

# Dense Quark Matter in Nature

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According to quantum chromodynamics (QCD), matter at ultra-high densities will take the form of a color-superconducting quark liquid, in which there is a condensate of Cooper pairs of quarks near the Fermi surface. I present a review of the physics of color superconductivity. I give particular attention to the recently proposed gapless CFL (gCFL) phase, which has unusual properties such as quasiquarks with a near-quadratic dispersion relation, and which may well be the favored phase of quark matter in the density range relevant to compact stars. I also discuss the effects of color superconductivity on the mass-radius relationship of compact stars, showing that one would have to fix the bag constant by other measurements in order to see the effects of color superconductivity. An additional parameter in the quark matter equation of state connected with perturbative corrections allows quark matter to imitate nuclear matter over the relevant density range, so that hybrid stars can show a mass-radius relationship very similar to that of nuclear matter, and their masses can reach  $1.9 M_{\odot}$ .

## §1. Introduction

One of the most striking features of QCD is asymptotic freedom: the force between quarks becomes arbitrarily weak as the characteristic momentum scale of their interaction grows larger. This immediately suggests that at sufficiently high densities and low temperatures, matter will consist of a Fermi sea of essentially free quarks, whose behavior is dominated by the high-momentum quarks that live at the Fermi surface.

However, over the last few years it has become clear that the phase diagram of QCD is much richer than this. In addition to the hadronic phase with which we are familiar and the quark gluon plasma (QGP) that is predicted to lie at temperatures above 170 MeV, there is a whole family of “color superconducting” phases, that are expected to occur at high density and low temperature.<sup>1)</sup> The essence of color superconductivity is quark pairing, driven by the BCS mechanism, which operates when there exists an attractive interaction between fermions at a Fermi surface. The QCD quark-quark interaction is strong, and is attractive in many channels, so we expect cold dense quark matter to *generically* exhibit color superconductivity. Moreover, quarks, unlike electrons, have color and flavor as well as spin degrees of freedom, so many different patterns of pairing are possible. This leads us to expect a rich phase structure in matter beyond nuclear density.

Color superconducting quark matter may occur naturally in the universe, in the cold dense cores of compact (“neutron”) stars, where densities are above nuclear density, and temperatures are of the order of tens of keV. (It might conceivably be possible to create it in future low-energy heavy ion colliders, such as the Japan Proton Accelerator Research Complex (J-PARC) or the Compressed Baryonic Matter facility

at GSI Darmstadt.) Up to now, most work on signatures has focussed on properties of color superconducting quark matter that would affect observable features of compact stars, and I will discuss some of these below.

## §2. Phase diagram of quark matter

In the real world there are two light quark flavors, the up ( $u$ ) and down ( $d$ ), with masses  $\lesssim 10$  MeV, and a medium-weight flavor, the strange ( $s$ ) quark, with mass  $\sim 150$  MeV. The strange quark therefore plays a crucial role in the phases of QCD, and we expect it to remain fully paired with the light flavors as long as  $\mu \gg M_s^2/\Delta$ , where  $\Delta$  is a gap parameter for the pairing of the strange quark. Figure 1 shows two conjectured phase diagrams for QCD. One panel is for small  $M_s^2/\Delta$ , in which case the strange quark's mass never breaks its pairing with the light flavors, so there is a direct transition from nuclear matter to color-flavor-locked (CFL) quark matter.<sup>2)</sup> In the CFL phase the strange quark participates symmetrically with the up and down quarks in Cooper pairing—this is described in more detail in §3.1. The other panel in Fig. 1 is for large  $M_s^2/\Delta$ , in which case the strange quark is too heavy to pair symmetrically with the light quarks at medium densities, then there will be an interval of some other phase or phases. These may well include the recently proposed gapless CFL phase,<sup>3)</sup> which will be described in §3.2, although other possibilities such as crystalline color superconductivity<sup>4)</sup> or some form of single-flavor pairing<sup>5),6)</sup> have been suggested.

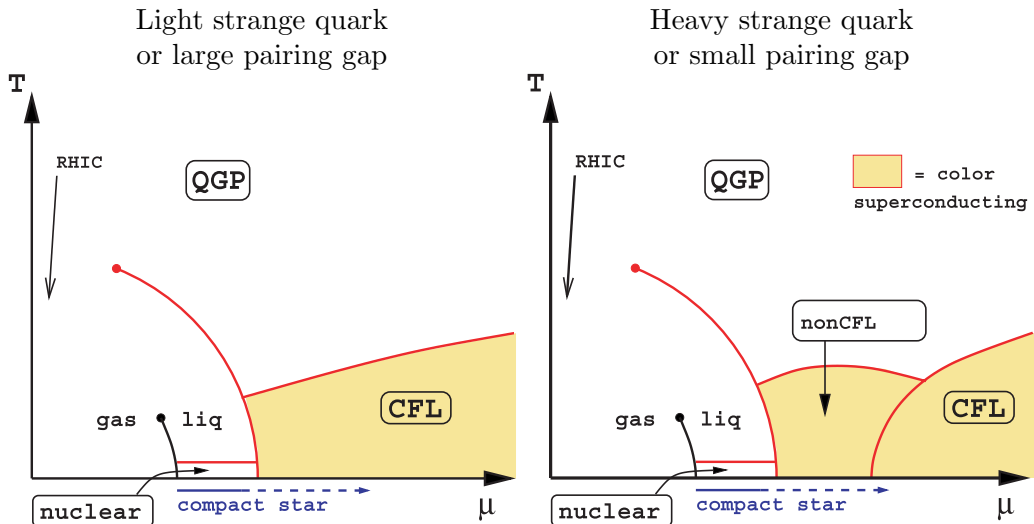


Fig. 1. Conjectured phase diagrams for QCD in the real world. For small  $M_s^2/\Delta$  there is a direct transition from nuclear matter to color-flavor locked color superconducting quark matter. For large  $M_s^2/\Delta$  there is an intermediate phase where the strange quark pairs in some other way. Depending on the strength of instanton interactions, the CFL phase may include  $K^0$  condensation.

