Dependency of the spectral reflectance curves of the Munsell color chips

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Abstract

The spectral reflectance curves of 433 chips in the Munsell Book of Color have been found to depend on only three components which account for 99.18% of the variance. It is suggested that this three-component dependency may be a characteristic of all organic pigments, including those in the retina, and thus explain the trichromatic nature of color vision.

Problem

The spectral reflectance curves for 433 color chips in the Munsell Book of Color were measured with a GE recording spectrophotometer by the National Bureau of Standards; these data, in matrix form (40 x 417—the rows giving the reflectance at 40 wavelengths at 10 mu intervals from 380 mu to 770 mu), have been made available by Nickerson (1957). The columns and rows of this matrix were thought to be independent, but the computer analysis, in connection with another problem, indicated that the matrix determinant was equal to zero; the nature of this reduction in rank was investigated.

Method and Results

A 40 x 150 matrix of reflectance data of 150 randomly selected Munsell chips (the capacity of the computer) was subjected to a linear component analysis by the centroid method; the first four components are shown in Table 1. This application to continuous curves is almost identical to that described by Simonds (1963). Component I extracted 92.72% of the cumulative variance, component II 97.25%, component III 99.18%, and component IV 99.68%: considering the reduction in rank, this is one of the highest extractions of variance on record. It follows that three scalar multipliers, M_1, M_2, M_3, may be assigned to each chip. When the first three components of Table 1 are weighted by their respective multipliers, for any chip, and each row summed, a reconstructed reflectance curve of high accuracy may be obtained for that chip. Examples of reconstructed curves with three components, and with four components, for two chips are given in Tables 2 and 3. Equally accurate reconstructions are obtained with other Munsell chips not included in this sample.

It is to be emphasized that these data are entirely physical; the three scalar multipliers do not correspond to the hue, value, and chroma of psychological specification. Given the psychological specification of a Munsell chip, the chip's reflectance curve cannot be derived; however, given the three scalar multipliers, the entire reflectance curve can be predicted with high accuracy.

Discussion

The dependence of the Munsell reflectance curves on just three components suggests that the Munsell chips were created from three pigments; the Munsell chips, in fact, were produced from a large number of pigments.

Even if the Munsell chips were mixed from three pigments, it seems impossible for a linear component analysis to recover almost completely the identity of the component pigments. The formulas which relate the reflectance curves of the pigment components to the reflectance curve of the pigment mixture are complex and involve exponents (Duncan, 1949). A linear component analysis might produce a rough first approximation, but it would seem unreasonable that three linear components would exhaust the variance almost completely.

The difficulty in explaining the dependence of the Munsell chips invites speculation that the dependency is a function of the molecular structure of organic pigments, and that all organic pigments and their mixtures may be expressed as the weighted sum of three

Table 1. Four Centroid Components of Munsell Reflectance Curves

Wavelength	I	II	III	IV
380	3.4425	-.8360	-.7880	-.1795
390	3.4776	-.8778	-.7860	-.2022
400	3.5005	-.9140	-.7828	-.2284
410	3.5145	-.9539	-.7776	-.2504
420	3.5241	-.9937	-.7645	-.2659
430	3.5366	-1.0525	-.7392	-.2642
440	3.5547	-1.1236	-.7088	-.2442
450	3.5662	-1.2012	-.6627	-.2071
460	3.5689	-1.2788	-.5812	-.1628
470	3.5561	-1.3469	-.4622	-.1126
480	3.5833	-1.3831	-.2469	-.0424
490	3.6180	-1.3811	.0084	.0418
500	3.6686	-1.3123	.2793	.1201
510	3.8025	-1.1329	.6102	.2621
520	3.9135	-.9279	.8886	.4000
530	3.9130	-.7835	1.0262	.4067
540	3.9128	-.6331	1.0998	.3534
550	3.9400	-.4659	1.1314	.2797
560	3.9357	-.2782	1.1241	.2146
570	4.0448	-.0745	1.1319	.0813
580	4.1749	.1705	1.0669	-.0877
590	4.4221	.4203	1.0056	-.1940
600	4.5508	.6630	.7182	-.3445
610	4.7270	.8696	.4748	-.3744
620	4.8407	1.0038	.2761	-.3799
630	4.8981	1.0793	.1468	-.3869
640	4.9321	1.1225	.0561	-.3923
650	4.9674	1.1465	-.0121	-.3868
660	5.0134	1.1584	-.0680	-.3667
670	5.0792	1.1616	-.1198	-.3191
680	5.1668	1.1610	-.1614	-.2403
690	5.2700	1.1553	-.1985	-.1285
700	5.3864	1.1413	-.2396	.0128
710	5.5036	1.1101	-.2863	.1719
720	5.6102	1.0559	-.3371	.3258
730	5.7045	.9929	-.3950	.4626
740	5.7744	.9323	-.4450	.5693
750	5.8231	.8912	-.4794	.6444
760	5.8541	.8672	-.5008	.6926
770	5.9119	.8482	-.5016	.7217

