## **Wolfgang Woess**

## Denumerable Markov Chains

Generating Functions
Boundary Theory
Random Walks on Trees

## **Contents**

Pr	efac	re	V		
In	Su	duction Immary	ix ix xiii		
1	Preliminaries and basic facts				
	Α	Preliminaries, examples	1		
	В	Axiomatic definition of a Markov chain	5		
	C	Transition probabilities in $n$ steps	12		
	D	Generating functions of transition probabilities	17		
2	Irreducible classes				
	Α	Irreducible and essential classes	28		
	В	The period of an irreducible class	35		
	C	The spectral radius of an irreducible class	39		
3	Recurrence and transience, convergence, and the ergodic theorem				
	A	Recurrent classes	43		
	В	Return times, positive recurrence, and stationary probability measures	47		
	C	The convergence theorem for finite Markov chains	52		
	D	The Perron–Frobenius theorem	57		
	E	The convergence theorem for positive recurrent Markov chains	63		
	F	The ergodic theorem for positive recurrent Markov chains	68		
	G	ho-recurrence	74		
4	Re	eversible Markov chains	78		
	Α	The network model	78		
	В	Speed of convergence of finite reversible Markov chains	83		
	C	The Poincaré inequality	93		
	D	Recurrence of infinite networks	102		
	E	Random walks on integer lattices	109		
5	Models of population evolution				
	Α	Birth-and-death Markov chains	116		
	В		131		
	C	Branching Markov chains	140		

## viii Contents

6	Ele	ements of the potential theory of transient Markov chains	153		
	Α	Motivation. The finite case	153		
	В	Harmonic and superharmonic functions. Invariant and excessive			
		measures	158		
	C	Induced Markov chains	164		
	D	Potentials, Riesz decomposition, approximation	169		
	E	"Balayage" and domination principle	173		
7	Th	e Martin boundary of transient Markov chains	179		
	Α	Minimal harmonic functions	179		
	В	The Martin compactification	184		
	C	Supermartingales, superharmonic functions, and excessive measures	191		
	D	The Poisson–Martin integral representation theorem	200		
	E	Poisson boundary. Alternative approach to the integral representation	209		
8	Mi	nimal harmonic functions on Euclidean lattices	219		
9	Ne	arest neighbour random walks on trees	226		
	A	Basic facts and computations	226		
	В	The geometric boundary of an infinite tree	232		
	C	Convergence to ends and identification of the Martin boundary	237		
	D	The integral representation of all harmonic functions	246		
	E	Limits of harmonic functions at the boundary	251		
	F	The boundary process, and the deviation from the limit geodesic	263		
	G	Some recurrence / transience criteria	267		
	H	Rate of escape and spectral radius	279		
So	luti	ons of all exercises	297		
Bi	Bibliography				
	Α	Textbooks and other general references	339		
	В	Research-specific references	341		
Li	List of symbols and notation				
In	dex		349		