}\hat{D})$ and $(\hat{B} + \hat{A}'\hat{C}\hat{D})$ when $\eta \leq 0.5$ and $\eta \leq 0.228$, respectively. The asymmetry between the two steering directions for the bipartition $(\hat{C} + \hat{A}'\hat{B}\hat{D})$ grows with increasing transmission efficiency, but no one-way property is observed in this case [Fig. 4(c)], since mode $\hat{C}$ and mode $\hat{D}$ can always steer each other independently. Quantitatively, the $(1+3)$- and $(3+1)$-mode steerability degrees are further enhanced in comparison to the $(1+2)$ and $(2+1)$ mode cases, even when the newly added mode alone cannot steer or be steered by the other party. We also confirm that the generalized CKW-type monogamy inequality $\mathcal{G}^{k\to(i,j,l)} - \mathcal{G}^{k\to i} - \mathcal{G}^{k\to j} - \mathcal{G}^{k\to l} \geq 0$ holds in this four-mode scenario, as shown in Fig. 4(d).

Finally, our experiment also validates for the first time general monogamy inequalities for Gaussian steerability with an arbitrary number of modes per party (type-IV) [37]. As a typical example of $(2+2)$-mode steering, our experimental results demonstrate that the steerability of $(\hat{A}'\hat{B} + \hat{C}\hat{D})$-mode partitions satisfies the following inequalities

$$\mathcal{G}^{A'B\to CD} - \mathcal{G}^{A'B\to C} - \mathcal{G}^{A'B\to D} \geq 0, \tag{5a}$$

$$\mathcal{G}^{CD\to A'B} - \mathcal{G}^{C\to A'B} - \mathcal{G}^{D\to A'B} \geq 0, \tag{5b}$$

as indicated in Fig. 4(d). We have verified that both these monogamy relations are also valid for all possible $(2+2)$-mode configurations in this cluster state. Note that, in general, Eq. (5b) can be violated on other classes of states [37].

In summary, the structure and sharing of EPR steering distributed over two-, three-, and four-mode partitions have been demonstrated and investigated quantitatively for a CV four-mode square Gaussian cluster state subject to asymmetric loss. By generating the cluster state deterministically and reconstructing its covariance matrix, we obtain a full steering characterization for all bipartite configurations. For general cases with arbitrary numbers of modes in each party, we quantify the bipartite steerability by Gaussian measurements, and provide experimental confirmation for four types of monogamy relations which bound the distribution of steerability among different modes, as summarized in Table I. Even though our state does not display genuine multipartite steering [39], several innovative features are observed, including the steerability of a group of two or three modes by a single mode, and the fact that a given mode of the state can be steered by its diagonal mode which is not directly coupled but cannot be jointly steered by its two directly coupled nearest neighbors.

Our work thus provides a concrete in-depth understanding of EPR steering and its monogamy in paradigmatic multipartite states such as cluster states. In turn, this can be useful to gauge the usefulness of these states for quantum communication technologies. For instance, secure CV teleportation with fidelity exceeding the no-cloning threshold requires two-way Gaussian steering [26], which arises in various partitions in our state, e.g. between $\hat{A}'$ and $\hat{B}$ for sufficiently large transmission efficiency [see Fig. 2(b)]. Furthermore, the amount of Gaussian steering directly bounds the secure key rate in CV 1sDI quantum key distribution and secret sharing [22,36,40]. Combined with a stronger initial squeezing level, the techniques used here could be adapted to demonstrate these protocols among many sites over lossy quantum channels.

This research was supported by the National Natural Science Foundation of China (Grants No. 11522433, No. 11622428, No. 61475092, and No. 61475006) and the Ministry of Science and Technology of China (Grants No. 2016YFA0301402 and No. 2016YFA0301302). X. S. thanks the program of Youth Sanjin Scholar; Q. H. thanks the Cheung Kong Scholars Programme (Youth) of China; G. A. thanks the European Research Council (ERC) Starting Grant GQCOP (Grant No. 637352) and the Foundational Questions Institute Physics of the Observer Programme (Grant No. FQXi-RFP-1601).