### §3. Review of color superconductivity

The essential physics of color superconductivity is the same as that underlying conventional superconductivity in metals.<sup>6)–8)</sup> As mentioned above, asymptotic freedom of QCD means that at sufficiently high density and low temperature, there is a Fermi surface of almost free quarks. The interactions between quarks near the Fermi surface are certainly attractive in some channels (quarks bind together to form baryons) and it was shown by Bardeen, Cooper, and Schrieffer (BCS)<sup>9)</sup> that if there is *any* channel in which the interaction is attractive, then there is a state of lower free energy than a simple Fermi surface. That state arises from a complicated coherent superposition of pairs of particles (and holes)—“Cooper pairs”.

Attractive interactions play a crucial role in the BCS mechanism for the formation of Cooper pairs. This can easily be understood in an intuitive way. The Helmholtz free energy is  $F = E - \mu N$ , where  $E$  is the total energy of the system,  $\mu$  is the chemical potential, and  $N$  is the number of fermions. The Fermi surface is defined by a Fermi energy  $E_F = \mu$ , at which the free energy is minimized, so adding or subtracting a single particle costs zero free energy. Now switch on a weak attractive interaction. It costs no free energy to add a pair of particles (or holes), and if they have the right quantum numbers then the attractive interaction between them will lower the free energy of the system. Many such pairs will therefore be created in the modes near the Fermi surface, and these pairs, being bosonic, will form a condensate. The ground state will be a superposition of states with all numbers of pairs, breaking the fermion number symmetry.

Since pairs of quarks cannot be color singlets, the resulting condensate will break the local color symmetry  $SU(3)_{\text{color}}$ . We call this “color superconductivity”. Note that the quark pairs play the same role here as the Higgs particle does in the standard model: the color-superconducting phase can be thought of as the Higgs phase of QCD.

#### 3.1. Three flavors: Color-flavor locking (CFL)

The favored pairing pattern at high densities, where the strange quark Fermi momentum is close to the up and down quark Fermi momenta, is “color-flavor locking” (CFL).<sup>2)</sup> This has been confirmed by both NJL<sup>2),10),11)</sup> and gluon-mediated interaction calculations.<sup>12)</sup> The CFL pairing pattern is

$$\begin{aligned} \langle q_i^\alpha q_j^\beta \rangle_{1PI} &\propto C \gamma_5 \left( (\kappa + 1) \delta_i^\alpha \delta_j^\beta + (\kappa - 1) \delta_j^\alpha \delta_i^\beta \right) \\ [SU(3)_{\text{color}}] \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset [U(1)_Q]} \times U(1)_B &\rightarrow \underbrace{SU(3)_{C+L+R}}_{\supset [U(1)_{\tilde{Q}}]} \times \mathbb{Z}_2. \end{aligned} \quad (3.1)$$

Color indices  $\alpha, \beta$  and flavor indices  $i, j$  run from 1 to 3, Dirac indices are suppressed, and  $C$  is the Dirac charge-conjugation matrix. The term multiplied by  $\kappa$  corresponds to pairing in the  $(\mathbf{6}_S, \mathbf{6}_S)$ , which although not energetically favored breaks no additional symmetries and so  $\kappa$  is in general small but not zero.<sup>2),12),13)</sup> The Kronecker deltas connect color indices with flavor indices, so that the condensate is not invari-

ant under color rotations, nor under flavor rotations, but only under simultaneous, equal and opposite, color and flavor rotations. Since color is only a vector symmetry, this condensate is only invariant under vector flavor+color rotations, and breaks chiral symmetry. The features of the CFL pattern of condensation are

- The color gauge group is completely broken. All eight gluons become massive. This ensures that there are no infrared divergences associated with gluon propagators.
- All the quark modes are gapped. The nine quasiquarks (three colors times three flavors) fall into an  $\mathbf{8} \oplus \mathbf{1}$  of the unbroken global  $SU(3)$ , so there are two gap parameters. The singlet has a larger gap than the octet.
- A rotated electromagnetism (“ $\tilde{Q}$ ”) survives unbroken. It is a combination of the original photon and one of the gluons.
- Two global symmetries are broken, the chiral symmetry and baryon number, so there are two gauge-invariant order parameters that distinguish the CFL phase from the QGP, and corresponding Goldstone bosons which are long-wavelength disturbances of the order parameter. When the light quark mass is non-zero it explicitly breaks the chiral symmetry and gives a mass to the chiral Goldstone octet, but the CFL phase is still a superfluid, distinguished by its baryon number breaking.
- The symmetries of the 3-flavor CFL phase are the same as those one might expect for 3-flavor hypernuclear matter,<sup>10)</sup> so it is possible that there is no phase transition between them.

