DENOISING OF IMAGES WITH MULTIPLICATIVE NOISE CORRUPTION

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ABSTRACT

Multiplicative noise is signal dependent and is difficult to be removed without impairing image details. It causes difficulties for many real world imaging applications. Previously, a hypothesis test based wavelet denoising algorithm had been proposed with promising results. In this paper, the algorithm has been further studied by fitting it into the framework of contourlet transform, an emerging two-dimensional technique for image processing and analysis. The developed contourlet denoising algorithm has been evaluated with standard test images, yielding successful results. It has also been demonstrated that the proposed algorithm outperformed the original wavelet based approach.

1. INTRODUCTION

Multiplicative noise is commonly found in many real world signal processing applications. Unlike additive noise, this kind of noise is much more difficult to be removed from the corrupted signal, mainly because of its multiplicative nature. Take an image as an example, when such noise is present in a bright area, it will be multiplied by high intensity values, thus its random variation will increase, or be "magnified." On the other hand, if the noise is introduced into a dark area, the change of the random variation may be much less significant. Since the noise variation greatly depends on the intensity levels of the image pixels being corrupted, it is not easy to establish an appropriate statistical model for the noise by simply examining the corrupted image.

To develop effective approaches for removing multiplicative noise, various filtering techniques have been proposed. Normally, they assume that statistical characteristics of the noise are available. Many researchers have also proposed wavelet based approaches for this filtering problem. Among them, the Variance Dependent Spatially Adaptive Multiplicative Denoising (VDSAMD) algorithm is a promising approach [1].

The VDSAMD algorithm is established on the basis of a statistical analysis of the wavelet coefficients. It takes each wavelet coefficient as a realization of a certain random process and estimates variances for each random process associated with the coefficients. By applying a hypothesis test to the estimated variances, this approach is capable of determining if a wavelet coefficient shares the same statistical characteristics with other coefficients in its neighborhood. If yes, then the wavelet coefficient under test is considered to fall in a region of smooth appearance, which may be filtered to remove the noise. Otherwise, the coefficient is considered to represent some image details and should be kept intact. This approach is adaptive in nature because the determination regarding each wavelet coefficient is made according to its local neighborhood only. Its effectiveness had been verified by simulation experiments.

In this paper, the denoising of images with multiplicative noise corruption has been further studied on the basis of the VDSAMD algorithm. In particular, an emerging two-dimensional transform for image processing – contourlet transform [2], has been adopted to replace the wavelet transform in VDSAMD, since image denoising for additive noise in the contourlet domain generated better results than in the wavelet domain [2,3]. This modified denoising algorithm is denoted as Contourlet-based VDSAMD, or C-VDSAMD. It has been implemented and tested with several standard images. The experimental results show that this modified approach outperformed the original VDSAMD in removing multiplicative noise from images.

The remaining of this paper is organized as follows: section 2 discusses the contourlet transform and the C-VDSAMD algorithm; section 3 presents experimental results and analysis; and section 4 gives conclusions about the study.

2. METHODOLOGY

As introduced above, the approach developed in this paper for denoising multiplicative noise is a hypothesis test based algorithm performed in the contourlet domain. In this section, the principles and the procedures related to the approach will be discussed briefly.

2.1 The Contourlet Transform

The contourlet transform is a 2-D transform technique recently developed for image representation and analysis [2]. Also referred to as the pyramidal directional filter bank, it consists of two filter banks. The first filter bank, known as the Laplacian pyramid, is utilized to generate a multiscale representation of an image of interest. Subsequently, the subband images from the multiscale decomposition are processed by a directional filter bank to reveal the directional details at each specific scale level. The output values from the second filter bank are called "contourlet coefficients." Any analysis performed with the contourlet coefficients is considered as in the "contourlet domain." The contourlet transform is illustrated in Figure 1.

The contourlet transform provides a multiscale and multidirectional representation of an image. Similar to the wavelet transform, it conforms to the multiresolution nature of the human visual systems. It is also easily adjustable for detecting fine details in any orientation at various scale levels, resulting in good potential for effective image analysis. Moreover, the decoupling of the multiscale decomposition (the Laplacian pyramid) from the multidirectional decomposition guarantees a flexible structure for image analysis. As demonstrated in [2,3], the contourlet transform is more powerful than the state of the art techniques such as the wavelet transform in characterizing natural images rich of directional details and smooth contours.