X. D. and Y. X. contributed equally to this work.

*qiongyihe@pku.edu.cn
†suxl@sxu.edu.cn

[1] E. Schrödinger, Discussion of probability relations between separated systems, Proc. Cambridge Philos. Soc. **31**, 555 (1935).
[2] A. Einstein, B. Podolsky, and N. Rosen, Can quantum-mechanical description of physical reality be considered complete?, Phys. Rev. **47**, 777 (1935).
[3] M. D. Reid, Demonstration of the Einstein-Podolsky-Rosen paradox using nondegenerate parametric amplification, Phys. Rev. A **40**, 913 (1989).
[4] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Steering, Entanglement, Nonlocality, and the Einstein-Podolsky-Rosen Paradox, Phys. Rev. Lett. **98**, 140402 (2007).
[5] E. Schrödinger, Die gegenwärtige Situation in der Quantenmechanik, Die Naturwissenschaften **23**, 807 (1935).
[6] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, Rev. Mod. Phys. **81**, 865 (2009).
[7] J. S. Bell, On the Einstein Podolsky Rosen paradox, Physics **1**, 195 (1964).
[8] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, Bell nonlocality, Rev. Mod. Phys. **86**, 419 (2014).
[9] S. J. Jones, H. M. Wiseman, and A. C. Doherty, Entanglement, Einstein-Podolsky-Rosen correlations, Bell nonlocality, and steering, Phys. Rev. A **76**, 052116 (2007).
[10] M. D. Reid, P. D. Drummond, W. P. Bowen, E. G. Cavalcanti, P. K. Lam, H. A. Bachor, U. L. Andersen, and G. Leuchs, Colloquium: The Einstein-Podolsky-Rosen paradox: From concepts to applications, Rev. Mod. Phys. **81**, 1727 (2009).
[11] E. G. Cavalcanti, S. J. Jones, H. M. Wiseman, and M. D. Reid, Experimental criteria for steering and the Einstein-Podolsky-Rosen paradox, Phys. Rev. A **80**, 032112 (2009).
[12] D. Cavalcanti and P. Skrzypczyk, Quantum steering: a review with focus on semidefinite programming, Rep. Prog. Phys. **80**, 024001 (2017).
[13] S. L. W. Midgley, A. J. Ferris, and M. K. Olsen, Asymmetric Gaussian steering: When Alice and Bob disagree; Phys. Rev. A **81**, 022101 (2010); S. P. Walborn, A. Salles, R. M. Gomes, F. Toscano, and P. H. S, Ribeiro, Revealing Hidden Einstein-Podolsky-Rosen Nonlocality, Phys. Rev. Lett. **106**, 130402 (2011); J. Schneeloch, C. J. Broadbent, S. P. Walborn, E. G. Cavalcanti, and J. C. Howell, Einstein-Podolsky-Rosen steering inequalities from entropic uncertainty relations, Phys. Rev. A **87**, 062103 (2013); J. Bowles, T. Vertesi, M. T. Quintino, and N. Brunner, One-way Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **112**, 200402 (2014); B. Opanchuk, L. Arnaud, and M. D. Reid, Detecting faked continuous-variable entanglement using one-sided device-independent entanglement witnesses, Phys. Rev. A **89**, 062101 (2014).
[14] Q. Y. He, Q. H. Gong, and M. D. Reid, Classifying Directional Gaussian Entanglement, Einstein-Podolsky-Rosen Steering, and Discord, Phys. Rev. Lett. **114**, 060402 (2015).
[15] I. Kogias, A. R. Lee, S. Ragy, and G. Adesso, Quantification of Gaussian Quantum Steering, Phys. Rev. Lett. **114**, 060403 (2015).
[16] L. Rosales-Zárate, R. Y. Teh, S. Kiesewetter, A. Brolis, K. Ng, and M. D. Reid, Decoherence of Einstein-Podolsky-Rosen steering, J. Opt. Soc. Am. B **32**, A82 (2015).
[17] V. Händchen, T. Eberle, S. Steinlechner, A. Samblowski, T. Franz, R. F. Werner, and R. Schnabel, Observation of one-way Einstein-Podolsky-Rosen steering, Nat. Photonics **6**, 596 (2012).
[18] S. Wollmann, N. Walk, A. J. Bennet, H. M. Wiseman, and G. J. Pryde, Observation of Genuine One-Way

Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **116**, 160403 (2016).

[19] K. Sun, X. J. Ye, J. S. Xu, X. Y. Xu, J. S. Tang, Y. C. Wu, J. L. Chen, C. F. Li, and G. C. Guo, Experimental Quantification of Asymmetric Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **116**, 160404 (2016).

[20] M. Tomamichel and R. Renner, Uncertainty Relation for Smooth Entropies, Phys. Rev. Lett. **106**, 110506 (2011).

[21] C. Branciard, E. G. Cavalcanti, S. P. Walborn, V. Scarani, and H. M. Wiseman, One-sided device-independent quantum key distribution: security, feasibility, and the connection with steering, Phys. Rev. A **85**, 010301 (2012).

[22] N. Walk, S. Hosseini, J. Geng, O. Thearle, J. Y. Haw, S. Armstrong, S. M. Assad, J. Janousek, T. C. Ralph, T. Symul, H. M. Wiseman, and P. K. Lam, Experimental demonstration of Gaussian protocols for one-sided device-independent quantum key distribution, Optica **3**, 634 (2016).

[23] T. Gehring, V. Händchen, J. Duhme, F. Furrer, T. Franz, C. Pacher, R. F. Werner, and R. Schnabel, Implementation of continuous-variable quantum key distribution with composable and one-sided-device independent security against coherent attacks, Nat. Commun. **6**, 8795 (2015).

[24] R. Gallego and L. Aolita, Resource Theory of Steering, Phys. Rev. X **5**, 041008 (2015).

[25] M. D. Reid, Signifying quantum benchmarks for qubit teleportation and secure quantum communication using Einstein-Podolsky-Rosen steering inequalities, Phys. Rev. A **88**, 062338 (2013).

[26] Q. He, L. Rosales-Zárate, G. Adesso, and M. D. Reid, Secure Continuous Variable Teleportation and Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **115**, 180502 (2015).

[27] C.-Y. Chiu, N. Lambert, Teh-Lu Liao, F. Nori, and C.-M. Li, No-cloning of quantum steering, npj Quantum Inf. **2**, 16020 (2016).

[28] M. Piani and J. Watrous, Necessary and Sufficient Quantum Information Characterization of Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **114**, 060404 (2015).

[29] S. Armstrong, M.Wang, R. Y. Teh, Q. H. Gong, Q. Y. He, J. Janousek, H. A. Bachor, M. D. Reid, and P. K. Lam, Multipartite Einstein-Podolsky-Rosen steering and genuine tripartite entanglement with optical networks, Nat. Phys. **11**, 167 (2015).

[30] D. Cavalcanti, P. Skrzypczyk, G. H. Aguilar, R. V. Nery, P. H. S. Ribeiro, and S. P. Walborn, Detection of entanglement in asymmetric quantum networks and multipartite quantum steering, Nat. Commun. **6**, 7941 (2015).

[31] C.-M. Li, K. Chen, Y.-N. Chen, Q. Zhang, Y.-A. Chen, and J.-W. Pan, Genuine High-Order Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **115**, 010402 (2015).

[32] V. Coffman, J. Kundu, and W. K. Wootters, Distributed Entanglement, Phys. Rev. A **61**, 052306 (2000).