In a real compact star we must require electromagnetic and color neutrality<sup>14),15)</sup> (possibly via mixing of oppositely-charged phases), allow for equilibration under the weak interaction, and include a realistic mass for the strange quark. These factors tend to pull apart the Fermi momenta of the different quark species, imposing an energy cost on cross-species pairing.

The requirement of neutrality penalizes the 2SC phase relative to the CFL phase. This can be shown by analyzing a generic expansion of the free energy in powers of  $m_s/\mu$ <sup>15)</sup> or by an NJL calculation<sup>16)</sup> that handles  $m_s \sim \mu$  and includes the coupling between the chiral condensate and quark condensate gap equations. The net result, assuming that mixed phases are excluded by the surface energy cost<sup>17)</sup> (see §4), is that there is no (or very little) density range in which 2SC is the phase with the lowest free energy: unpaired or CFL-paired quark matter is generally favored over 2SC.

### 3.2. Gapless CFL ( $gCFL$ )

Introducing a strange quark mass leads to new color-superconducting phases. The dimensionless parameter that expresses the effect of the strange quark mass is  $M_s^2/(\mu\Delta)$ , which tells us how the pairing gap  $\Delta$  of the strange quark compares with the amount by which the strange quark mass is trying to separate the strange quark Fermi surface from those of the light quarks. In reality, both  $M_s$  and  $\Delta$  will depend on  $\mu$ , but in the rest of this section we will treat  $M_s$  as a parameter, and discuss the effects of varying  $M_s^2/\mu$ , invoking model-independent arguments as well as results from an NJL model in which the pairing gap at  $\mu = 500$  MeV and  $M_s = 0$

is  $\Delta_0 = 25$  MeV.

At very high densities the strange quark mass is small relative to the chemical potential ( $M_s^2/\mu \ll \Delta_{CFL}$ ) and it may, depending on the size of instanton effects,<sup>18)</sup> induce a flavor rotation of the CFL condensate known as “kaon condensation”,<sup>19)</sup> which breaks isospin. As one reduces the density (increasing  $M_s^2/\mu$ ) the strange quark mass becomes more important, and it now seems<sup>3)</sup> that there is a smooth transition into a gapless CFL phase “gCFL” at  $M_s^2/\mu = 2\Delta_{CFL}$ . This will be discussed in more detail below. At lower densities things become complicated. The strange quark mass and the requirements of color and electric neutrality impose a free energy cost for keeping the Fermi momenta of different flavors locked together so that they can pair with each other, and when  $M_s^2/\mu$  is large this cost is too great to be compensated by the resultant pairing energy. One might expect the strange quark to decouple first, leading to “2SC” pairing between up and down only,<sup>10),20)</sup> but neutrality constraints disfavor this.<sup>15)</sup> However, we do not expect the favored phase in this region to be rigorously unpaired. There may be a range of  $M_s^2/\mu$  in which the different flavors are able to maintain some pairing by switching from BCS pairing to “LOFF” crystalline pairing, involving only part of the Fermi surface<sup>4),21),22)</sup> (see §4). Another possibility is that each flavor simply pairs with itself.<sup>5),6)</sup> At some point that cannot be predicted with current techniques, there will be a transition from quark matter to baryonic matter.

The gapless CFL phase, then, is the “medium-high density” phase of QCD. It involves a more complicated pairing pattern than (3.1),

$$\langle \psi_a^\alpha C \gamma_5 \psi_b^\beta \rangle \sim \Delta_1 \epsilon^{\alpha\beta 1} \epsilon_{ab1} + \Delta_2 \epsilon^{\alpha\beta 2} \epsilon_{ab2} + \Delta_3 \epsilon^{\alpha\beta 3} \epsilon_{ab3}. \quad (3.2)$$

Here  $\psi_a^\alpha$  is a quark of color  $\alpha = (r, g, b)$  and flavor  $a = (u, d, s)$ ; the condensate is a Lorentz scalar, antisymmetric in Dirac indices, antisymmetric in color (the channel with the strongest attraction between quarks), and consequently antisymmetric in flavor. The  $\Delta_i$  parameterize the pairing as follows:

- The  $rd$  and  $gu$  quarks pair with gap parameter  $\Delta_3$ .
- The  $bu$  and  $rs$  quarks pair with gap parameter  $\Delta_2$ .
- The  $gs$  and  $bd$  quarks pair with gap parameter  $\Delta_1$ .
- The  $ru$ ,  $gd$  and  $bs$  quarks pair among each other in a fashion involving all three gap parameters  $\Delta_1$ ,  $\Delta_2$  and  $\Delta_3$ .

As with CFL pairing, there is a “rotated electromagnetism” generated by a mixture  $\tilde{Q}$  of the photon and a gluon. However, unlike CFL, which is a  $\tilde{Q}$  insulator, gCFL has some  $\tilde{Q}$ -charged gapless modes (assuming we include electrons as well as quark matter), and is therefore a  $\tilde{Q}$  conductor. At the continuous phase transition between CFL and gCFL, there is a simultaneous partial unpairing of the  $gs$  and  $bd$  quarks, and the  $bu$  and  $rs$  quarks (see Fig. 2). The corresponding gaps  $\Delta_1$  and  $\Delta_2$  drop ( $\Delta_1$  much more rapidly) as  $M_s^2/\mu$  increases, while the gap parameter  $\Delta_3$  for the  $rd$  and  $gu$  quarks, which remain robustly paired, rises.