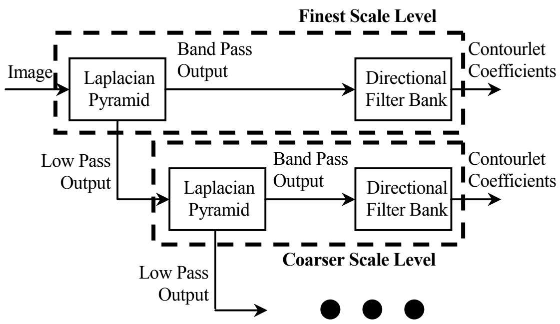

Figure 1. An illustration of the contourlet transform.

2.2 The C-VDSAMD Algorithm

A common approach for image denoising is to convert the noisy image into a transform domain such as the wavelet domain, and then compare the transform coefficients with a fixed threshold. Given the inflexibility of this approach, a hypothesis test based denoising algorithm in the wavelet domain, that is, the VDSAMD algorithm, has been proposed [1]. This algorithm performs a hypothesis test to determine if a pixel is corrupted or not, thus not depending on a fixed threshold. In this paper, this algorithm has been extended into the contourlet domain. The resulted C-VDSAMD algorithm follows similar processes of the VDSAMD, with a more sophisticated windowing scheme for variance estimation and hypothesis test.

Assume that contourlet coefficients at each pixel position are realizations of zero mean Gaussian distributions, that is, $C_{i,j} \sim N(0, \sigma_{i,j}^2)$. The variance for each Gaussian distribution may be estimated from contourlet coefficients of a local neighborhood consisting of N coefficients. By discarding the maximum and minimum among the N values to avoid any possible outliers, the variance at position (i, j) is estimated as $\hat{\sigma}_{i,j}^2 = \frac{1}{N-2}\sum_{n=1}^{N-2} C_n^2$, where C_n denotes a particular contourlet coefficient in the selected neighborhood.

Obviously, the estimated variance conforms to a chi-square distribution, or $\hat{\sigma}_{i,j}^2 \sim \chi^2(N-2)$, if the contourlet coefficients follow the same distribution. Then the ratio of two such variance estimates would conform to an F-distribution [4]:

$$R = \frac{\hat{\sigma}_{i,j}^2}{\hat{\sigma}_{k,l}^2} \sim F(N-2, N-2).$$

This provides the basis for the hypothesis test for noise detection in the contourlet domain.

Figure 2 is the block diagram for the C-VDSAMD algorithm. As the first step, a noisy image is transformed into the contourlet domain by the contourlet decomposition. Then, for each coefficient, variance for the associated Gaussian distribution is estimated. Subsequently, hypothesis tests are performed with the variance image. For each variance, a series of F-tests are carried out to determine if the associated contourlet coefficient is statistically equal to its neighbors. If it is, then it is determined to fall into a smooth region and may be processed for noise suppression. If it is not, then it should stand for an image feature pixel and be preserved. Afterwards, the processed contourlets are utilized to reconstruct the image, which is the final denoised output.

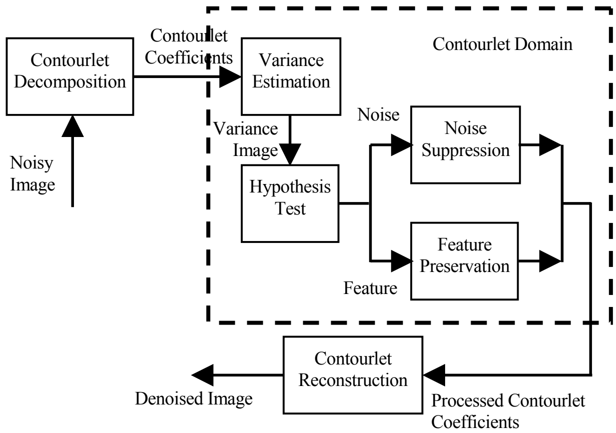

Figure 2. The block diagram for the C-VDSAMD algorithm.