[33] M. D. Reid, Monogamy inequalities for the Einstein-Podolsky-Rosen paradox and quantum steering, Phys. Rev. A **88**, 062108 (2013).

[34] S-W. Ji, M. S. Kim, and H. Nha, Quantum steering of multimode Gaussian states by Gaussian measurements: Monogamy relations and the Peres conjecture, J. Phys. A **48**, 135301 (2015).

[35] G. Adesso and R. Simon, Strong subadditivity for log-determinant of covariance matrices and its applications, J. Phys. A **49**, 34LT02 (2016).

[36] Y. Xiang, I. Kogias, G. Adesso, and Q. Y. He, Multipartite Gaussian steering: monogamy constraints and quantum cryptography applications, Phys. Rev. A **95**, 010101(R) (2017).

[37] L. Lami, C. Hirche, G. Adesso, and A. Winter, Schur Complement Inequalities for Covariance Materices and Monogamy of Quantum Correlations, Phys. Rev. Lett. **117**, 220502 (2016).

[38] S. Cheng, A. Milne, M. J. W. Hall, and H. M. Wiseman, Volume monogamy of quantum steering ellipsoids for multiqubit systems, Phys. Rev. A **94**, 042105 (2016).

[39] Q. Y. He and M. D. Reid, Genuine Multipartite Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **111**, 250403 (2013).

[40] I. Kogias, Y. Xiang, Q. Y. He, and G. Adesso, Unconditional security of etanglement-based continuous variable quantum secret sharing, Phys. Rev. A **95**, 012315 (2017).

[41] R. Raussendorf and H. J. Briegel, A One-way Quantum Computer, Phys. Rev. Lett. **86**, 5188 (2001).

[42] P. Walther, K. J. Resch, T. Rudolph, E. Schenck, H. Weinfurter, V. Vedral, M. Aspelmeyer, and A. Zeilinger, Experimental one-way quantum computing, Nature (London) **434**, 169 (2005).

[43] N. C. Menicucci, P. van Loock, M. Gu, C. Weedbrook, T. C. Ralph, and M. A. Nielsen, Universal Quantum Computation with Continuous-Variable Cluster States, Phys. Rev. Lett. **97**, 110501 (2006).

[44] P. van Loock, Examples of Gaussian cluster computation, J. Opt. Soc. Am. B **24**, 340 (2007).

[45] M. Gu, C. Weedbrook, N. C. Menicucci, T. C. Ralph, and P. van Loock, Quantum computing with continuous-variable clusters, Phys. Rev. A **79**, 062318 (2009).

[46] J. Zhang, G. Adesso, C. Xie, and K. Peng, Quantum Teamwork for Unconditional Multiparty Communication with Gaussian States, Phys. Rev. Lett. **103**, 070501 (2009).

[47] S. Muralidharan and P. K. Panigrahi, Quantum-information splitting using multipartite cluster states, Phys. Rev. A **78**, 062333 (2008).

[48] Y. Qian, Z. Shen, G. He, and G. Zeng, Quantum-cryptography network via continuous-variable graph states, Phys. Rev. A **86**, 052333 (2012).

[49] H.-K. Lau and C. Weedbrook, Quantum secret sharing with continuous-variable cluster states, Phys. Rev. A **88**, 042313 (2013).

[50] J. Zhang and S. L. Braunstein, Continuous-variable Gaussian analog of cluster state, Phys. Rev. A **73**, 032318 (2006).

[51] P. van Loock, C. Weedbrook, and M. Gu, Building Gaussian cluster states by linear optics, Phys. Rev. A **76**, 032321 (2007).

[52] X. L. Su, Y. P. Zhao, S. H. Hao, X. Jia, C. Xie, and K. Peng, Experimental preparation of eight-partite cluster state for photonic qumodes, Opt. Lett. **37**, 5178 (2012).