The difference in behavior of  $\Delta_1$ ,  $\Delta_2$  and  $\Delta_3$  reflects the ways in which different quarks experience the stress of an increasing  $M_s$ , and the requirement of neutrality. To understand this better, let us study their dispersion relations.

For a pair of massless quarks that pair with gap parameter  $\Delta$ , the dispersion

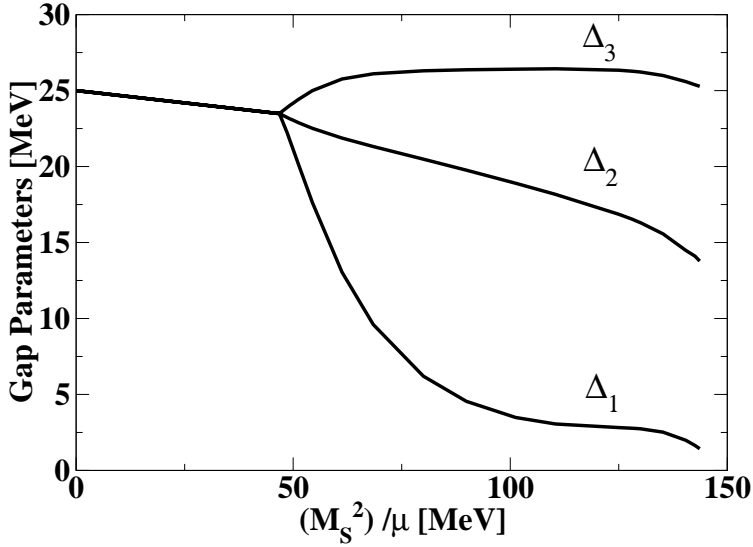


Fig. 2. Gap parameters  $\Delta_3$ ,  $\Delta_2$ , and  $\Delta_1$  as a function of  $M_s^2/\mu$  for  $\mu = 500$  MeV, in an NJL model where  $\Delta_0 = 25$  MeV.<sup>3)</sup> There is a second order phase transition between the CFL phase and the gapless CFL phase at  $M_s^2/\mu = 2\Delta$ .

relation is

$$E(p) = \left| \delta\mu \pm \sqrt{(p - \bar{\mu})^2 + \Delta^2} \right|, \quad (3.3)$$

where the individual chemical potentials of the quarks are  $\bar{\mu} \pm \delta\mu$ . As long as the chemical potentials pulling the two species apart are not too strong, Cooper pairing occurs in all modes

$$\text{pairing criterion: } |\delta\mu| < \Delta. \quad (3.4)$$

However when this condition is violated there are gapless ( $E = 0$ ) modes at momenta

$$p_{\text{gapless}} = \bar{\mu} \pm \sqrt{\delta\mu^2 - \Delta^2} \quad (3.5)$$

and there is no pairing in the “blocking” or “breached pairing” region between these momenta<sup>4), 21), 23)–26)</sup> The pairing criterion (3.4) can be interpreted as saying that the free energy cost  $2\Delta$  of breaking a Cooper pair of two quarks  $a$  and  $b$  is greater than the free energy  $2\delta\mu$  gained by emptying the  $a$  state and filling the  $b$  state (assuming that  $\delta\mu$  pushes the energy of the  $a$  quark up and the  $b$  quark down).<sup>27)</sup>

We now apply these ideas to quark matter. To impose neutrality, we introduce an electrostatic potential  $\mu_e$  coupled to  $Q_e$  which is the *negative* of the electric charge, and chemical (color-electrostatic) potentials  $\mu_3$  and  $\mu_8$  coupled to the diagonal color generators. To take into account the leading effect of the strange quark mass, we introduce an effective chemical potential  $-M_s^2/(2\mu)$  for the strange quarks. The splittings of the various pairs are then as given in the middle column of Table I. We do not discuss the  $ru$ ,  $gd$ , and  $bs$  quarks that pair in a  $3 \times 3$  block, because they do not play a role in defining the boundaries of the CFL and gCFL phases.

First consider just the quark matter, with no electrons (i.e. send the electron mass to infinity). In this case there is a range of allowed  $\mu_e$  at each  $M_s^2/\mu$ , because

Table I. Chemical potential splittings ( $\mu_{1,2} = \bar{\mu} \pm \delta\mu$ ) for the  $2 \times 2$  pairing blocks. The middle column is for the general case, with chemical potentials coupled to negative electric charge ( $\mu_e$ ) and to the diagonal color generators ( $\mu_3, \mu_8$ ). To obtain the last column we fix  $\mu_3$  and  $\mu_8$  as functions of  $\mu_e$  so that varying  $\mu_e$  corresponds to varying  $\mu_{\tilde{Q}}$ .