Both the variance estimation and the hypothesis test are performed within a local neighborhood of a certain location. Such neighborhoods are determined by a set of window templates, which are illustrated in Figure 3. These window templates were designed to fit for the multi-directional nature of the contourlet transform. They are more complicated than the templates utilized in the VDSAMD algorithm [1], where only horizontal, vertical, and diagonal directions are considered.

For variance estimation, the window template used for a specific directional subband should show a matching orientation. On the other hand, for hypothesis test, the window template used for a certain directional subband should represent a "perpendicular" orientation, which means that the orientation of this template is obtained by rotating the template used for variance estimation at the same location by 90 degrees in the clockwise direction. As an example, for the first directional subband from a four-directional contourlet transform, the first template in the top row of the figure should be used for variance estimation, while the third template of the same row should be used for hypothesis test, or noise detection.

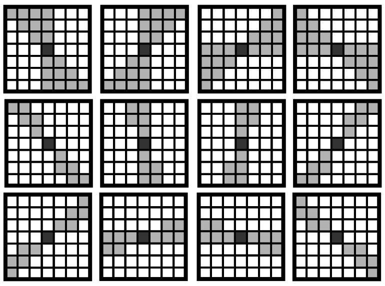

Figure 3. Window templates used for variance estimation and hypothesis test (noise detection). The four templates from the first row are utilized for four-directional contourlet decomposition. The remaining eight templates are utilized for eight-directional contourlet decomposition.

Figures 4 and 5 illustrate how to perform the noise detection for a given variance estimate at the location (i, j). To do this, two quantities are needed. One is the "Adjacent Maximum," or the maximum of the adjacent variance estimates around the position within the window template. The other is the "Neighboring Minimum," or the minimum of the two median values obtained from the two sub-windows within the template, separated by the target pixel point. As shown in Figure 4, for a four-directional decomposition, *Adjacent Maximum = Max(V1,V2,V3,V4)*; while for an eight-directional decomposition, *Adjacent Maximum = Max(V1,V2)*. For both cases, *Neighboring Minimum = Min(Median1, Median2)*.

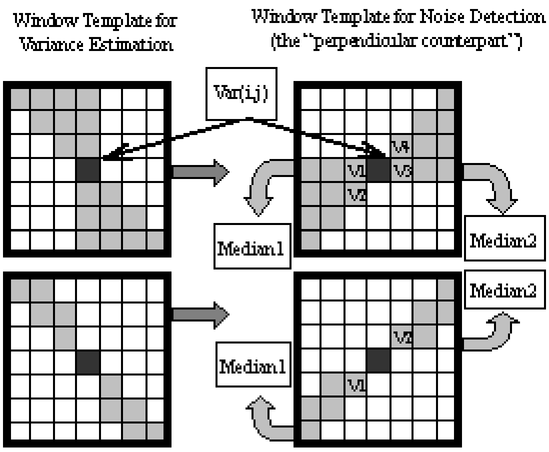

Figure 4. An illustration of how to apply window templates to noise detection. The two templates on the top are for a four-directional decomposition, while the two on the bottom are for an eight-directional decomposition.

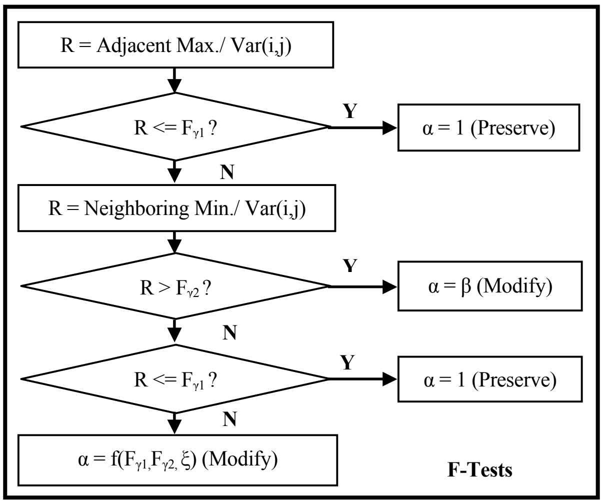

Figure 5. The flow chart for the F-tests used for noise detection.

