Denoising of complex MRI data by wavelet-domain filtering: Application to high b-value diffusion weighted imaging

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Introduction

The recent interest in high b-value diffusion MRI, e.g. for q-space imaging, has emphasised the noise sensitivity of the acquired data [1]. At high b-values the signal is influenced by the rectified noise floor in magnitude MR images, associated with the Rician noise distribution. Furthermore, noise is a well-documented problem in the assessment of diffusion anisotropy by, for example, the fractional anisotropy (FA) index [1, 2]. In this study, appropriate filtering in the wavelet domain [3] is proposed as a non-parametric tool for noise reduction in quantitative diffusion MRI.

Methods

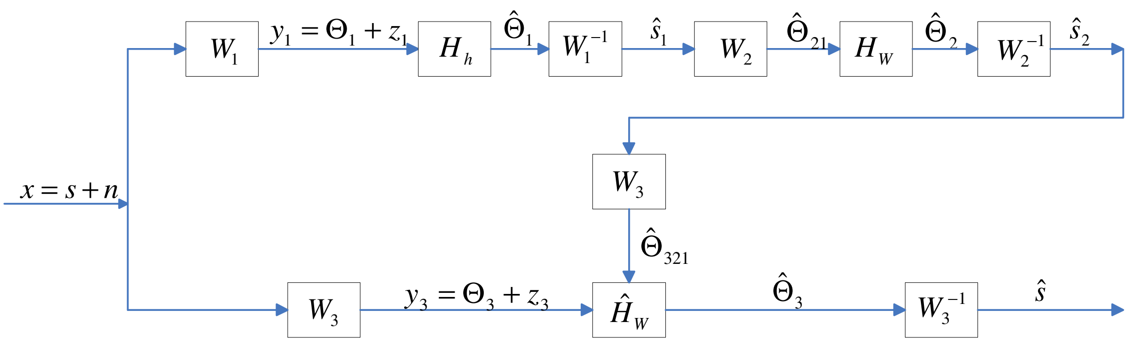

Figure 1: Schematic overview of the filtering procedure. Symbols are briefly explained in the text.

The real and imaginary parts of complex MRI data in the image domain (i.e. after Fourier transform of k-space data) were filtered, before construction of the magnitude image. The filtering procedure is illustrated in Fig. 1 above. W denotes a wavelet transform and W^{-1} denotes an inverse wavelet transform (different indices refer to the use of different wavelet families). The original signal is $x = s + n$ (true signal plus noise) and $y_1 = W_1 x_1$, $\Theta_1 = W_1 s_1$ and $z_1 = W_1 n_1$. H_h is a hard threshold filter that provides the initially filtered signal $\hat{s}_1 = W_1^{-1} H_h W_1 x$. The hard threshold filter is given by Eq. 1:

$$h_h(i, j) = \begin{cases} 1, & \text{if } |y(i, j)| > \rho\sigma \\ 0, & \text{otherwise} \end{cases} \quad (Eq.1)$$

where y is the wavelet coefficient, ρ is an empiric threshold factor and σ is standard deviation of the noise (determined from the finest-scale wavelet coefficients).

Thereafter, a Wiener-like filter H_W was constructed from the $\hat{\Theta}_{21} = W_2 \hat{s}_1$ data (being the best approximation to true signal at this point) and applied to the hard-threshold filtered data. The Wiener-like filter is, in principle, given by Eq. 2. To achieve additional noise reduction, the estimated signal value $\hat{s}_2$ (after the first Wiener-like filtering) was used to construct a new Wiener-like filter $\hat{H}_W$ that was applied to the original noisy signal x, giving the final denoised estimate $\hat{s}$. The above filter approach was applied to simulated images (SNR=15 at b=0, 30 images with b-values 0 - 6000 s/mm^2) as well as to experimental data.

$$h_w(i, j) = \frac{|\hat{\theta}(i, j)|^2}{|\hat{\theta}(i, j)|^2 + \sigma^2} \quad (Eq.2)$$

Results

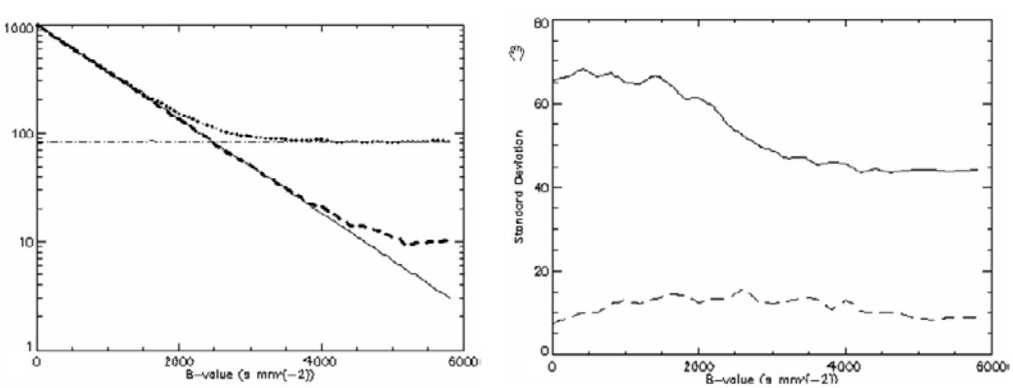

Figure 2. *Left:* Signal versus b-value from a homogeneous ROI in simulated images (ADC=1.0×10^{-9} m^2/s). The dotted curve is the original signal before denoising and the dashed curve shows the corresponding data after the noise reduction. The solid line is the theoretical, noise-free relationship. *Right:* ROI standard deviation (SD) versus b-value before (solid curve) and after (dashed curve) noise reduction. The non-filtered SD shows the characteristic decrease when the noise distribution transfers from Gaussian to Rician.

Discussion

The described wavelet-based noise-reduction algorithm considerably reduced the noise floor as well as the standard deviation. A specific advantage with the present approach, compared with previously proposed wavelet-domain filtering of the complex k-space data, is that image artefacts caused by filtering-induced phase errors in k-space data are avoided.

References: [1] Jones & Basser, MRM 52, 2004, 979; [2] Skare et al., JMR 147, 2000, 340; [3] Ghael et al., Proc. SPIE, San Diego, July 1997.