quark pair	$\delta\mu_{\text{eff}}$	$\delta\mu_{\text{eff}}$ in electronless CFL
<i>rd-gu</i>	$\frac{1}{2}(\mu_e + \mu_3)$	$\mu_e$
<i>rs-bu</i>	$\frac{1}{2}(\mu_e + \frac{1}{2}\mu_3 + \mu_8 - M_s^2/(2\mu))$	$\mu_e - M_s^2/(2\mu)$
<i>bd-gs</i>	$\frac{1}{2}(\frac{1}{2}\mu_3 - \mu_8 + M_s^2/(2\mu))$	$M_s^2/(2\mu)$

the CFL-paired quark matter is a  $\tilde{Q}$ -insulator, with a free energy that is independent of  $\mu_{\tilde{Q}}$  as long as all  $\tilde{Q}$ -charged modes are gapless. This means that one can vary  $\mu_e$ , keeping  $\mu_3 = \mu_e$  and  $\mu_8 = \frac{1}{2}(\mu_e - M_s^2/\mu)$ , and the free energy of the quark matter will not change, as long the pairing criterion (3.4) is obeyed by all quark pairs. The resultant chemical potential splittings as a function of  $\mu_e$  are given in the last column of Table I. In Fig. 3 we show the lines that bound the areas where the pairing criterion (3.4) is obeyed, for each pair in Table I. We see that CFL matter exists in a wedge, between the *rd-gu* unpairing line and the *rs-bu* unpairing line, up to a critical value of  $M_s^2/\mu$ , where the *gs-bd* pairs break. From Table I we can see that the *bd-gs* is vertical because the *bd* and *gs* are  $\tilde{Q}$ -neutral, so their splitting does not depend on  $\mu_e$ . Above the critical  $M_s^2/\mu$ , the *bd* and *gs* are unpaired, and there is a region in which *rd-gu* and *rs-bu* pairing is maintained.

Now include the electrons. In the CFL region, the system is forced to  $\mu_e = 0$

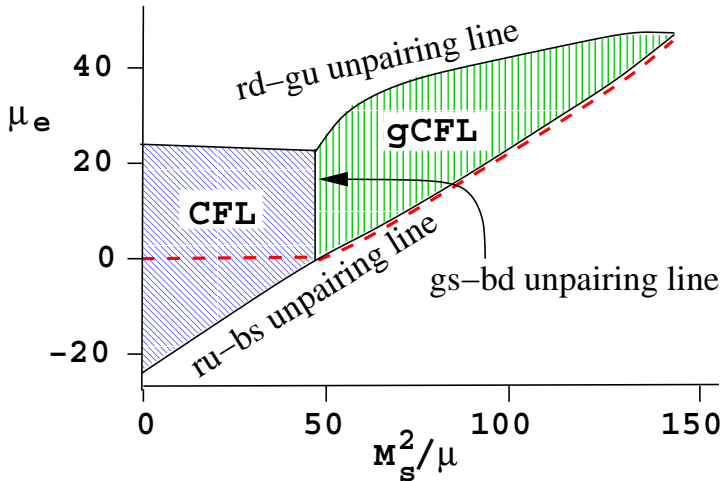


Fig. 3. Unpairing lines and phases for the same model as used in Fig. 2. If electrons are neglected, then the upper and lower curves bound the region of  $\mu_e$  where CFL or gCFL solutions are found. Between the curves the quark matter is a  $\tilde{Q}$ -insulator. Taking electrons into account, the correct solution is the dashed line: in the CFL phase  $\mu_e = 0$ , and gCFL corresponds to values of  $\mu_e$  below but very close to the *ru-bs* unpairing line, which is a  $\tilde{Q}$ -conductor because of the unpaired  $\tilde{Q}$ -charged *ru-bs* quasi-quarks.

(dashed line in Fig. 3).<sup>27)</sup> However, at the transition point to gCFL, where the  $gs$ - $bd$  pairs break, we find that the neutrality requirement forces us over the line where  $rs$ - $bu$  pairs also begin to break. The result is that as  $M_s^2/\mu$  increases further, the system maintains neutrality by staying close to the  $rs$ - $bu$ -unpairing line, where there is a narrow blocking region in which there are unpaired  $bu$  quarks. Their charge is cancelled by a small density of electrons.

We see that gCFL quark matter is a conductor of  $\tilde{Q}$  charge, since it has gapless  $\tilde{Q}$ -charged quark modes, as well as electrons. The  $rd$  and  $gu$  quarks, which are insensitive to the strange quark mass, remain robustly paired, and the  $\tilde{Q}$ -neutral  $bd$  and  $gs$  quarks develop a large blocking region, as the system moves far beyond their unpairing line. The neutrality requirement naturally keeps the system close to the  $rs$ - $bu$ -unpairing line, so these quarks have a very narrow blocking region. We can see the effect of this by taking  $p - \bar{\mu} \ll \Delta$  with  $\delta\mu \approx \Delta$ , and expanding Eq. (3.3): the dispersion relation for these quarks is almost quadratic, with a very high density of states at the lowest energies.

We conclude that gCFL quark matter is likely to have very different transport properties from CFL quark matter. The occurrence of many gapless fermionic modes will certainly affect cooling, and the fact that some of them are  $\tilde{Q}$ -charged will affect the conductivity and hence the behavior of magnetic fields. It remains to be seen what astrophysical signatures flow from these properties, and this question is being actively studied. In the rest of these proceedings we will discuss observable properties of compact stars, however we will not try to treat the complications of the gCFL phase: we will assume that the pairing is strong enough or the strange quark is light enough so that quark matter always occurs in the CFL phase.

#### §4. Compact star transport phenomenology

The high density and relatively low temperature required to produce color superconducting quark matter may be attained in compact stars. Typical compact stars have masses close to  $1.4M_\odot$ , and are believed to have radii of order 10 km.