Table 2. Reconstruction of Spectral Reflectance, Munsell Chip R 6/6, Three and Four Components

Wavelength	Measured	Reconstruction 3	Reconstruction 4
380	31.1 %	29.3 %	30.0 %
390	31.1	29.2	30.1
400	31.0	29.1	30.1
410	30.7	28.8	30.0
420	29.1	28.5	29.7
430	28.1	28.0	29.1
440	26.9	27.3	28.4
450	26.0	26.5	27.4
460	25.3	25.4	26.1
470	24.3	24.1	24.5
480	23.6	22.9	23.1
490	23.0	22.0	21.8
500	22.8	21.7	21.3
510	22.7	23.0	22.0
520	23.1	24.6	22.9
530	23.1	25.2	23.6
540	23.5	26.2	24.9
550	25.0	27.9	26.9
560	27.4	29.6	29.0
570	31.5	32.5	32.6
580	38.2	36.3	37.3
590	44.9	41.3	42.7
600	50.5	46.2	48.3
610	54.8	51.0	53.2
620	57.4	54.3	56.5
630	58.8	56.2	58.4
640	59.4	57.3	59.6
650	60.0	58.3	60.4
660	60.3	59.1	61.1
670	60.7	60.0	60.8
680	61.4	61.1	62.4
690	61.8	62.2	62.9
700	62.2	63.4	63.4
710	62.7	64.5	63.6
720	63.0	65.2	63.6
730	63.3	65.8	63.5
740	63.6	66.2	63.3
750	63.9	66.5	63.1
760	64.2	66.7	63.1
770	64.4	67.0	63.3

Three Components: $M_1 = .09597$, $M_2 = .09192$, $M_3 = -.04994$
Four Components: $M_1 = .09623$, $M_2 = .09210$, $M_3 = -.04637$, $M_4 = -.05169$

Table 3. Reconstruction of Spectral Reflectance, Munsell Chip N 4/ , Three and Four Components

Wavelength	Measured	Reconstruction 3	Reconstruction 4
380	13.5%	11.2%	11.6%
390	13.3	11.4	11.8
400	13.0	11.5	12.0
410	12.9	11.7	12.2
420	12.7	11.8	12.4
430	12.6	12.0	12.6
440	12.5	12.2	12.8
450	12.5	12.4	12.9
460	12.4	12.6	13.0
470	12.2	12.8	13.0
480	12.0	13.0	13.0
490	11.9	13.1	13.0
500	11.9	13.1	12.9
510	12.0	13.2	12.6
520	12.0	13.0	12.1
530	12.1	12.7	11.8
540	12.2	12.4	11.7
550	12.2	12.1	11.6
560	12.2	11.6	11.3
570	12.1	11.4	11.5
580	12.1	11.2	11.7
590	12.1	11.3	12.1
600	12.0	11.0	12.2
610	12.0	11.0	12.2
620	11.9	10.9	12.1
630	12.0	10.9	12.1
640	11.9	10.8	12.0
650	11.9	10.9	12.0
660	11.9	10.9	12.0
670	11.9	11.1	12.1
680	11.9	11.3	12.1
690	11.8	11.6	12.0
700	11.8	12.0	12.0
710	11.8	12.3	11.9
720	11.9	12.8	11.9
730	11.9	13.2	11.9
740	11.9	13.5	11.9
750	11.9	13.7	11.9
760	11.9	13.8	11.9
770	11.9	14.0	12.0

Three Components: $M_1 = .02728$, $M_2 = -.02342$, $M_3 = .00219$
Four Components: $M_1 = .02743$, $M_2 = -.02332$, $M_3 = .00412$, $M_4 = -.02792$

component curves. The reflectance curves of organic pigments seem always to have a characteristic shape and belong to a family of curves; the curves, with respect to one another, seem to be neither random nor chaotic. The curves seem to be correlated. It is also true that an organic pigment may be identified uniquely by inspecting four spectrophotometric curves of the pigment dissolved in four different media (Pratt, 1947).

The problem of the generality of the dependency of organic pigments will be tested empirically but it is perhaps interesting to consider its consequences. If the general dependency should be sustained, the pigments of the eye would also be dependent on the same set of primary components. The color response curves of the retina (uncorrected for ocular media) would then be represented by the first three componets of Table 1, and a plot should be significant. These components plot as a near-perfect helix (it is best to plot axes II and III, and visualize I), where complementary wavelengths seem to be opposite each other. It is odd that the Munsell reflection data should collapse to rank three, and perhaps odder still that the vectors plot in a structure so elegant.

References

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