[53] M. Chen, N. C. Menicucci, and O. Pfister, Experimental Realization of Multipartite Entanglement of 60 Modes of a Quantum Optical Frequency Comb, Phys. Rev. Lett. **112**, 120505 (2014).

[54] S. Yokoyama, R. Ukai, S. C. Armstrong, C. Sornphiphatphong, T. Kaji, S. Suzuki, J.-i. Yoshikawa, H. Yonezawa, N. C. Menicucci, and A. Furusawa, Optical generation of ultra-large-scale continous-variable cluster states, Nat. Photonics **7**, 982 (2013).

[55] Y. Wang, X. Su, H. Shen, A. Tan, C. Xie, and K. Peng, Toward demonstrating controlled-X operation based on continuous-variable four-partite cluster states and quantum teleporters, Phys. Rev. A **81**, 022311 (2010).

[56] R. Ukai, N. Iwata, Y. Shimokawa, S. C. Armstrong, A. Politi, J.-i. Yoshikawa, P. van Loock, and A. Furusawa, Demonstration of Unconditional One-Way Quantum Computations for Continuous Variables, Phys. Rev. Lett. **106**, 240504 (2011).

[57] R. Ukai, S. Yokoyama, J. I. Yoshikawa, P. van Loock, and A. Furusawa, Demonstration of a Controlled-Phase Gate for Continuous-Variable One-Way Quantum Computation, Phys. Rev. Lett. **107**, 250501 (2011).

[58] X. Su, S. Hao, X. Deng, L. Ma, M. Wang, X. Jia, C. Xie, and K. Peng, Gate sequence for continuous variable one-way quantum computation, Nat. Commun. **4**, 2828 (2013).

[59] P. van Loock and S. L. Braunstein, Multipartite Entanglement for Continuous Variables: A Quantum Teleportation Network, Phys. Rev. Lett. **84**, 3482 (2000); , Greenberger-Horne-Zeilinger nonlocality in phase space, Phys. Rev. A **63**, 022106 (2001).

[60] H. Shen, X. Su, X. Jia, and C. Xie, Quantum communication network utilizing quadripartite entangled states of optical field, Phys. Rev. A **80**, 042320 (2009).

[61] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.118.230501 for details of the experimental setup, preparation and verification of the cluster state, measurement of the covariance matrix, and additional figures. The Supplemental Material contains additional references [62–65].

[62] Y. Zhou, X. Jia, F. Li, C. Xie, and K. Peng, Experimental generation of 8.4 dB entangled state with an optical cavity involving a wedged type-II nonlinear crystal, Opt. Express **23**, 4952 (2015).

[63] X. Su, A. Tan, X. Jia, J. Zhang, C. Xie, and K. Peng, Experimental Preparation of Quadripartite Cluster and Greenberger-Horne-Zeilinger Entangled States for Continuous Variables, Phys. Rev. Lett. **98**, 070502 (2007).

[64] P. van Loock and A. Furusawa, Detecting genuine multipartite continuous-variable entanglement, Phys. Rev. A **67**, 052315 (2003).

[65] G. Adesso and F. Illuminati, Entanglement in continuous-variable systems: recent advances and current perspectives, J. Phys. A **40**, 7821 (2007).

[66] S. Steinlechner, J. Bauchrowitz, T. Eberle, and R. Schnabel, Strong Einstein-Podolsky-Rosen steering with unconditional entangled states, Phys. Rev. A **87**, 022104 (2013).

[67] M. Wang, Y. Xiang, Q. Y. He, and Q. H. Gong, Detection of quantum steering in multipartite continuous-variable Greenberger-Horne-Zeilinger-like states, Phys. Rev. A **91**, 012112 (2015).

[68] S.-W. Ji, J. Lee, J. Park, and H. Nha, Quantum steering of Gaussian states via non-Gaussian measurements, Sci. Rep. **6**, 29729 (2016).