Color superconductivity affects the equation of state at order  $(\Delta/\mu)^2$ . It also gives mass to excitations around the ground state: it opens up a gap at the quark Fermi surface, and makes the gluons massive. One would therefore expect it to have a profound effect on transport properties, such as mean free paths, conductivities and viscosities. Various observable consequences are under investigation.

- *r-mode spindown.* The *r*-mode is a bulk flow in a rotating star that, if the viscosity is low enough, radiates away energy and angular momentum in the form of gravitational waves. One can rule out certain models for compact stars on the grounds that they have such low damping that they could not support the high rotation rates observed in pulsars: *r*-mode spindown would have slowed them down. Madsen<sup>28)</sup> has shown that for a compact star made *entirely* of quark matter in the CFL phase, even a gap as small as  $\Delta = 1$  MeV is ruled out by observations of millisecond pulsars. It remains to extend this calculation to the more generic picture of a quark matter core surrounded by a nuclear mantle.



- *Interfaces and mixed phases.* These were studied in Ref. 17), and it was found that a mixed phase only occurs if the surface tension of the interface is less than about  $40 \text{ MeV/fm}^2 = 0.2 \times (200 \text{ MeV})^3$ , a fairly small value compared to the relevant scales  $\Lambda_{\text{QCD}} \approx 200 \text{ MeV}$ ,  $\mu \sim 400 \text{ MeV}$ . A sharp nuclear-quark interface will have an energy-density discontinuity across it, which will affect gravitational waves emitted in mergers, and also the  $r$ -mode spectrum and the damping forces to which  $r$ -modes are subject.
- *Crystalline pairing (the “LOFF” phase).* This is expected to occur when two different types of quark have sufficiently different Fermi momenta that BCS pairing cannot occur.<sup>4)</sup> This is a candidate for the intermediate “non-CFL” phase of Fig. 1, where the strange quark mass, combined with requirements of weak equilibrium and charge neutrality, gives each quark flavor a different Fermi momentum. The phenomenology of the crystalline phase has not yet been worked out, but recent calculations using Landau-Ginzburg effective theory indicate that the favored phase may be a face-centered cubic crystal,<sup>22)</sup> with a reasonably large binding energy. This raises the interesting possibility of glitches in quark matter stars.
- *Cooling by neutrino emission.* The cooling rate is determined by the heat capacity and emissivity, both of which are sensitive to the spectrum of low-energy excitations, and hence to color superconductivity. CFL quark matter, where all modes are gapped, has a much smaller neutrino emissivity and heat capacity than nuclear matter, and hence the cooling of a compact star is likely to be dominated by the nuclear mantle rather than the CFL core.<sup>29)–31)</sup> Other phases such as 2SC or LOFF give large gaps to only some of the quarks. Their cooling would proceed quickly, then slow down suddenly when the temperature fell below the smallest of the small weak-channel gaps. This behavior should be observable.<sup>32)</sup>

### §5. Mass-radius relationship for compact stars

Although the effects of color superconductivity on the quark matter equation of state are subdominant, they may have a large effect on the mass-radius relationship. The reason for this is that the pressure of quark matter relative to the hadronic vacuum contains a constant (the “bag constant”  $B$ ) that represents the cost of dismantling the chirally broken and confining hadronic vacuum,

$$p = (1 - c) \frac{3}{4\pi^2} \mu^4 - \frac{3}{4\pi^2} m_s^2 \mu^2 + \frac{3}{\pi^2} \Delta^2 \mu^2 - B. \quad (5.1)$$

If the bag constant is large enough so that nuclear matter is favored (or almost favored) over quark matter at  $\mu \sim 320 \text{ MeV}$ , then the bag constant and  $\mu^4$  terms almost cancel, so if we can fix the bag constant by other means then the strange quark mass  $m_s$  and color superconducting gap  $\Delta$  may have a large effect on the equation of state and hence on the mass-radius relationship of a compact star.<sup>33)</sup>

### 5.1. $M(R)$ at fixed bag constant

In Ref. 34) Sanjay Reddy and I explored the effect of quark pairing on the  $M$ - $R$  relationship at fixed values of the bag constant that are consistent with nuclear phenomenology. Figure 4 shows the mass-radius curve for the bag model of dense matter, in which there is competition between a nuclear matter phase and a quark matter phase. The nuclear matter was described either by the APR98 equation of state.<sup>35)</sup> The quark matter equation of state was essentially that of equation (5-1), but we included the full (free-quark) correction due to the strange quark mass. The coefficient of the  $\Delta^2\mu^2$  term is the one appropriate to CFL color superconductivity involving all nine colors/flavors of the quarks. We used physically reasonable values of the bag constant  $B^{1/4} = 180$  MeV ( $B = 137$  MeV/fm<sup>3</sup>) and strange quark mass  $m_s = 200$  MeV. We set the parameter  $c$  to zero: its effects will be discussed below.