The two quantities are compared with the target variance estimate through a series of hypothesis tests, or F-tests, as described in Figure 5. Here, the determined parameter α is then multiplied to the original contourlet coefficients to obtain the processed coefficients for image reconstruction, and the equation $f(F_{\gamma_1}, F_{\gamma_2}, \xi)$ is calculated as:

$$f(F_{\gamma_1}, F_{\gamma_2}, \xi) = \beta + (1-\beta)\left\{\frac{1-e^{\xi((R-F_{\gamma_1})/(F_{\gamma_2}-F_{\gamma_1}))}}{1-e^{\xi}}\right\}, \xi \neq 0$$

or $$f(F_{\gamma_1}, F_{\gamma_2}) = \beta + (1-\beta)\left\{\frac{R-F_{\gamma_1}}{F_{\gamma_2}-F_{\gamma_1}}\right\}, \xi = 0.$$

3. EXPERIMENTAL RESULTS AND ANALYSIS

To evaluate the performance of the C-VDSAMD algorithm for removing multiplicative noise from images, simulation experiments have been carried out with three standard test images: "Lena," "Barbara," and "Peppers" [5]. All these gray scale images are of 256 x 256 pixels, cropped from the 512 x 512 original ones, as shown in Figure 6. In the experiments, the three images were corrupted with Rayleigh multiplicative noise. These noisy images were then processed by the proposed algorithm with the parameter settings as shown in Table 1. Here, β and ξ are the parameters controlling the degree of attenuation of the contourlet coefficients. γ_1 and γ_2 are the confidence levels involved in the F-tests. For comparison purpose, the same parameter values as reported in [1] were utilized here.

Figure 6. Test images utilized in the simulation experiments (from left to right): "Lena," "Barbara," and "Peppers."

For the contourlet decomposition, three scale levels were utilized. The number of directions for the directional filter bank at each scale level (from the finest to the coarsest level) was set to 8, 8, and 4, respectively. The filters adopted were the "9-7" bi-orthogonal filters. For an objective evaluation of the denoising performance, the Peak Signal-to-Noise Ratio (PSNR) was calculated for each denoising experiment as:

$$PSNR = 10\log_{10}\left\{\frac{255^2 \cdot N \cdot M}{\sum_{N,M} error^2}\right\}$$

These calculated PSNR values are presented in Table 2. Example images are also provided in Figure 7 to demonstrate the visual quality of the denoised images.

Table 1. Parameter settings for the simulation experiments.

Parameters	β	ξ	γ_1	γ_2
Scale Level 1	0	0	0.98	0.52
Scale Level 2	0	0	0.92	0.52
Scale Level 3	0	0	0.84	0.52

Table 2. Denoising results (PSNR) for the standard test images.

Images	Noisy (dB)	C-VDSAMD (dB)	VDSAMD (dB)
Lena	13.26	17.30	16.83
Barbara	13.36	17.32	16.79
Peppers	12.60	17.02	16.54

Figure 7. Denoising results for "Barbara". The images are: (top row, from left to right) the original, the noisy (PSNR = 13.36dB); and (bottom row, from left to right) the denoised with the proposed C-VDSAMD algorithm (PSNR = 17.32dB), and the denoised with the VDSAMD algorithm (PSNR = 16.79dB).

From Table 2, it can be observed that for all three test images, the proposed C-VDSAMD algorithm achieved good results. It outperformed the wavelet based VDSAMD algorithm in all three cases, as well. Another observation is that the performance of the algorithm was consistent across the whole test data set.

A further examination of the visual appearance of the denoised images verified that the C-VDSAMD algorithm outperformed its wavelet counterpart. As illustrated in Figure 7, the C-VDSAMD denoised image provided better smoothness than the VDSAMD algorithm. At the same time, it preserved the details of the image as well as the other did, if not better. Such observation was consistent with all the denoised images, just as was the case with the objective PSNR values. This improvement of the denoising performance is reasonably explained by the fact that the contourlet transform is more powerful than the wavelet transform in representing directional details.

4. CONCLUSIONS

In this paper, the removal of multiplicative noise from images has been discussed. A hypothesis test based contourlet denoising algorithm, denoted as the C-VDSAMD algorithm, has been developed. This algorithm has been evaluated with several standard images, generating successful denoising results. The performance of the algorithm has also been compared with that of the original VDSAMD algorithm. The comparison indicated that the proposed algorithm outperformed the other one in terms of both the objective PSNR values and the subjective visual quality assessment.

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