Curves for unpaired ( $\Delta = 0$ ) and color-superconducting ( $\Delta = 100$  MeV) quark matter are shown. At these values the stars are typically “hybrid”, containing both quark matter and nuclear matter. The solid lines in Fig. 4 correspond to stars that either have no QM at all, or a sharp transition between NM and QM: the core is made of quark matter, which is the favored phase at high pressure, and at some radius there is a transition to nuclear matter, which is favored at low pressure. The transition pressure is sensitive to  $\Delta$ , for reasons discussed earlier. The dashed lines are for stars that contain a mixed NM-QM phase. In all cases we see that light, large stars consist entirely of nuclear matter. When the star becomes heavy enough, the

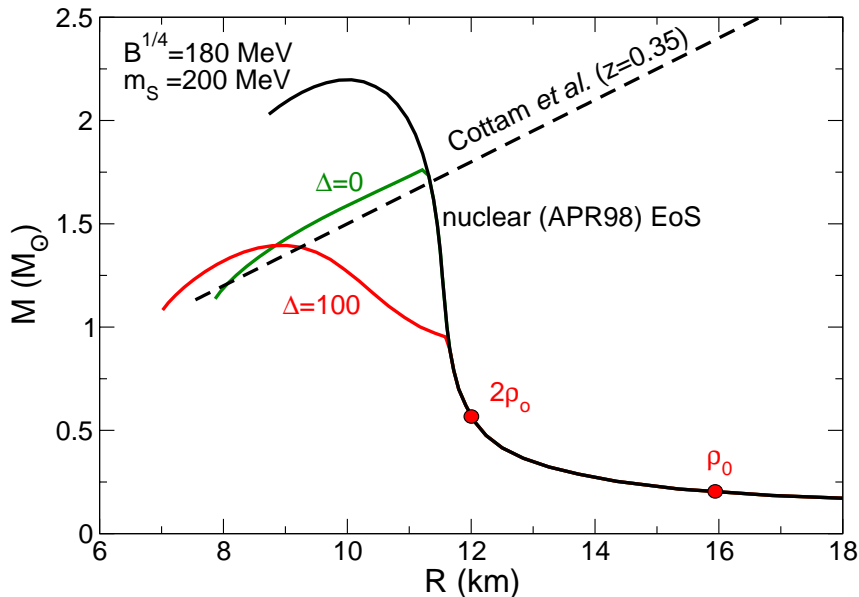


Fig. 4. Mass-radius relationships for APR98 nuclear matter, competing with quark matter with fixed bag constant  $B^{1/4} = 180$  MeV and  $m_s = 200$  MeV, either unpaired ( $\Delta = 0$ ) or CFL color-superconducting ( $\Delta = 100$  MeV). The dots labeled  $\rho_0$  and  $2\rho_0$  on the nuclear matter mass-radius curve indicate that the central density at these locations correspond to nuclear and twice nuclear saturation density respectively.

central pressure rises to a level where QM occurs in the core. As can be seen from the figure the transition density is very sensitive to  $\Delta$ . The line labeled “Cottam *et al.*” indicates the constraint obtained by recent measurements of the redshift on three spectral lines from EXO0748-676.<sup>36)</sup>

### 5.2. $M(R)$ with non-free quarks: quark matter mimicking nuclear matter

In Ref. 34) we kept the bag constant fixed, assuming that it could be fixed by other observations, and we treated the quark matter as free quarks with a pairing energy. It is interesting to see what happens when we relax these assumptions, since the bag constant is not easily measured, and even after taking pairing into account we expect remaining QCD interactions between the quarks in the Fermi sea.

To allow for effects of quark interactions beyond Cooper pairing, we follow the parameterization of Fraga *et al.*,<sup>37)</sup> who find that the  $\mathcal{O}(\alpha_s^2)$  pressure for three unpaired flavors over the relevant range of  $\mu$  is well-described by a bag-model-inspired form given by

$$P_{\alpha_s^2}(\mu) = \frac{3}{4\pi^2} a_{\text{eff}} \mu^4 - B_{\text{eff}}, \quad a_{\text{eff}} \equiv 1 - c. \quad (5.2)$$

They find  $a_{\text{eff}} \approx 0.63$  ( $c \approx 0.37$ ), but admit that at the density of interest for compact star physics the QCD coupling is strong, and there is no reason to expect the leading order calculation to be accurate. We therefore take their result as indicating that it is reasonable to treat  $c$  as an additional parameter in the quark matter equation of state, as shown in Eq. (5.1), and we proceed to study its effects on the mass-radius relationship of compact stars.

To see how closely quark matter can mimic nuclear matter, we will not treat the bag constant as fixed, but tune it to keep the physics as constant as possible. Thus when we compare, say,  $\Delta = 0$  (non-color-superconducting) quark matter with CFL ( $\Delta = 50$ ) quark matter, we set the bag constant in each case so that the transition from nuclear to quark matter occurs at a given density of nuclear matter. This effectively “subtracts out” the part of any variation in  $\Delta$  that simply corresponds to a renormalization of the bag constant, which is in any case very poorly known.

We explore the effect of a color superconducting gap  $\Delta$  and perturbative correction  $c$  on the mass-radius relationship. We fix the bag constant by requiring that the nuclear to quark matter phase transition occur at nuclear matter baryon density  $\rho = 1.5n_{\text{sat}}$ . The resultant  $M(R)$  curves are shown in Fig. 5. The noticeable features of the plots are

1. *Increasing  $c$  makes the stars smaller and lighter.*

In our parameterization the stars resulting from quark matter equations of state without perturbative correction ( $c = 0$ , blue lines) are smaller and lighter.

2. *Color superconductivity acts like a change in the bag constant.*

In Ref. 34) we showed that at fixed bag constant, color superconductivity has a strong effect on the mass-radius relationship of compact stars. Here, by comparing the dashed lines with the dotted lines in Fig. 5, we see that it is difficult to distinguish the effect of color superconductivity from a change in the bag constant. In Fig. 5, as we vary parameters  $c$  and  $\Delta$  of the quark matter equation of state, the bag constant is tuned to maintain a constant value of the

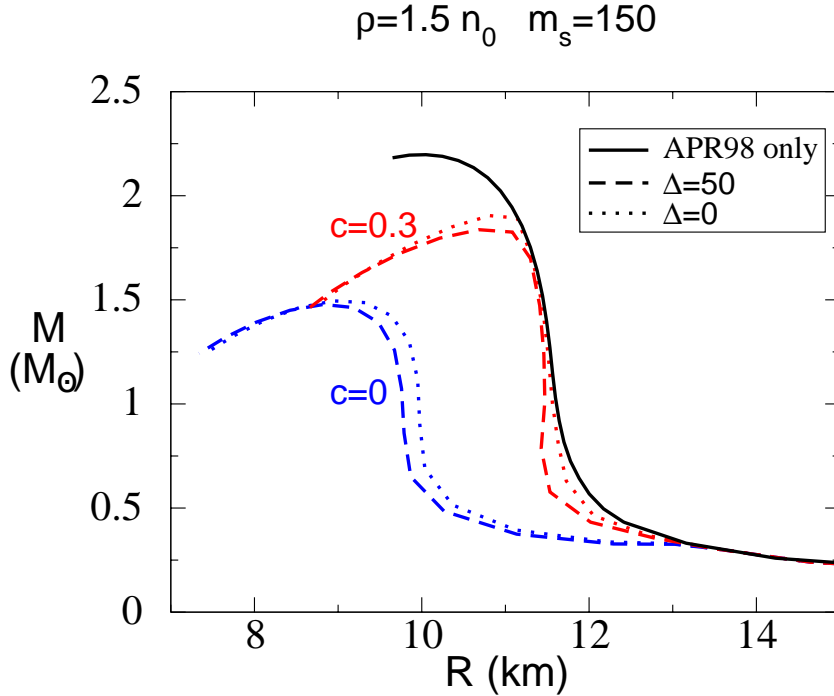


Fig. 5.  $M(R)$  relationship for APR98 nuclear matter with various quark matter equations of state. The strange quark is light, and the bag constant is tuned so that the nuclear matter to quark matter transition occurs at 1.5 times nuclear saturation density. Dotted lines are unpaired quark matter, dashed lines are CFL with gap of 50 MeV. Note how the curve for CFL quark matter with perturbative correction but no color superconductivity ( $c = 0.3$ ,  $\Delta = 0$ ; dotted) closely follows the pure nuclear curve up to  $M \approx 1.9 M_\odot$ .

nuclear density at the transition to quark matter, and in this situation color superconductivity only makes a small difference to the mass-radius relationship.

3. *Quark matter with  $c \approx 0.3$  looks just like APR98 nuclear matter.*

The stars with  $c = 0.3$  have mass-radius relationships that are very similar to the pure nuclear APR98 matter. In fact, for the case where there are perturbative corrections but no color superconductivity the equations of state ( $p(\mu)$ ) are so similar that our program found a series of phase transitions back and forth between CFL and APR98 up to  $\mu = 546$  MeV (baryon density  $\rho = 5.4n_{\text{sat}}$ ). This is why the  $c = 0.3$  dotted curve lies almost exactly on top of the solid black (APR98) curve, even though there was a phase transition from APR98 to CFL at  $\rho = 1.5n_{\text{sat}}$  (which is first attained when the APR98 star reaches a mass of  $0.315 M_\odot$ ,  $R = 13.3$  km).

5.3.  *$M(R)$  measurements and quark matter*

We can now ask what significance mass and radius measurements will have for the presence of quark matter, and particularly color-superconducting quark matter, in compact stars.

- *What would rule out quark matter?*

From Fig. 5 we see that an observed mass  $M \gtrsim 2 M_{\odot}$  would be inconsistent with the star containing quark matter obeying the equation of state that we have studied here. However, we emphasize that by introducing the parameter  $c$  and setting it to a reasonable value  $c \approx 0.3$  we have increased the mass range for hybrid stars, moving the upper limit from its old value around  $1.6 M_{\odot}$  up to about  $1.9 M_{\odot}$ .

- *What would indicate the presence of quark matter?*

This is difficult. Regions of  $M$ - $R$  space that cannot be reached by any nuclear matter equation of state also cannot be reached by hybrid NM-QM equations of state. It is clear from Figs. 5 and 4 that hybrid stars are smaller than Walecka or APR98 nuclear matter stars, with radii of around 10 km at  $M \approx 1.4 M_{\odot}$ . But there are many other suggested nuclear equations of state, and the flattening of the  $M(R)$  curve that appears in our plots to be characteristic of quark matter may easily be mimicked by kaon condensation in nuclear matter.<sup>38)</sup>

Obviously the region of pure quark matter objects which lie at very low mass and radius (“strangelets”) is not attainable by nuclear matter, but the existence of such objects, unlike that of compact stars, remains a matter of speculation.

- *What would indicate the presence of color superconducting quark matter?*

This is more difficult. Even if we found an  $M(R)$  characteristic of quark matter, we would need an independent determination of the bag constant to claim that it was color-superconducting.

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